Image Quantization, Halftoning, and Dithering

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Overview

- Images
  - What is an image?
- Quantization
  - Halftoning
  - Dithering

What is an Image?

- An image is a 2D rectilinear array of pixels

Continuous image

Digital image

A pixel is a sample, not a little square

Image Acquisition

- Pixels are samples from continuous function
  - Photoreceptors in eye
  - CCD cells in digital camera
  - Rays in virtual camera

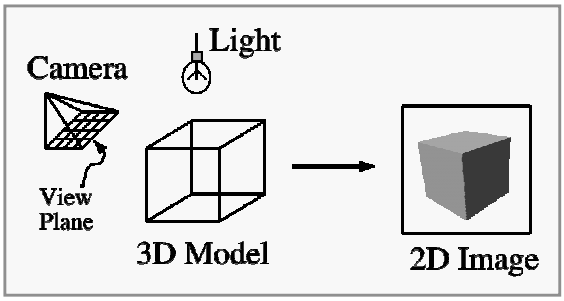


Image Resolution

- Intensity resolution
  - Each pixel has only “Depth” bits for colors/intensities
- Spatial resolution
  - Image has only “Width” x “Height” pixels
- Temporal resolution
  - Monitor refreshes images at only “Rate” Hz

Typical Resolutions

	Width x Height	Depth	Rate
NTSC	640 x 480	8	30
Workstation	1280 x 1024	24	75
Film	3000 x 2000	12	24
Laser Printer	6600 x 5100	1	-

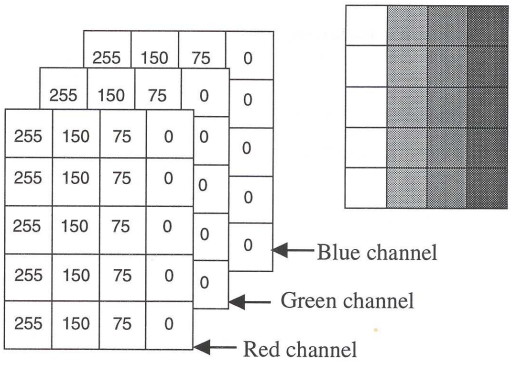
Sources of Error

- Intensity quantization
  - Not enough intensity resolution
- Spatial aliasing
  - Not enough spatial resolution
- Temporal aliasing
  - Not enough temporal resolution

$$E^2 = \sum_{(x,y)} \left(I(x,y) - P(x,y)\right)^2$$

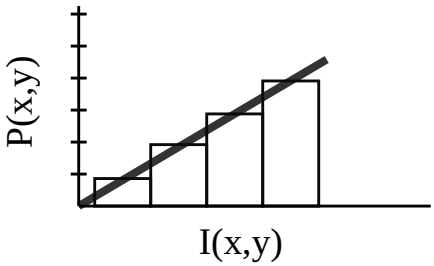
Quantization

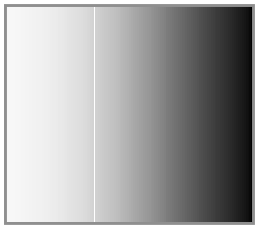
- Artifacts due to limited intensity resolution
  - Frame buffers have limited number of bits per pixel
  - Physical devices have limited dynamic range



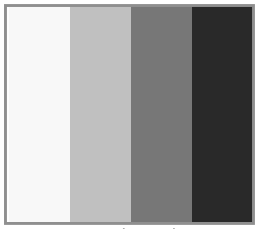
Uniform Quantization

$$P(x, y) = \text{trunc}(I(x, y) + 0.5)$$





I(x,y)



P(x,y)
(4 bits per pixel)

Uniform Quantization

- Images with decreasing bits per pixel:

8 bits

4 bits

2 bits

1 bit

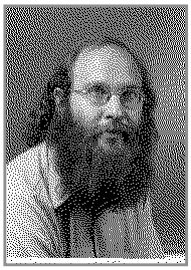
Notice contouring

Halftoning and Dithering

- Distribute errors among pixels
  - Exploit spatial integration in our eye
  - Display greater range of perceptible intensities

Original
(8 bits)

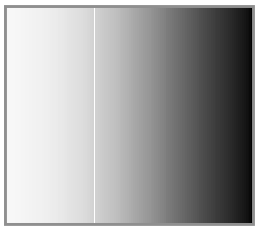
Uniform
Quantization
(1 bit)



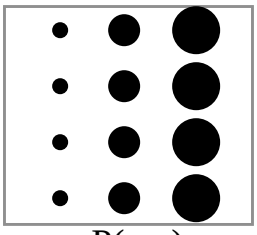
Floyd-Steinberg
Dither
(1 bit)

Classical Halftoning

- Use dots of varying size to represent intensities
  - Area of dots proportional to intensity in image

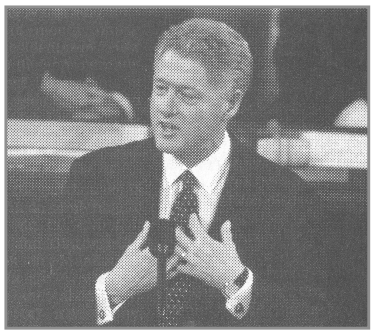


I(x,y)

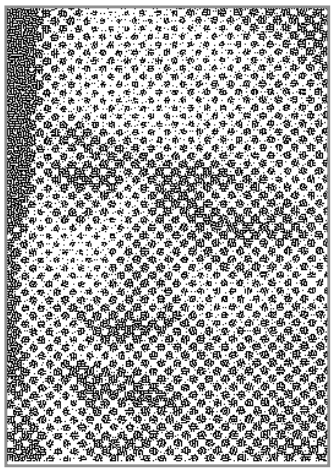


P(x,y)

Classical Halftoning

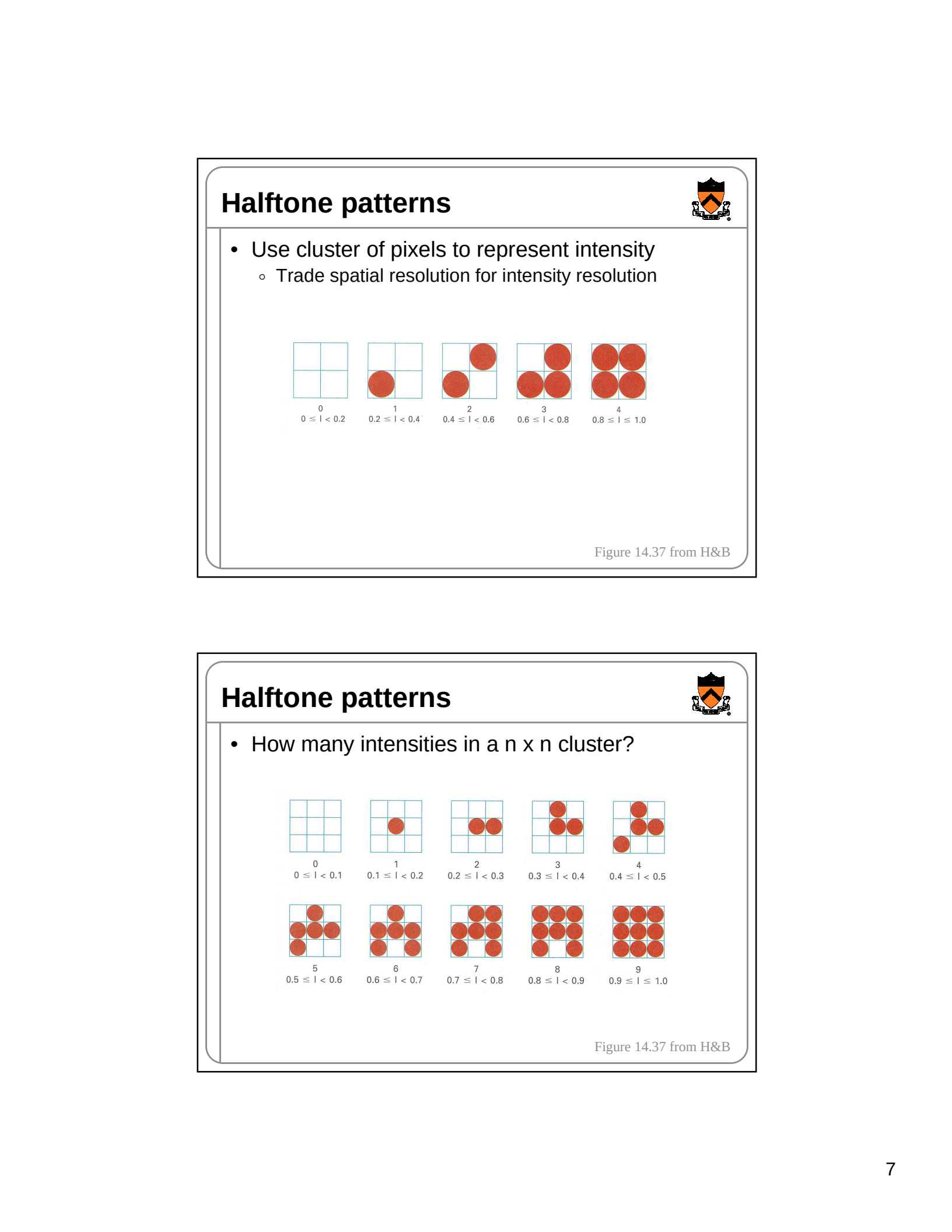


Newspaper Image



From New York Times, 9/21

Halftone patterns



- Use cluster of pixels to represent intensity
  - Trade spatial resolution for intensity resolution

Figure 14.37 from H&B

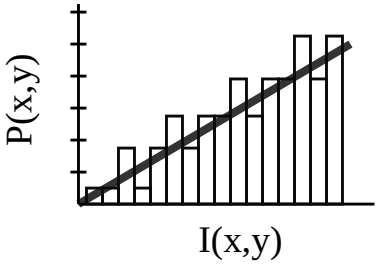
Halftone patterns

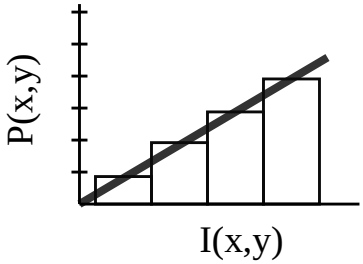
- How many intensities in a n x n cluster?

Figure 14.37 from H&B

Random Dither

- Randomize quantization errors
  - Errors appear as noise





$$P(x, y) = \text{trunc}(I(x, y) + \text{noise}(x,y) + 0.5)$$

Random Dither

Original
(8 bits)

Uniform
Quantization
(1 bit)

Random
Dither
(1 bit)

Ordered Dither

- Pseudo-random quantization errors
  - Matrix stores pattern of threshholds

```
i = x mod n
j = y mod n
e = I(x,y) - trunc(I(x,y))
if (e > D(i,j))
        P(x,y) = ceil(I(x, y))
else
        P(x,y) = floor(I(x,y))
```

$$D_2 = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$$

Ordered Dither

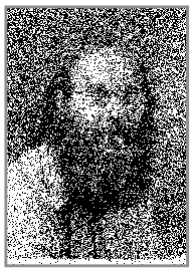
- Bayer's ordered dither matrices

$$D_n = \begin{bmatrix} 4D_{n/2} + D_2(1,1)U_{n/2} & 4D_{n/2} + D_2(1,2)U_{n/2} \\ 4D_{n/2} + D_2(2,1)U_{n/2} & 4D_{n/2} + D_2(2,2)U_{n/2} \end{bmatrix}$$

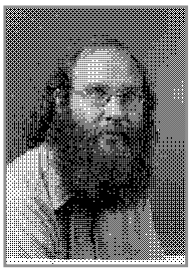
$$D_2 = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \qquad D_4 = \begin{bmatrix} 15 & 7 & 13 & 5 \\ 3 & 11 & 1 & 9 \\ 12 & 4 & 14 & 6 \\ 0 & 8 & 2 & 10 \end{bmatrix}$$

Ordered Dither

Original
(8 bits)



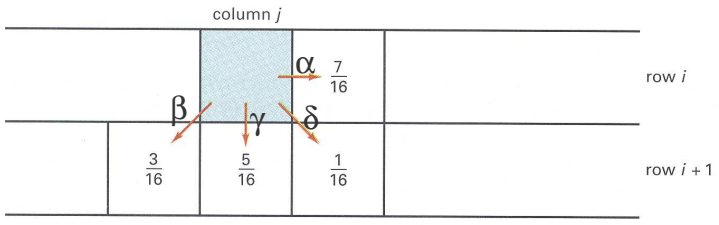
Random
Dither
(1 bit)



Ordered
Dither
(1 bit)

Error Diffusion Dither

- Spread quantization error over neighbor pixels
  - Error dispersed to pixels right and below



$$\alpha + \beta + \gamma + \delta = 1.0$$

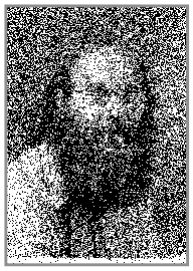
Figure 14.42 from H&B

Floyd-Steinberg Algorithm

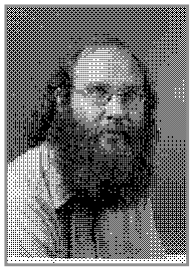
```
for (x = 0; x < width; x++) {
        for (y = 0; y < height; y++) {
                P(x,y) = trunc(I(x,y) + 0.5)
                e = I(x,y) - P(x,y)
                I(x,y+1) += α*e;
                I(x+1,y-1) += β*e;
                I(x+1,y) += γ*e;
                I(x+1,y+1) += δ *e;
        }
}
```

Error Diffusion Dither

Original
(8 bits)



Random
Dither
(1 bit)



Ordered
Dither
(1 bit)

Floyd-Steinberg
Dither
(1 bit)

Summary

- Images have limited intensity resolution
  - Quantization
  - Halftoning
  - Dithering
- Images have limited spatial and temporal resolutions
  - More on this Friday