Light Transport and the Rendering Equation

COS 526: Advanced Computer Graphics

PRINCETON UNIVERSITY

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Course Outline

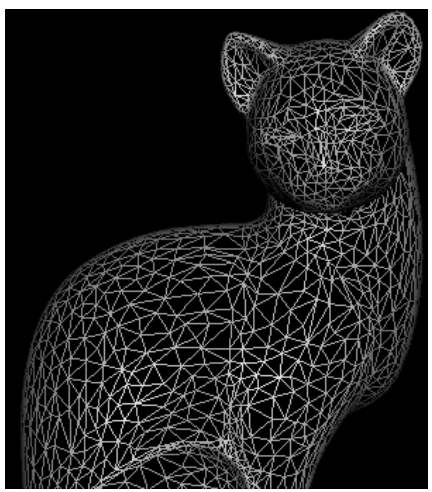
- 3D Graphics Pipeline

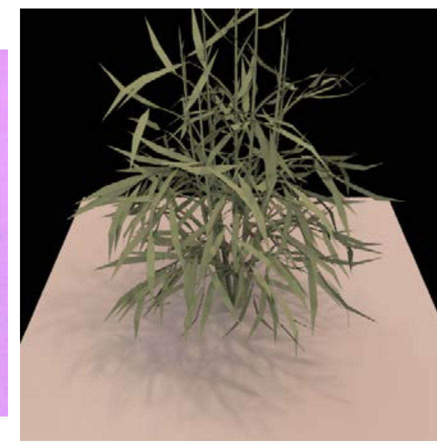
Modeling

(Creating 3D Geometry)

Rendering

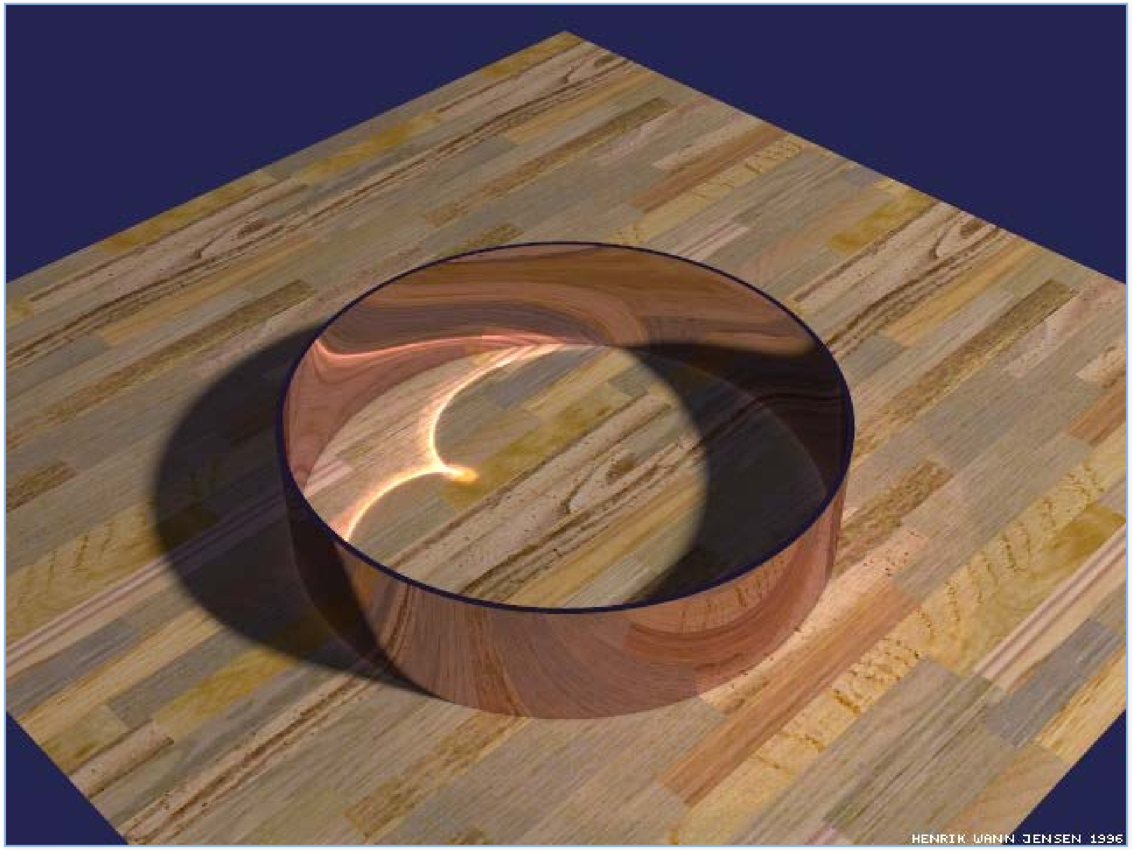
(Creating, shading images from geometry, lighting, materials)





Goal

- Synthesize image of a 3D scene accounting for all paths of light transport (including indirect illumination)

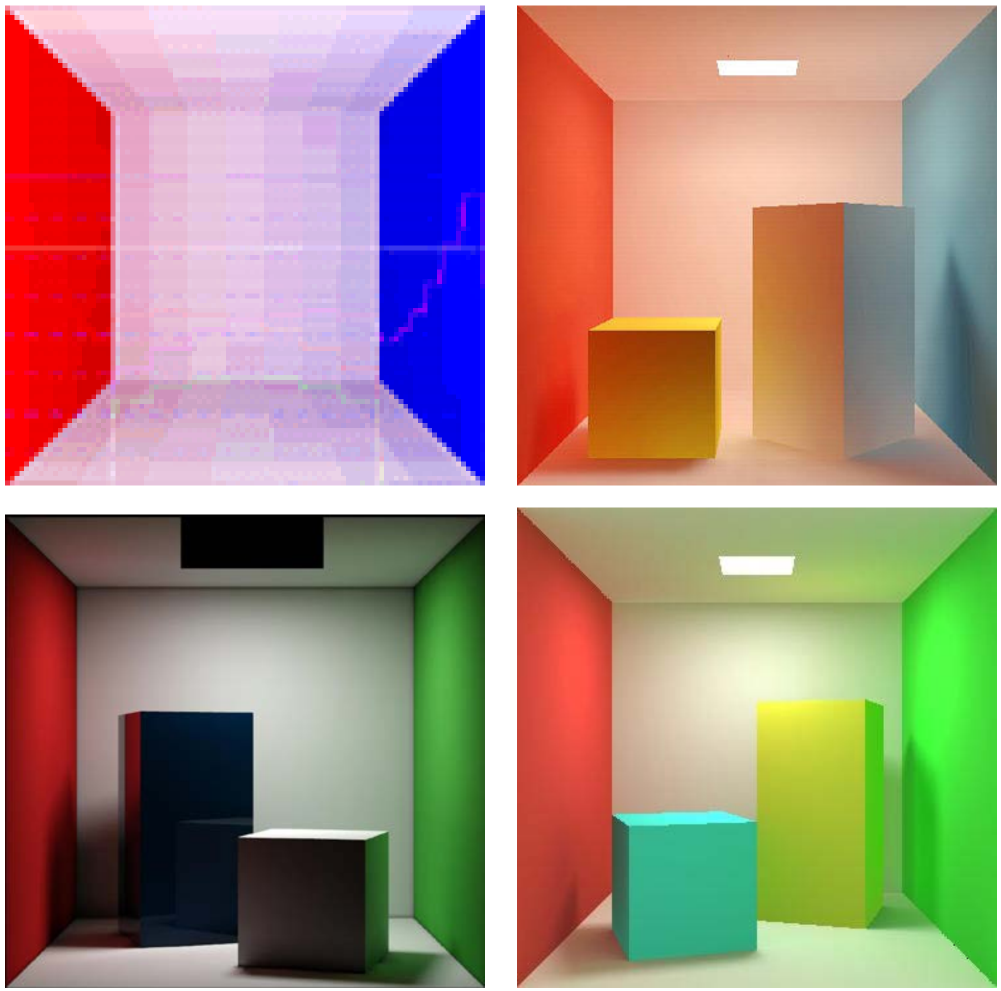


Rendering Challenges

- OpenGL and interactive rendering systems typically model *direct illumination*
- Simple ray tracers add *specular interreflection*
- Modeling all light transport through a scene (*global illumination*) requires accounting for:
  - Diffuse interreflection
  - Caustics
  - Volume scattering
  - etc.

Diffuse Interreflection

- Diffuse interreflection, color bleeding [Cornell Box]



Radiosity

Caustics

- Caustics: Focusing through specular surface

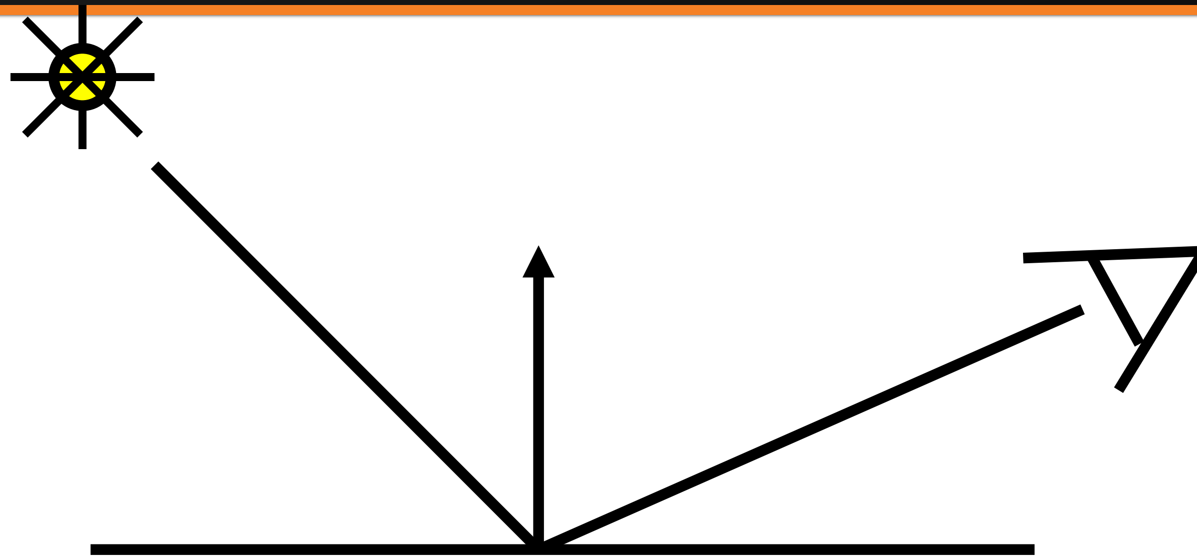
Overview of lecture

- *Theory* for all global illumination methods (ray tracing, path tracing, radiosity)
- We derive *Rendering Equation* [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
- Discuss existing approaches as special cases

Outline

- *Reflectance Equation*
- *Global Illumination*
- *Rendering Equation*
  - As a general Integral Equation and Operator
  - Approximations (Ray Tracing, Radiosity)
  - Surface Parameterization (Standard Form)

Reflection Equation



$$L_r(x,\omega_r) = L_e(x,\omega_r) + L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \cdot n)$$

Reflected Light (Output Image)

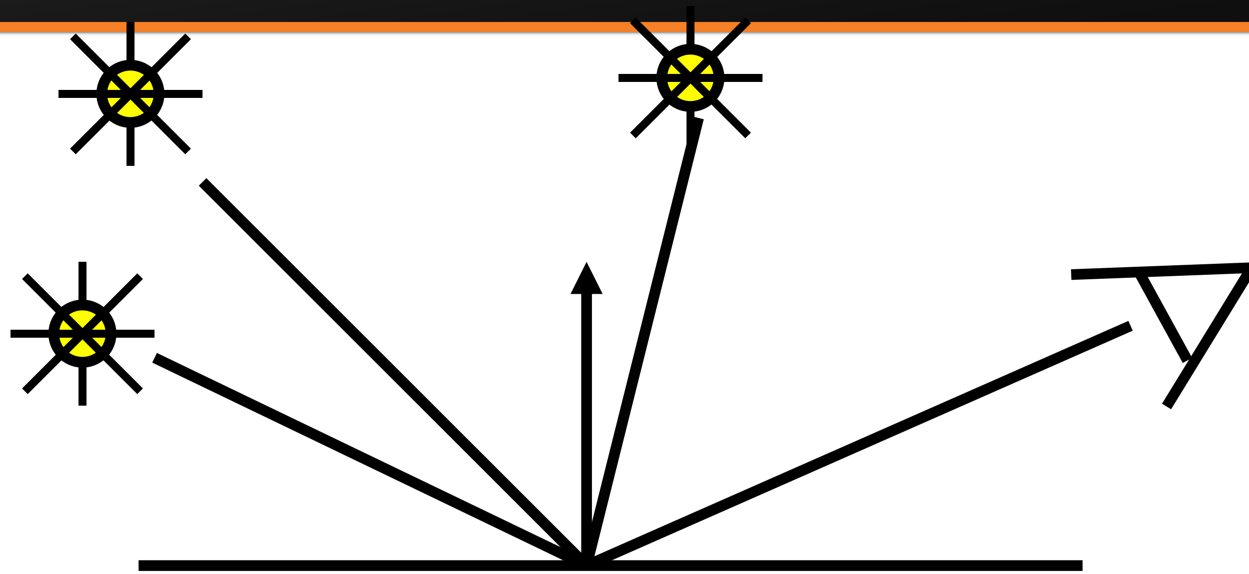
Emission

Incident Light (from light source)

BRDF

Cosine of Incident angle

Reflection Equation



Sum over all light sources

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \sum L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \cdot n)$$

Reflected Light (Output Image)

Emission

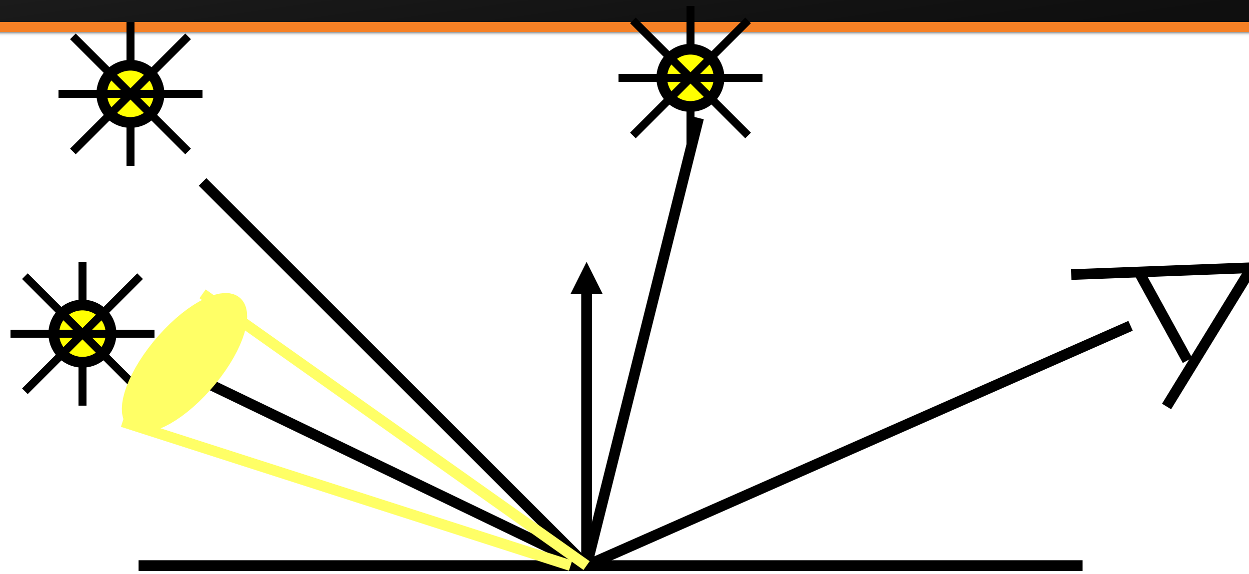
Incident Light (from light source)

BRDF

Cosine of Incident angle

Reflection Equation

ω_i



Replace sum with integral

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_i(x,\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i \, d\omega$$

Reflected Light (Output Image)

Emission

Incident Light (from light source)

BRDF

Cosine of Incident angle

Environment Maps

- Light as a function of direction, from entire environment
- Captured by photographing a chrome steel or mirror sphere
- Accurate only for one point, but distant lighting same at other scene locations (typically use only one env. map)

Blinn and Newell 1976, Miller and Hoffman, 1984
Later, Greene 86, Cabral et al. 87

Environment Maps

- Environment maps widely used as lighting representation
- Many modern methods deal with offline and real-time rendering with environment maps
- Image-based complex lighting + complex BRDFs

The Challenge

- Computing reflectance equation requires knowing the incoming radiance from surfaces

- But determining incoming radiance requires knowing the reflected radiance from surfaces

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Rendering Equation (Kajiya 86)

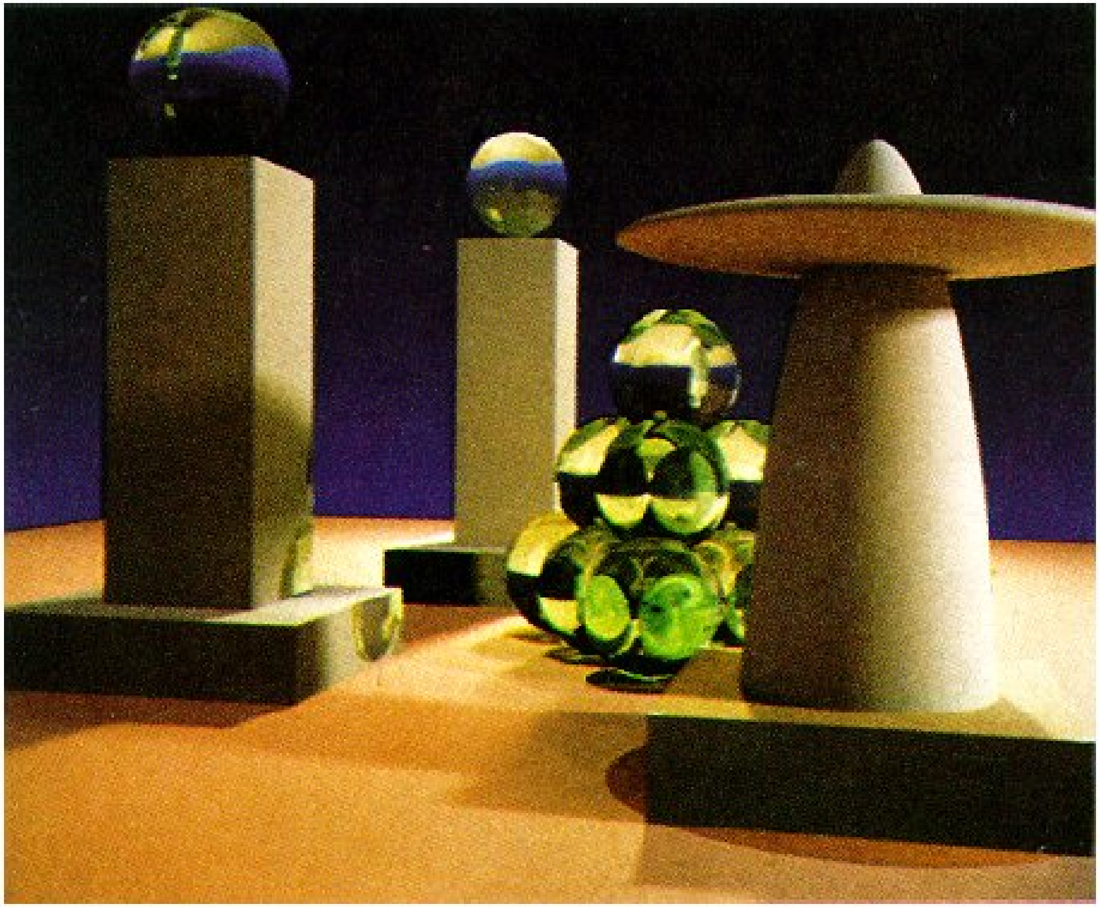
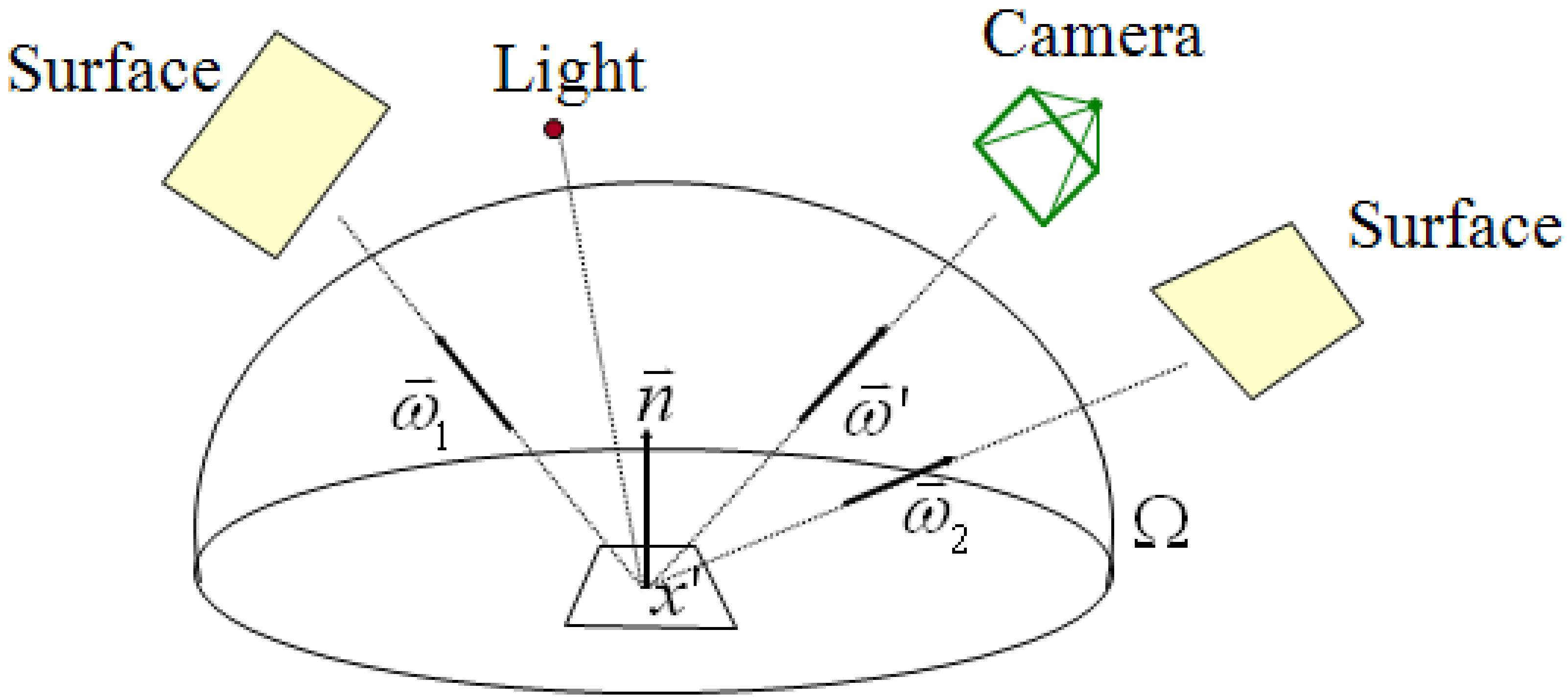


Figure 6. A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygon.

Rendering Equation (1)

Compute radiance in outgoing direction by integrating reflections over all incoming directions

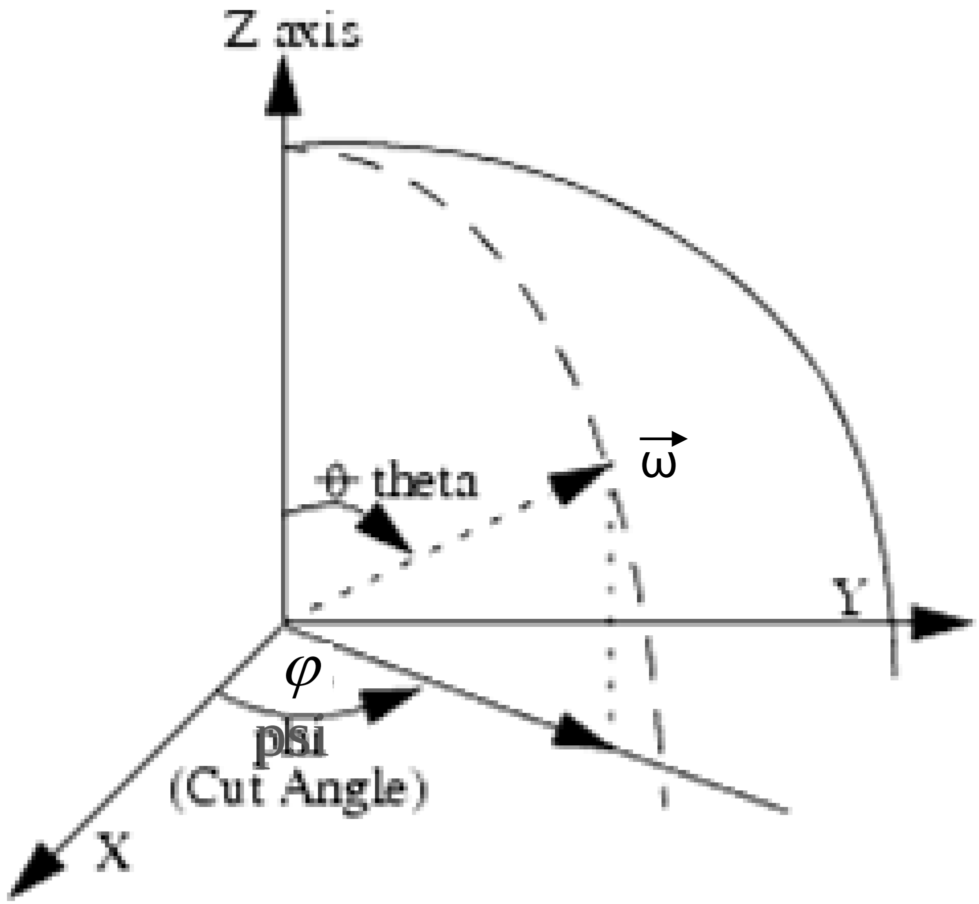


$$L_o(x', \bar{\omega}') = L_e(x', \bar{\omega}') + \int_{\Omega} f_r(x', \bar{\omega}, \bar{\omega}') L_i(x', \bar{\omega})(\bar{\omega} \bullet \bar{n}) d\bar{\omega}$$

What is $\vec{\omega}$?

$\vec{\omega}$ is a direction

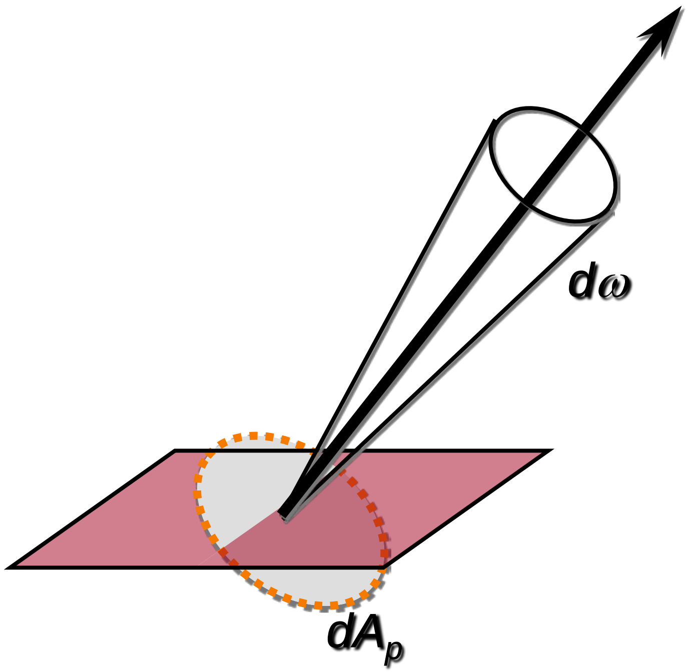
- 2-dimensional: (φ,θ)



What is L?

Radiance = power emitted from a surface in a direction

- Power $d\Phi$ per unit area dA_p per unit solid angle $d\omega$



$$L = \frac{d\Phi}{dA_p \, d\vec{\omega}}$$

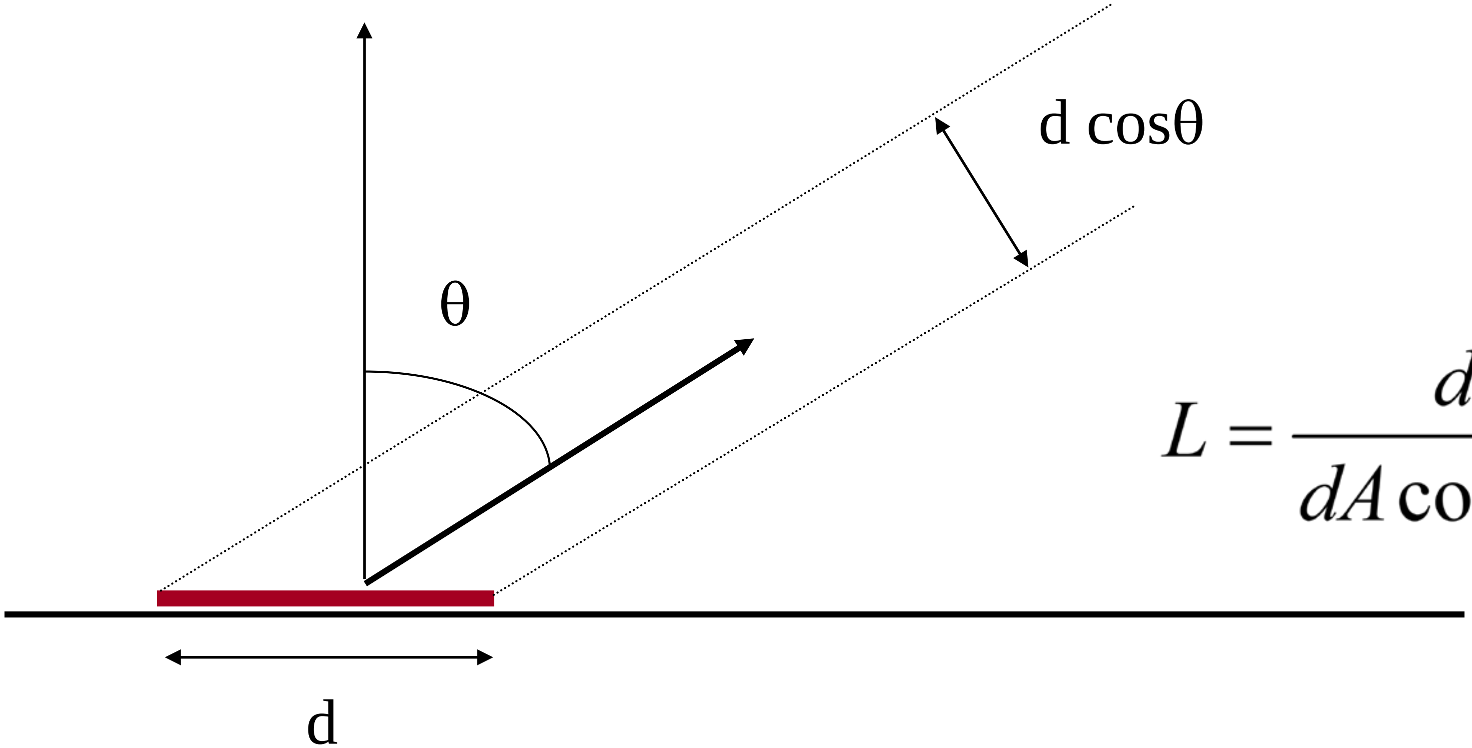
A_p = area perpendicular to given direction

$$L = \frac{d\Phi}{dA \cos\theta \;\; d\vec{\omega}}$$

Digression – Why Cosine Term?

Foreshortening is by cosine of angle.

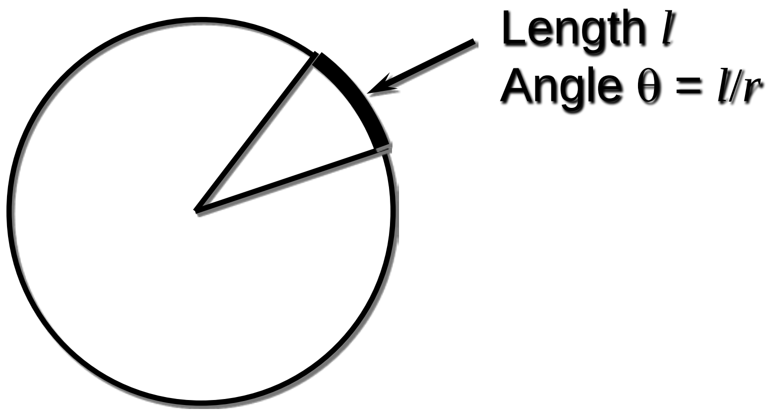
Radiance gives energy by *effective* surface area.



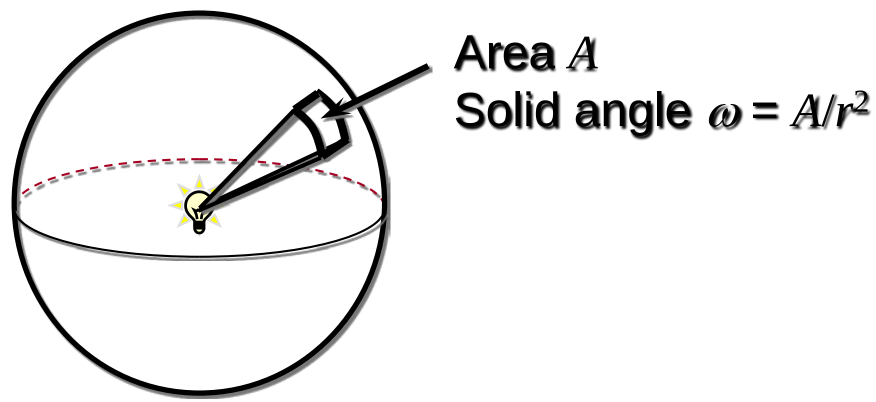
$$L = \frac{d\Phi}{dA \cos\theta \, d\vec{\omega}}$$

Digression – What is Solid Angle?

Angle in radians



Solid angle in steradians



Digression – Why Radiance?

Radiance doesn't change with distance

- Therefore it's the quantity we want to measure in a path tracer

Radiance is proportional to what a sensor (camera, eye) measures.

- Therefore it's what we want to output

Digression – What Units?

Light is a form of energy

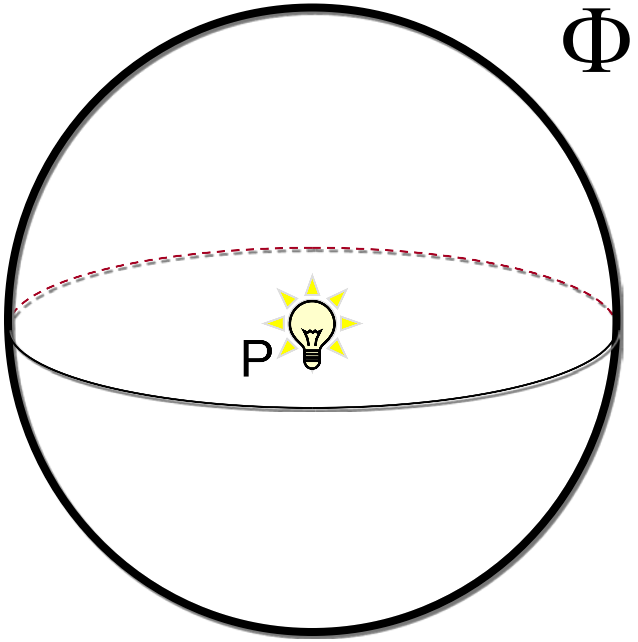
- Measured in Joules (J)

Power: energy per unit time

- Measured in Joules/sec = Watts (W)
- Also called Radiant Flux (Φ)

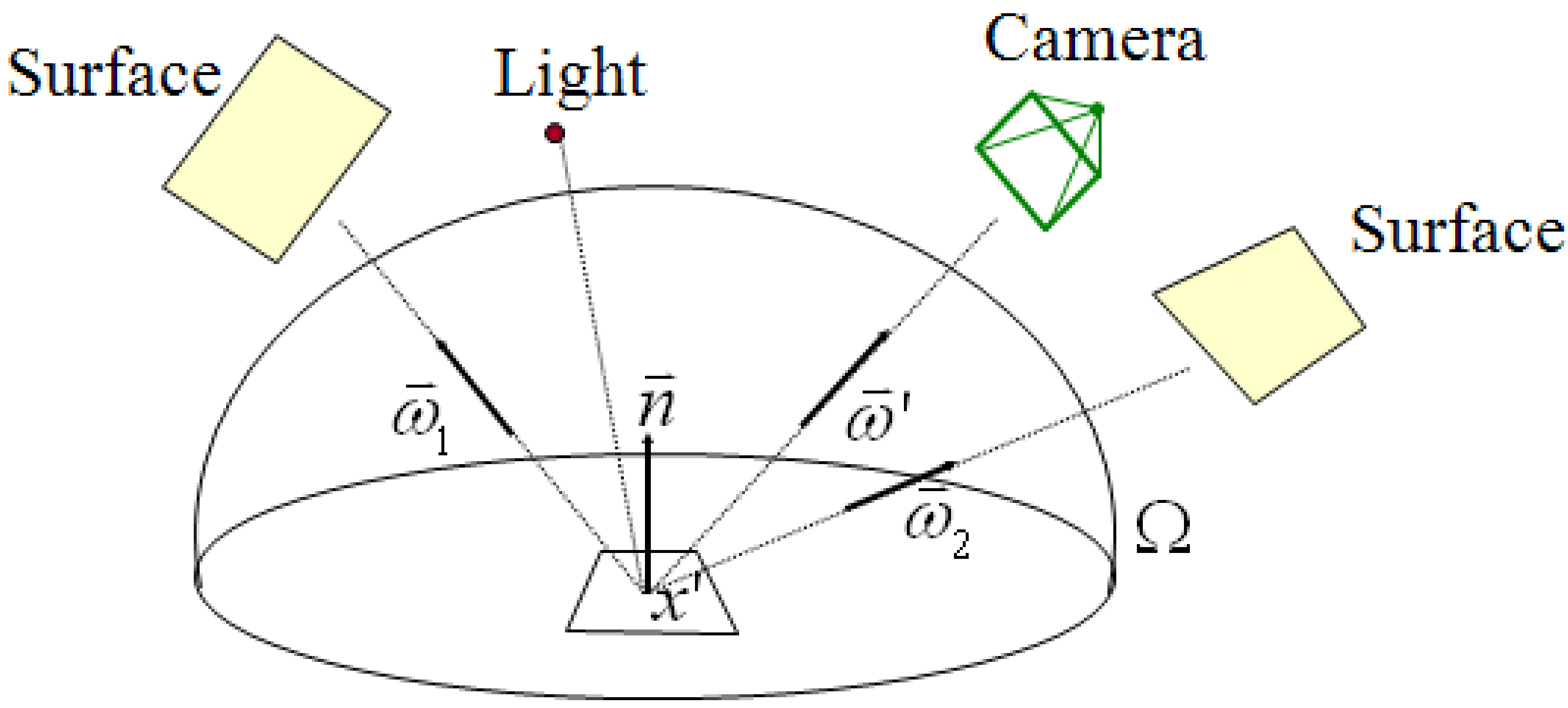
Radiance:

- Measured in $W/m^2/sr$



Rendering Equation (1) … Again

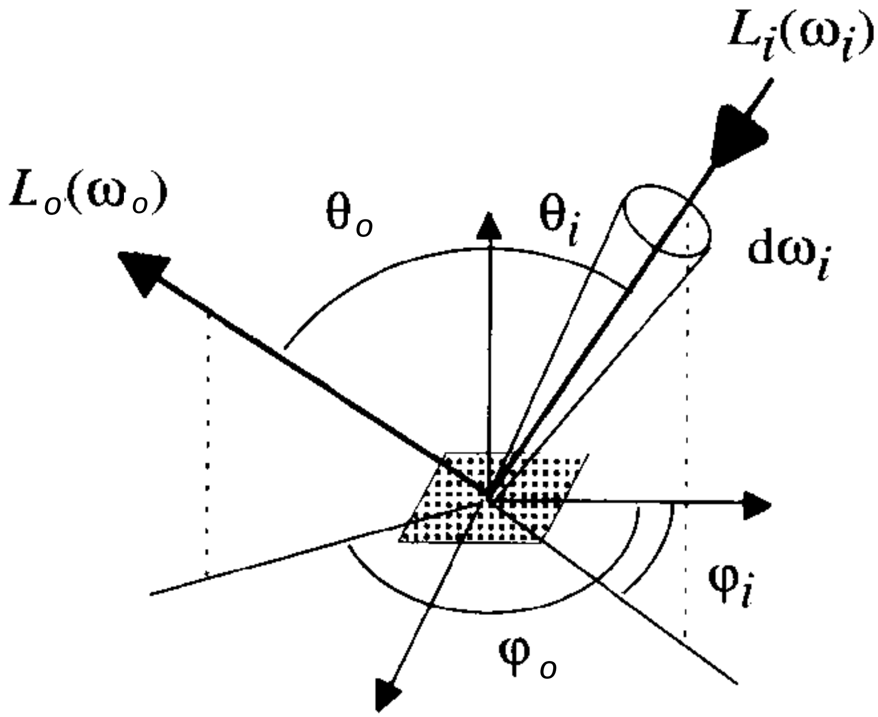
Compute radiance in outgoing direction by integrating reflections over all incoming directions



$$L_o(x',\bar{\omega}') = L_e(x',\bar{\omega}') + \int_{\Omega} f_r(x',\bar{\omega},\bar{\omega}')L_i(x',\bar{\omega})(\bar{\omega} \bullet \bar{n})d\bar{\omega}$$

What is f_r?

Bidirectional Reflectance Distribution Function (f_r) = fraction of irradiance E_i in incoming direction $\vec{\omega}_i$ reflected in outgoing direction $\vec{\omega}_o$



$$f_r(\omega_i \rightarrow \omega_o) = \frac{L_o(\omega_o)}{E_i(\omega_i)}$$

$$E_i(\vec{\omega}_i) \equiv L_i(\vec{\omega}_i)\cos\theta_i d\omega$$

$$f_r(\vec{\omega}_i \rightarrow \vec{\omega}_o) \equiv \frac{L_o(\vec{\omega}_o)}{L_i(\vec{\omega}_i)\cos\theta_i d\omega_i}$$

What is f_r?

BRDF (f_r) is usually a 4-dimensional function for each of three frequencies:

$$f_r(\theta_i, \varphi_i, \theta_o, \varphi_o) = \frac{L_o(\theta_o, \varphi_o)}{E_i(\theta_i, \varphi_i)}$$

BRDF (f_r) is an intrinsic property of a surface material

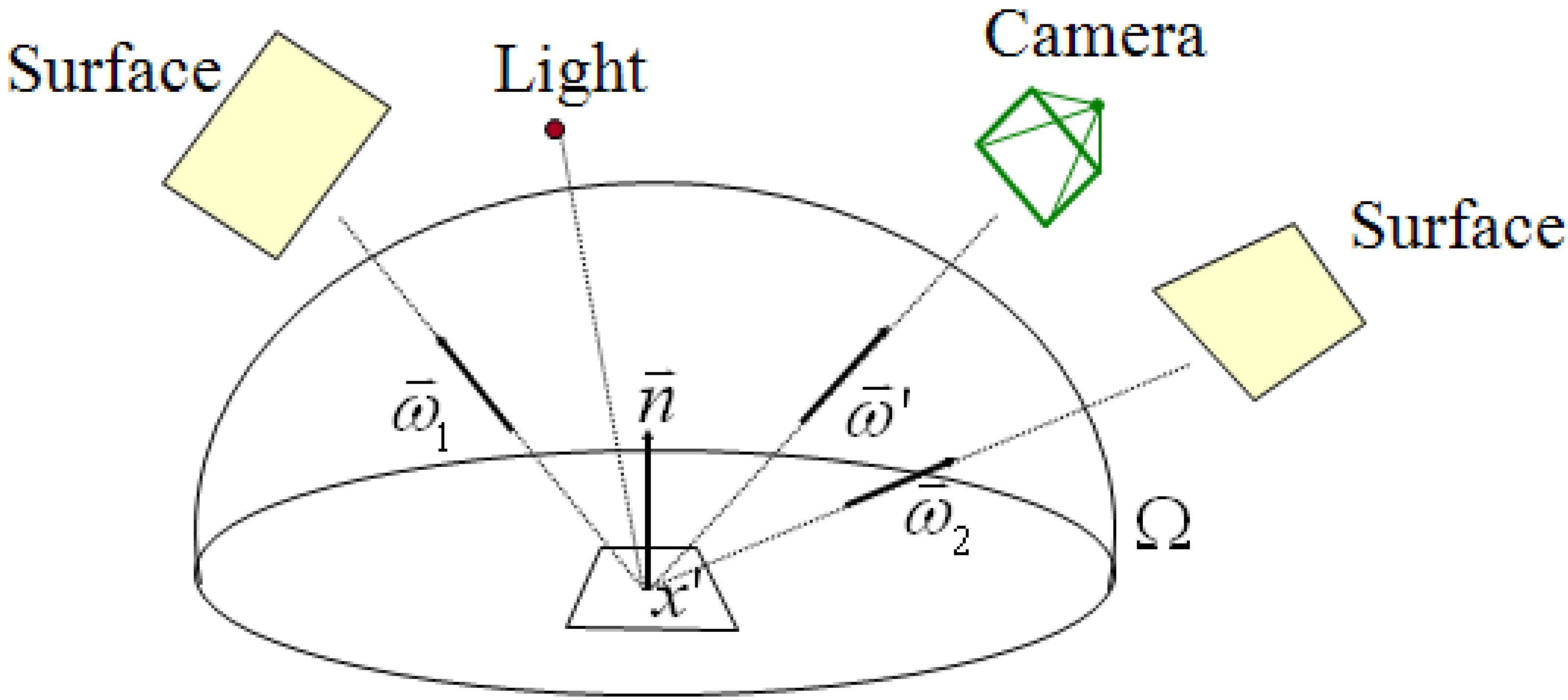
- Provided as part of material with scene description

Outgoing radiance and incoming irradiance are proportional to one another:

$$dL_o(\vec{\omega}_o) \propto dE_i(\vec{\omega}_i)$$

Rendering Equation (1) … Again

Compute radiance in outgoing direction by integrating reflections over all incoming directions



$$L_o(x', \bar{\omega}') = L_e(x', \bar{\omega}') + \int_{\Omega} f_r(x', \bar{\omega}, \bar{\omega}') L_i(x', \bar{\omega})(\bar{\omega} \bullet \bar{n}) d\bar{\omega}$$

Rendering Equation as Integral Equation

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) \boxed{f(x,\omega_i,\omega_r)\cos\theta_i d\omega_i}$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

$$l(u) = e(u) + \int l(v) \boxed{K(u,v)dv}$$

Kernel of equation

Linear Operator Theory

- Linear operators act on functions like matrices act on vectors or discrete representations

$$h(u) = \left(M \circ f\right)(u)$$

M is a linear operator.
f and h are functions of u

- Basic linearity relations hold

a and b are scalars
f and g are functions

$$M \circ \left(af + bg\right) = a\left(M \circ f\right) + b\left(M \circ g\right)$$

- Examples include integration and differentiation

$$\left(K \circ f\right)(u) = \int k(u,v)\, f(v)\, dv$$

$$\left(D \circ f\right)(u) = \frac{\partial f}{\partial u}(u)$$

Linear Operator Equation

$$l(u) = e(u) + \int l(v) \boxed{K(u,v)} dv$$

Kernel of equation
Light Transport Operator

$$L = E + KL$$

Can be discretized to a simple matrix equation
[or system of simultaneous linear equations]
(L, E are vectors, K is the light transport matrix)

Solving the Rendering Equation

- Too hard for analytic solution, numerical methods
- Approximations, that compute different terms, accuracies of the rendering equation
- Two basic approaches are ray tracing, radiosity. More formally, Monte Carlo and Finite Element
- Monte Carlo techniques sample light paths, form statistical estimate (example, path tracing)
- Finite Element methods discretize to matrix equation

Solving the Rendering Equation

- General linear operator solution. Within raytracing:
- General class numerical ***Monte Carlo*** methods
- Approximate set of all paths of light in scene

$$L = E + KL$$

$$IL - KL = E$$

$$(I - K)L = E$$

$$L = (I - K)^{-1}E$$

Binomial Theorem

$$L = (I + K + K^2 + K^3 + ...)E$$

$$L = E + KE + K^2E + K^3E + ...$$

Term n corresponds to n bounces of light

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

Direct Illumination
on surfaces

Global Illumination
(One bounce indirect)
[Mirrors, Refraction]

(Two bounce indirect)
[Caustics etc]

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

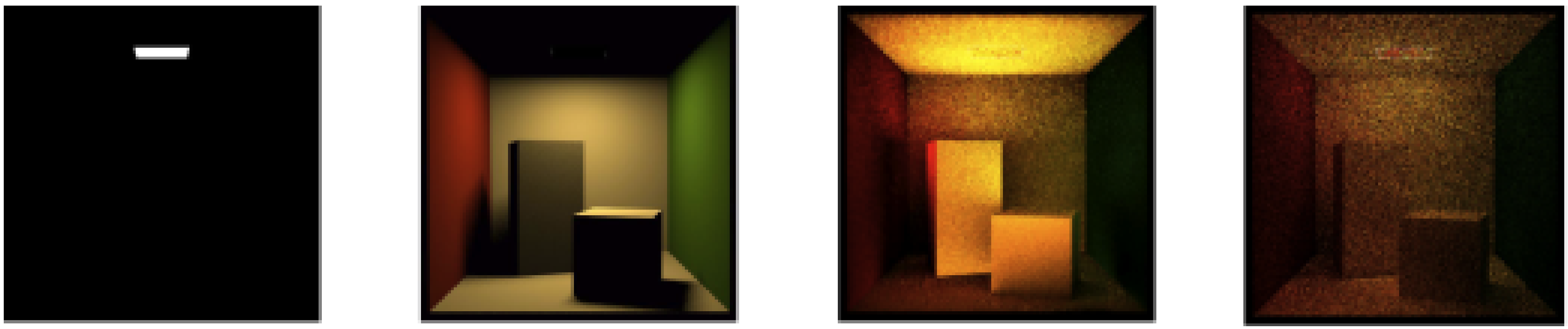
Direct Illumination
on surfaces

OpenGL Shading

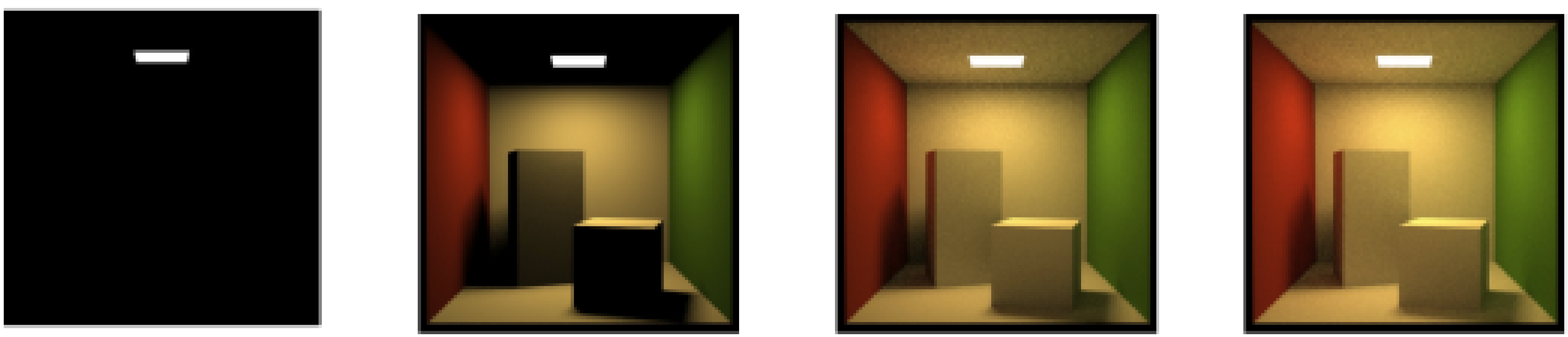
Global Illumination
(One bounce indirect)
[Mirrors, Refraction]

(Two bounce indirect)
[Caustics etc]

Successive Approximation



L_e | $K \circ L_e$ | $K \circ K \circ L_e$ | $K \circ K \circ K \circ L_e$



L_e | $L_e + K \circ L_e$ | $L_e + \cdots K^2 \circ L_e$ | $L_e + \cdots K^3 \circ L_e$

Outline

- *Reflectance Equation*
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- *Rendering Equation*
  - As a general Integral Equation and Operator
  - Approximations (Ray Tracing, Radiosity)
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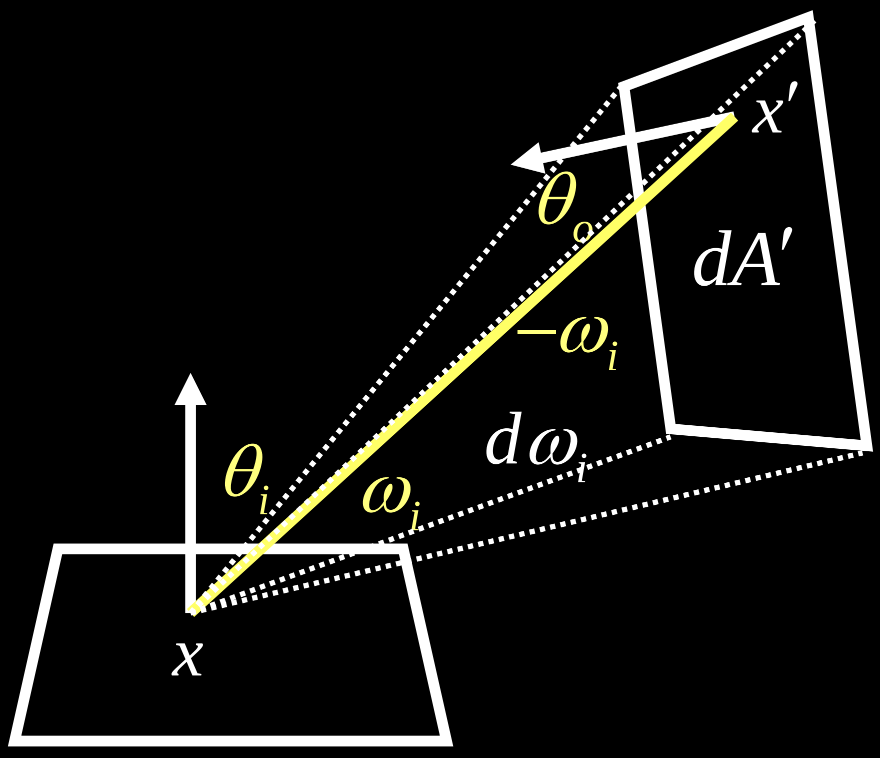
Change of Variables

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r)\boxed{\cos\theta_i d\omega_i}$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)



$$d\omega_i = \frac{dA' \cos\theta_o}{|x-x'|^2}$$

Change of Variables

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\cos\theta_i d\omega_i}$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all } x' \text{ visible to } x} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\frac{\cos\theta_i \cos\theta_o}{|x-x'|^2} dA}$$

$$d\omega_i = \frac{dA' \cos\theta_o}{|x-x'|^2}$$

$$G(x,x') = G(x',x) = \frac{\cos\theta_i \cos\theta_o}{|x-x'|^2}$$

Rendering Equation: Standard Form

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\cos\theta_i \, d\omega_i}$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all } x' \text{ visible to } x} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\frac{\cos\theta_i \cos\theta_o}{|x-x'|^2} \, dA}$$

Domain integral awkward. Introduce binary visibility fn V

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all surfaces } x'} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) G(x,x') V(x,x') \, dA'$$

$$d\omega_i = \frac{dA' \cos\theta_o}{|x-x'|^2}$$

$$G(x,x') = G(x',x) = \frac{\cos\theta_i \cos\theta_o}{|x-x'|^2}$$

Radiosity Equation

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all surfaces } x'} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) G(x,x') V(x,x')\, dA'$$

Drop angular dependence (diffuse Lambertian surfaces)

$$L_r(x) = L_e(x) + f(x) \int_S L_r(x') G(x,x') V(x,x')\, dA'$$

Change variables to radiosity (B) and albedo (ρ)

$$B(x) = E(x) + \rho(x) \int_S B(x') \frac{G(x,x') V(x,x')}{\pi}\, dA'$$

Expresses conservation of light energy at all points in space

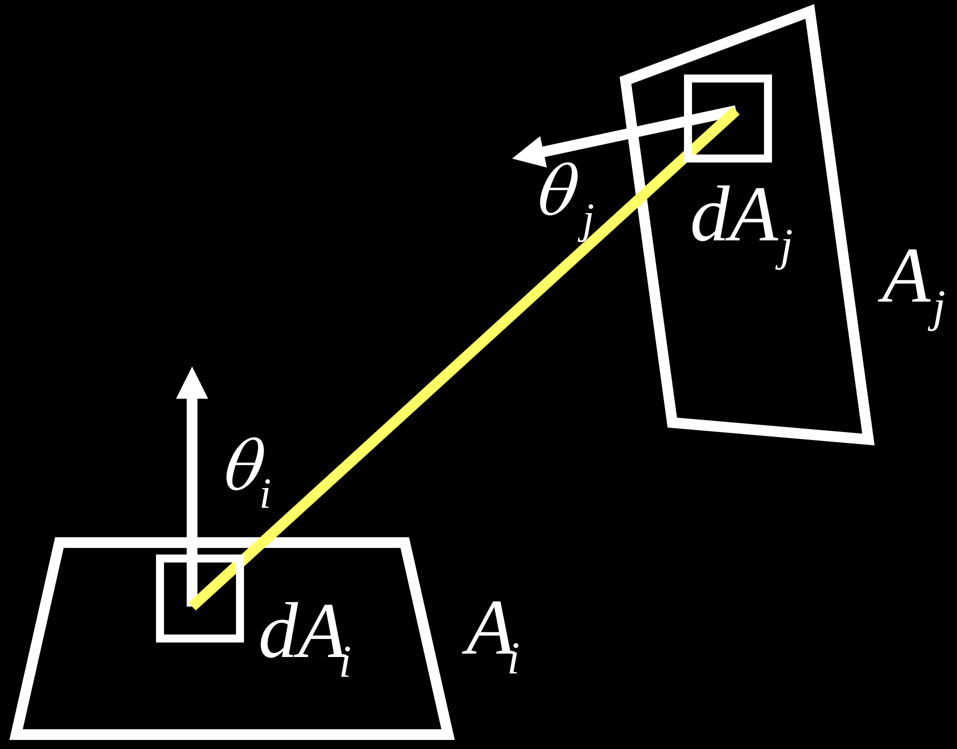
Discretization and Form Factors

$$B(x) = E(x) + \rho(x) \int_S B(x') \frac{G(x,x')V(x,x')}{\pi} dA'$$

$$B_i = E_i + \rho_i \sum_j B_j F_{j \to i} \frac{A_j}{A_i}$$

F is the ***form factor.*** It is dimensionless and is the fraction of energy leaving the entirety of patch j (*multiply by area of* j to get total energy) that arrives anywhere in the entirety of patch i (*divide by area of i* to get energy per unit area or radiosity).

Form Factors

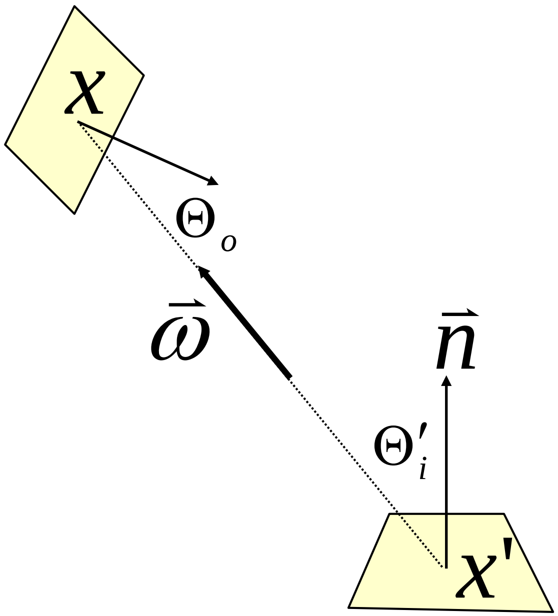


$$A_i F_{i\rightarrow j} = A_j F_{j\rightarrow i} = \iint \frac{G(x,x')V(x,x')}{\pi} dA_i dA_j$$

$$G(x,x') = G(x',x) = \frac{\cos\theta_i \cos\theta_o}{|x-x'|^2}$$

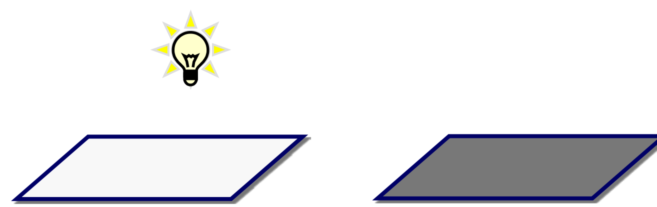
What is V(x,x')G(x, x')?

Irradiance at x in direction $\vec{\omega}$ as a fraction of radiance leaving x' in direction $\vec{\omega}$



$$G(x, x') = \frac{\cos\Theta_i' \cos\Theta_o}{\left\| x - x' \right\|^2}$$

Move surface away from light:
Inverse square law: E ~ $1/r^2$



Tilt surface away from light:
Cosine law: E ~ $\mathbf{n} \cdot \mathbf{l}$

Matrix Equation

$$B_i = E_i + \rho_i \sum_j B_j F_{j \to i} \frac{A_j}{A_i}$$

$$A_i F_{i \to j} = A_j F_{j \to i} = \iint \frac{G(x, x') V(x, x')}{\pi} dA_i dA_j$$

$$B_i = E_i + \rho_i \sum_j B_j F_{i \to j}$$

$$B_i - \rho_i \sum_j B_j F_{i \to j} = E_i$$

$$\sum_j M_{ij} B_j = E_i \qquad MB = E \qquad M_{ij} = I_{ij} - \rho_i F_{i \to j}$$

Summary

- *Theory* for all global illumination methods (ray tracing, path tracing, radiosity)
- We derive *Rendering Equation* [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
- Discuss existing approaches as special cases