CS 126 Lecture T3: Formal Languages

Outline

- **Introduction**
- Defining grammar
- Type 3 grammars
- Type 2 grammars
- Chomsky hierarchy
- Conclusions

Where We Are

- T1
  - Simplest language generators: regular expressions
  - Simplest language recognizer: FSAs
- T2: more powerful machines
  - FSA, NFSA
  - PDA, NPDA
  - TM
- **T3: more powerful languages associated with the more powerful machines**
- T4:
  - Nature of most powerful machines
  - Languages that no machine can ever deal with

Review: Formal Languages

- Formal definitions
  - An **alphabet**: a finite set of symbols
  - A **string**: a finite sequence of symbols from the alphabet
  - A **language**: a (potentially infinite) set of strings over an alphabet
- Intriguing topic: **finite representation** of a language
  - How?
    - + language **generators** (a set of rules for producing strings)
    - + language **recognizers**

Languages and Automata

Given a RE, can construct a FSA that recognizes any string matching the RE

Given a FSA, can construct a RE that matches the strings recognized by the FSA

Thus, FSA's and RE's computationally equivalent

More powerful machines than FSAs:

- PDA: add pushdown stack, nondeterminism
- LBA: use (linear-bounded) tape
- Turing machine: use infinite tape

What languages do these machines recognize?

Why Learn Languages?

Concrete applications:

- understanding computability
- compiler implementation
- language recognition/translation
- models of physical world

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Grammars

- Generate words in a formal language by a process of replacing symbols systematically

Four elements:
- nonterminals
- terminals
- start symbol
- productions

NONTERMINAL symbols
- "local variables" for internal use
- notation: <name>

TERMINAL symbols
- set of characters that appear in the words
- "alphabet" of the language
- ex: {0,1} or ASCII

START symbol
- one particular nonterminal

PRODUCTIONS
- replacement rules
- ordered pairs of strings of symbols
- notation (ex.): a<B>c -> <D>e<F>
- LHS must have at least one nonterminal

Example 1

Nonterminal:

<sentence> <subject> <verb> <object>

Terminal:

horse dog cat saw heard the

Start:

<sentence>

Example 1 (cont.)

Productions:

```
<sentence> -> <subject><verb><object>
<subject>  -> the horse
<subject>  -> the dog
<subject>  -> the cat
<object>   -> the horse
<object>   -> the dog
<object>   -> the cat
<verb>     -> saw
<verb>     -> heard
```

"Words" in the language:

the horse saw the dog

the dog heard the cat

the cat saw the horse

Example 2

Nonterminal:

<stmnt> <selection-stmnt> <expression>, etc.

Terminal:

if while else switch ( ), etc.

Start:

<translation-unit>

Example 2 (cont.)

Productions:

```
<stmnt>
   -> <selection-stmnt>| <iteration-stmt> | ...
<selection-stmnt>
   -> if ( <expression> ) <stmnt>
   -> if ( <expression> ) <stmnt> else <stmnt>
<iteration-stmnt>
   -> while ( <expression> ) <stmnt>
   -> do <stmnt> while ( <expression> ) ;
```

```
if (x == 0) {
   while (y == 1) {
      if (z > 0) {
         s = z * 2;
      }
   }
}
```

Each box is one production.

Example 3

Nonterminal, Start: <pal>

Terminals: 0, 1

<pal> -> 0<pal>0 | 1<pal>1

<pal> -> 0 | 1

<pal> ->

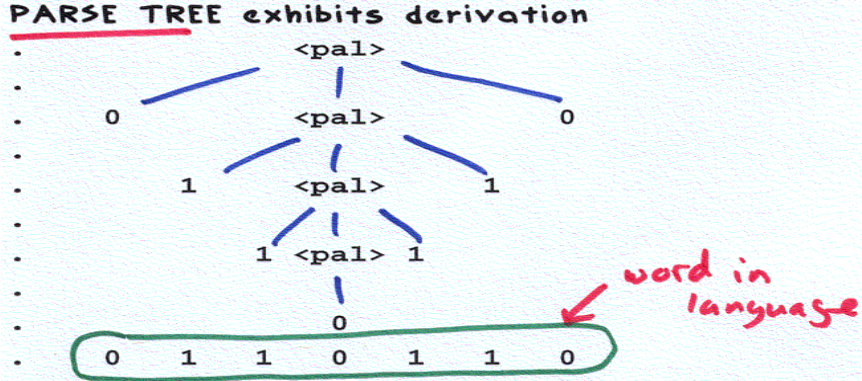
Example 3 (cont.)

DERIVATION: apply replacement rules until no nonterminals left

<pal> -> 0<pal>0 -> 01<pal>10

-> 011<pal>110 -> 0110110

PARSE TREE exhibits derivation



Grammar GENERATES language set of all strings that could be derived

0, 1, 00, 11, 010, 101, 000, 111, 0000, 00100

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Type 3 Grammars

Restrict all productions to be of the form

<X> -> x

<X> -> <Y>x

Ex:

<A> -> <z>0

<z> -> <A>1

<z> ->

generates alternating sequence of 0's and 1's

Type 3 Grammars and Regular Expressions

THM: Type 3 grammars are equivalent to REs

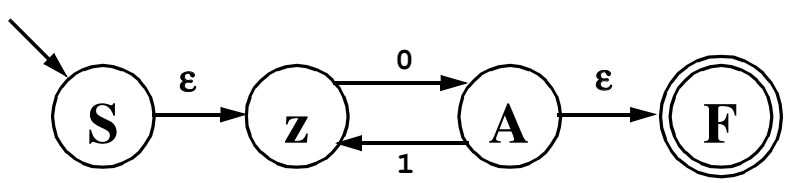
proof sketch:

- given a Type 3 grammar, construct an FSA that recognizes any string in the language generated by the grammar
- given an FSA, construct a Type 3 grammar that generates the strings recognized by the FSA

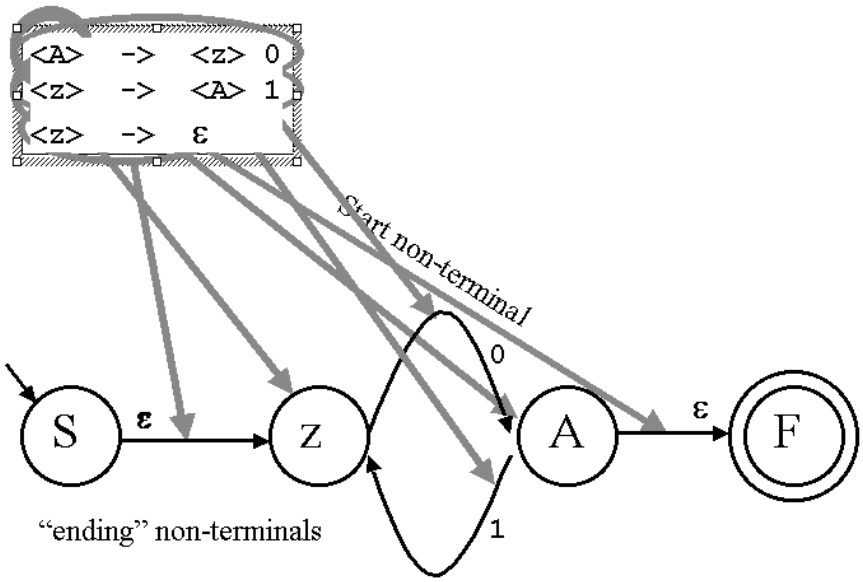
[FSA states correspond to nonterminals]

Example

```
<A> -> <z>0
<z> -> <A>1
<z> -> ε
```



Demo: Construction of Equivalent FSA



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Context-Free Grammars

Restrict all productions to allow only a single nonterminal on the LHS

Ex:

```
<pal> -> 0<pal>0
<pal> -> 1<pal>1
<pal> -> 0
<pal> -> 1
<pal> ->
```

Ex:

grammar for C

Also called CONTEXT-FREE grammars

Much more descriptive than regular expressions

Example Context-Free Grammar

Productions:

```
<stmnt>
   -> <selection-stmnt>| <iteration-stmt> | ...
<selection-stmnt>
   -> if ( <expression> ) <stmnt>
   -> if ( <expression> ) <stmnt> else <stmnt>
<iteration-stmnt>
   -> while ( <expression> ) <stmnt>
   -> do <stmnt> while ( <expression> ) ;
```

```
if (x == 0) {
   while (y == 1) {
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         s = z * 2;
      }
   }
}
```

Each box is one production.

How to Recognize Context-Free Grammars

Q. How does the FSA have to be augmented to recognize strings from languages generated by context-free grammars?

C compiler

A. Add a "memory" capability (as expected) *and* add power of nondeterminism (!)

Still limited: can't generate strings with equal numbers of a's, b's, and c's

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Type 1 Grammars (Context-Sensitive) and Type 0 Grammars

Type 1 (context-sensitive) grammars:

add productions of the type

[X]<Y>[Z] -> [X]y[Z]

where [X] and [Z] represent any sequence of terminals and nonterminals

Type 0 grammars: no restrictions

Chomsky Hierarchy, Languages and Automata

- Essential correspondence between languages and automata

Regular (Type 3)
- finite-state machine

Context-free (Type 2)
- nondeterministic pushdown automata

Context-sensitive (Type 1)
- linear bounded automata

Recursive (Type 0)
- Turing machines

Language Expressiveness and Machine Power

- One-to-one correspondence to machines persists through the hierarchy

Each type is more descriptive than the previous

Each machine is more powerful than the previous

Are there limits to machine "power"?

Are there languages that no machine can recognize?

[stay tuned]

T3.8

Example Language Outside Chomsky Hierarchy: Lindenmayer Systems

Apply productions simultaneously

Ex:

```
0 -> 1 [ 0 ] 1 [ 0 ] 0
1 -> 1 1
```

Reproduction

Growth

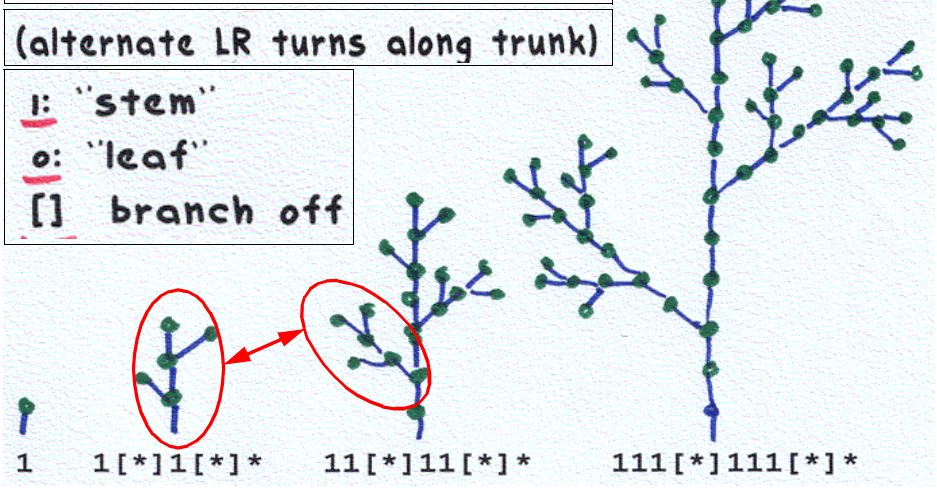
Start with 0

At stage i, apply rules to each symbol in string form stage i-1

```
0
1 [     0     ]1 [     0     ]     0
11[1[0]1[0]0]11[1[0]1[0]0]1[0]1[0]0
```

Graftal Plants: Lindenmayer in 2D

"Production" rules:
add one to each segment of my trunk;
replace each branch with myself of prev generation



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More on Languages, Machines, and Life

- The fine art of describing infinite beauty and variety with finite representation
  - Shakespearean sonnets and machines that can identify authorship?
    (Is intelligence/art nothing but a set of rules, with some non-determinism thrown in?)
  - DNA: the language of life
    (Is life nothing but a set of rules, with some non-determinism thrown in?)
  - .....

What We Have Learned Today

- What is a grammar?
- What is a type 3 grammar? type 2? type 1? type 0?
- How to construct an FSA from a type 3 grammar?
- What is the Chomsky Hierarchy?
- What type of automata recognize which grammars?