3D Acquisition and Registration

COS 526: Advanced Computer Graphics

PRINCETON UNIVERSITY

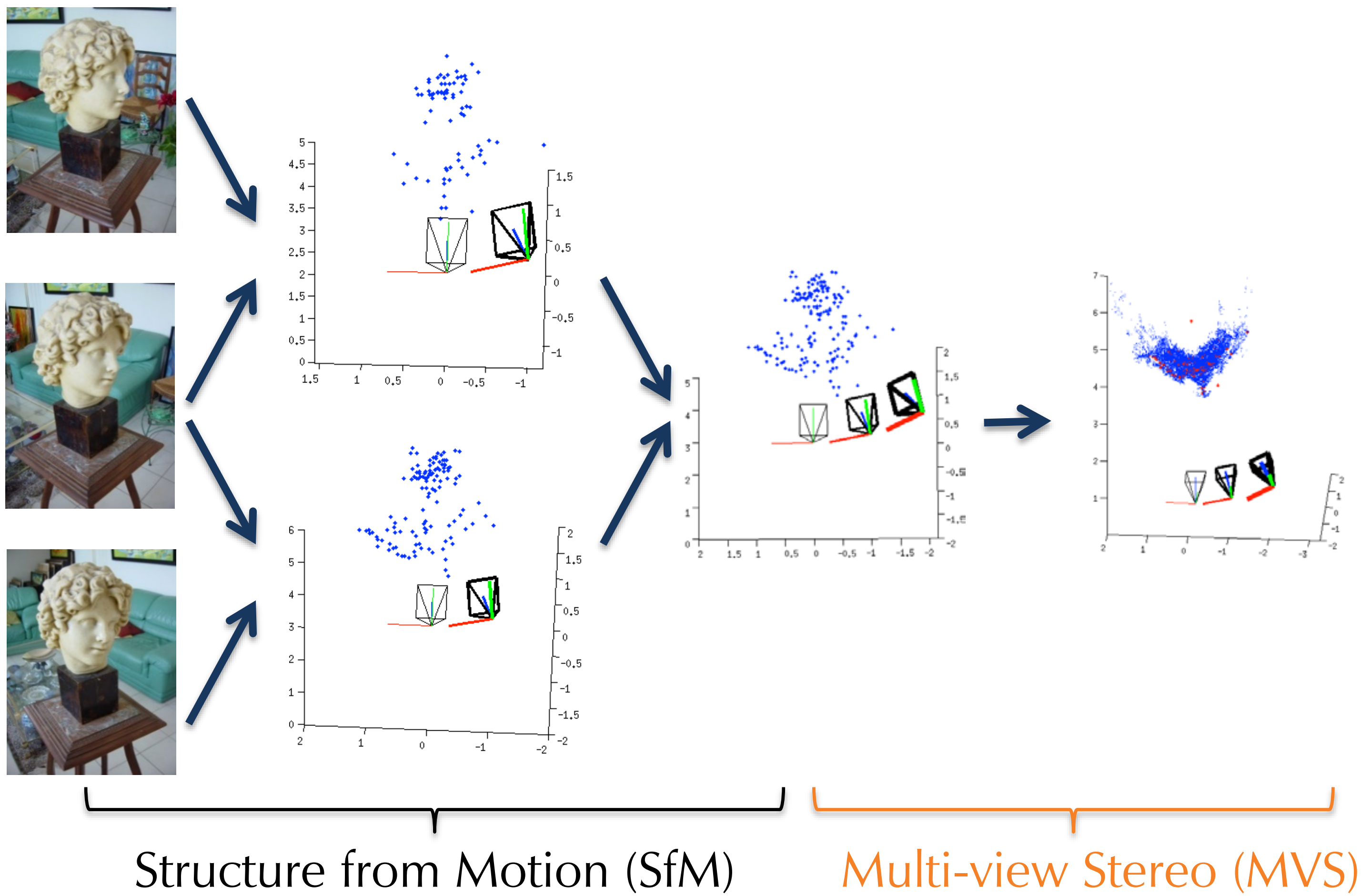
Slide credits: Tom Funkhouser

Point Cloud Acquisition

- Passive
  - Structure from motion
- Active
  - Touch probes
  - Optical scanning
    - Time of flight
    - Triangulation
      - Laser
      - Structured light

Structure from Motion

- Solve for 3D structure from correspondences



Structure from Motion

- Advantages:
  - Demonstrated for large photo collections
- Disadvantages:
  - Pixel correspondences required

Touch Probes

- Capture points on object with tracked tip of probe
  - Physical contact with the object
  - Manual or computer-guided

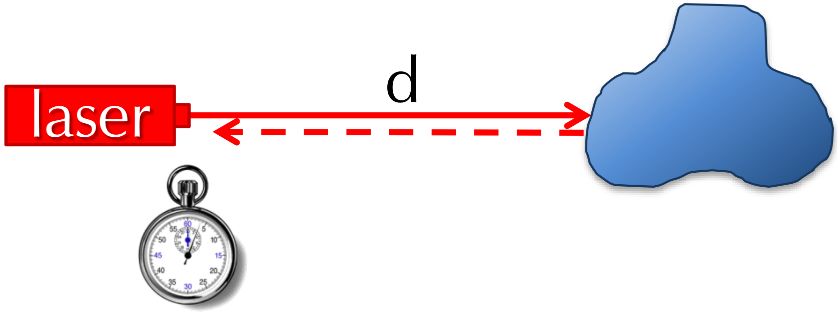
Touch Probes

- Advantages:
  - Can be very precise
  - Insensitive to appearance

- Disadvantages:
  - Slow, small scale
  - Can't use on fragile objects

Time of Flight Laser Scanning

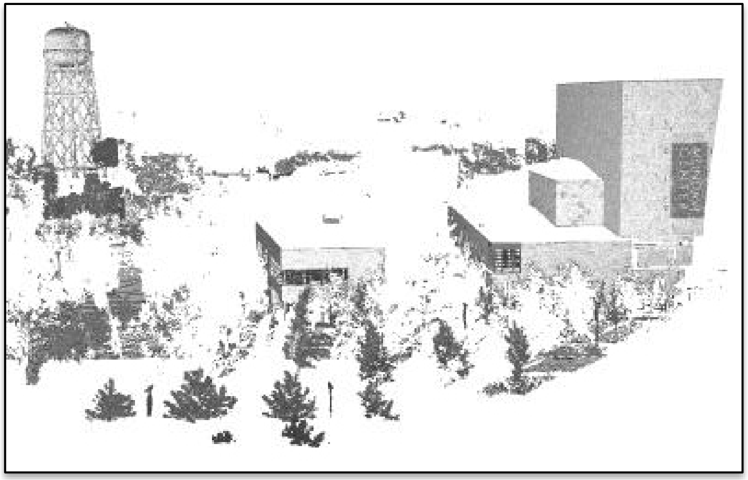
- Measures the time it takes the laser beam to hit the object and come back
  - e.g., LIDAR

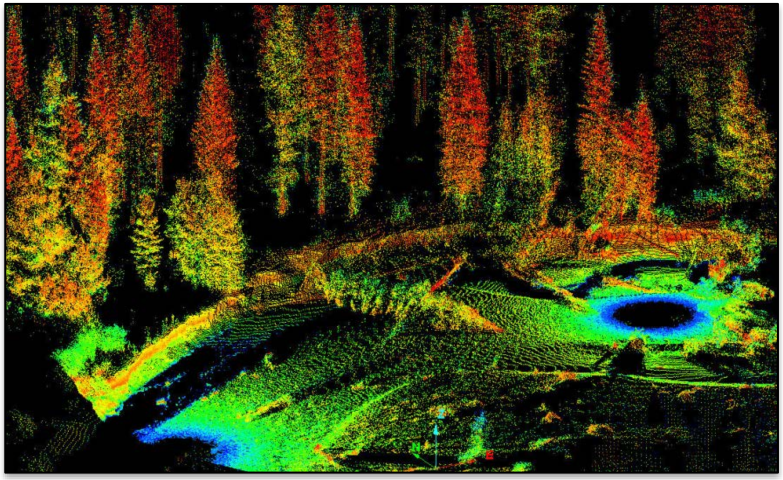


$$d = 0.5\, t \cdot c$$

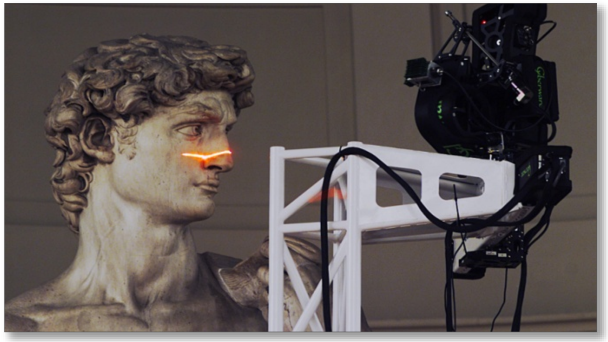
Time of Flight Laser Scanning

- Advantages
  - Accommodates large range – up to several miles (suitable for buildings, landscapes)
- Disadvantages
  - Lower accuracy (~ several mm)

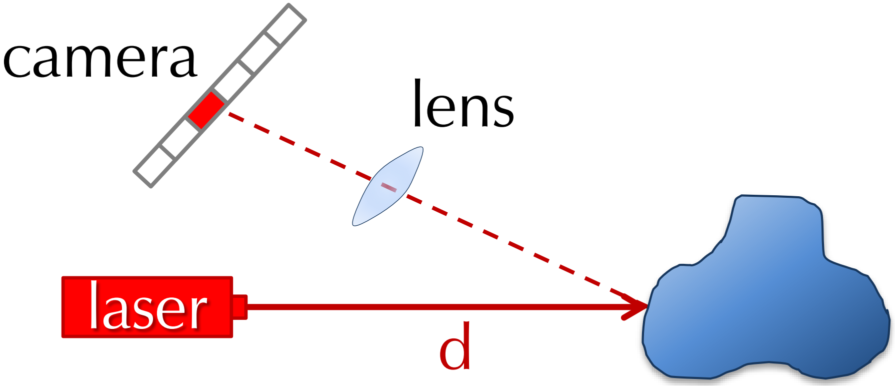




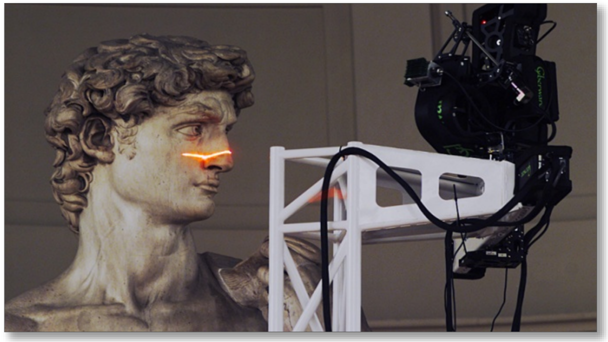
Triangulation Laser Scanning



- System includes calibrated laser beam and camera
  - Laser dot is photographed
- The location of the dot in the image allows triangulation: tells distance to the object



Triangulation Laser Scanning



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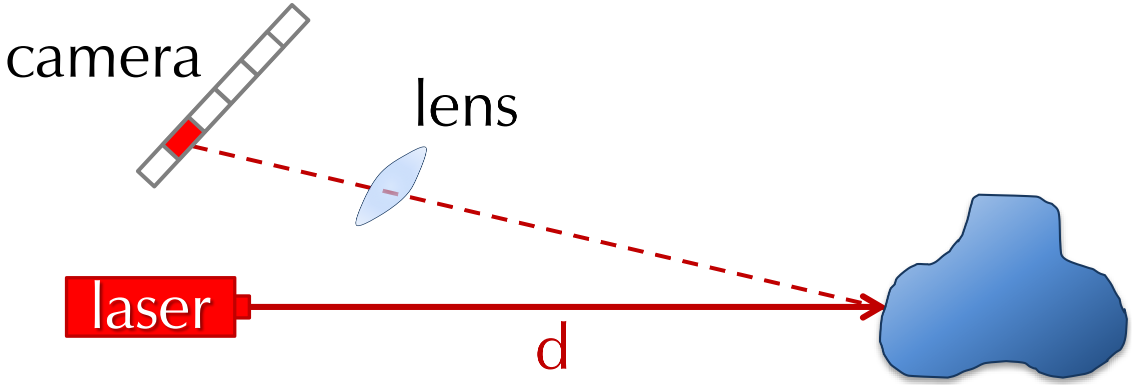
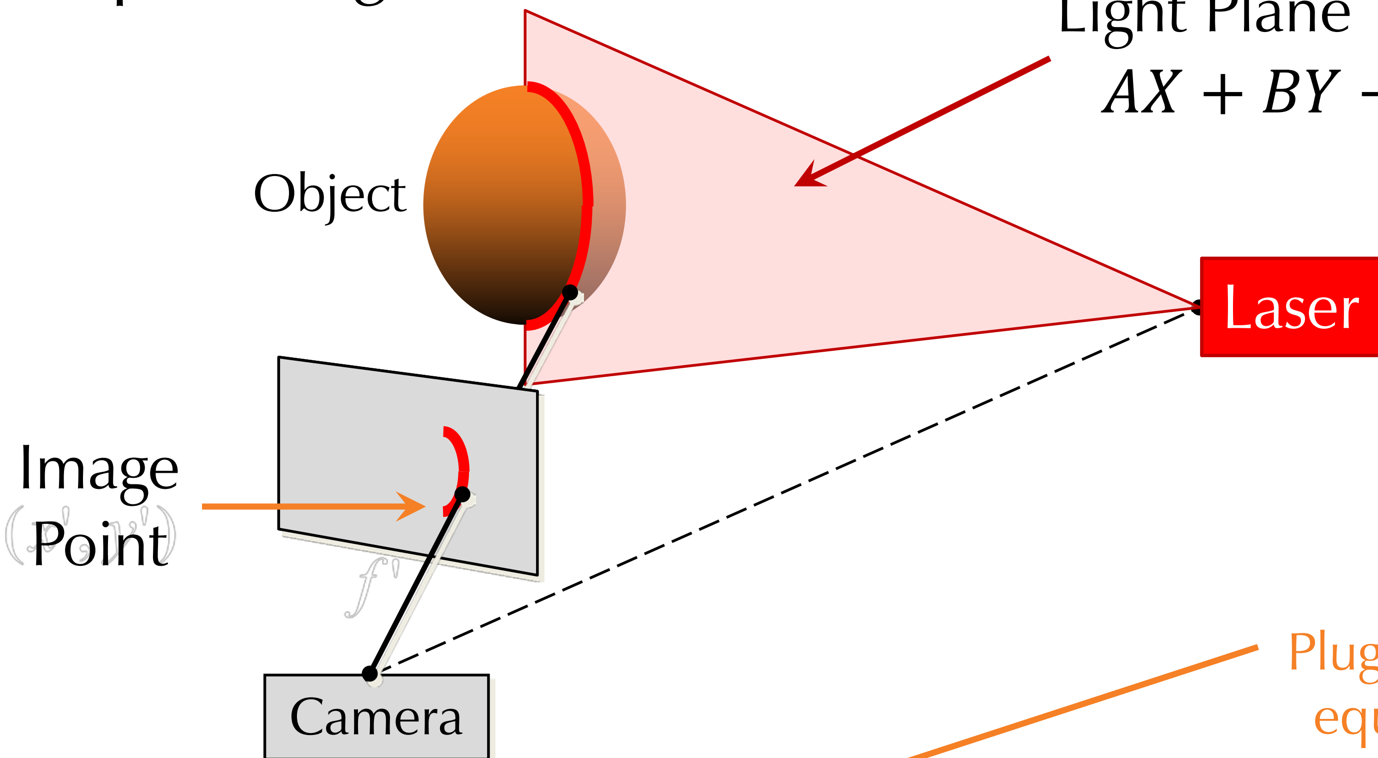


Triangulation Laser Scanning

- Stripe triangulation



Light Plane

$$AX + BY + CZ + D = 0$$

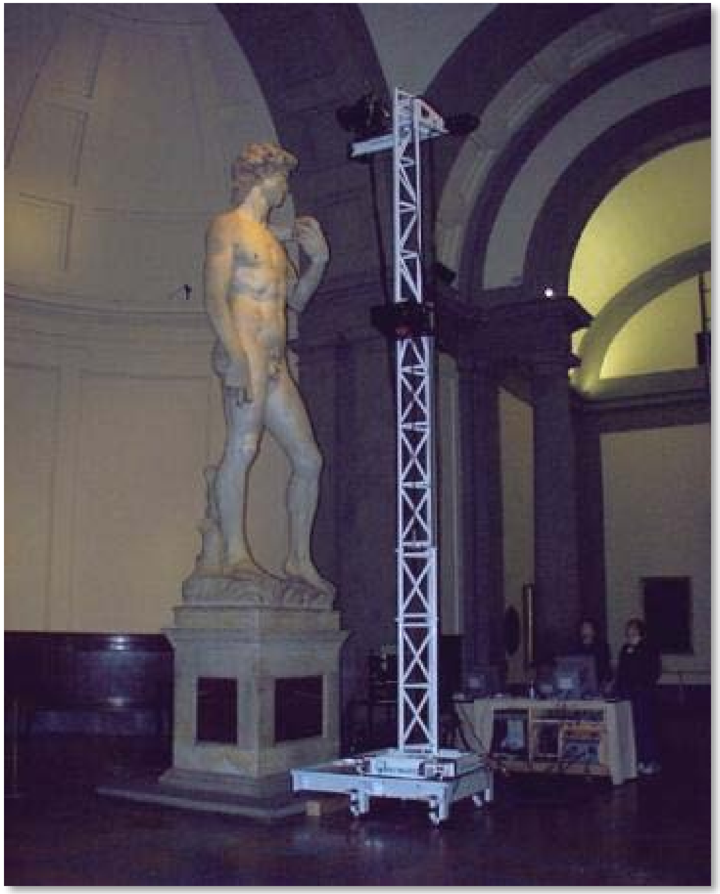
Plug X, Y into plane equation to get Z

$$X = x'Z / f' \qquad Z = \frac{-Df'}{Ax' + By' + Cf'}$$

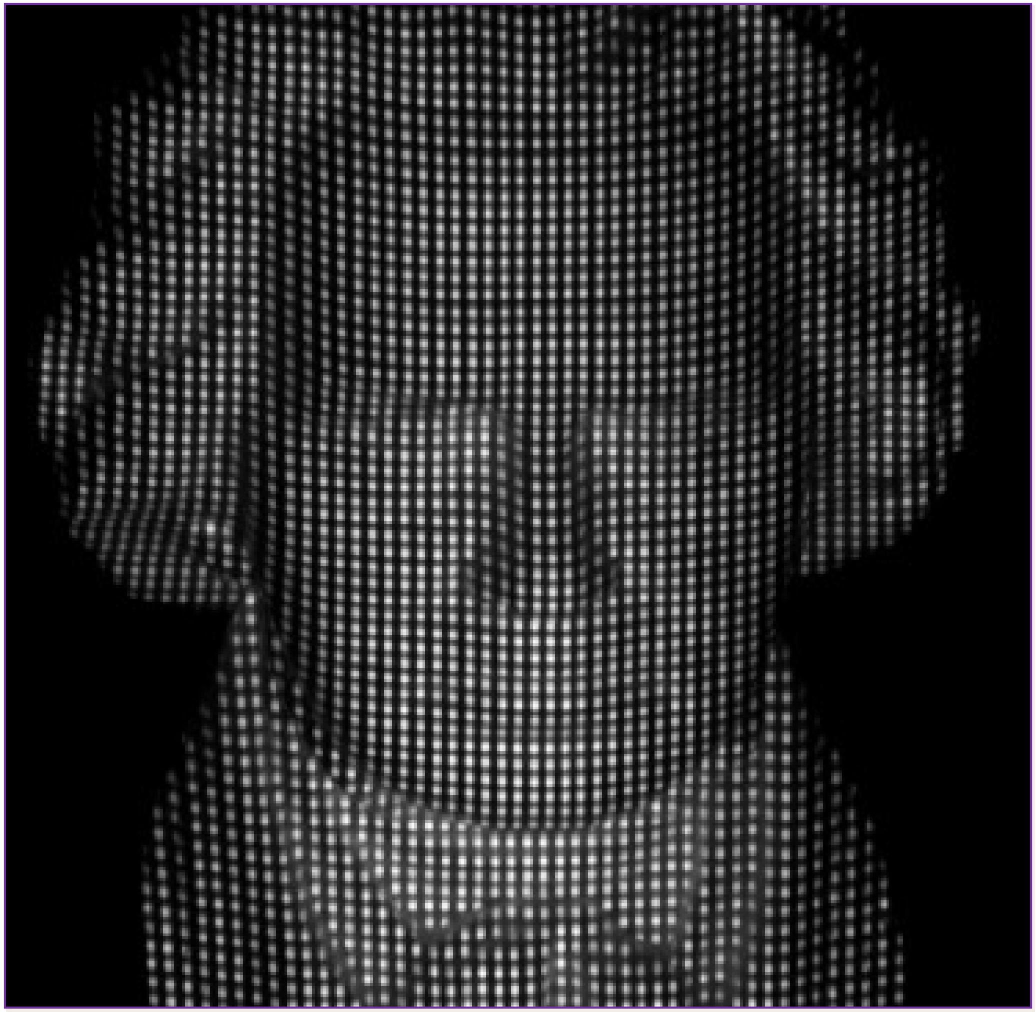
$$Y = y'Z / f'$$

Triangulation Laser Scanning

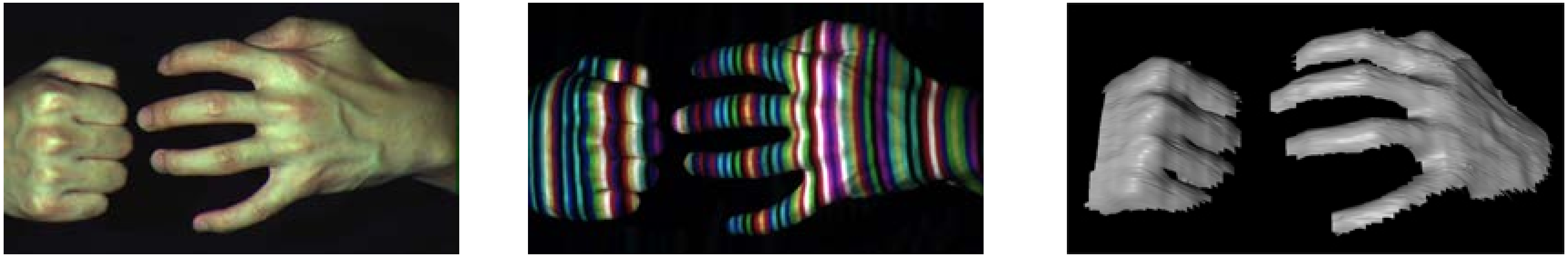
- Advantages
  - Very precise (tens of microns)
- Disadvantages
  - Small distances (meters)
  - Inaccessible regions

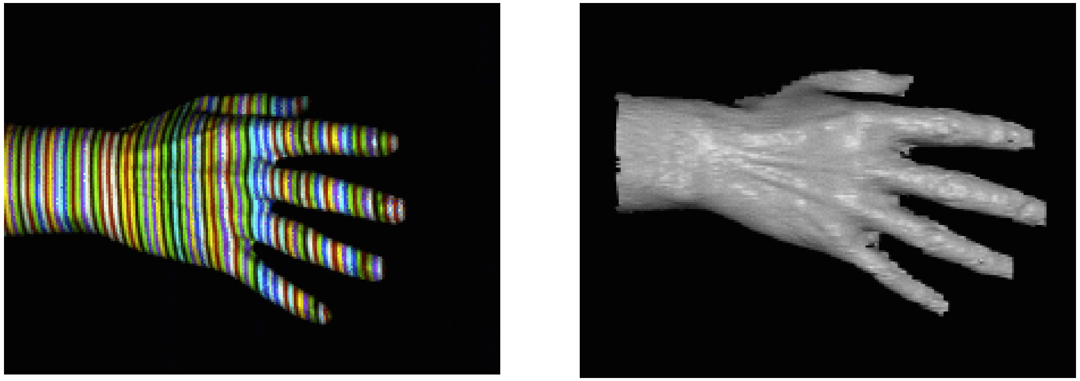


Multi-Stripe Triangulation



Color-Coded Stripe Triangulation

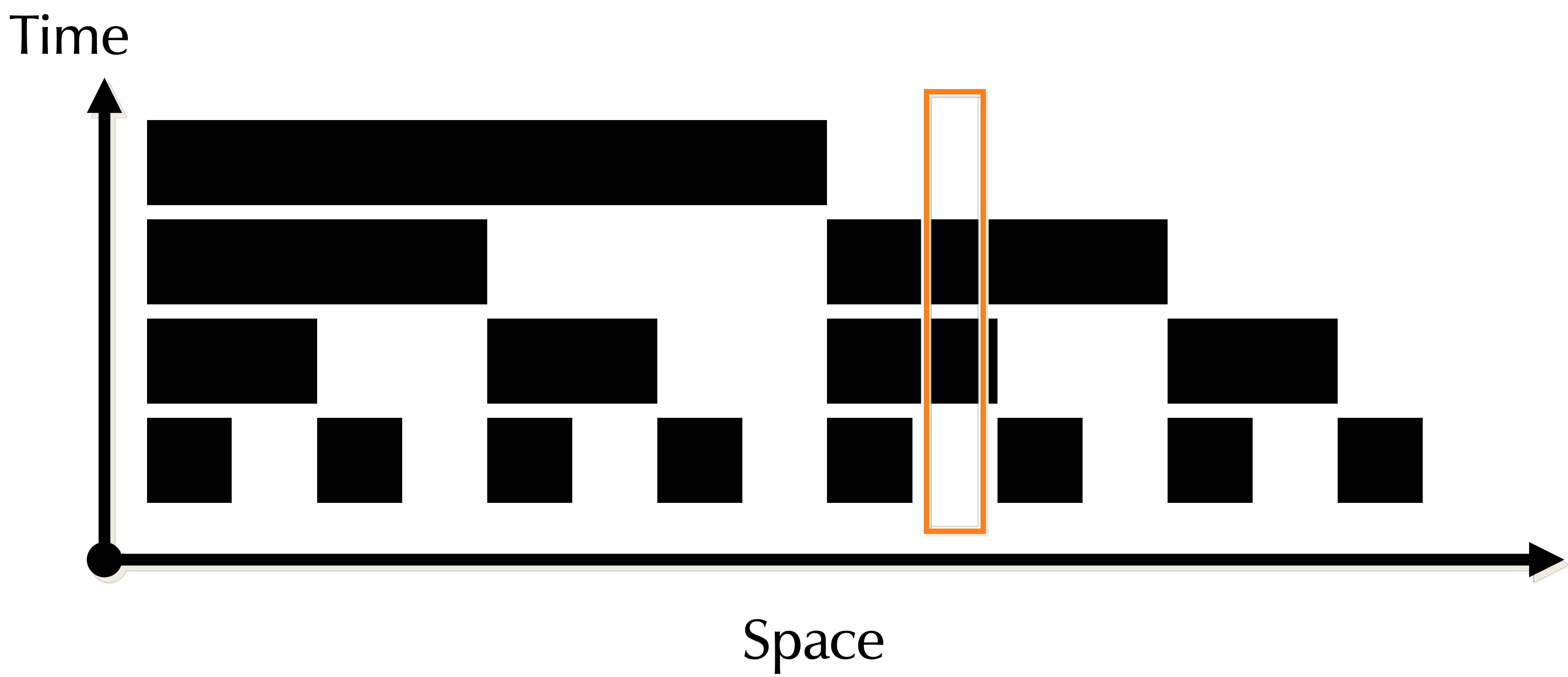




Zhang et al, 3DPVT 2002

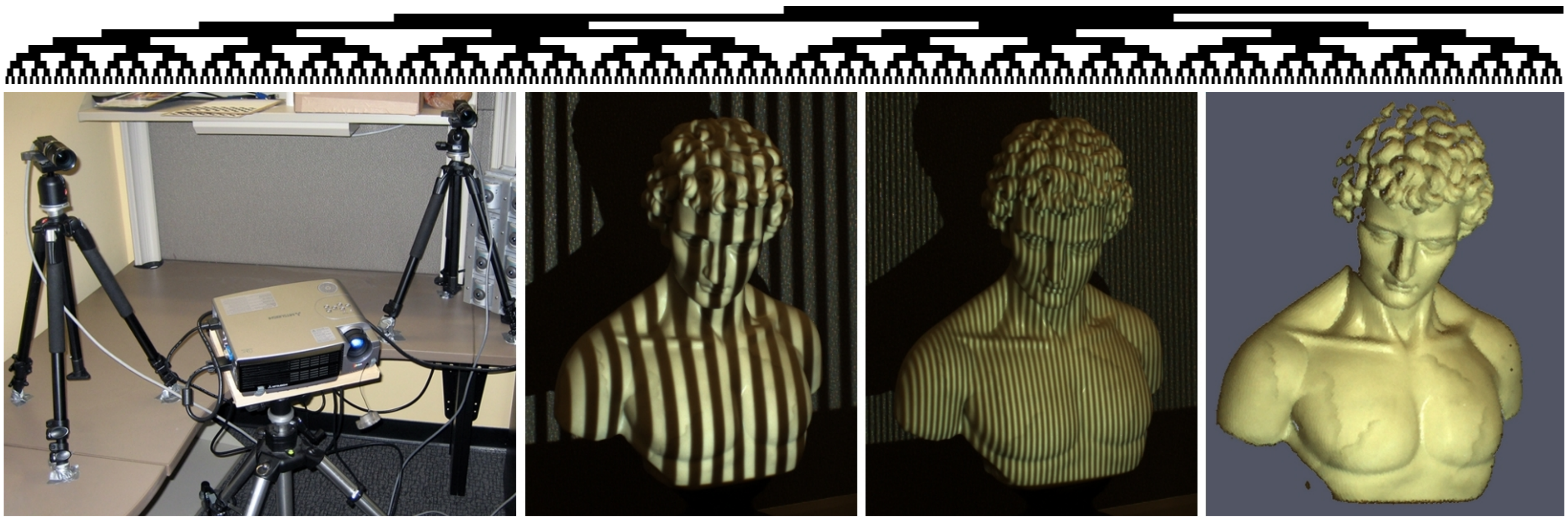
Time-Coded Stripe Triangulation

Assign each stripe a unique illumination code over time

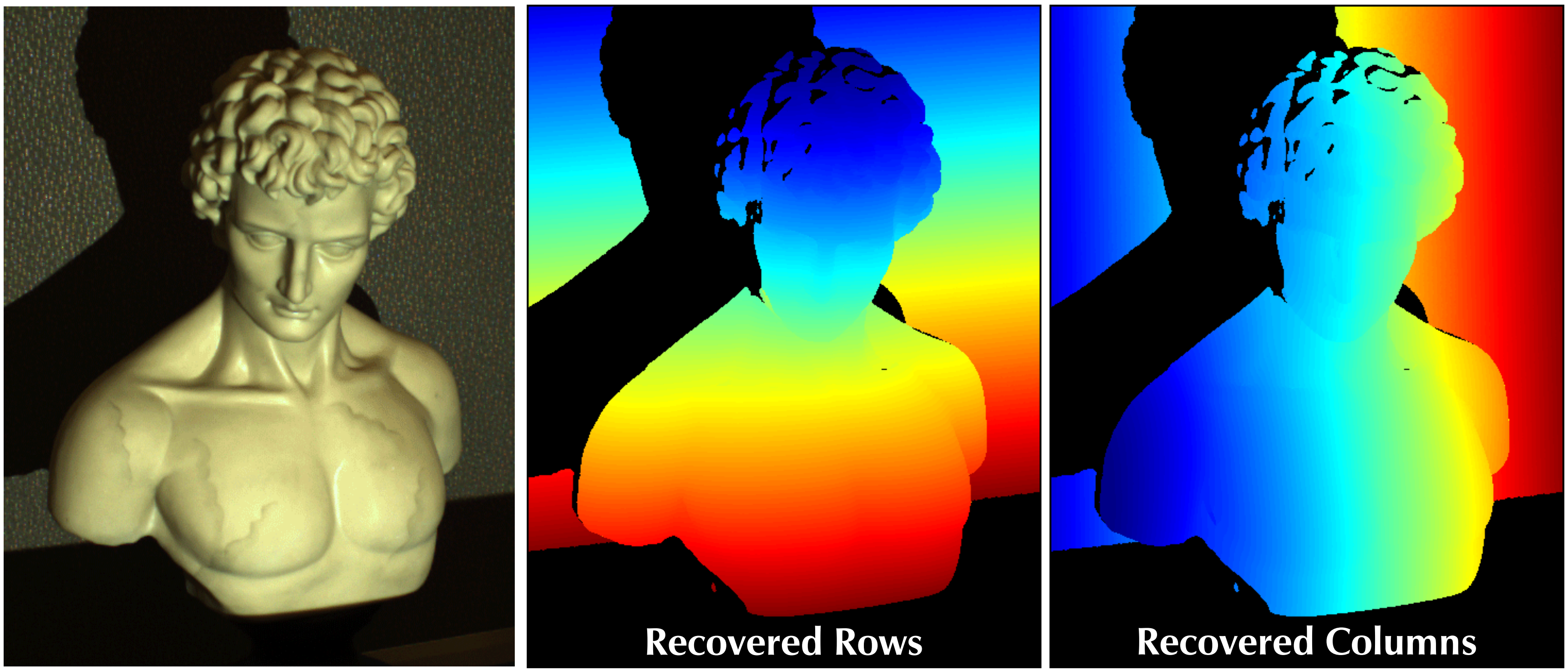


[Posdamer 82]

Time-Coded Stripe Triangulation

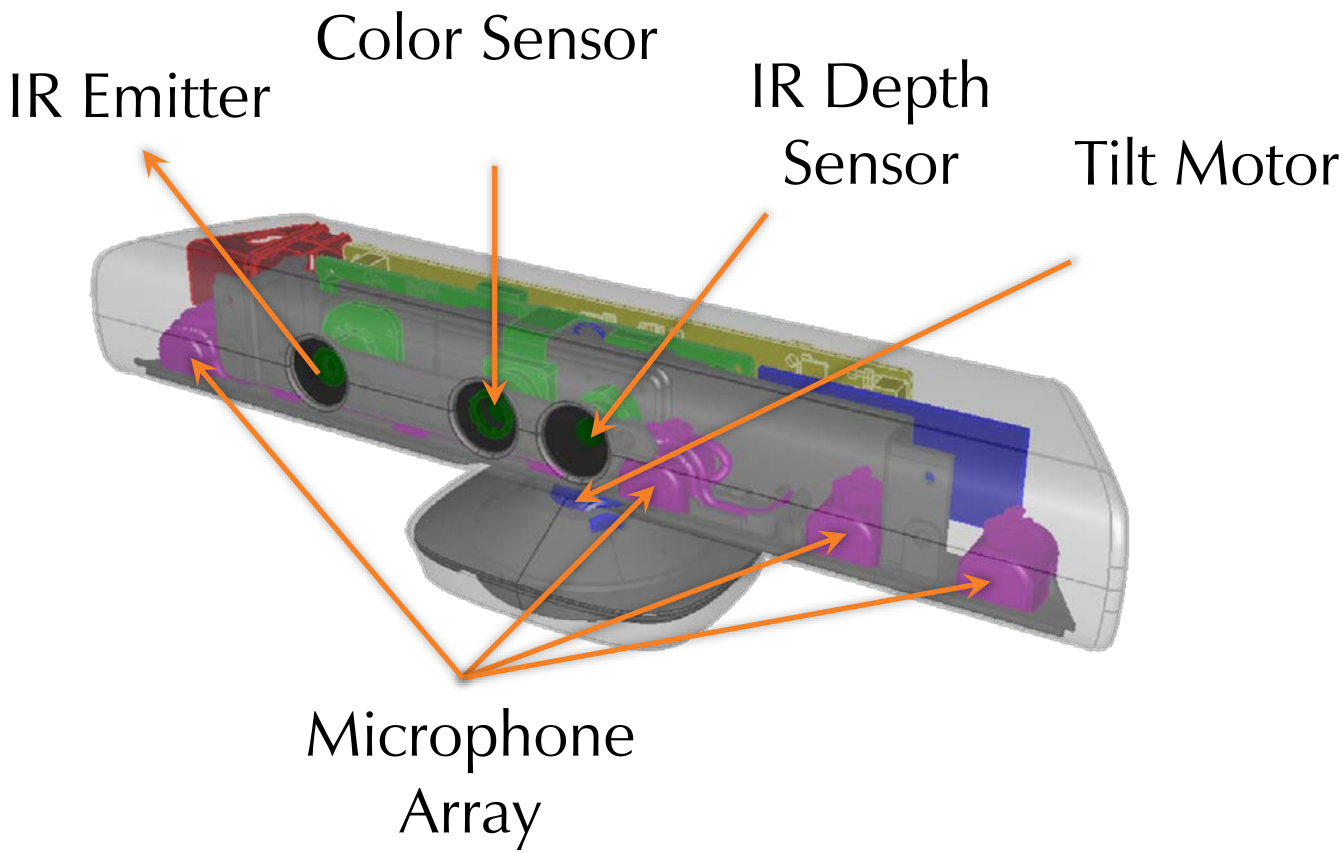


Time-Coded Stripe Triangulation



3D Reconstruction using Structured Light [Inokuchi 1984]

Structured Light Scanning: Kinect



Structured Light Scanning: Kinect

Projected IR Pattern

Structured Light Scanning: Kinect

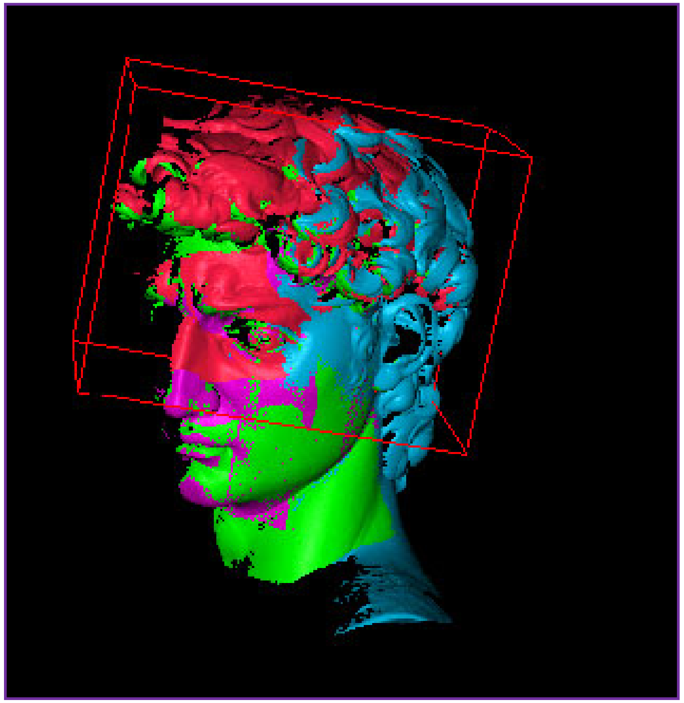
Depth Map

RGB Image

Structured Light Scanning

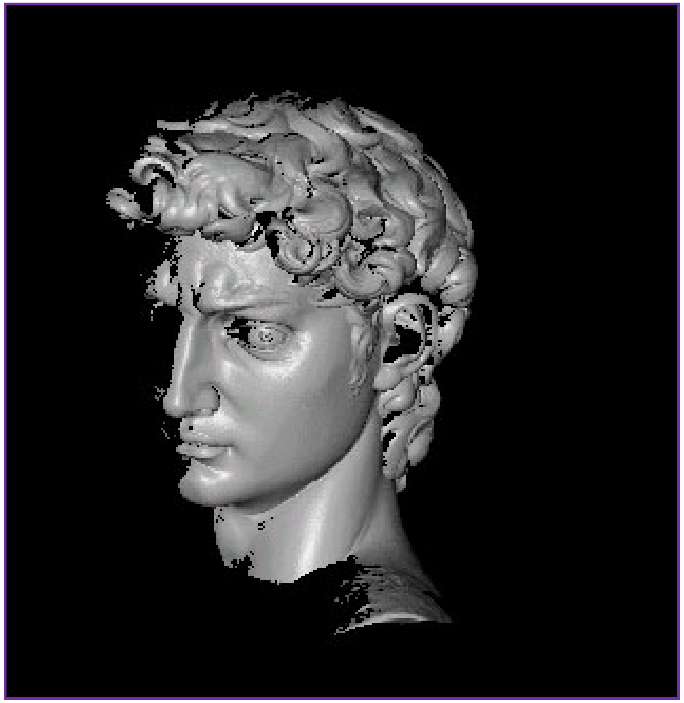
- Advantages:
  - Very fast – 2D pattern at once
- Disadvantages:
  - Prone to noise
  - Indoor only

Range Processing Pipeline



- Steps
  1. manual initial alignment
  2. ICP to one existing scan
  3. automatic ICP of all overlapping pairs
  4. global relaxation to spread out error
  5. merging using volumetric method

Range Processing Pipeline



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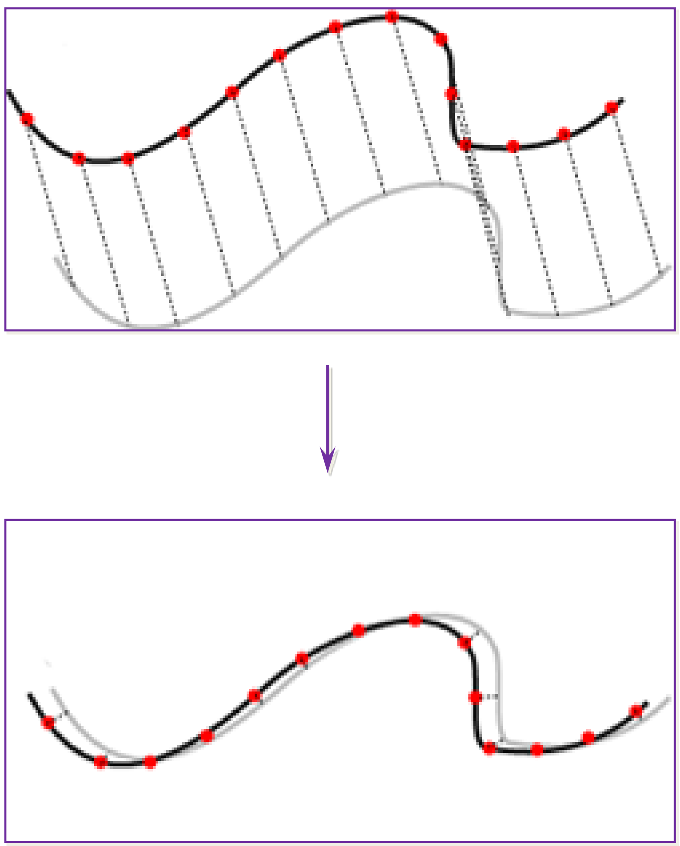

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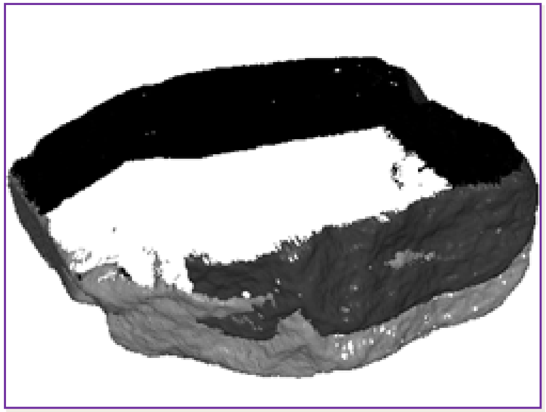

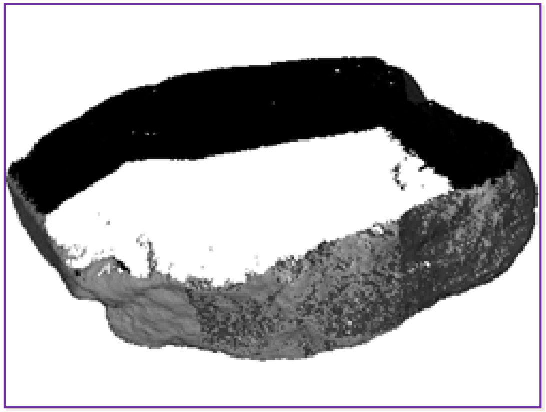

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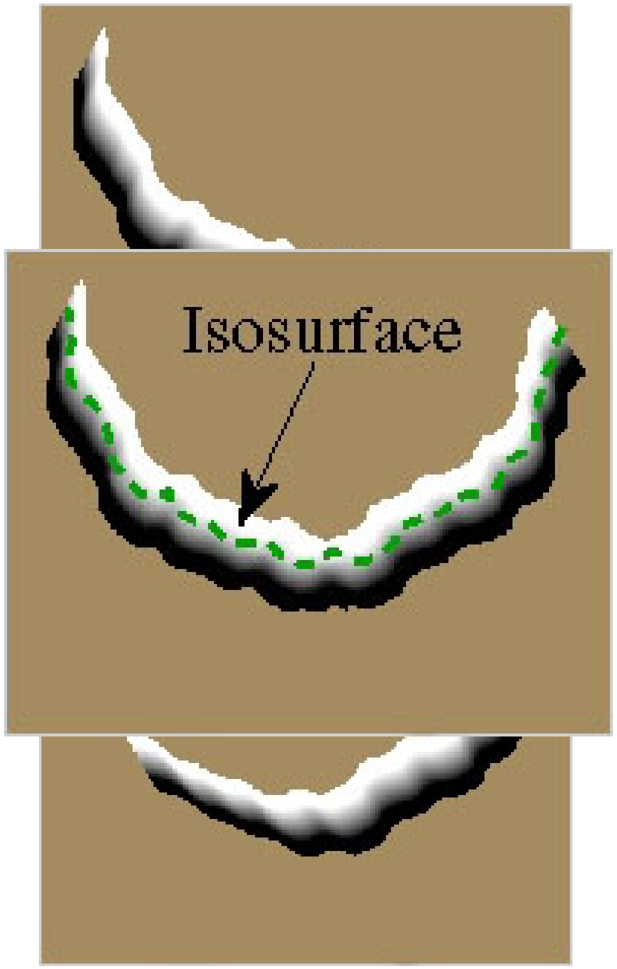

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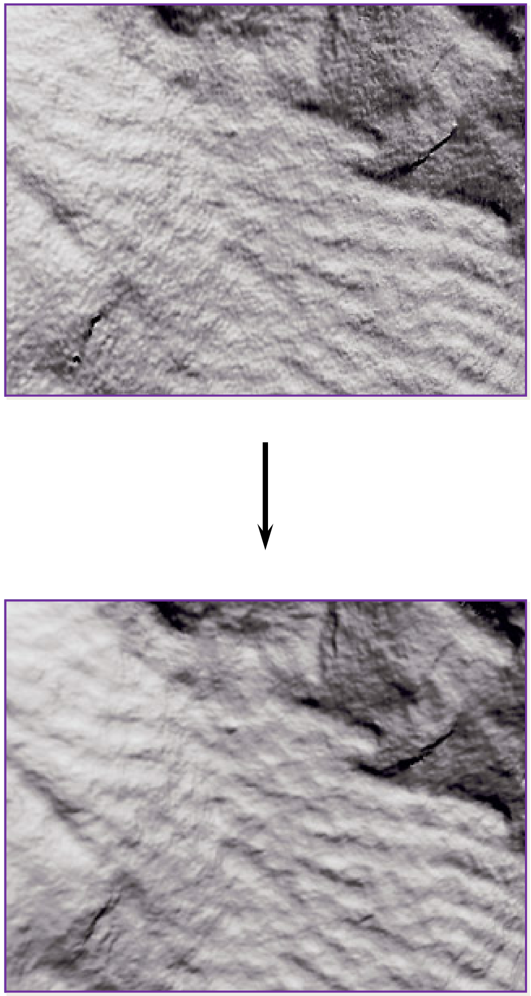

- Steps
  1. manual initial alignment
  2. ICP to one existing scan
  3. automatic ICP of all overlapping pairs
  4. global relaxation to spread out error
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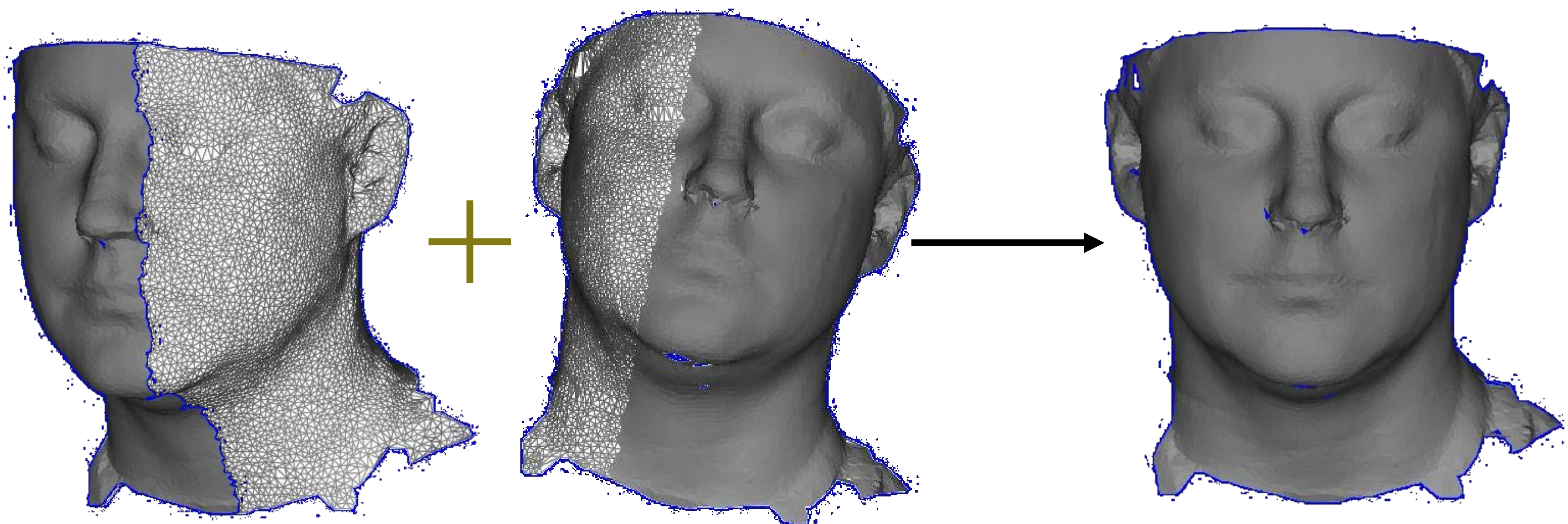
3D Registration: Goal

- Given two partially overlapping scans, compute the transformation that makes them lie on top of each other

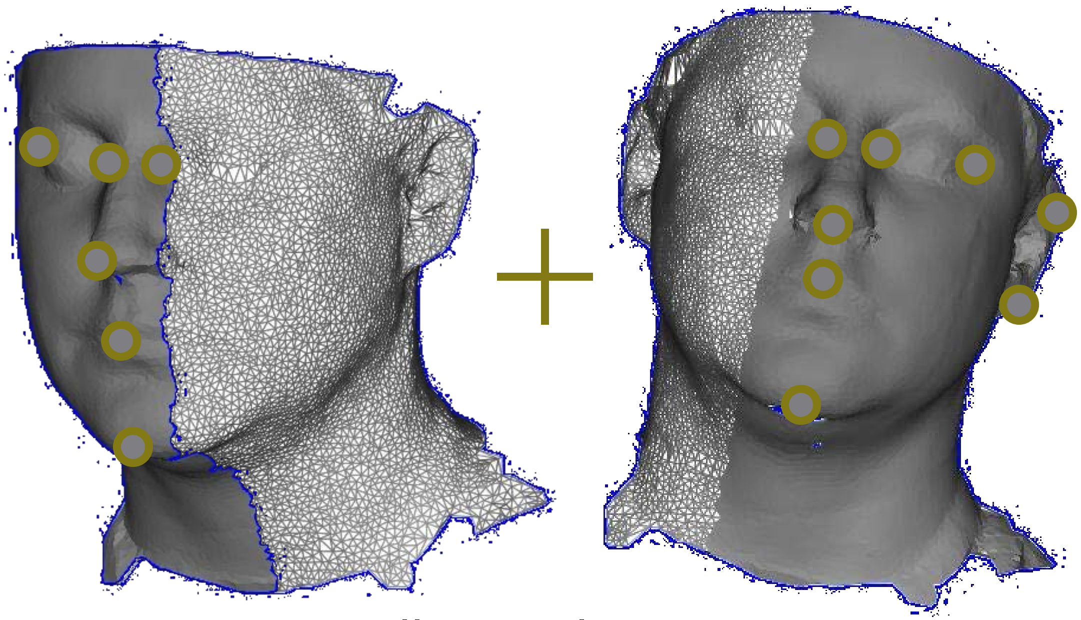


Partially Overlapping Scans

Aligned Scans

General Approach

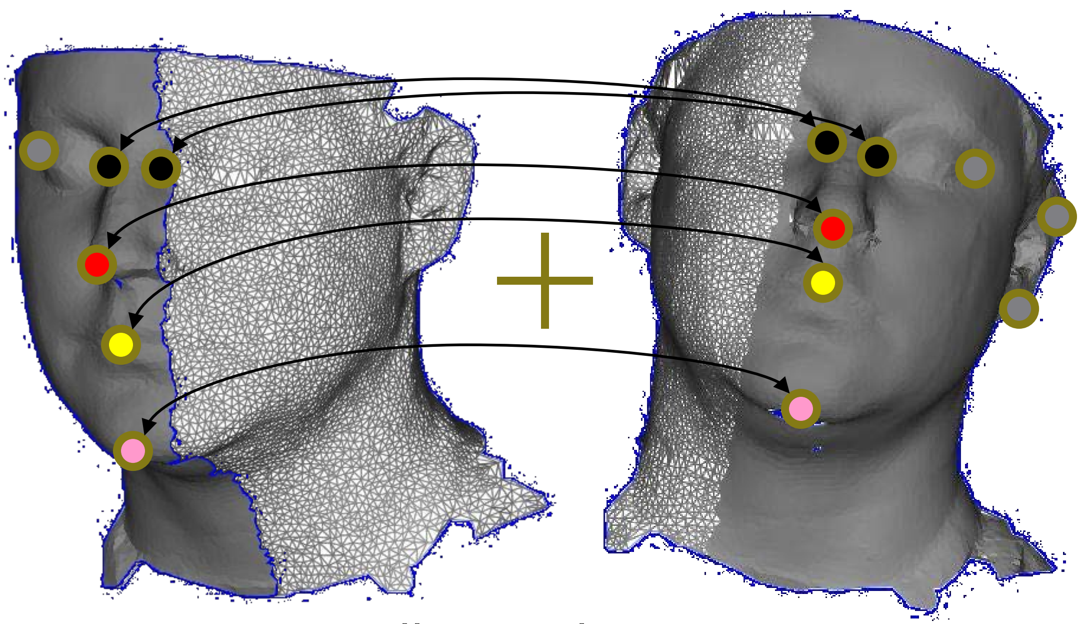
1. Find feature points



Partially Overlapping Scans

General Approach

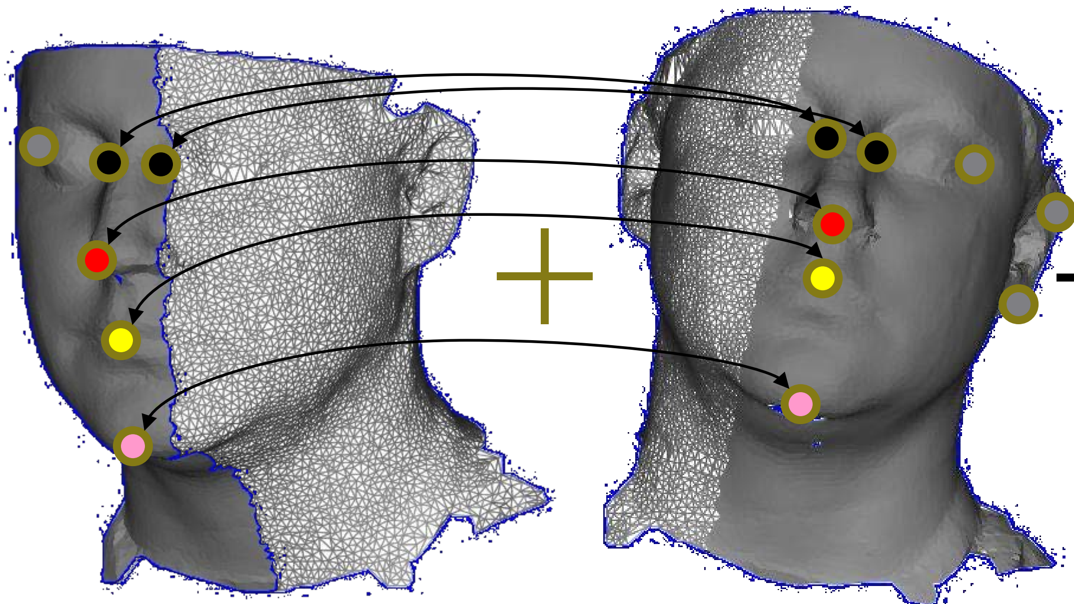
1. Find feature points
2. Establish correspondences



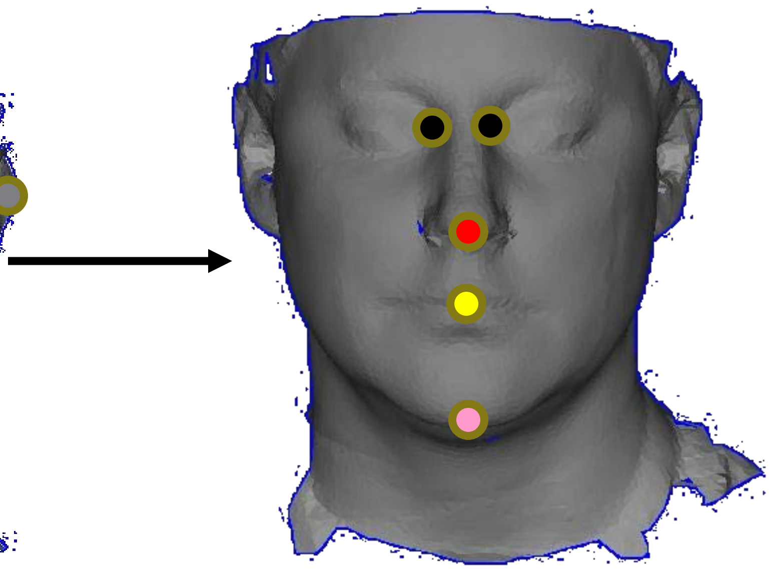
Partially Overlapping Scans

General Approach

1. Find feature points
2. Establish correspondences
3. Compute the aligning transformation



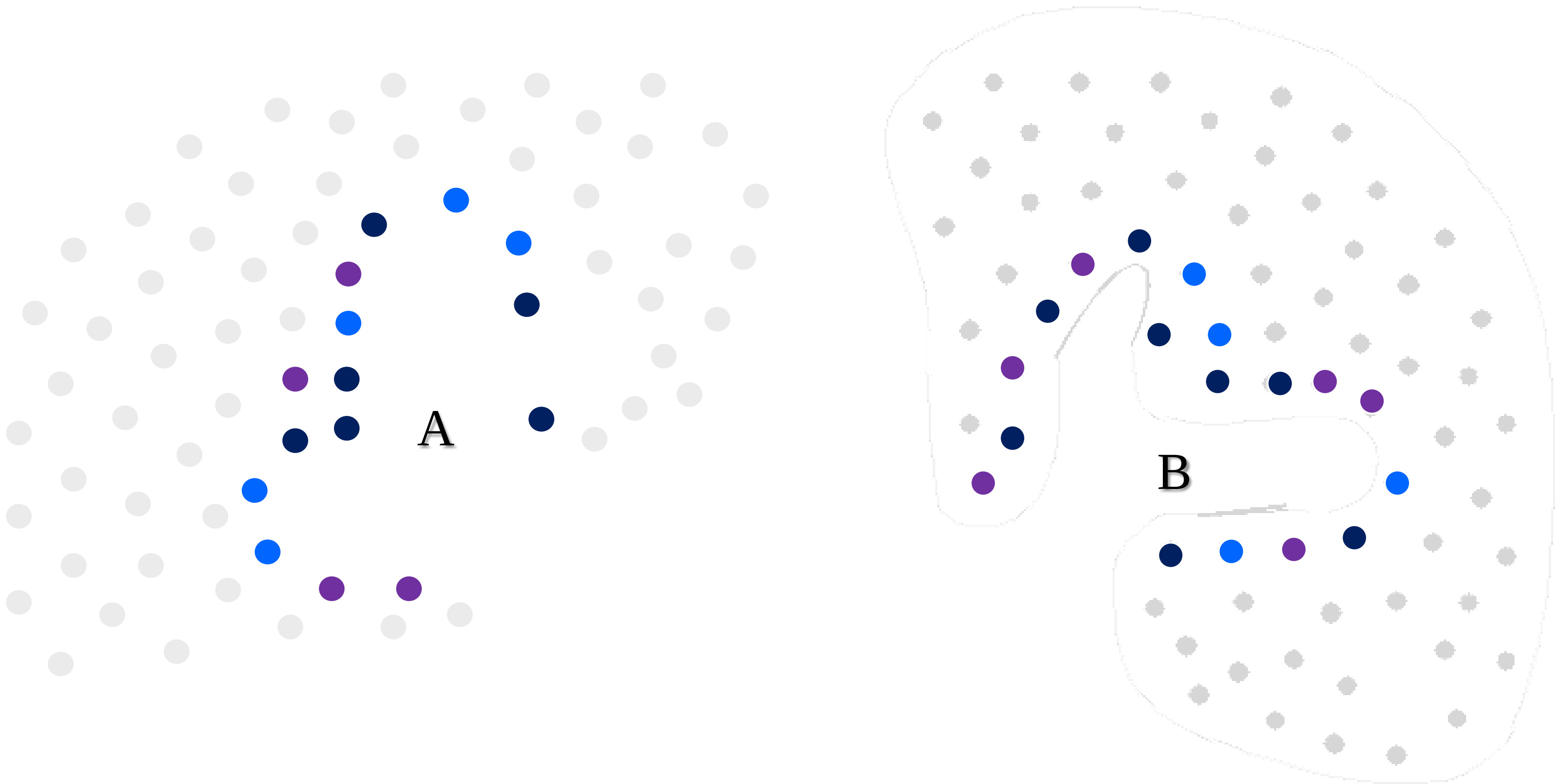
Partially Overlapping Scans



Aligned Scans

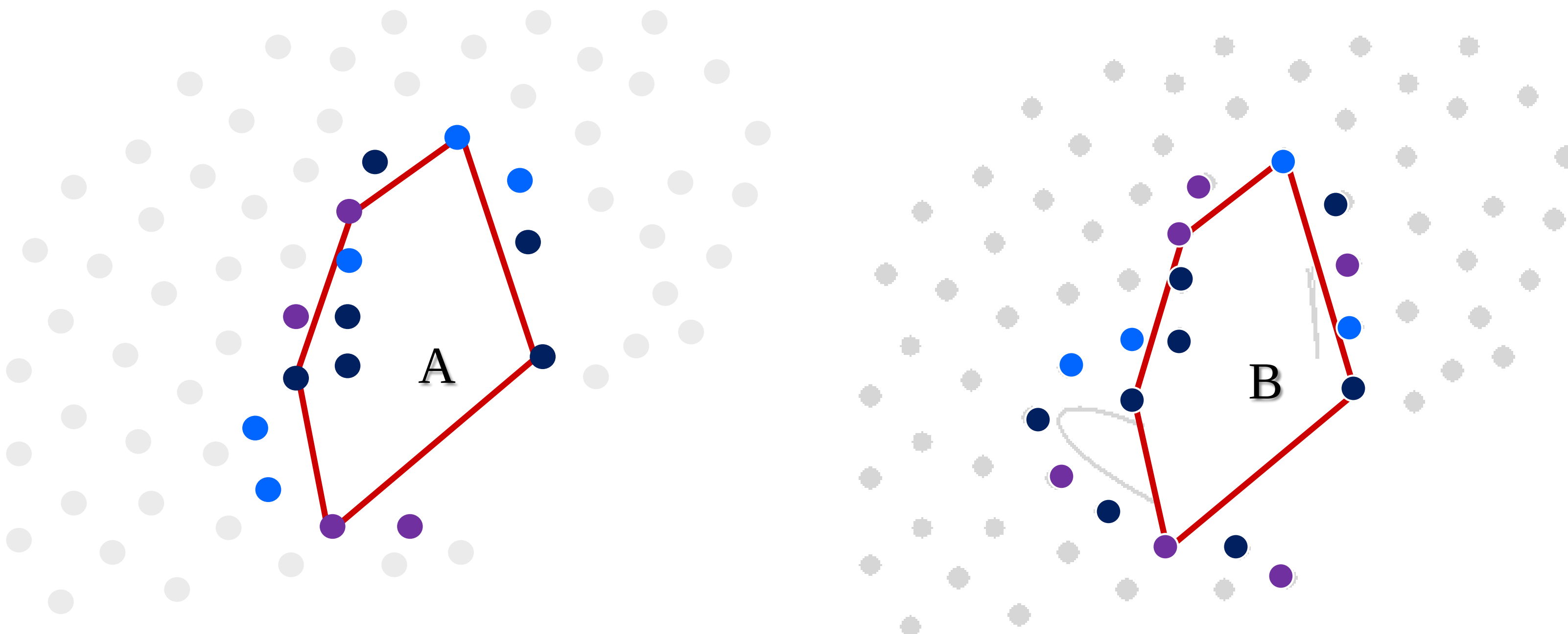
Problem

- Most problems require aligning a subset of features



Observation I

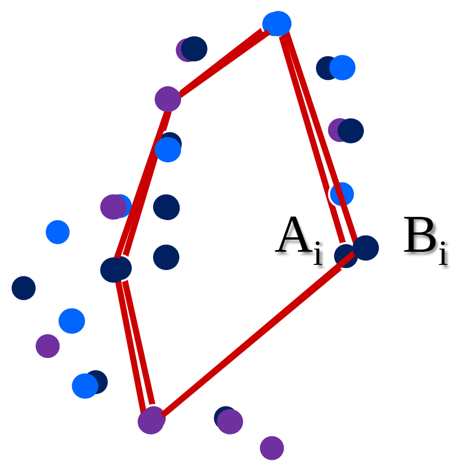
- Calculating the aligning transformation is usually easy if correspondences are known (proposed)



$$RMSD(A,B)=\sqrt{\sum_{i=1}^{N}(A_i-B_i)^2}$$

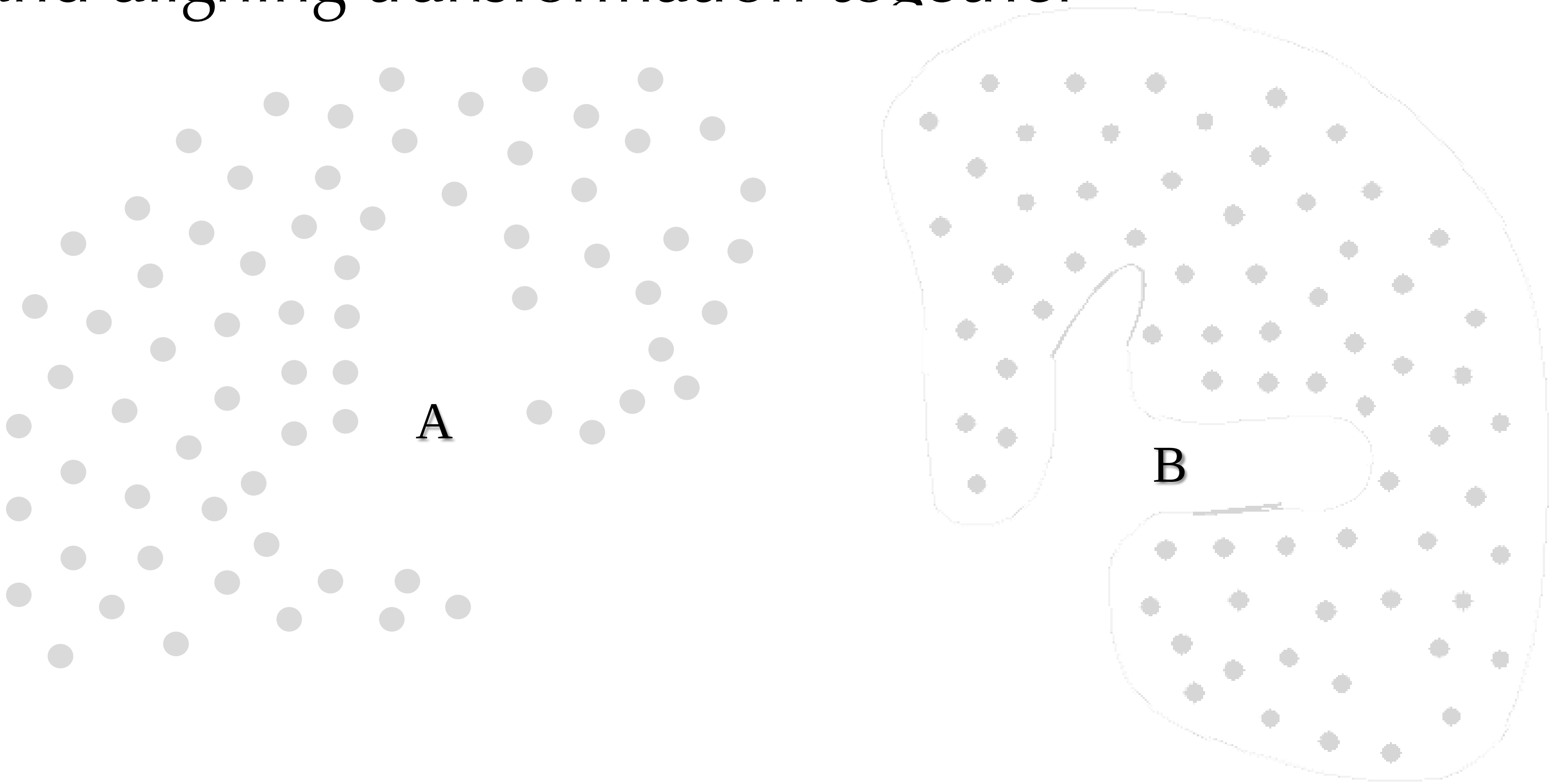
Observation II

- Calculating the correspondences is usually easy if the aligning transformation is known (proposed)



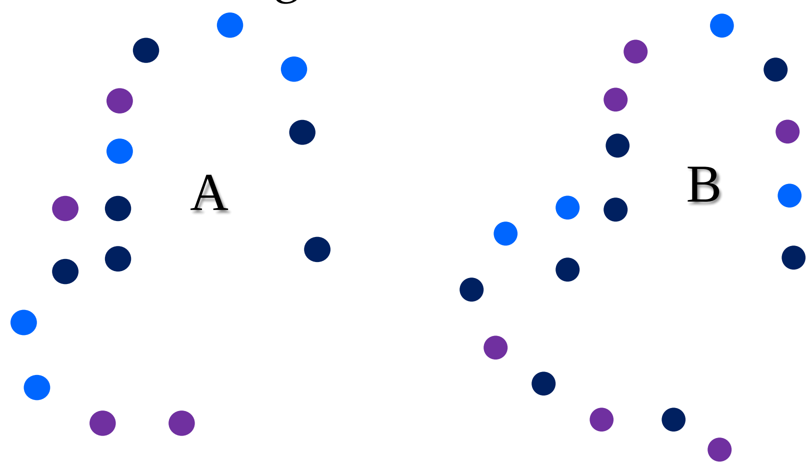
Challenge

- The challenge is to discover the correspondences and aligning transformation together



Brute Force Search

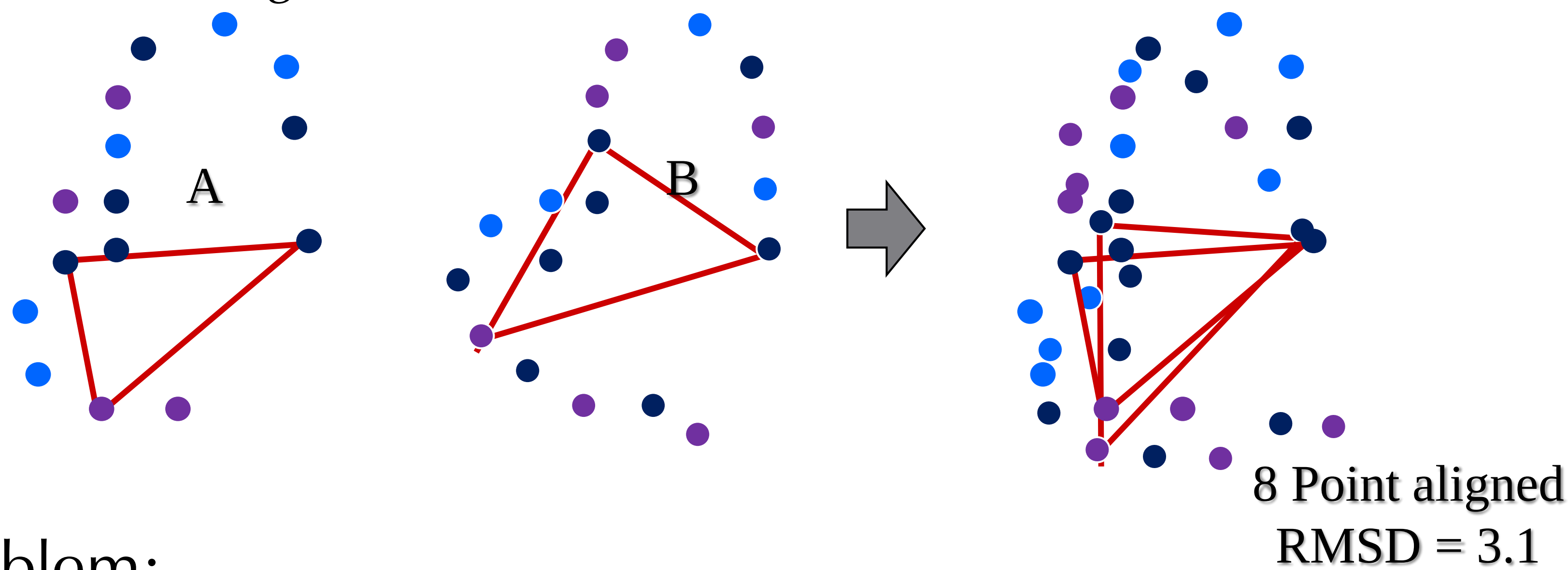
- Simple method:
  - Try all possible sets of point correspondences
  - Score the alignment for each one



- Problem:
  - $O(n^m)$ possible sets of m correspondences among n points

Brute Force Search

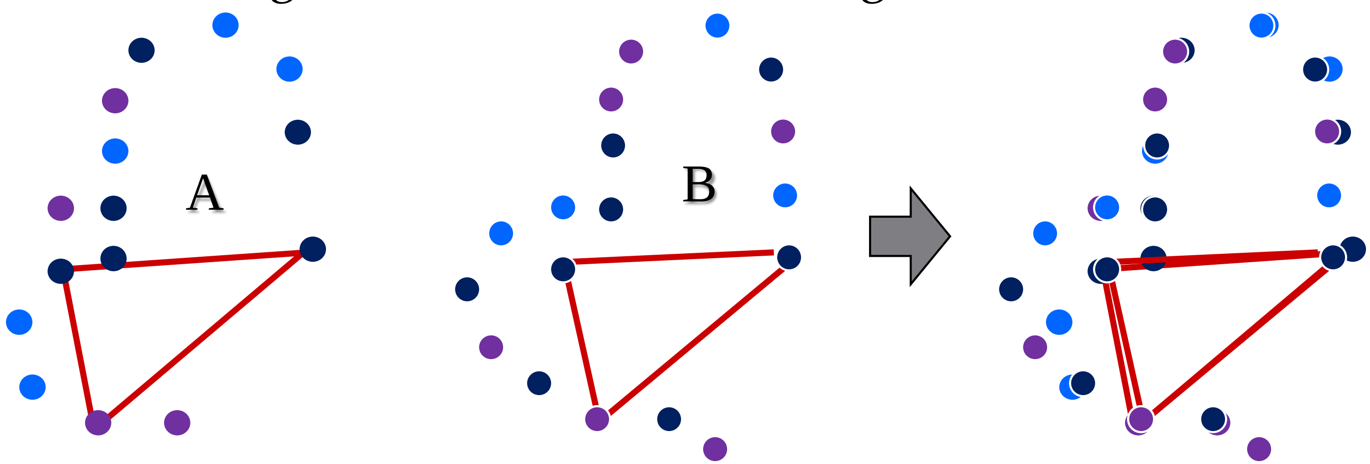
- Simple method:
  - Try all possible sets of point correspondences
  - Score the alignment for each one



- Problem:
  - $O(n^m)$ possible sets of m correspondences among n points

Brute Force Search

- Simple method:
  - Try all possible sets of point correspondences
  - Score the alignment for each one (e.g., RMSD)

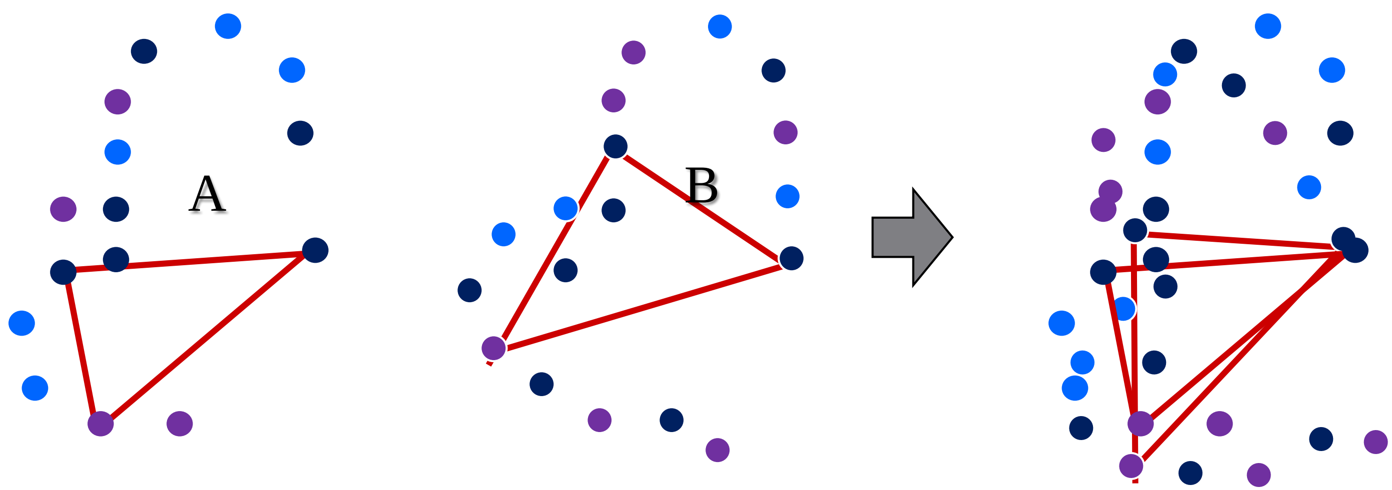


All points aligned
RMSD = 0.2

- Problem:
  - $O(n^m)$ possible sets of m correspondences among n points

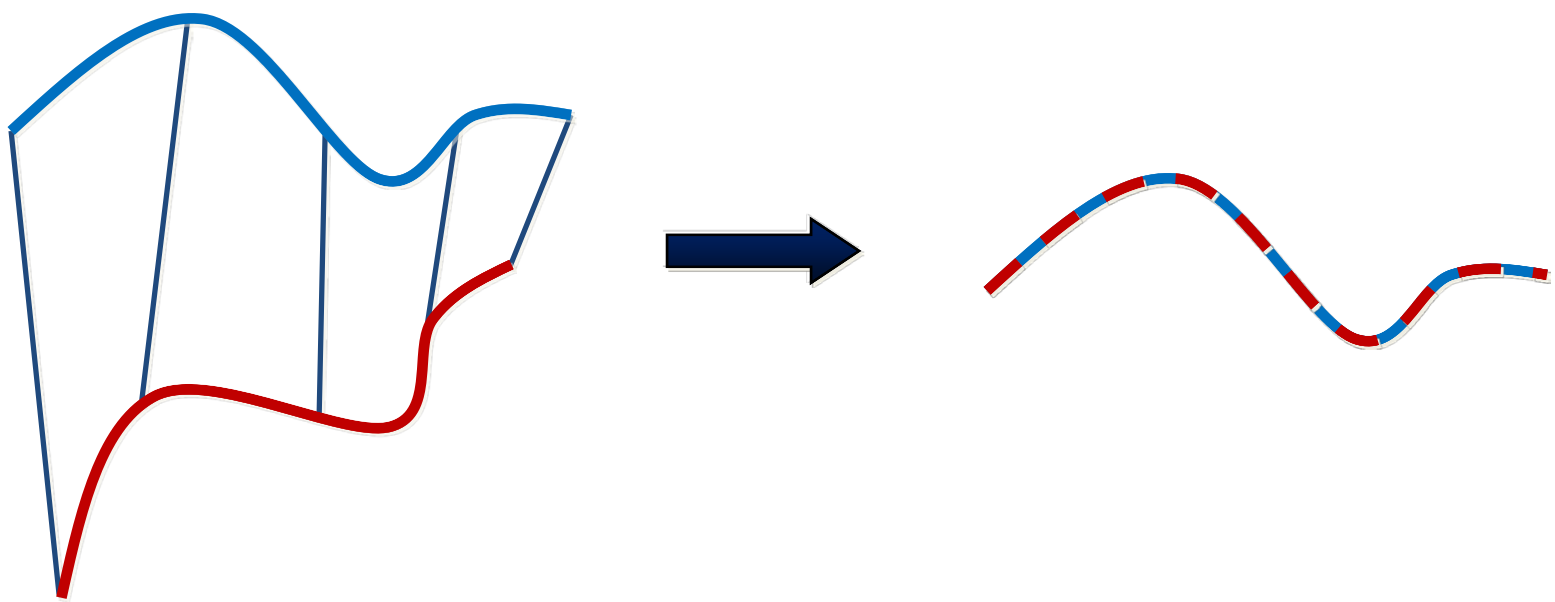
RANSAC

- Randomly sample set of possible correspondences
  - Randomly generate a small set of point correspondences
  - Compute the aligning transformation for correspondences
    Score how well other points align after that transformation
  - Remember the best transformation



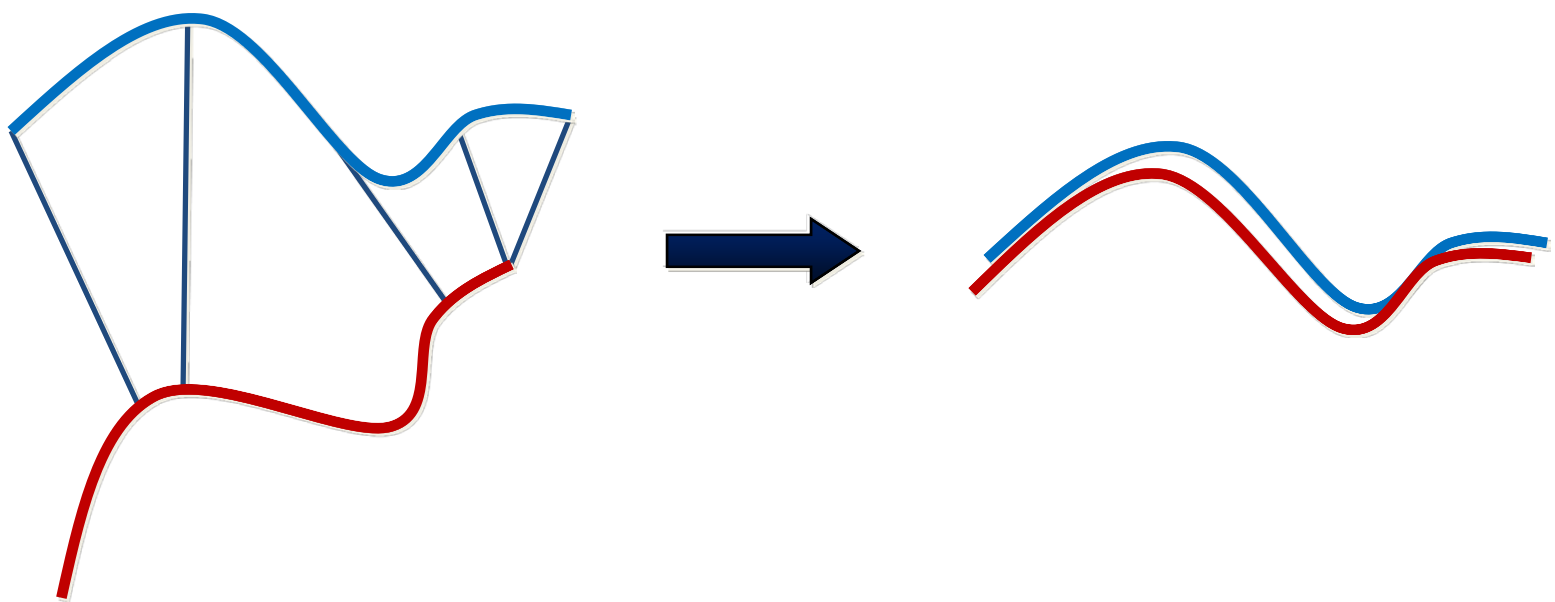
ICP: Motivation

- If correct correspondences are known, can find correct relative rotation/translation



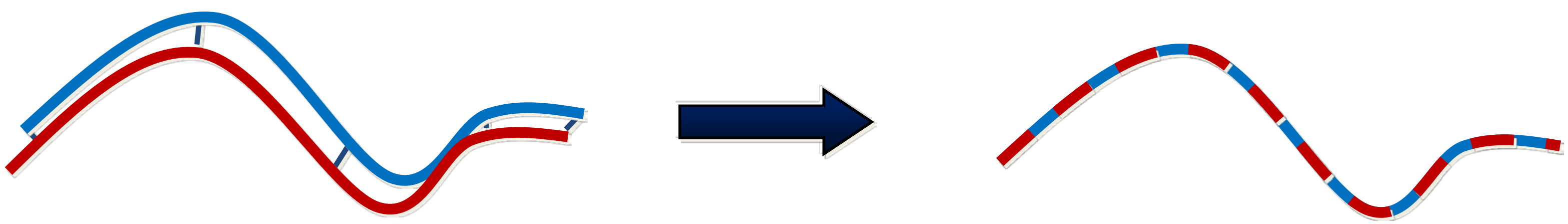
ICP: Motivation

- How to find correspondences?
  - Assume closest points correspond!



ICP: Motivation

- … and iterate to find alignment
  - Iterative Closest Points (ICP) [Besl & McKay 92]
- Converges if starting position "close enough"



ICP Algorithm

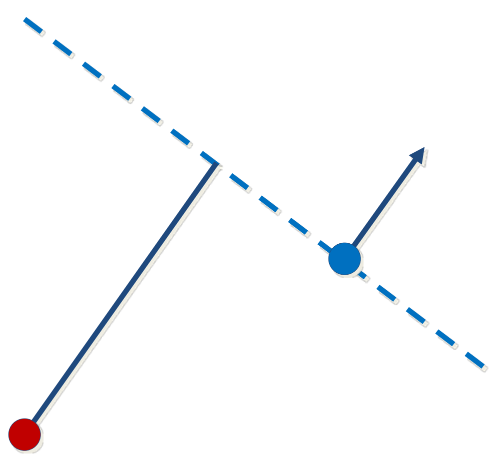
- Select e.g. 1000 random points
- Match each to closest point on other scan, using data structure such as k-d tree
- Reject pairs (e.g. with distance > k times median)
- Construct error function:

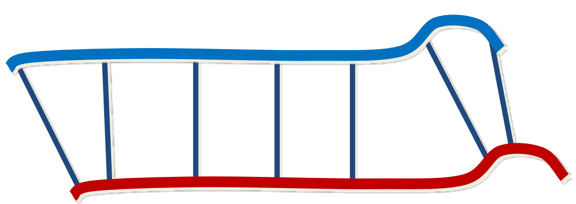
$$E = \sum \left| \mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i \right|^2$$

- Minimize (closed form solution in [Horn 87])

Point-to-Plane Error Metric

- Using point-to-plane distance instead of point-to-point lets flat regions slide along each other [Chen & Medioni 91]
  - In practice, much faster convergence





Point-to-Plane Error Metric

- Error function:

$$E = \sum \left[(\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i) \cdot \mathbf{n}_i \right]^2$$

where R is a rotation matrix, t is translation vector

Point-to-Plane Error Metric

- Linearize rotations
  - (i.e. assume that $\sin\theta \approx \theta$, $\cos\theta \approx 1$):

$$R = R_x R_y R_z$$
$$\approx \begin{pmatrix} 1 & -r_z & r_y \\ r_z & 1 & -r_x \\ -r_y & r_x & 1 \end{pmatrix}$$

$$Rp_i \cdot n_i \approx r \cdot (p_i \times n_i)$$

Point-to-Plane Error Metric

- Error function:

$$E = \sum \left[(\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i) \cdot \mathbf{n}_i \right]^2$$

  where R is a rotation matrix, t is translation vector

- After linearization:

$$E \approx \sum \left((\mathbf{p}_i - \mathbf{q}_i) \cdot \mathbf{n}_i + \mathbf{r} \cdot (\mathbf{p}_i \times \mathbf{n}_i) + \mathbf{t} \cdot \mathbf{n}_i \right)^2, \qquad \text{where } \mathbf{r} = \begin{pmatrix} \mathbf{r_x} \\ \mathbf{r_y} \\ \mathbf{r_z} \end{pmatrix}$$

- Result: overconstrained linear system

Point-to-Plane Error Metric

- Overconstrained linear system

$$\mathbf{A}\mathbf{x} = \mathbf{b},$$

$$\mathbf{A} = \begin{pmatrix} \leftarrow & \mathbf{p}_1 \times \mathbf{n}_1 & \rightarrow & \leftarrow & \mathbf{n}_1 & \rightarrow \\ \leftarrow & \mathbf{p}_2 \times \mathbf{n}_2 & \rightarrow & \leftarrow & \mathbf{n}_2 & \rightarrow \\ & \vdots & & & \vdots & \end{pmatrix}, \qquad \mathbf{x} = \begin{pmatrix} r_x \\ r_y \\ r_z \\ t_x \\ t_y \\ t_z \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} -(\mathbf{p}_1 - \mathbf{q}_1) \cdot \mathbf{n}_1 \\ -(\mathbf{p}_2 - \mathbf{q}_2) \cdot \mathbf{n}_2 \\ \vdots \end{pmatrix}$$

- Solve using least squares

$$\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$$

$$\mathbf{x} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$