Mesh Processing and Analysis

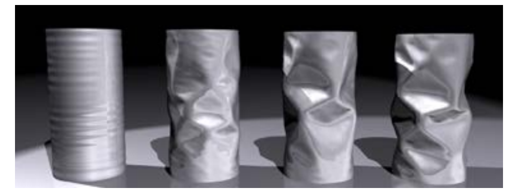
COS 526: Advanced Computer Graphics

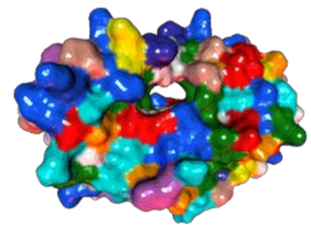
PRINCETON UNIVERSITY

Slide credits: Tom Funkhouser

Digital Geometry Processing

- Processing of 3D surfaces
  - Creation, acquisition
  - Storage, transmission
  - Editing, animation, simulation
  - Manufacture
  - Analysis
- Applications
  - Movies, games
  - Computer-aided design
  - Medicine, biology
  - Art, history
  - All fields with 3D data





Sweldens

Mesh Processing Tasks

Polygonal Mesh Processing

- Storage
  - Compression
  - Transmission
- Analysis
  - Parameterization
  - Differential geometry
  - Feature detection
  - Segmentation
- Editing
  - Smoothing, sharpening, etc.
  - Deformation
  - Completion

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Lossy Compression
(Simplification)

Garland

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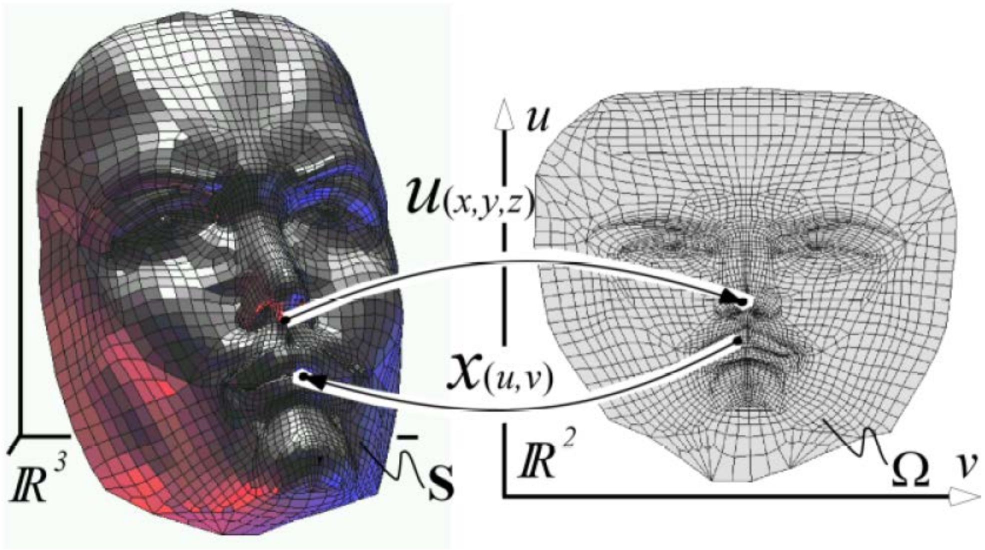

Sheffer

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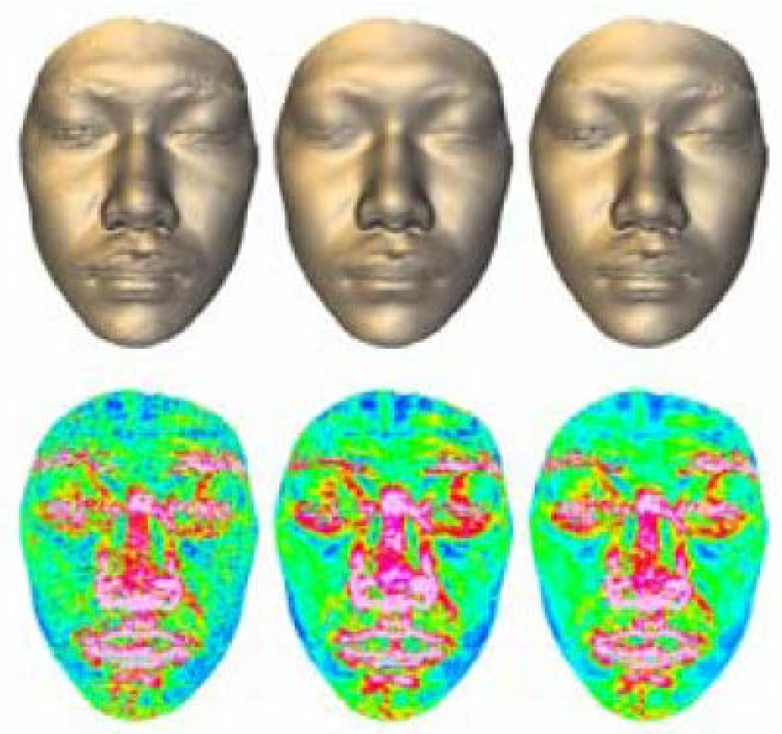

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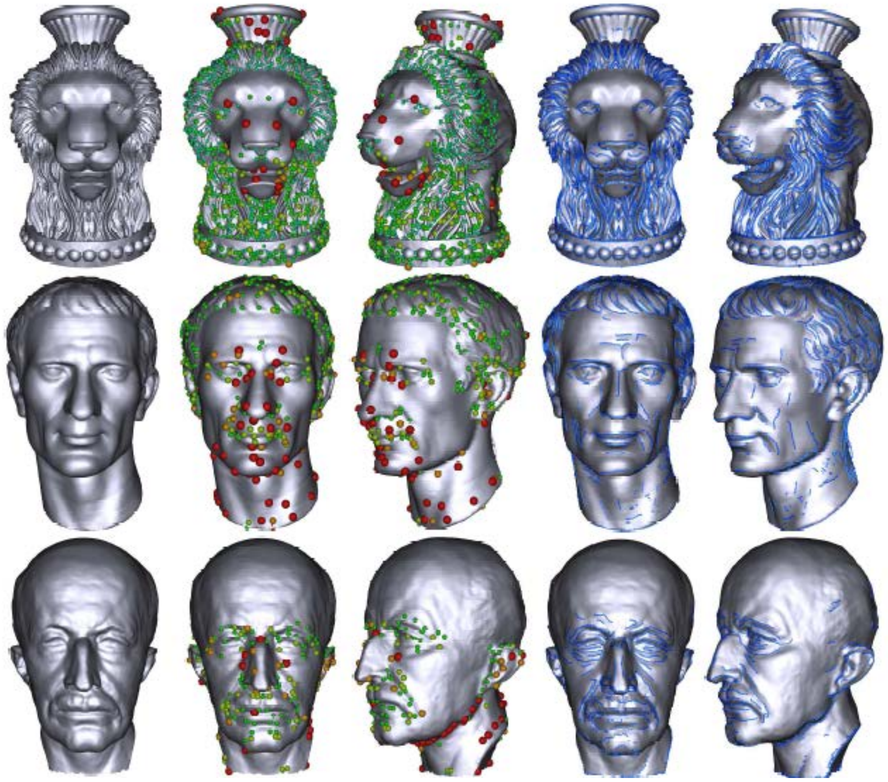

Novatnek et al.

Polygonal Mesh Processing

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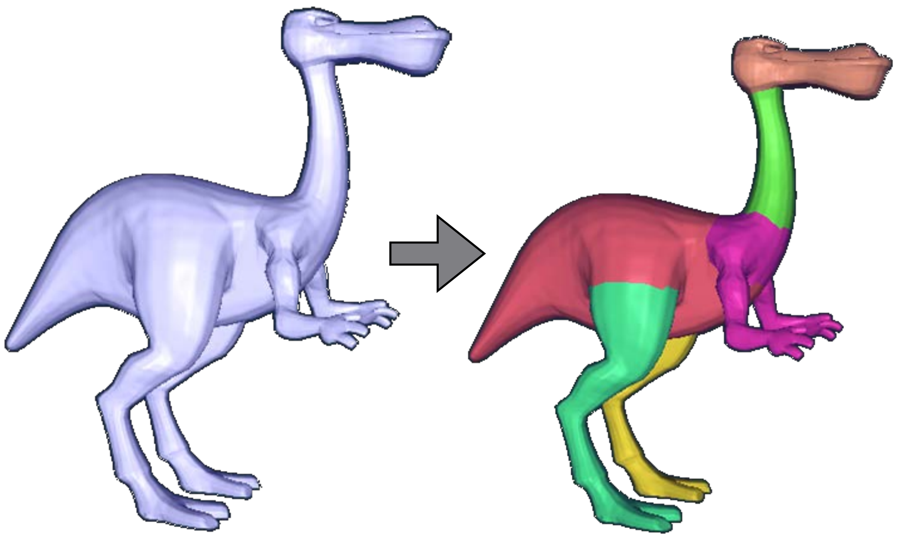

Katz & Tal

Polygonal Mesh Processing

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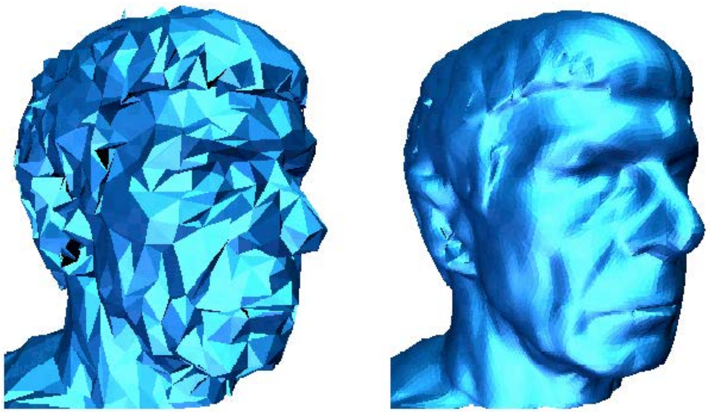
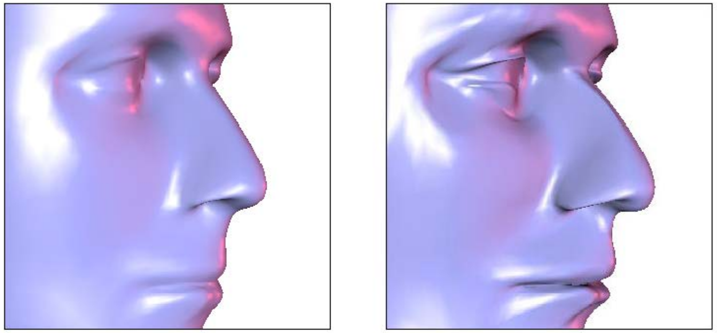


Smoothing



Sharpening

Desbrun

Polygonal Mesh Processing

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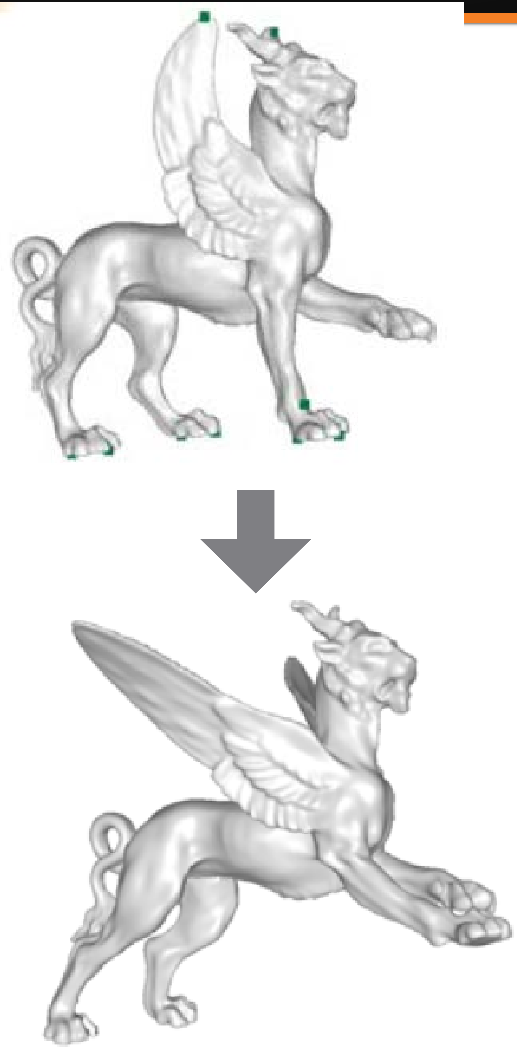

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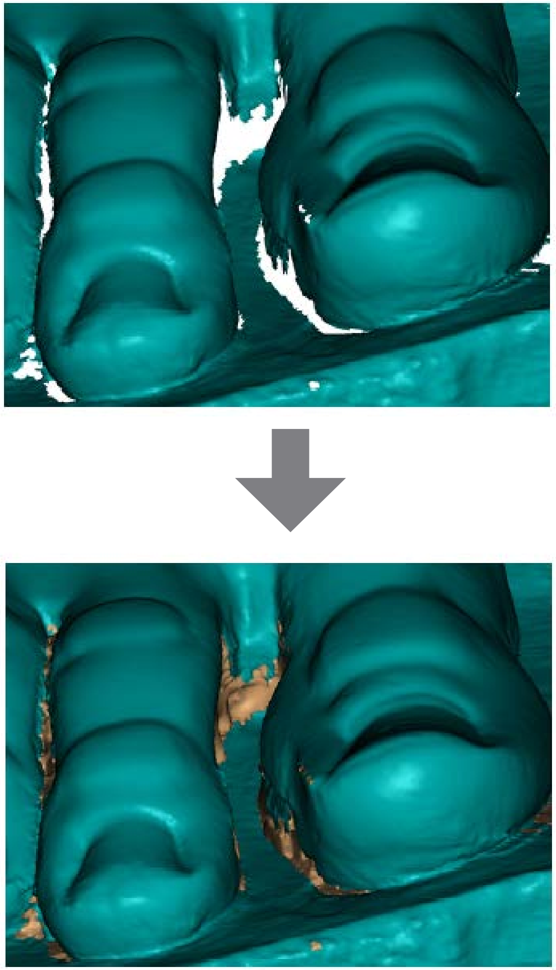
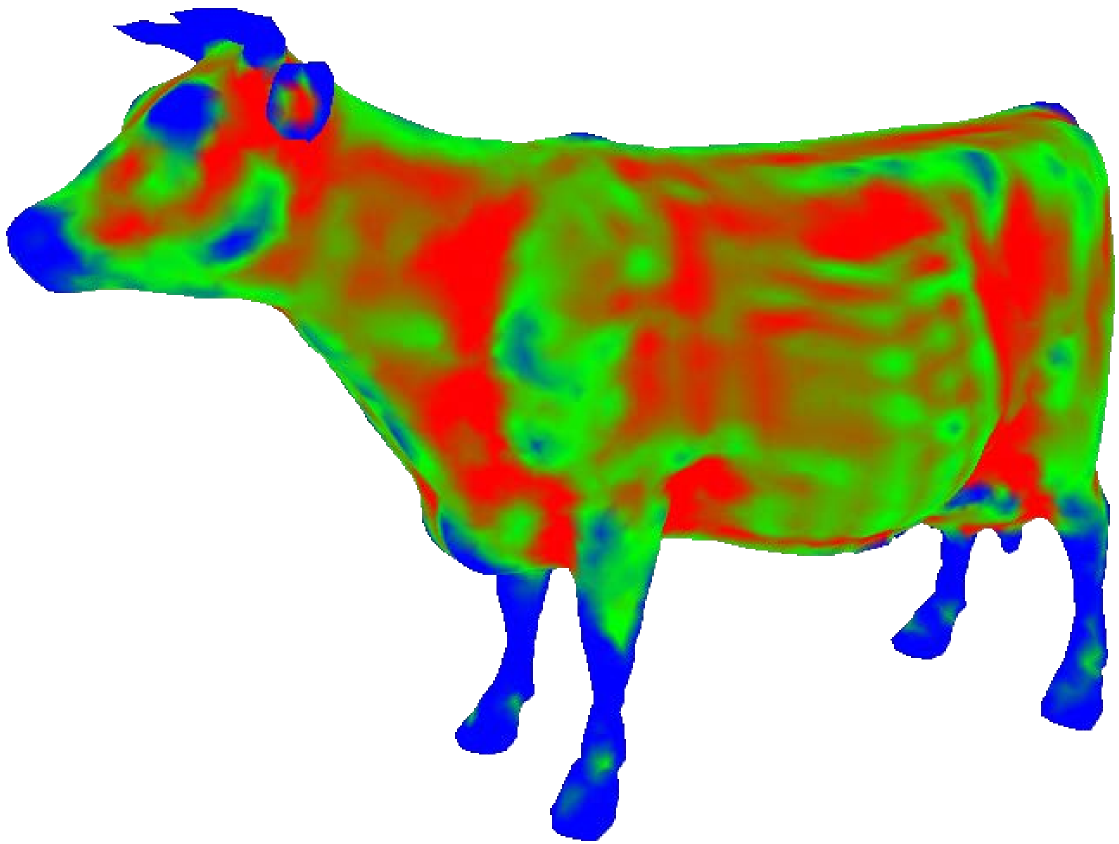


Podolak

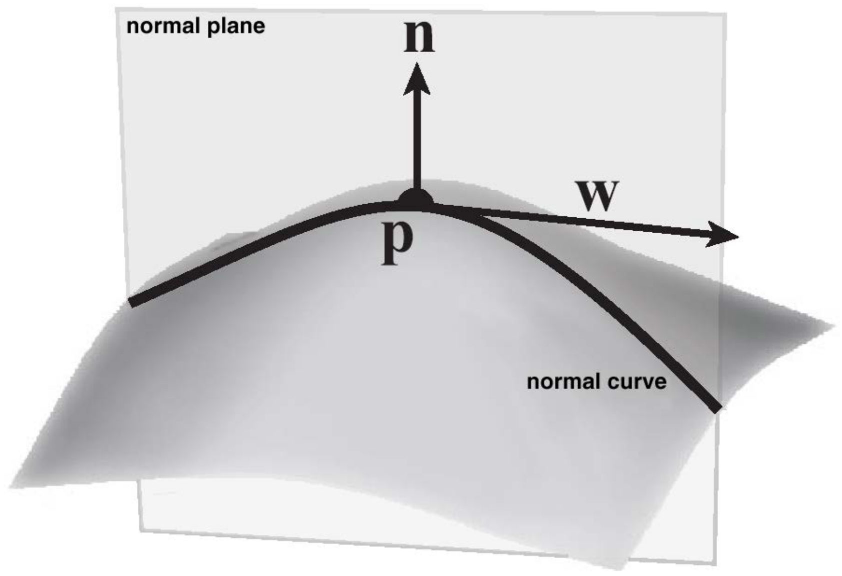
Mesh Analysis: Surface Properties

Curvature



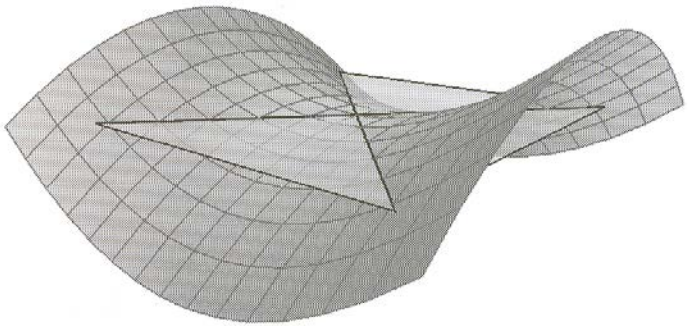
Curvature

- Curvature κ of a curve is reciprocal of radius of circle that best approximates it
- Defined at a point **p** in a direction **w**
- *Line has* $\kappa = 0$

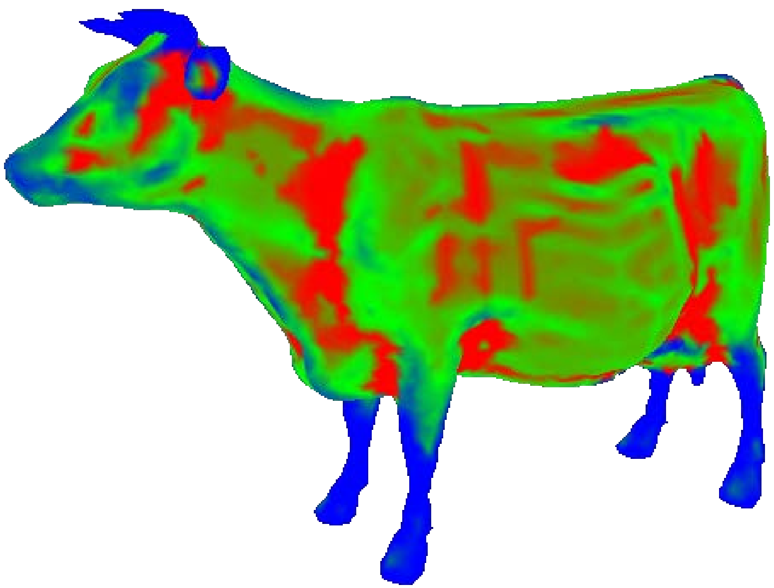


Principal Curvatures

- The curvature at a point varies between some minimum and maximum – these are the *principal curvatures* κ_1 and κ_2
- They occur in the *principal directions* d_1 and d_2, which are perpendicular to each other

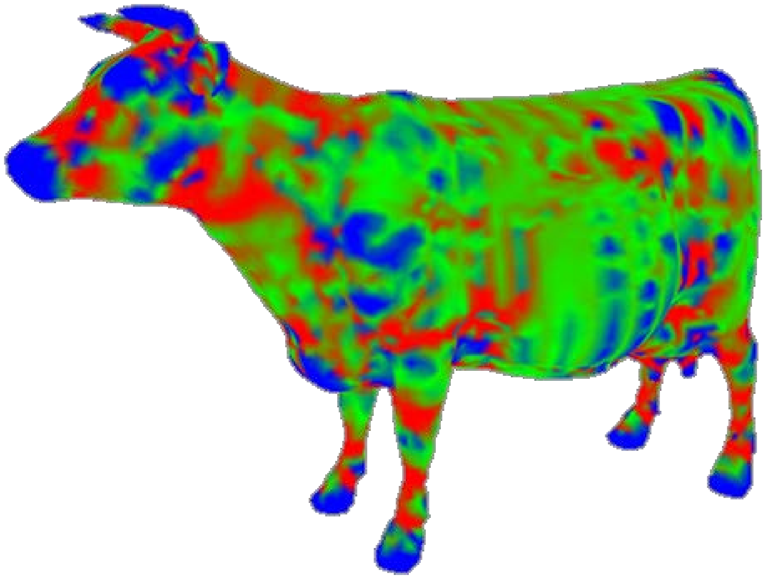


Principal Curvatures



Minimum Curvature

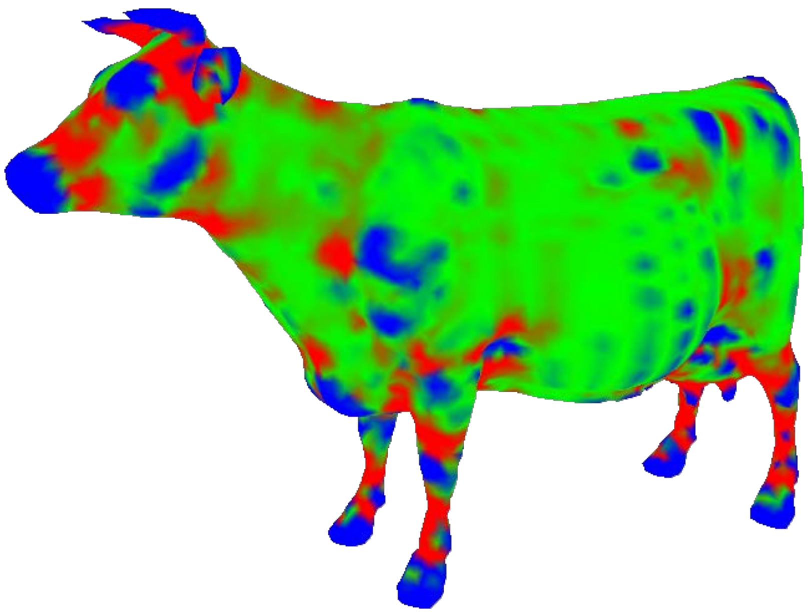
κ_1



Maximum Curvature

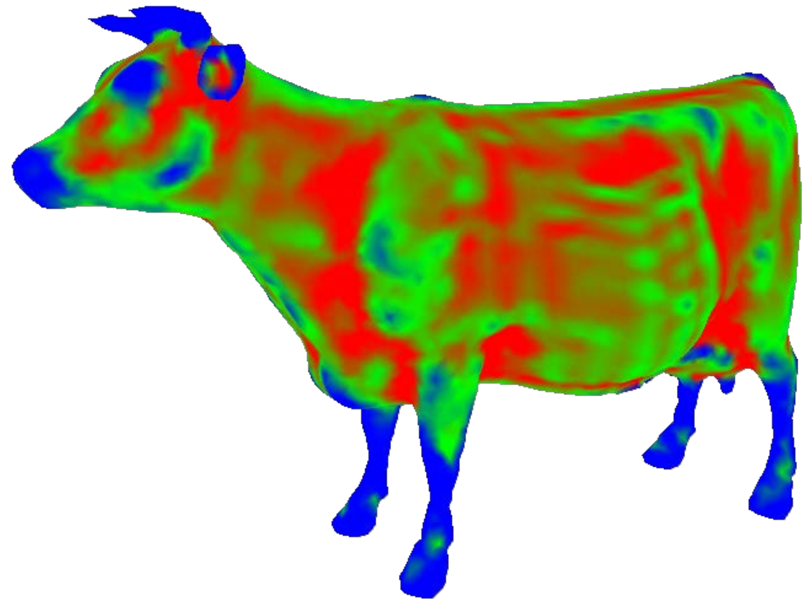
κ_2

Gaussian and Mean Curvature



Gaussian Curvature

$K = \kappa_1 \kappa_2$



Mean Curvature

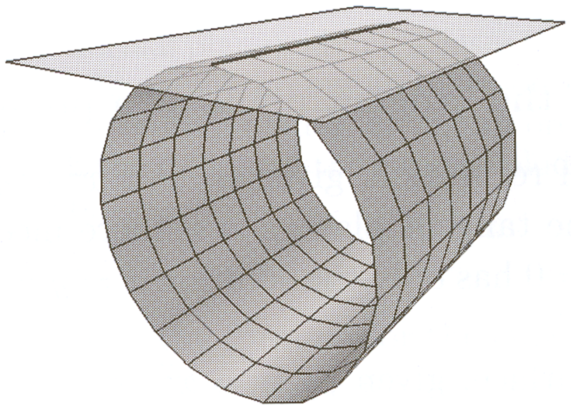
$H = \frac{1}{2} (\kappa_1 + \kappa_2)$

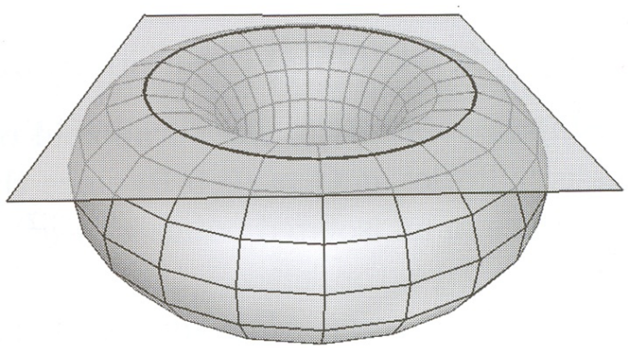
What Does Curvature Tell Us?

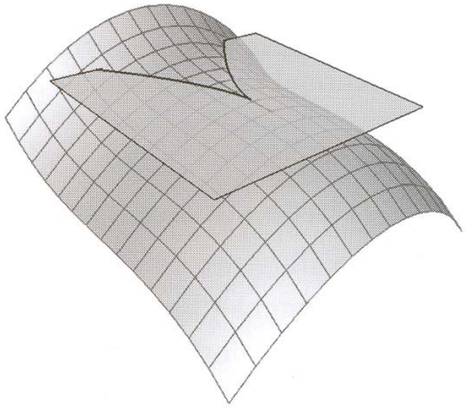
- Planar points:
  - Zero Gaussian curvature and zero mean curvature
  - Tangent plane intersects surface at infinitely-many points

What Does Curvature Tell Us?

- Parabolic points:
  - Zero Gaussian curvature, non-zero mean curvature
  - Tangent plane intersects surface along a curve

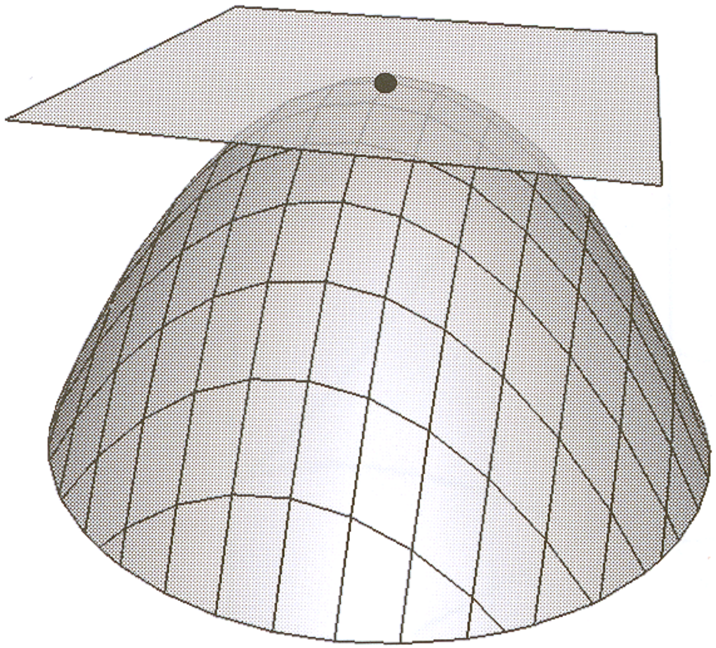






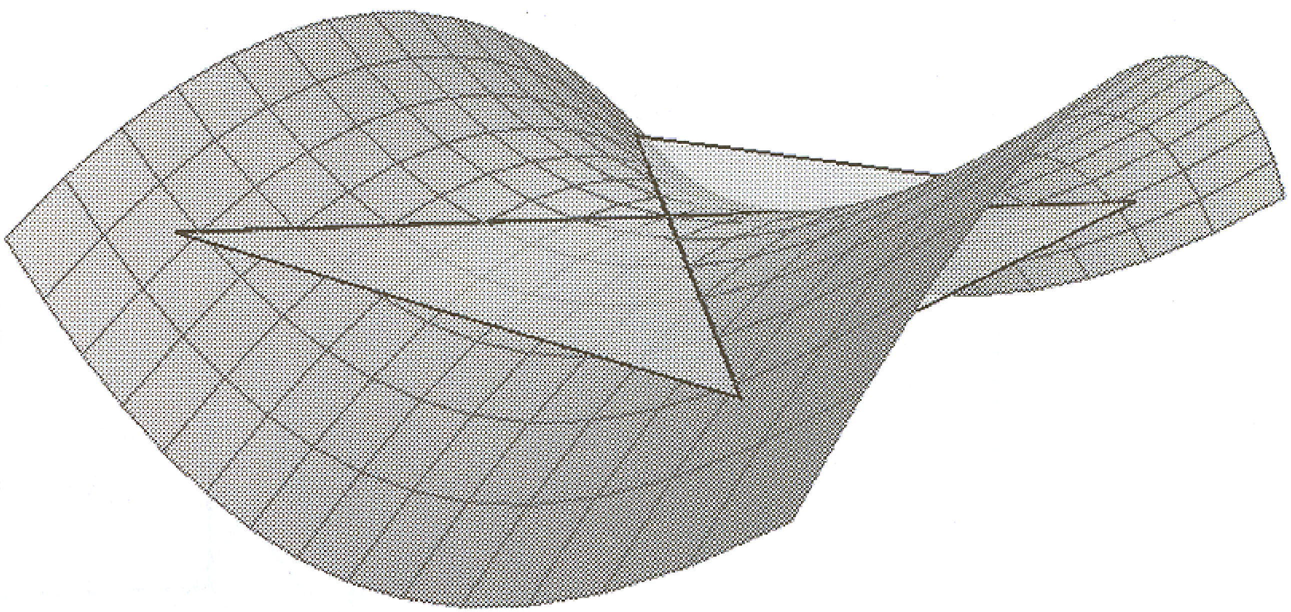
What Does Curvature Tell Us?

- Elliptical points:
  - Positive Gaussian curvature
  - Convex/concave depending on sign of mean curvature
  - Tangent plane intersects surface at 1 point



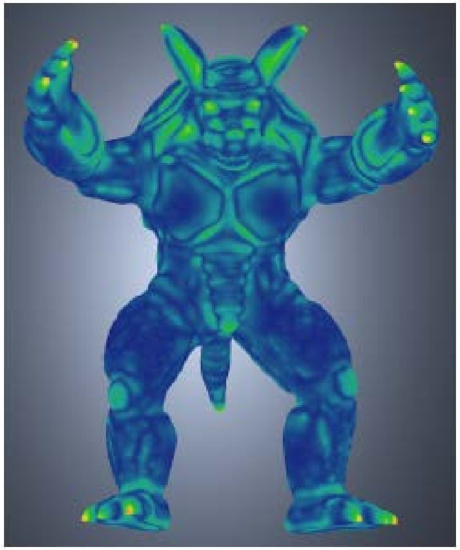
What Does Curvature Tell Us?

- Hyperbolic points:
  - Negative Gaussian curvature
  - Tangent plane intersects surface along 2 curves



What Does Curvature Tell Us?

- Mesh Saliency:
  - Motivated by models of perceptual salience
  - Difference between mean curvature blurred with σ and blurred with 2σ



Lee05

Principal Component Analysis (PCA)

- Based on covariance of points {q}: sum of qq^T
  - Analyze eigenvalues and eigenvectors of M (via SVD)
  - Eigenvectors are Principal Axes

$$\mathbf{M} = \frac{1}{n}\sum_{i=1}^{n}\begin{bmatrix} q_i^x q_i^x & q_i^x q_i^y & q_i^x q_i^z \\ q_i^y q_i^x & q_i^y q_i^y & q_i^y q_i^z \\ q_i^z q_i^x & q_i^z q_i^y & q_i^z q_i^z \end{bmatrix}$$

Covariance Matrix

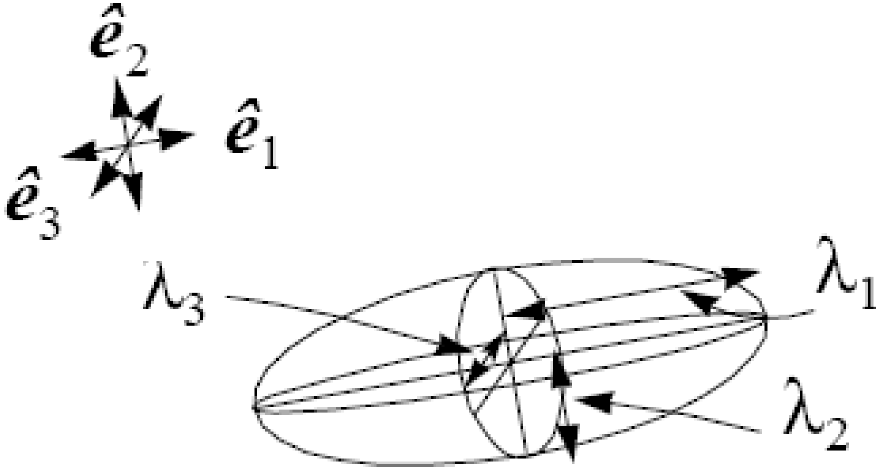
$$\mathbf{M} = \mathbf{USU}^t$$

$$\mathbf{S} = \begin{bmatrix} \lambda_a & 0 & 0 \\ 0 & \lambda_b & 0 \\ 0 & 0 & \lambda_c \end{bmatrix} \quad \mathbf{U} = \begin{bmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{bmatrix}$$

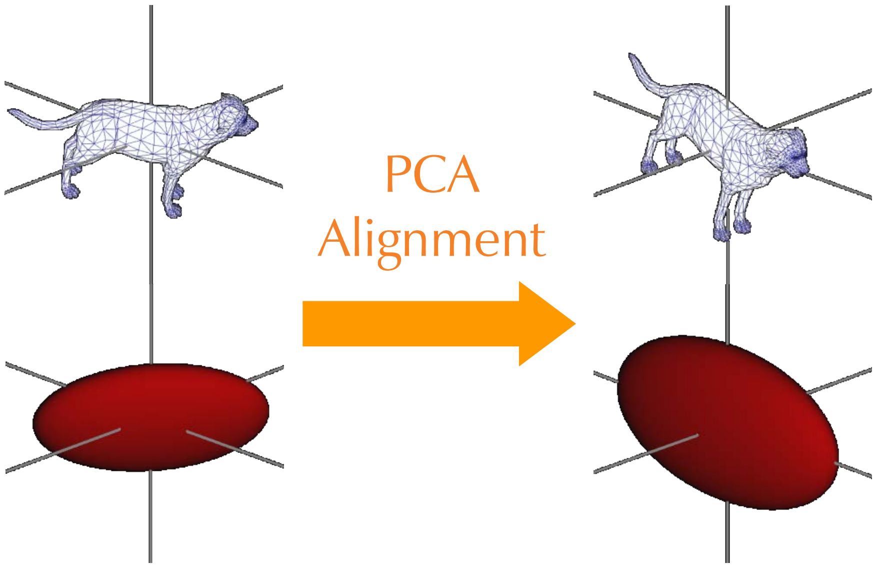
Eigenvalues & Eigenvectors

Global PCA

- Eigenvectors are "Principal Axes of Inertia"
- Eigenvalues are variances of the point distribution in those directions



- Useful for alignment

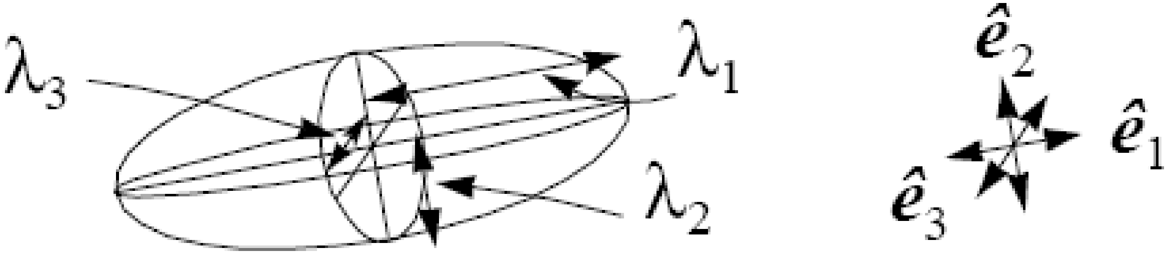


Local PCA

- Provides estimate of normal direction
  - Eigenvector (principal axis) associated with smallest eigenvalue

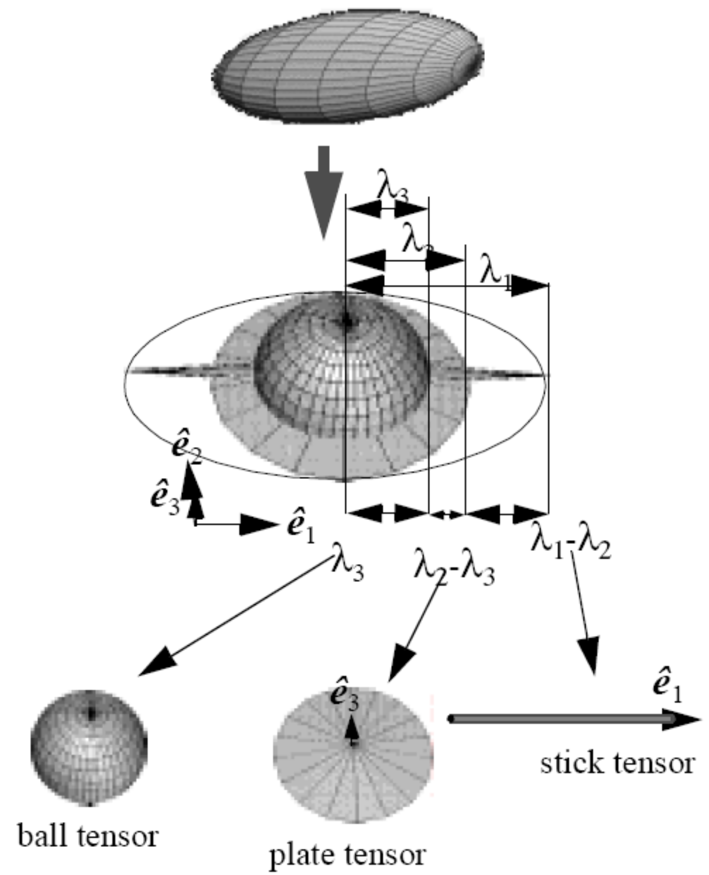
Local PCA

- Helps us construct a local coordinate frame for every point
  - Map $\hat{e}_1$ to X axis
  - Map $\hat{e}_2$ to Y axis
  - Map $\hat{e}_3$ to Z axis



Local PCA

- Helps differentiate nearly plane-like, from stick-like, from sphere-like, etc.



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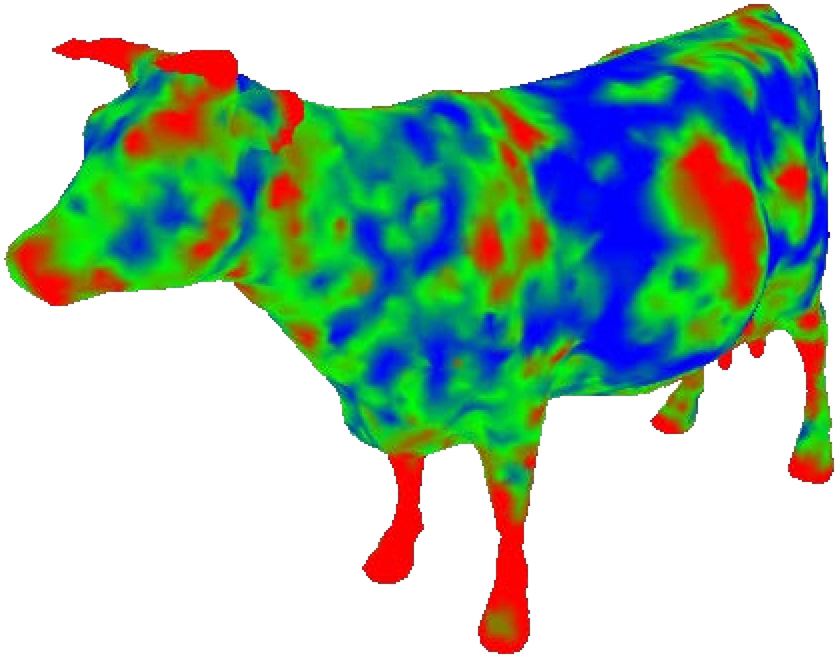
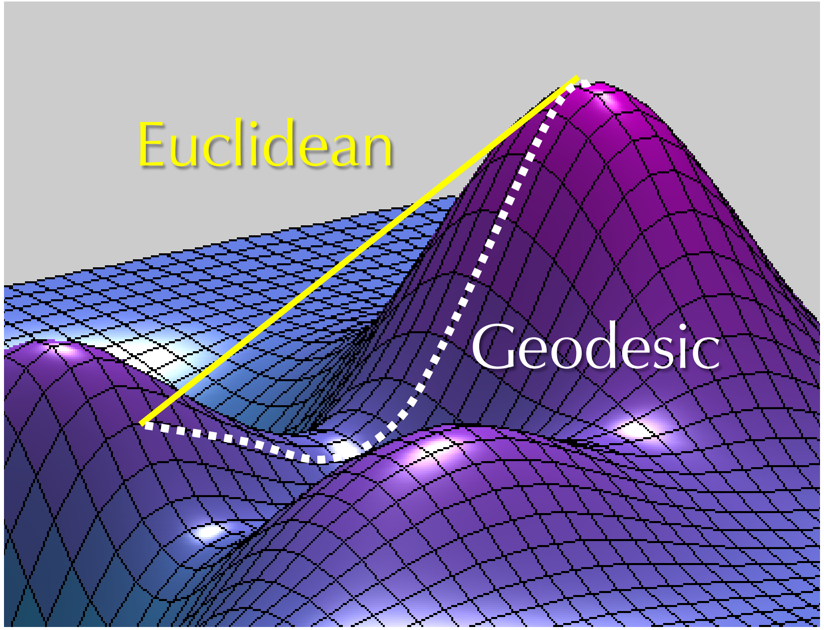


$\lambda_2 / (\lambda_1+\lambda_2+\lambda_3)$

Statistics of Distances

- Distances can be along surface (geodesic) or as a crow flies (Euclidean)

Geodesic distance to point

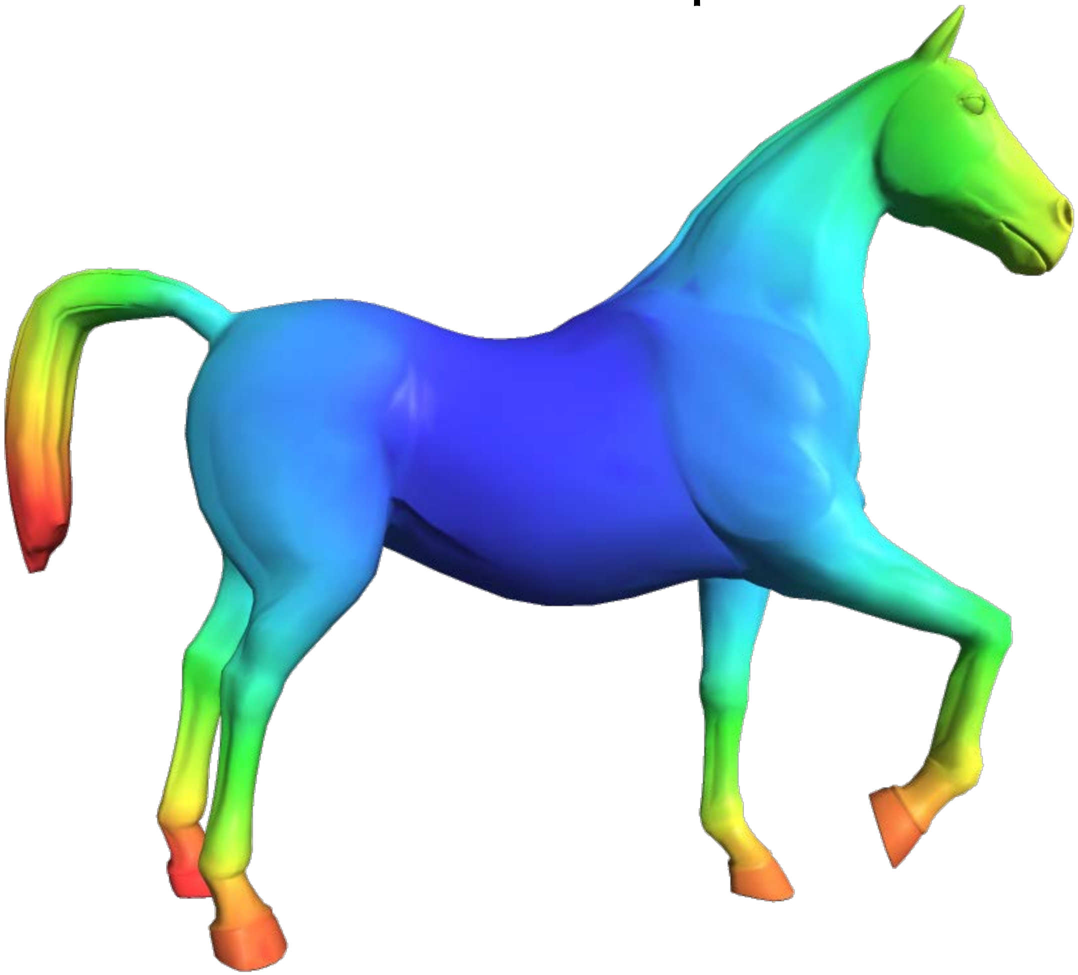


Geodesic vs. Euclidean distance

Shamir

Statistics of Distances

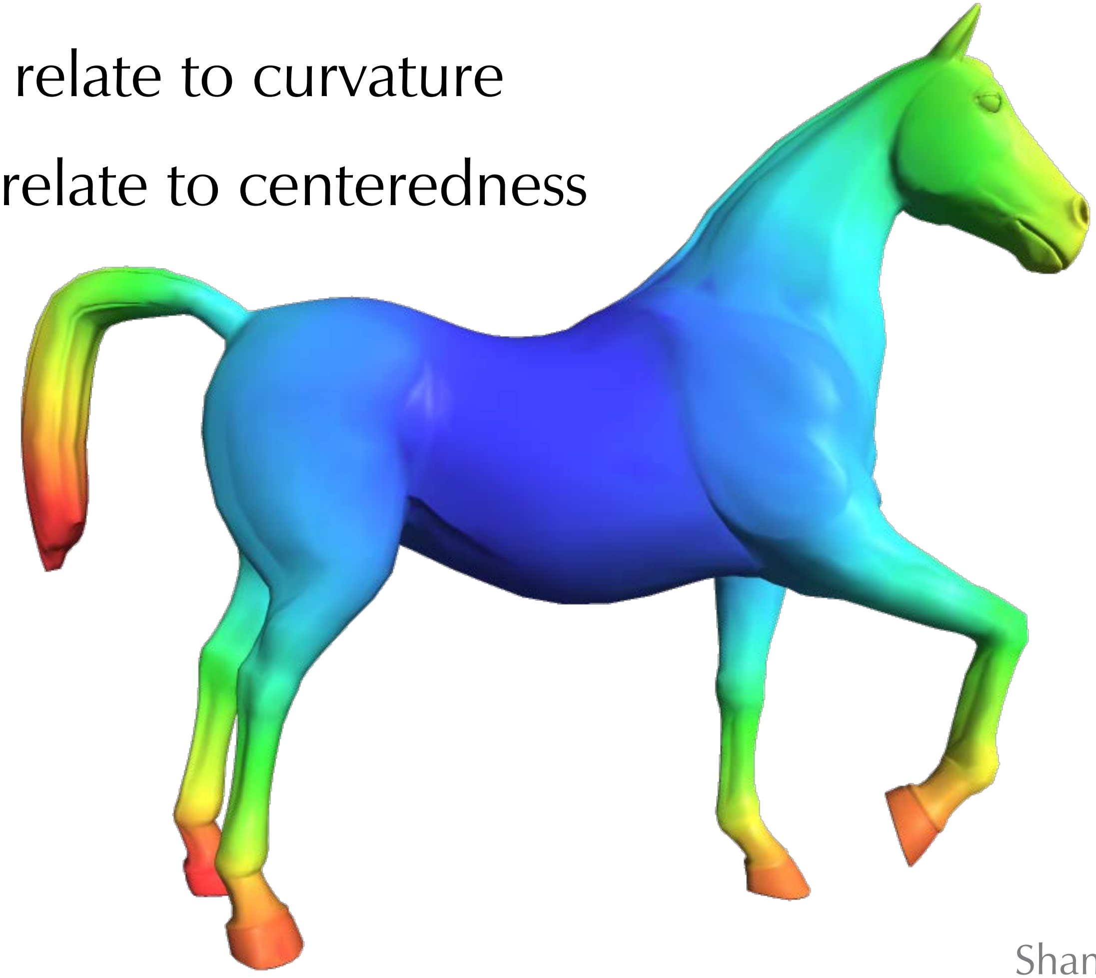
- Average geodesic distance to other points on surface



Shamir

What Do Statistics of Distance Tell Us?

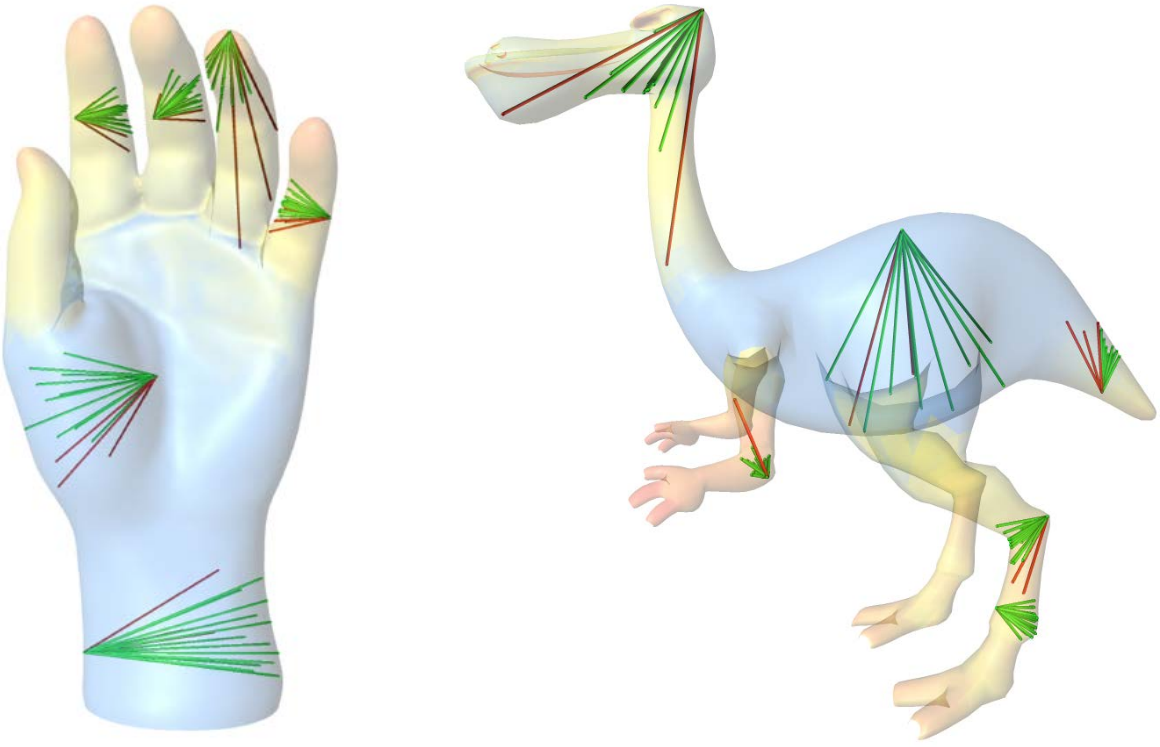
- Histograms of geodesic distances
  - Small distances relate to curvature
  - Long distances relate to centeredness



Shamir

Shape Diameter Function

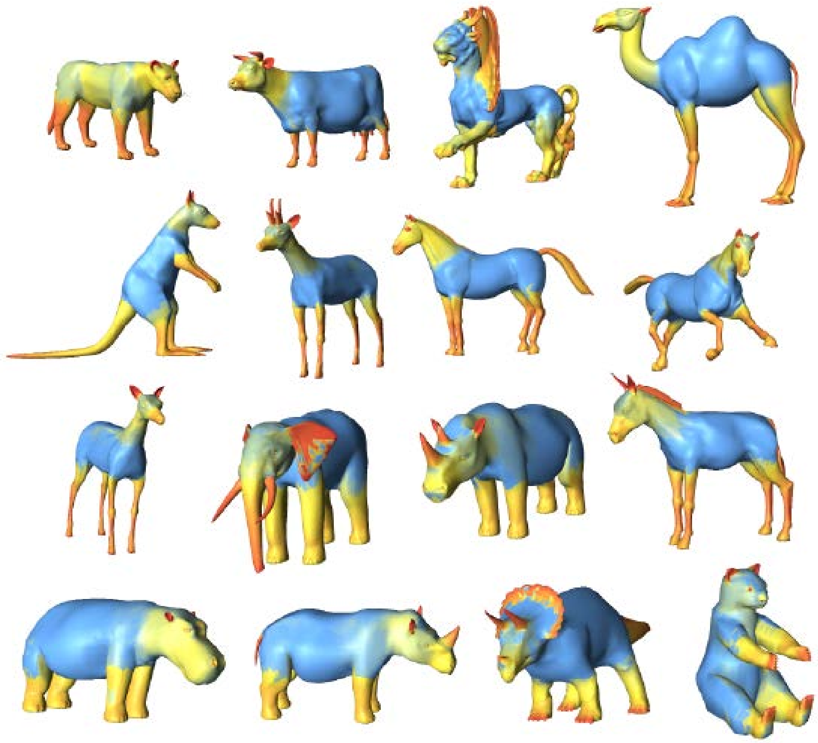
- Median distance along sampling of rays through interior



Shamir

Shape Diameter Function

- Distinguish between thin and thick parts in a model
- Sharp changes often correlate with part boundaries



Shamir

Mesh Analysis: Applications

Mesh Analysis

- Feature detection
- Segmentation
- Labeling
- Registration
- Matching
- Retrieval
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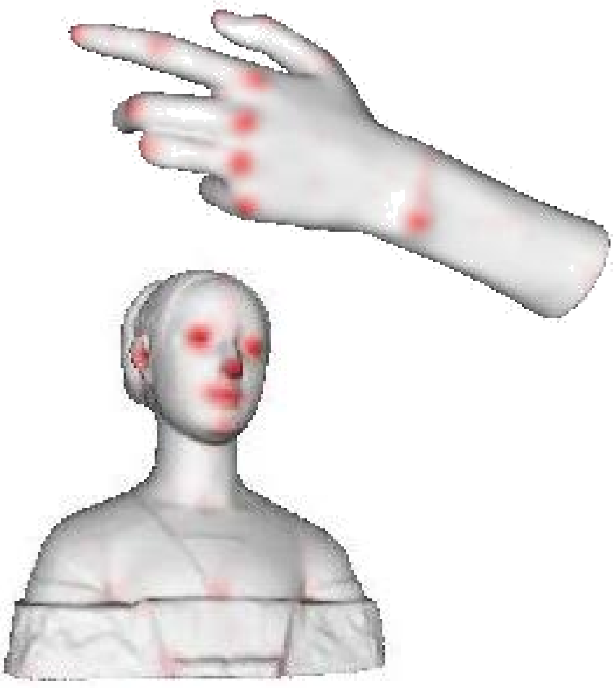

Schelling Points

"How can we find significant geometric features robustly?"

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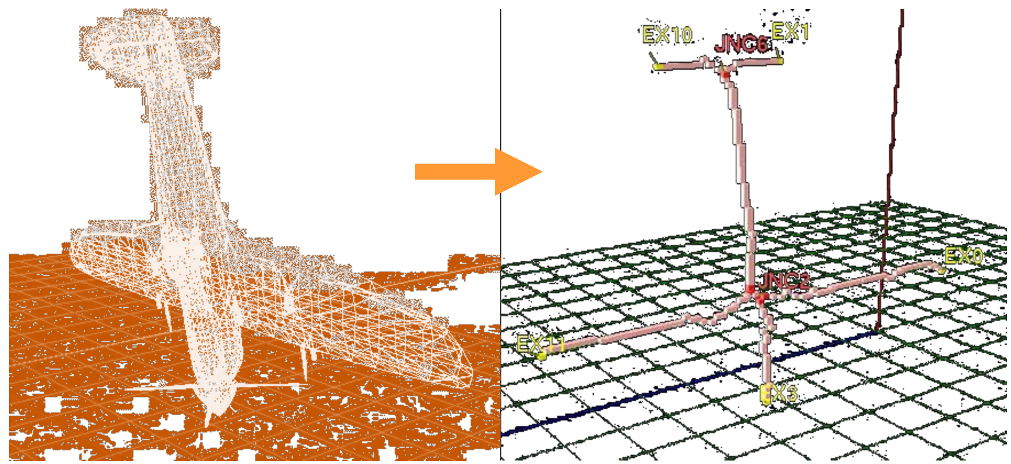

Input Mesh

Skeletal Graph

"How can we decompose a 3D model into its parts?"

Mesh Analysis

Images courtesy of
Ayellet Tal, Technion &
Princeton University

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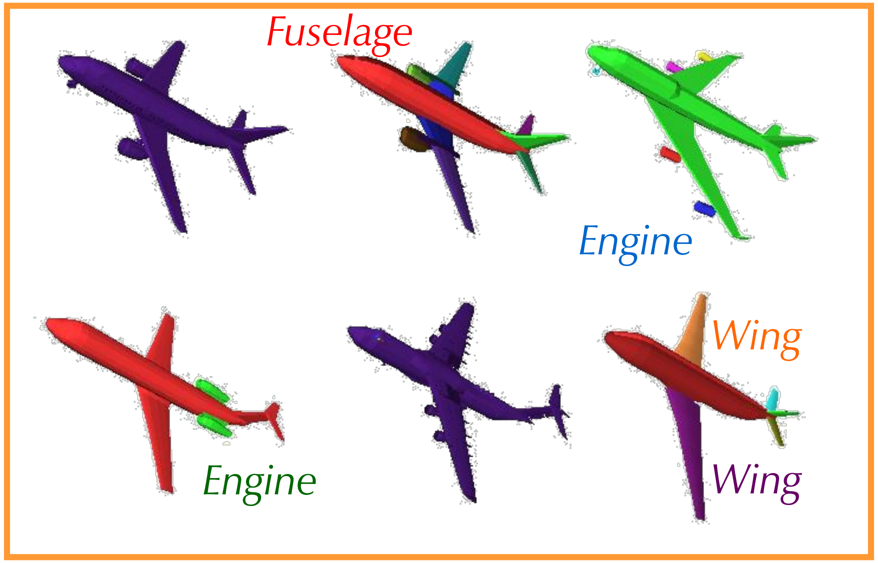

Semantic Labels

(Golovinskiy, Lee, et al.)

"How can we decompose a 3D model into its parts?"

Mesh Analysis

Images courtesy of
Emil Praun

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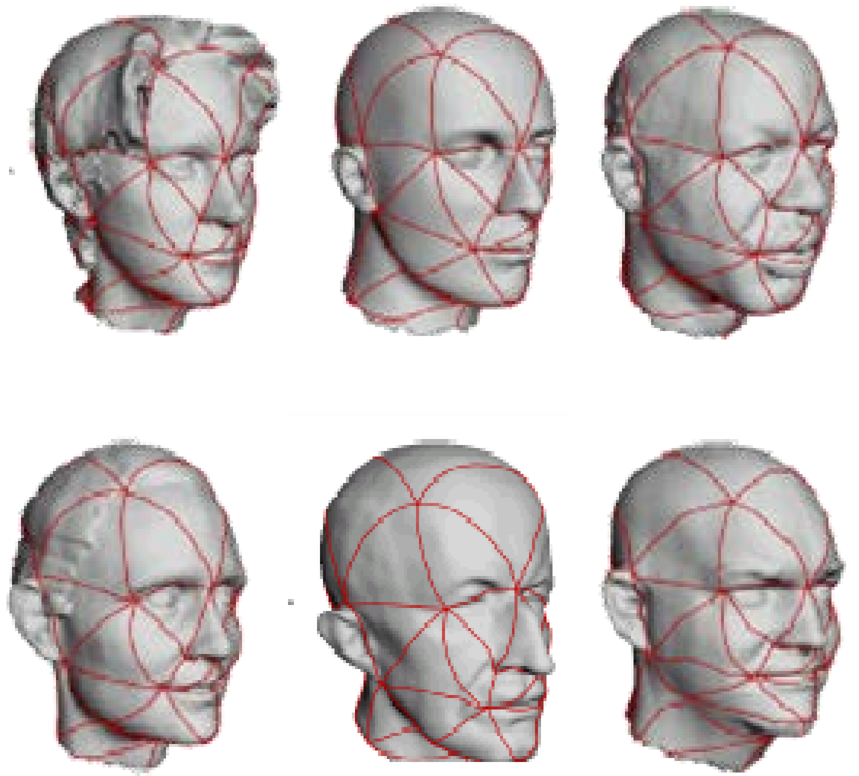

"How can we align features of 3D models?"

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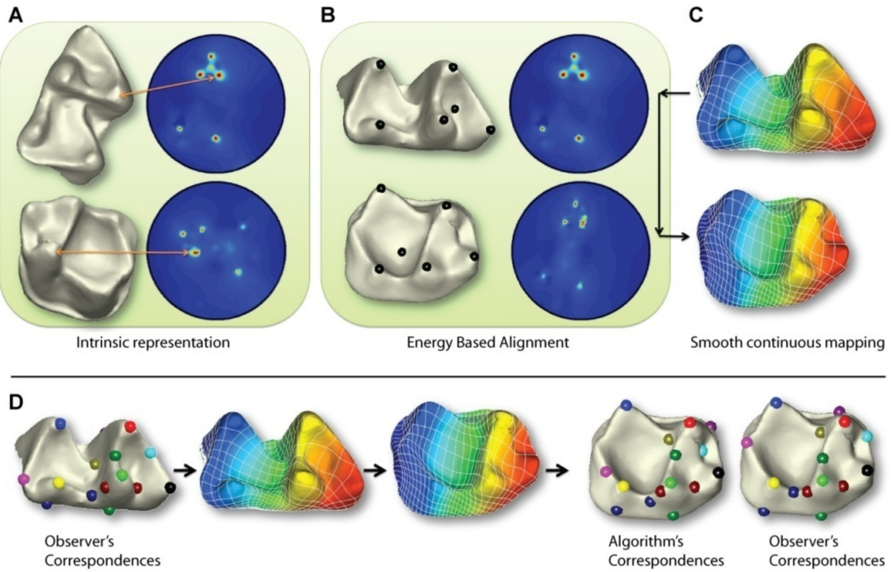

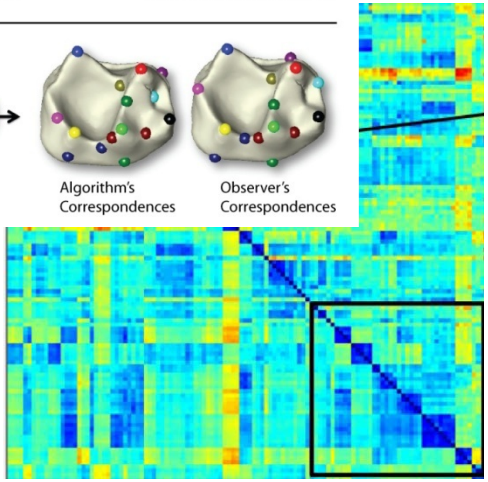

"How can we compute a measure of geometric similarity?"

Mesh Analysis

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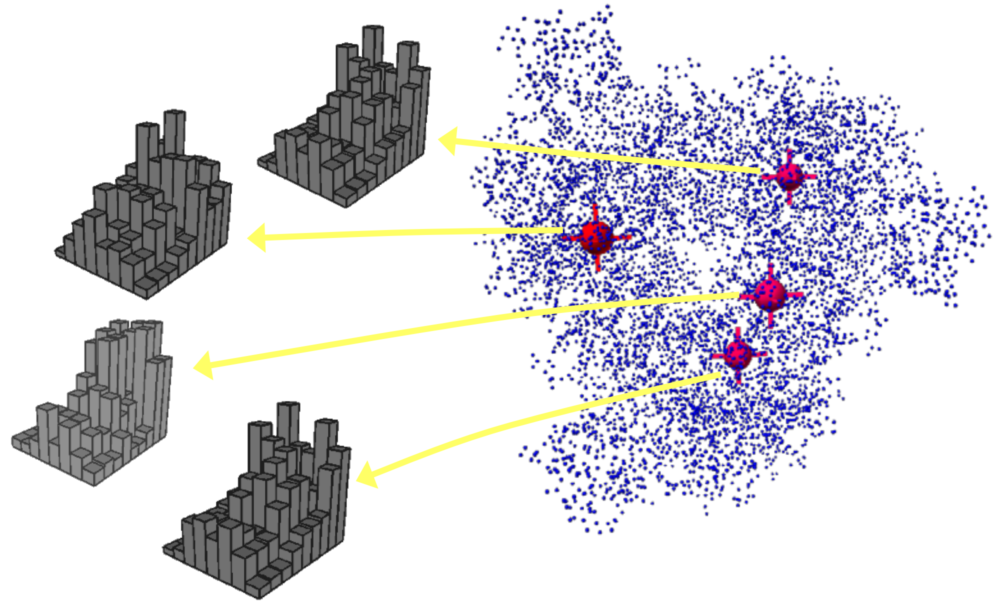

Harmonic Shape Descriptors

"How can we find similar 3D shapes in a database?"

Mesh Analysis

Images courtesy of Florida State Univ.

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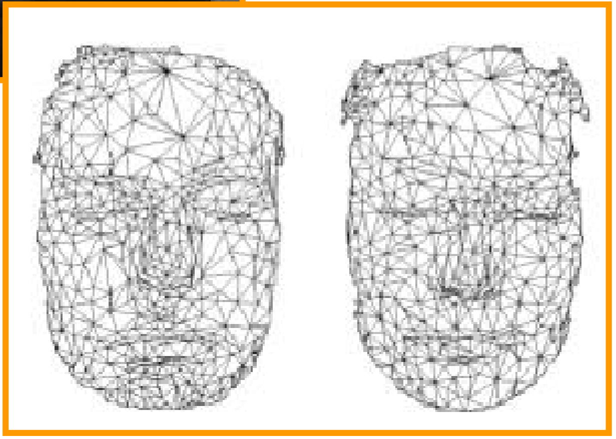

"How can we find a given 3D model in a large database?"

Mesh Analysis

Images courtesy of
Darpa E3D Project

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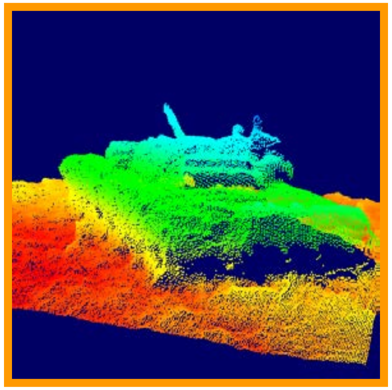
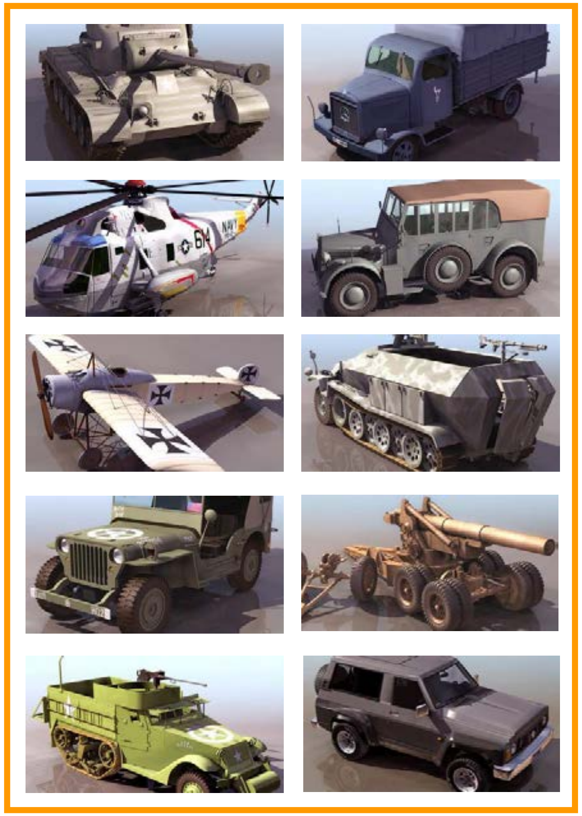


Query



Classes

"How can we determine the class of a 3D model?"

Mesh Analysis

Images courtesy of Viewpoint

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"How can we learn classes of 3D models automatically?"

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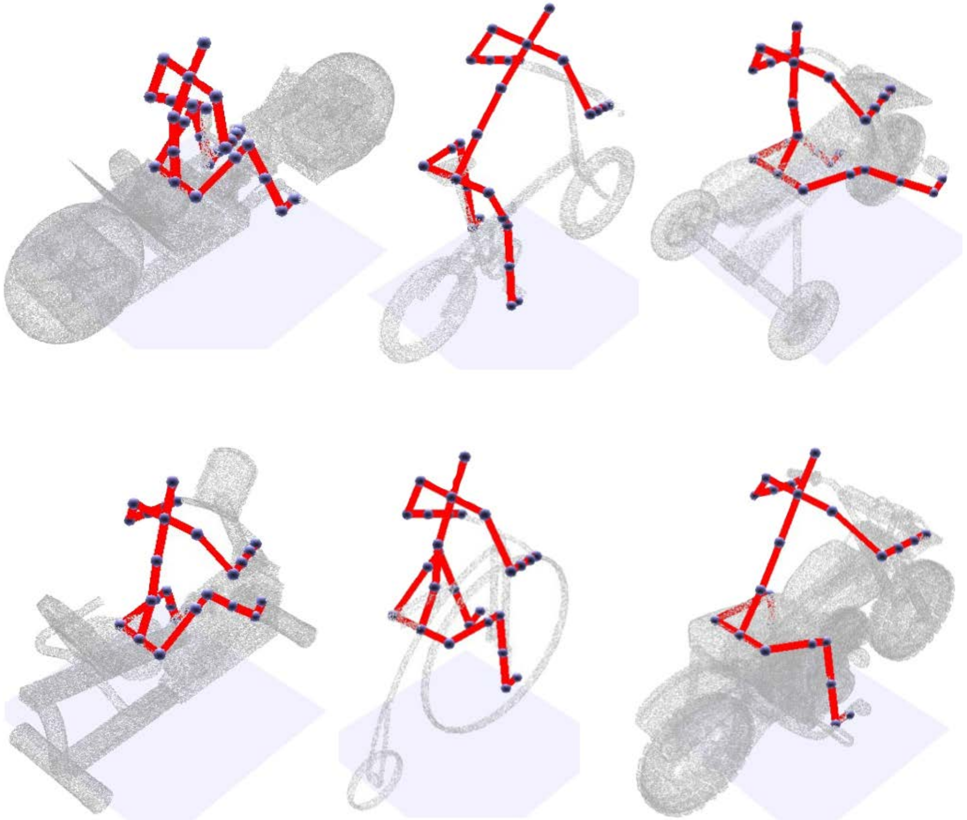

"Can we predict how an object might be used?"