Introduction to Machine Learning - COS 324

Homework Assignment 1

Due date: The minute after 11:59pm on 26th Sept. Electronic submisisons only.

IMPORTANT:

1. Consulting with other students from this course is allowed. If you do so, clearly state whom you consulted with for each problem separately.
2. Searching the internet or literature for solutions is **prohibited**.

1. (Monty Hall problem.) You are given three curtains to choose from. Behind two of them sits goat while behind the third there is a state-of-the-art computer with a TPU. You do not know which one is which. You pick one curtain at random. Clearly, behind at least one of the two remaining curtains resides a goat. This curtain (or one of two) is revealed to you.

   (a) Would you now change your choice in order to maximize your chance of getting a TPU? Prove your claim.

   (b) Perform a simulation of at least 10,000 repeated experiments in order to verify your answer from the previous section. Submit your python code and an output of the results using the function `bar()` from the python package `matplotlib.pyplot`

2. Recall the weighted majority algorithm and its mistake bound.

   (a) Assume that you know the number of mistakes that the best expert is going to make. Call it $m^\star$. Describe changes to the WM algorithm such that the difference between the number of mistakes it makes, denoted m, and twice that of the best expert in indsight, is bounded by

   $$m - 2m^\star \leq 4\sqrt{m^\star \log n} + 4 \log n \quad .$$

   (b) **Bonus Question.** Show how to modify the WM algorithm such that it is guaranted to make at most $2m^\star + 16\sqrt{m^\star \log n} + 16 \log n$, that is,

   $$m - 2m^\star \leq 16\sqrt{m^\star \log n} + 16 \log n \quad ,$$

   even without the knowledge of $m^\star$ in advance. Prove this mistake bound.