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2.3 Partitioning Demos

- *Sedgewick 2-way partitioning*
- *Dijkstra 3-way partitioning*
- *Bentley–McIlroy 3-way partitioning*
- *dual-pivot partitioning*

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2.3 Partitioning Demos

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Quicksort partitioning demo

Repeat until `i` and `j` pointers cross.

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K	R	A	T	E	L	E	P	U	I	M	Q	C	X	O	S
lo	i														j

stop i scan because a[i] >= a[lo]

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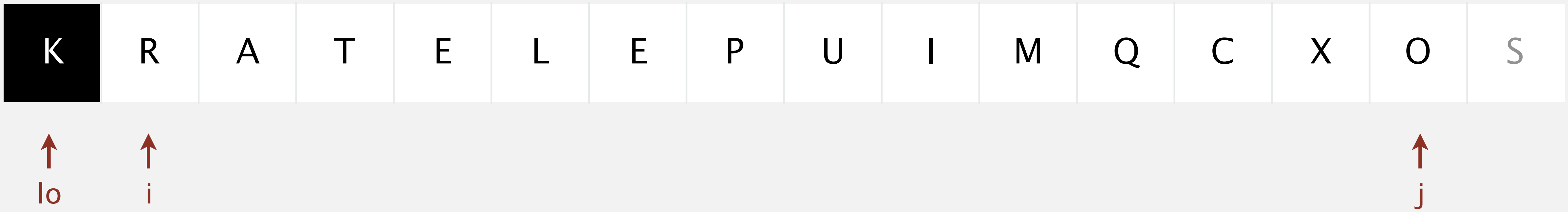

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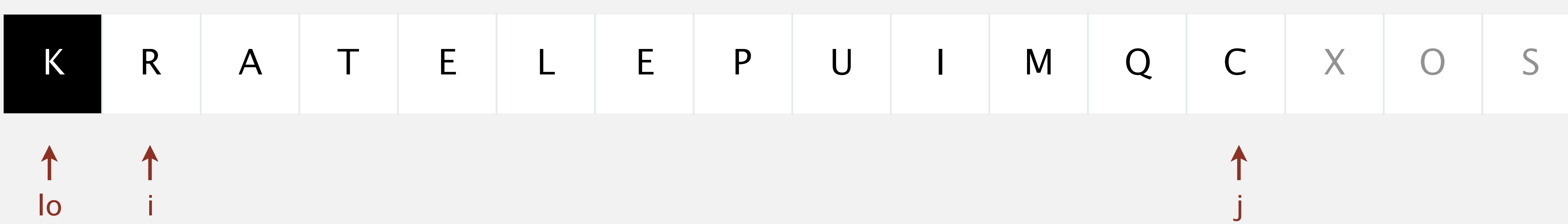

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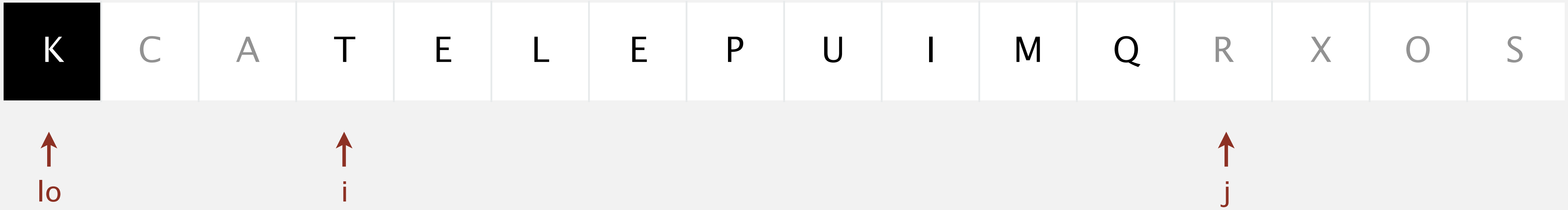

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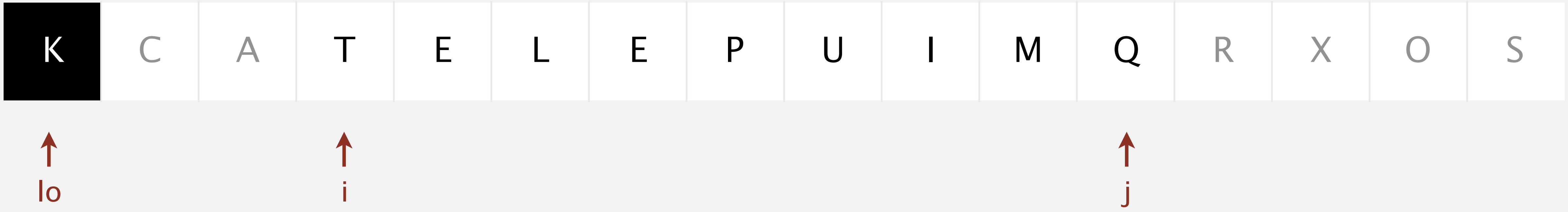

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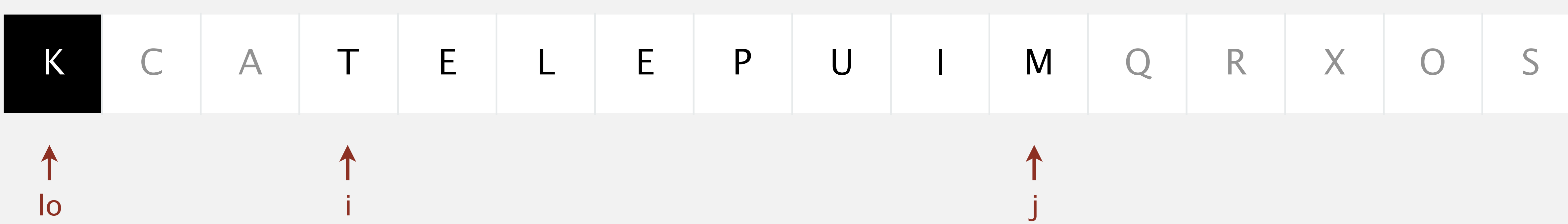

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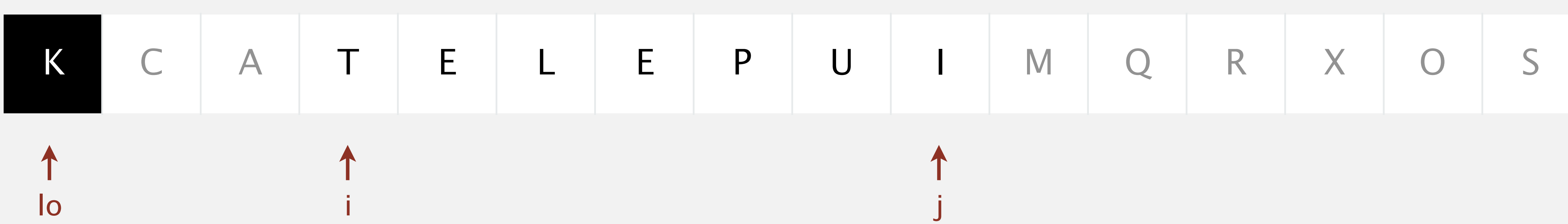

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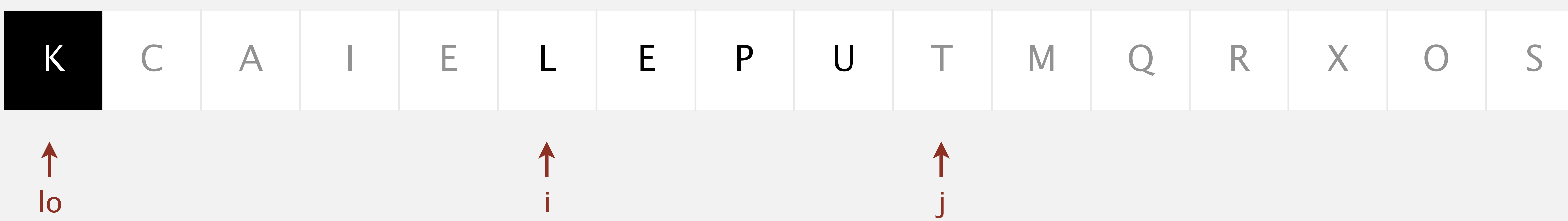

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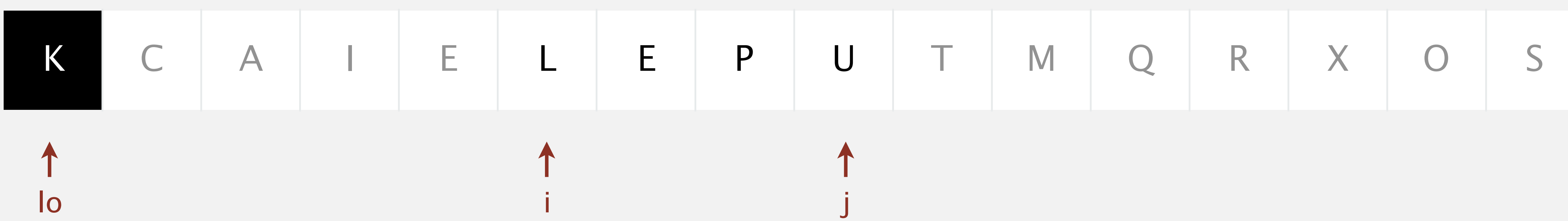

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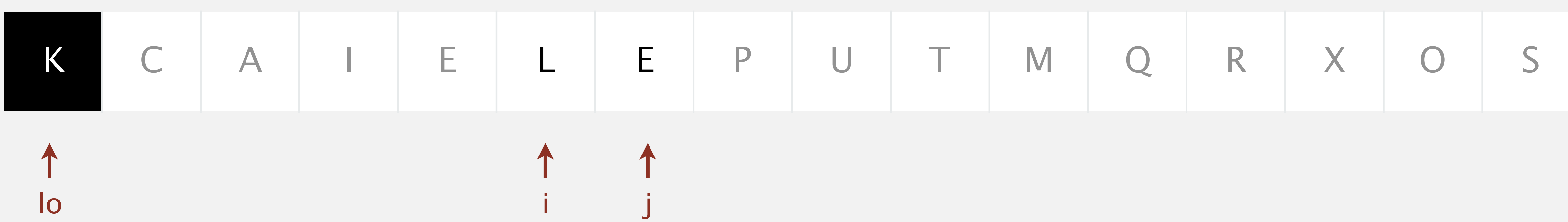

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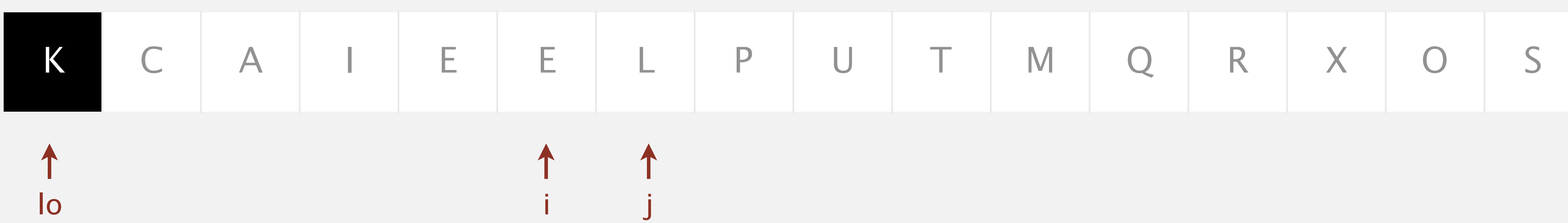

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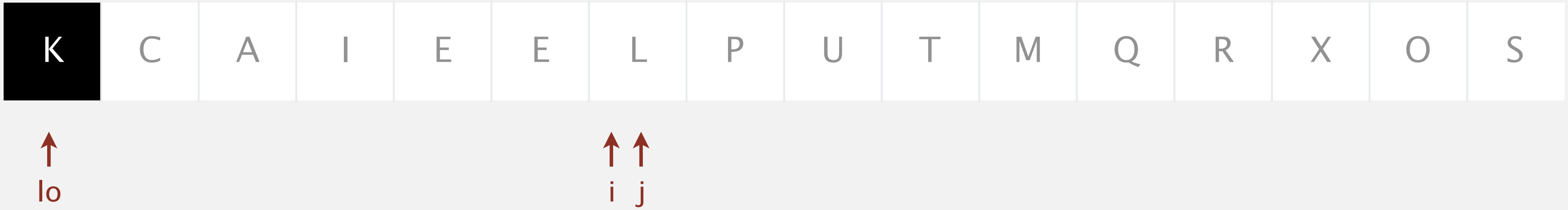

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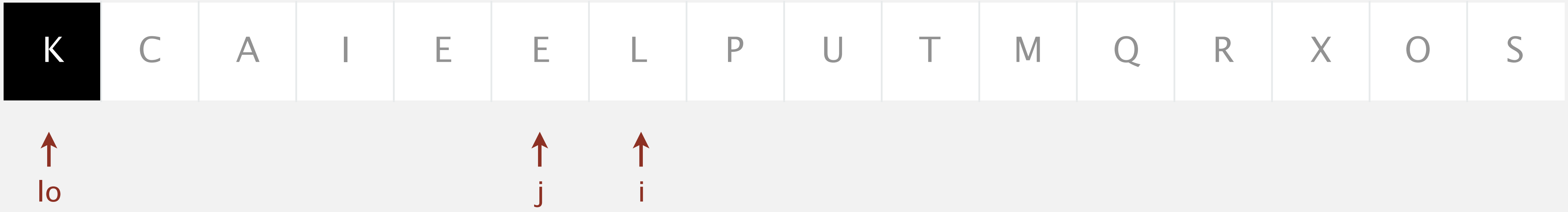

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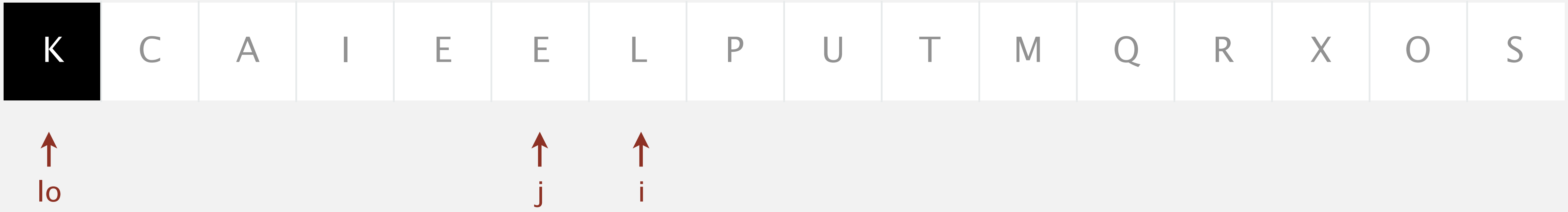
When pointers cross.

- Exchange `a[lo]` with `a[j]`.



pointers cross: exchange a[lo] with a[j]

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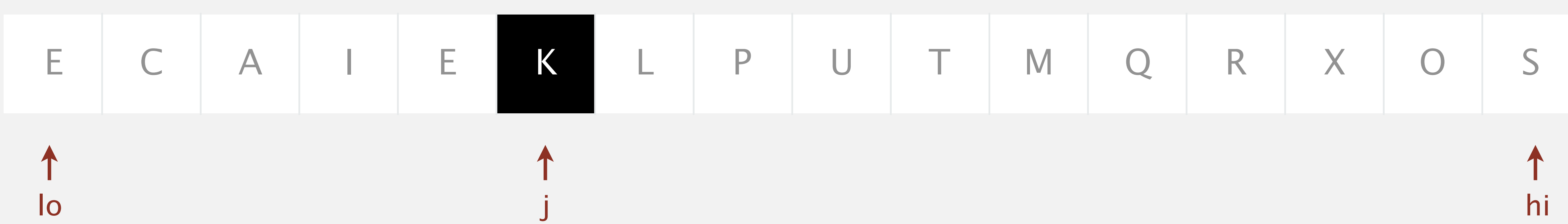

partitioned!

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2.3 Partitioning Demos

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Dijkstra 3-way partitioning demo

- Let `v` be partitioning item `a[lo]`.
- Scan `i` from left to right.
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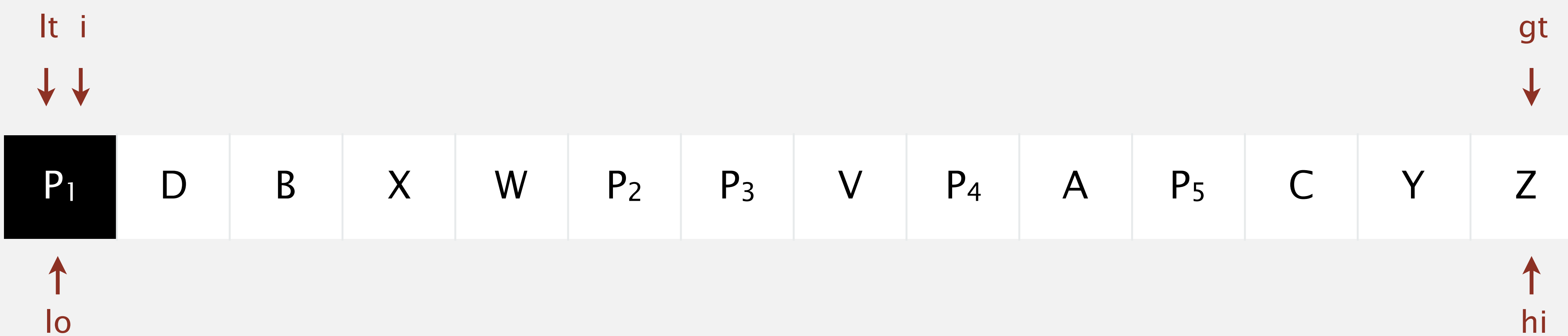

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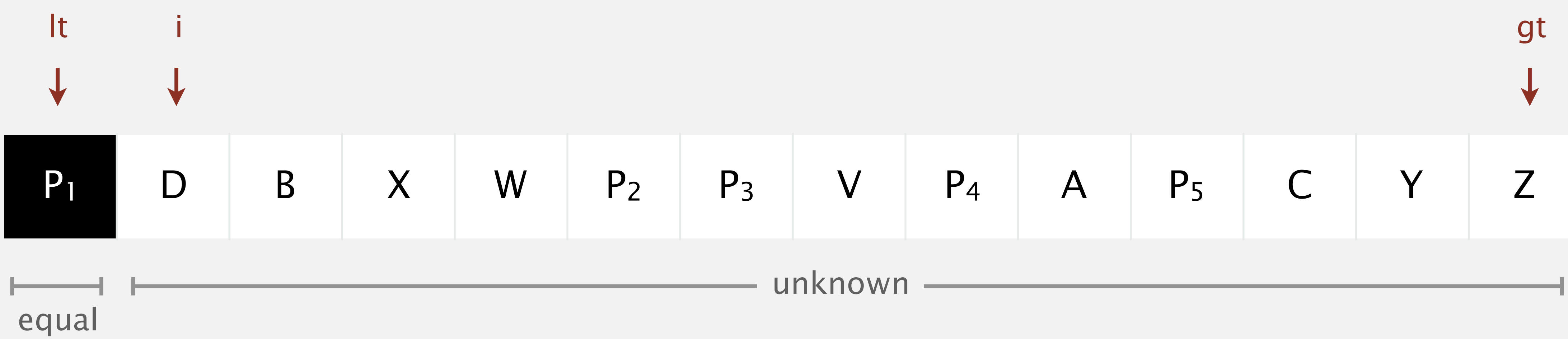

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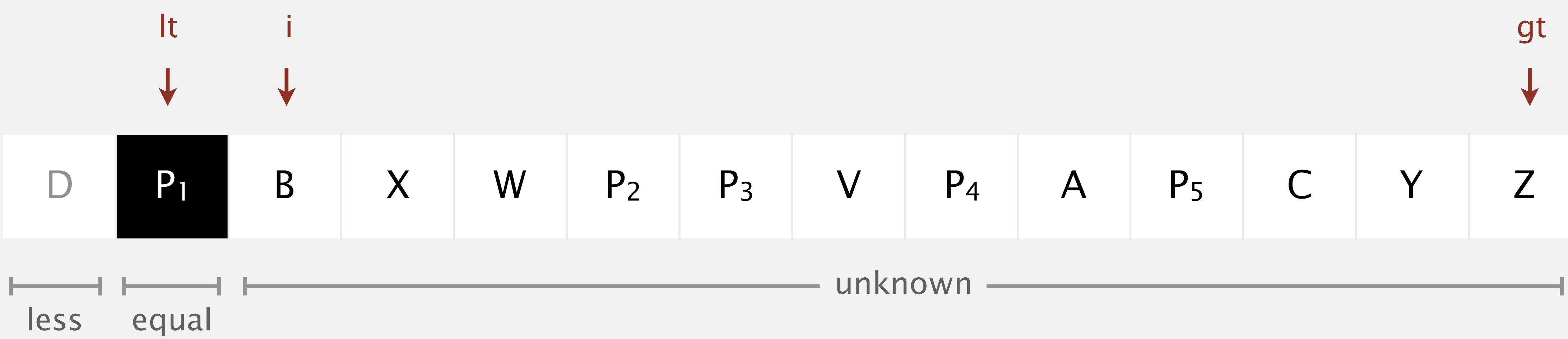

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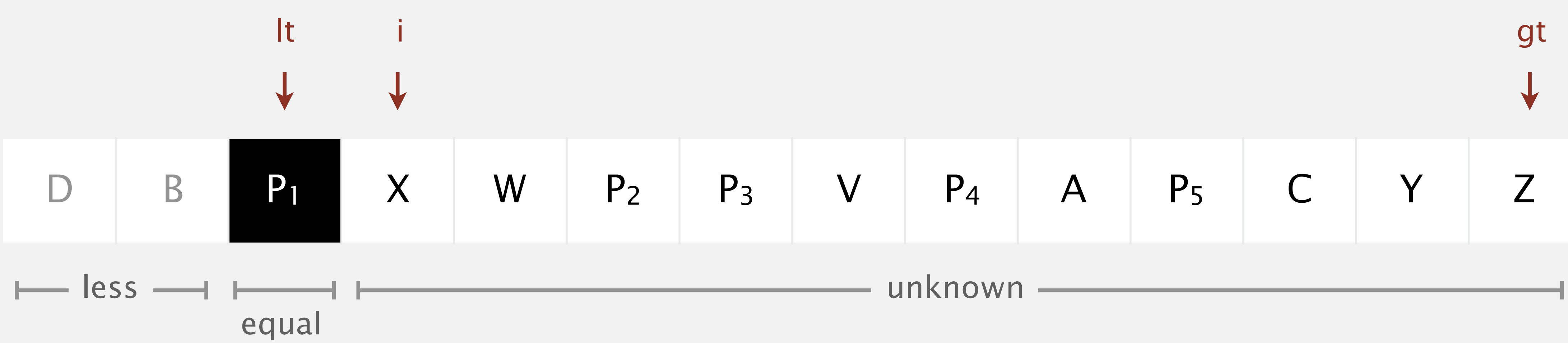

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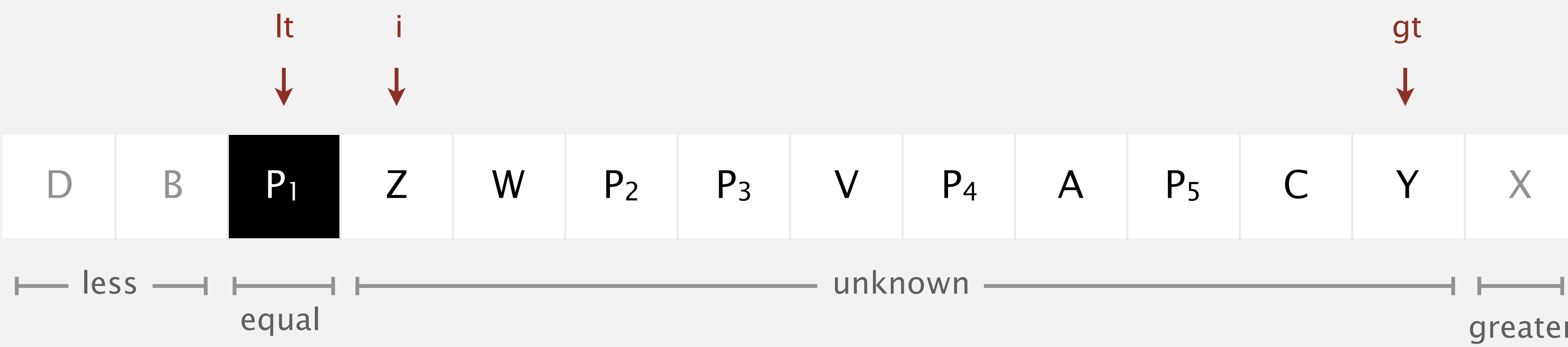

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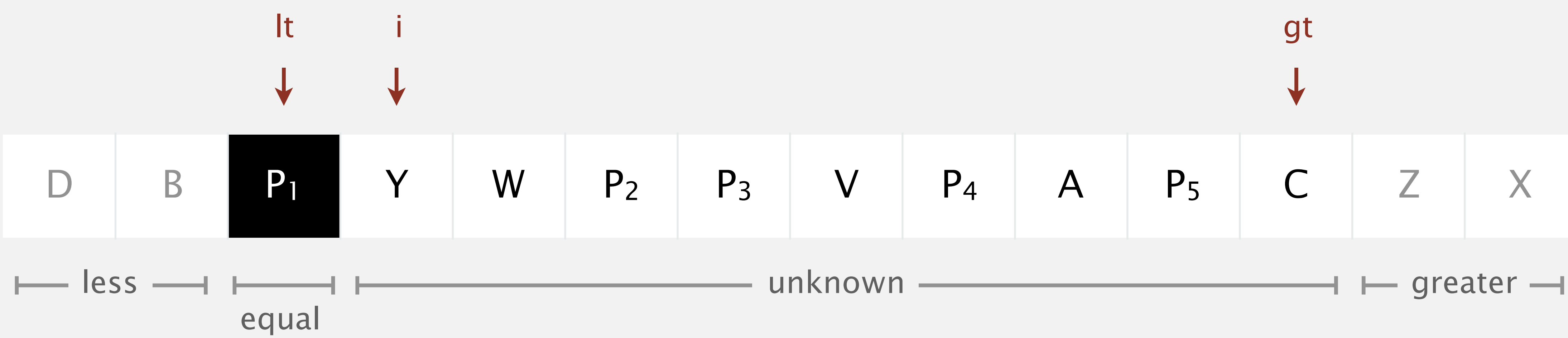

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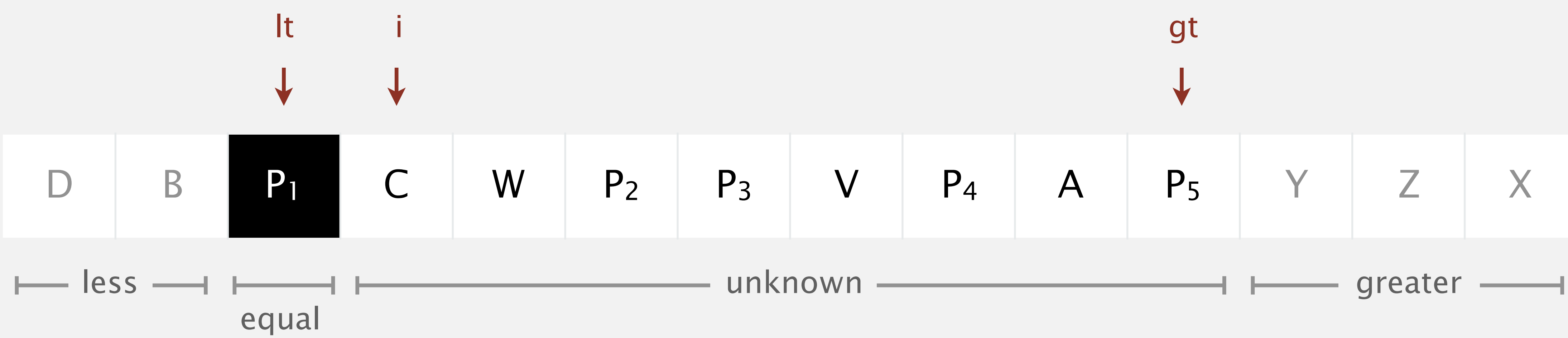

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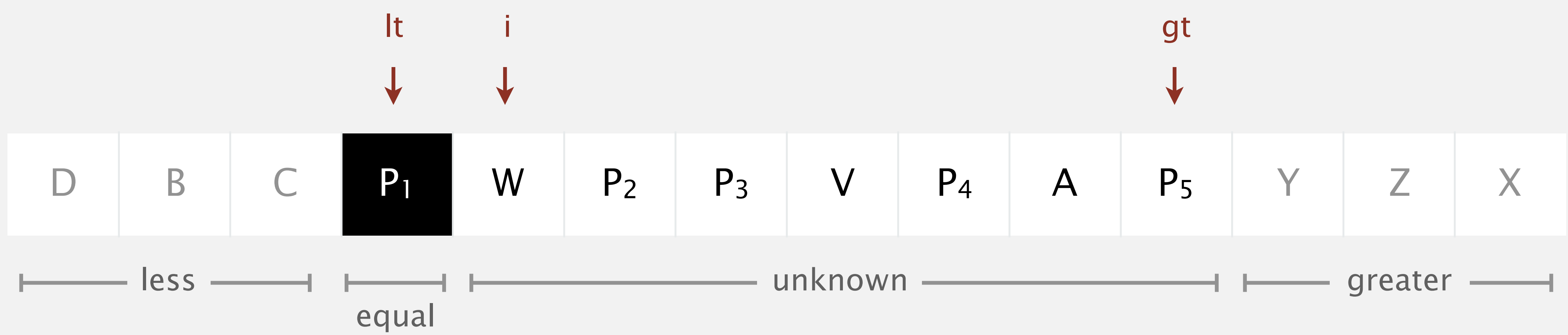

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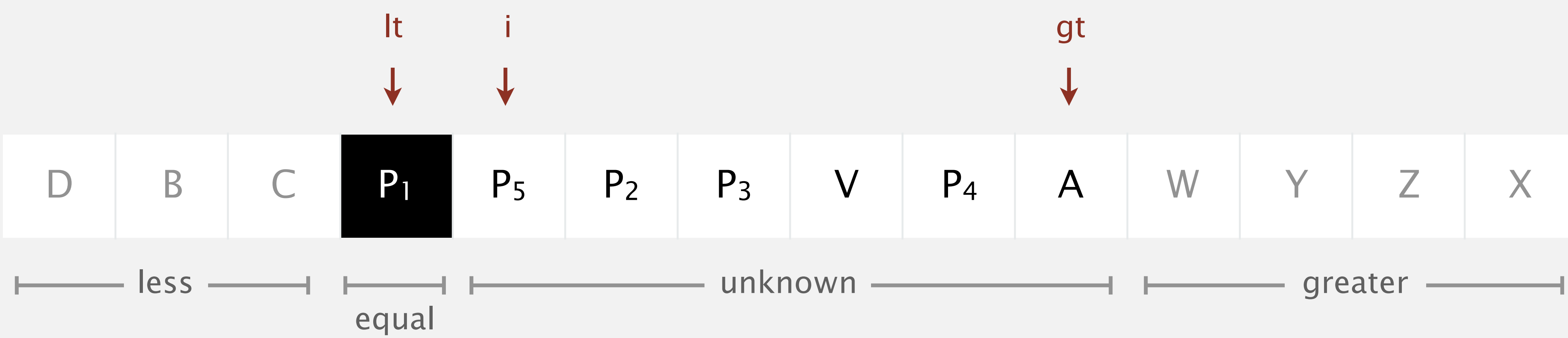

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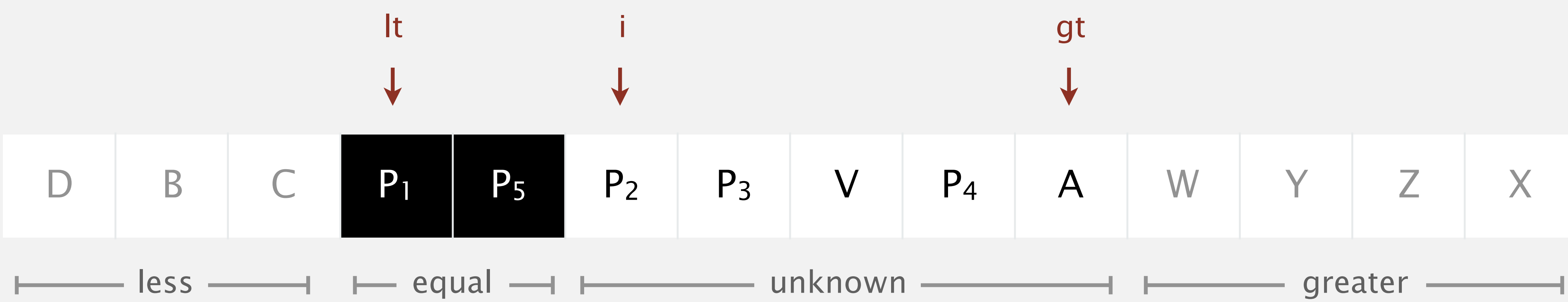

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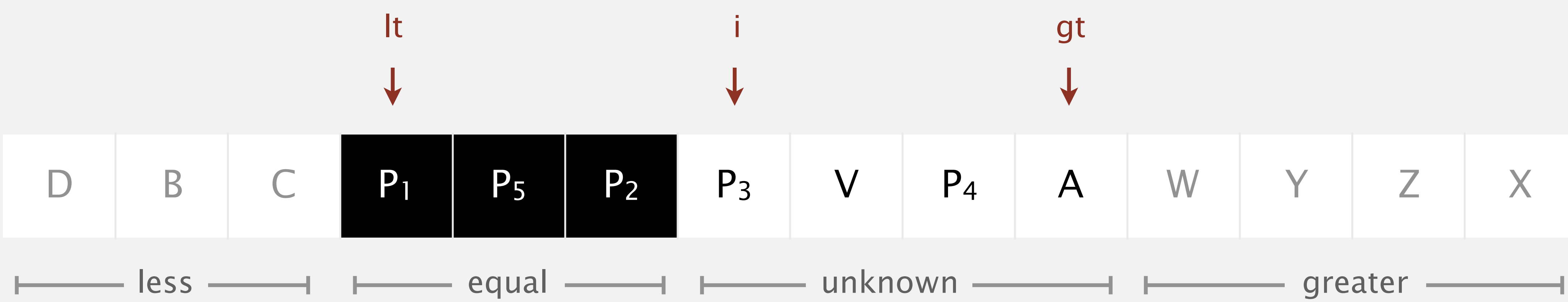

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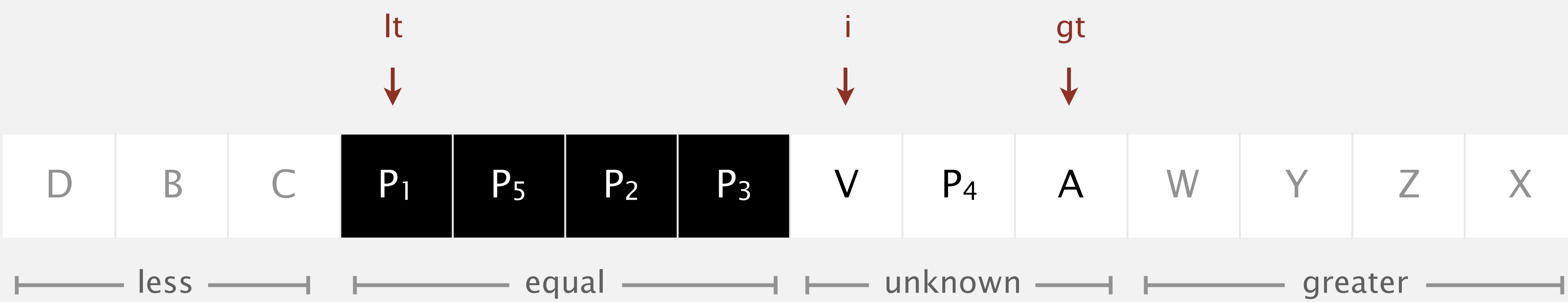

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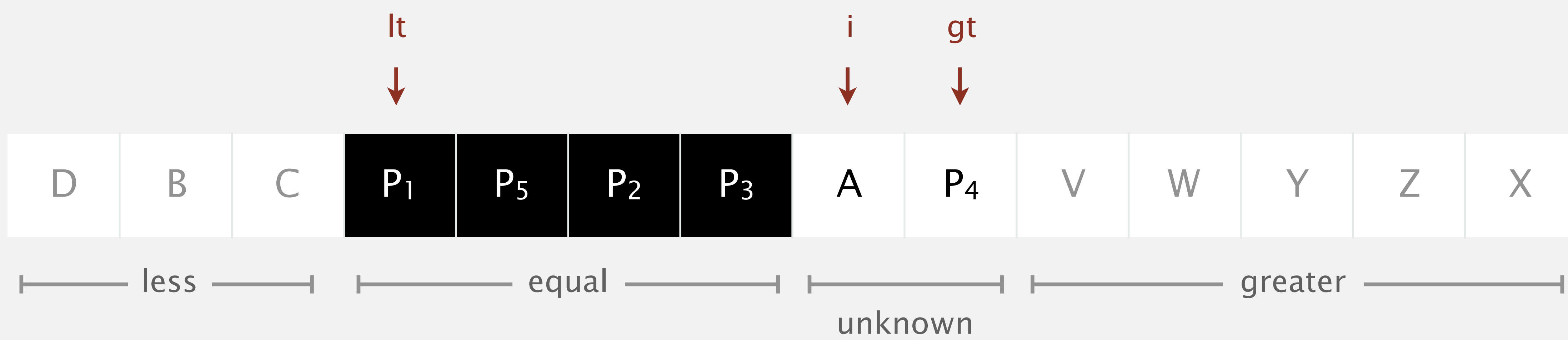

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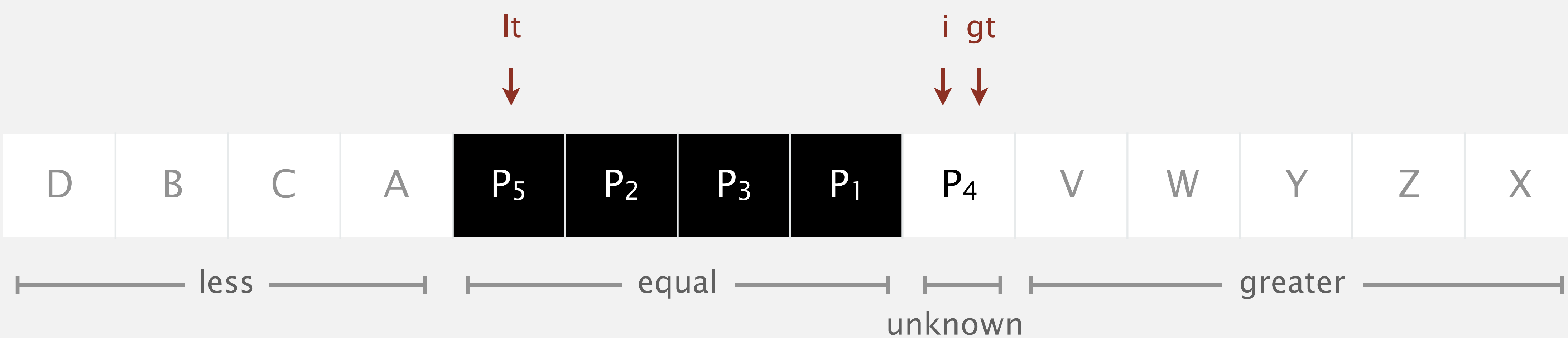

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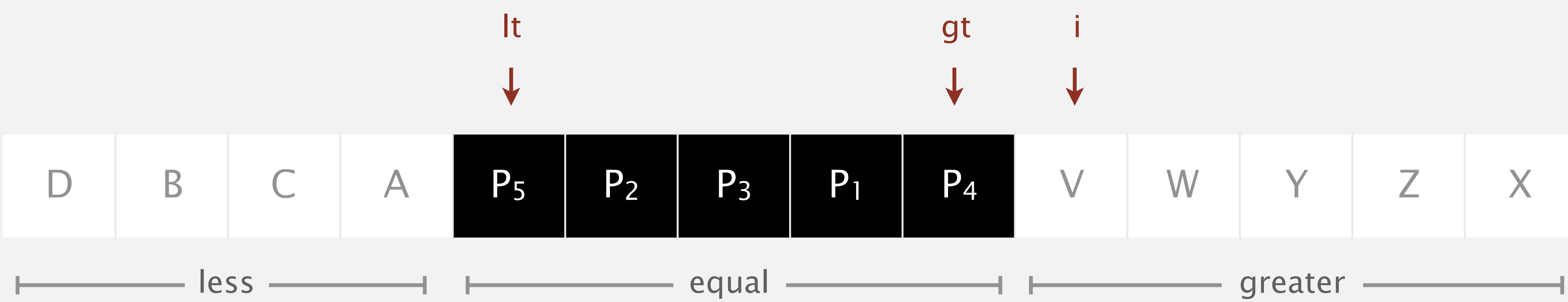

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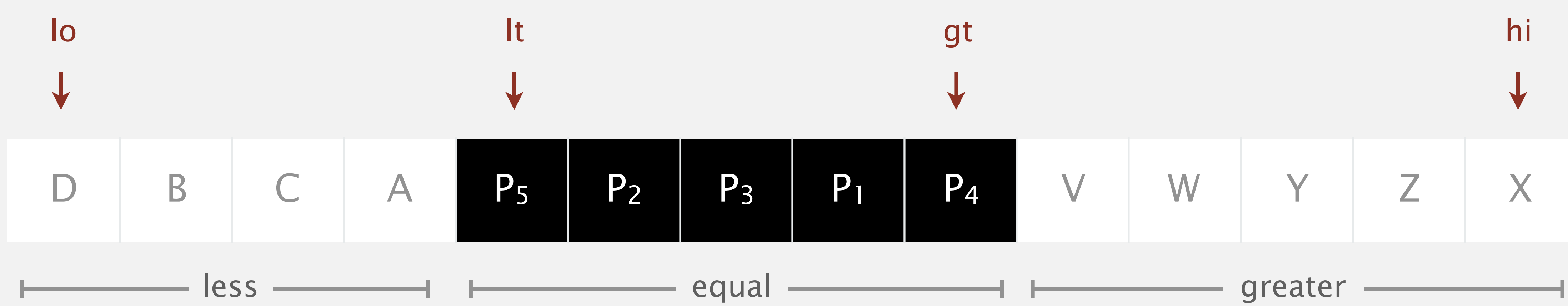

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Bentley–McIlroy 3-way partitioning demo

Phase I. Repeat until `i` and `j` pointers cross.

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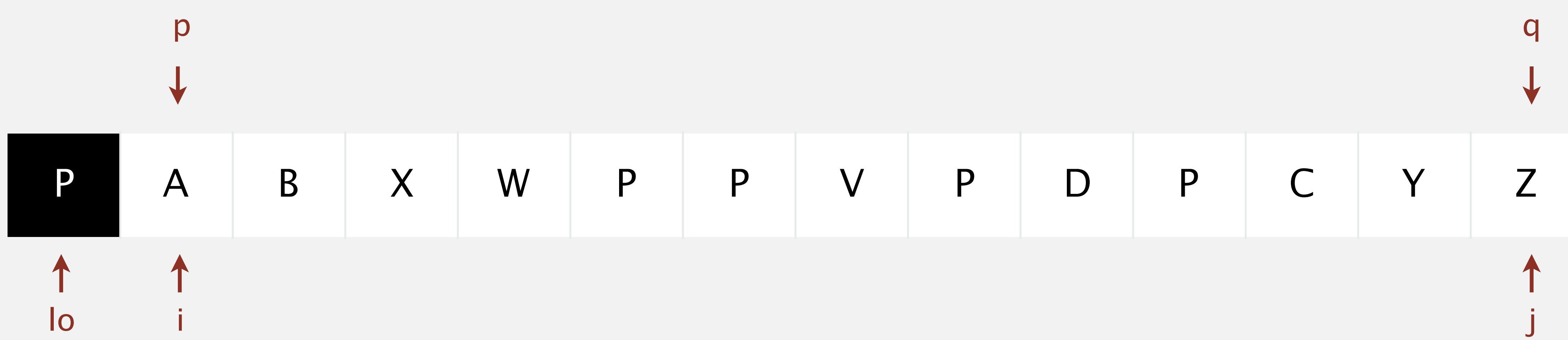
- Scan `j` from right to left so long as `(a[j] > a[lo])`.
- Exchange `a[i]` with `a[j]`.
- If `(a[i] == a[lo])`, exchange `a[i]` with `a[p]` and increment `p`.
- If `(a[j] == a[lo])`, exchange `a[j]` with `a[q]` and decrement `q`.



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- Scan `j` from right to left so long as `(a[j] > a[lo])`.
- Exchange `a[i]` with `a[j]`.
- If `(a[i] == a[lo])`, exchange `a[i]` with `a[p]` and increment `p`.
- If `(a[j] == a[lo])`, exchange `a[j]` with `a[q]` and decrement `q`.

Bentley–McIlroy 3-way partitioning demo

Phase I. Repeat until `i` and `j` pointers cross.

- Scan `i` from left to right so long as `(a[i] < a[lo])`.
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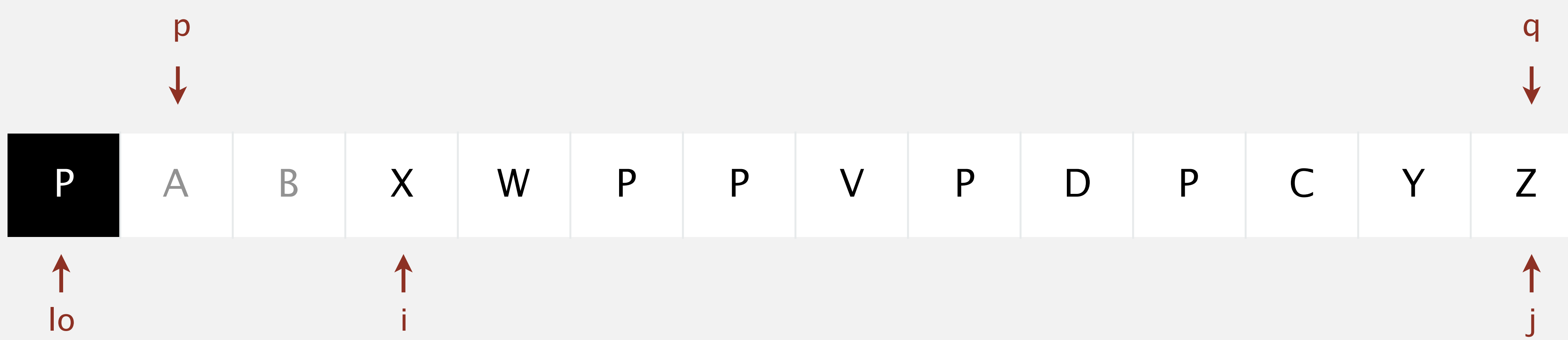

Bentley–McIlroy 3-way partitioning demo

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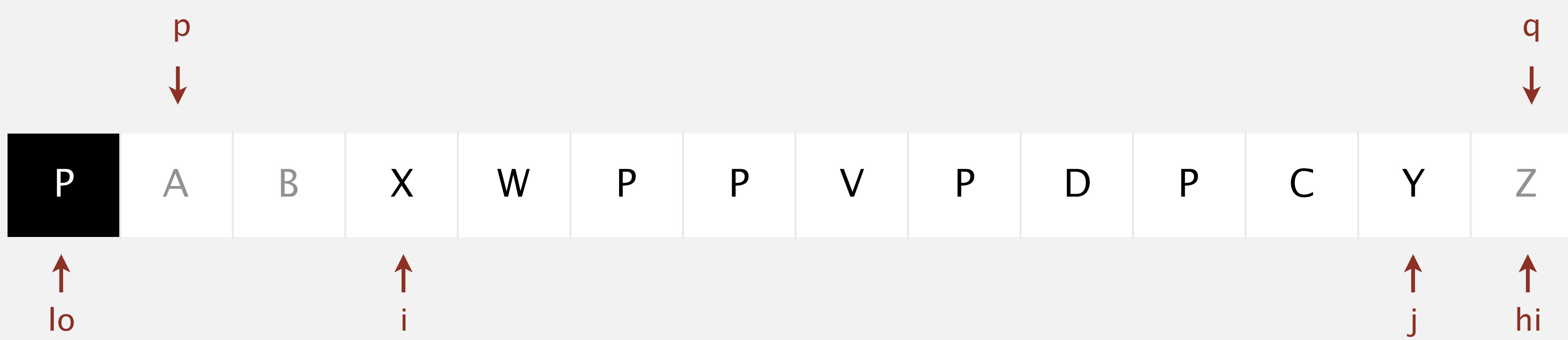

Bentley–McIlroy 3-way partitioning demo

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- Scan `i` from left to right so long as `(a[i] < a[lo])`.
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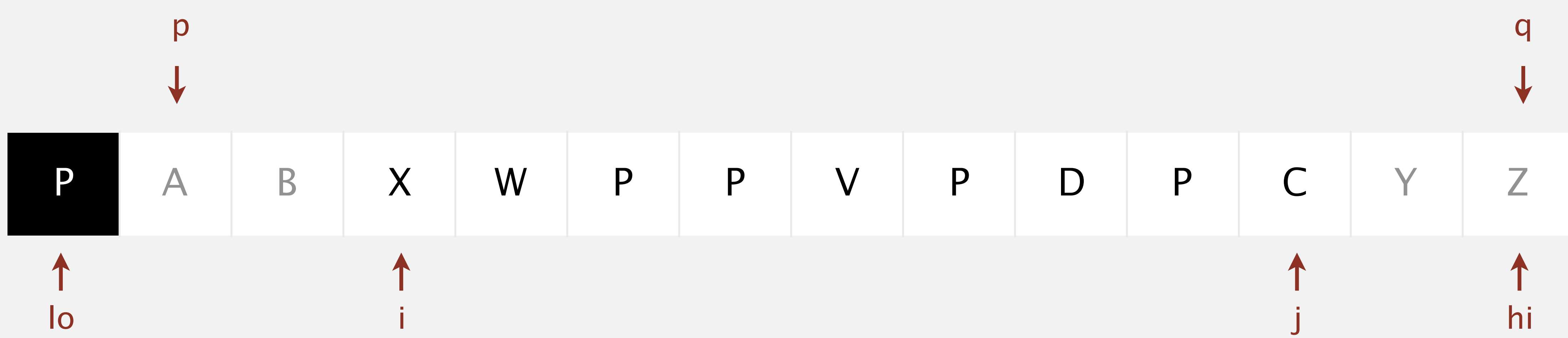

exchange `a[i]` with `a[j]`

Bentley–McIlroy 3-way partitioning demo

Phase I. Repeat until `i` and `j` pointers cross.

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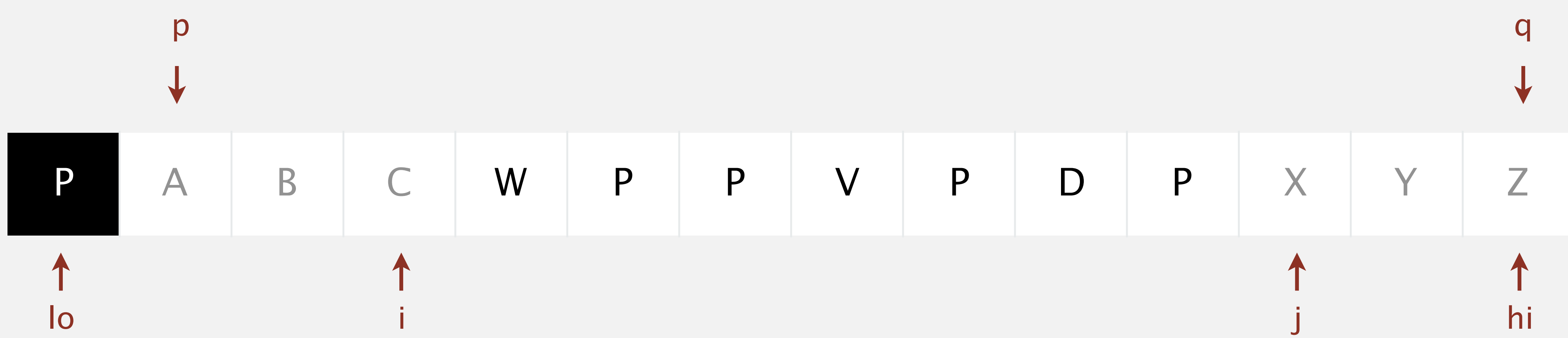

Bentley–McIlroy 3-way partitioning demo

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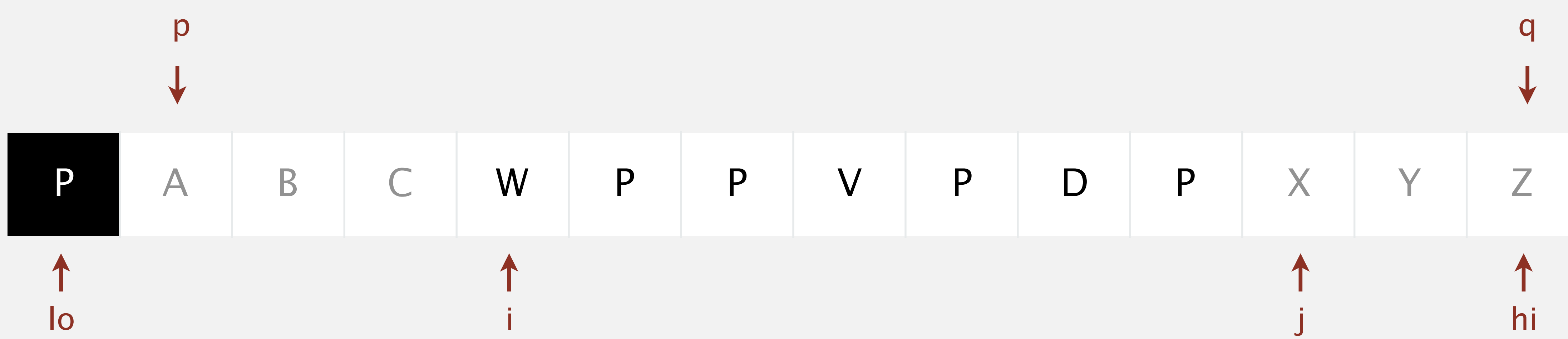

Bentley–McIlroy 3-way partitioning demo

Phase I. Repeat until `i` and `j` pointers cross.

- Scan `i` from left to right so long as `(a[i] < a[lo])`.
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- Exchange `a[i]` with `a[j]`.
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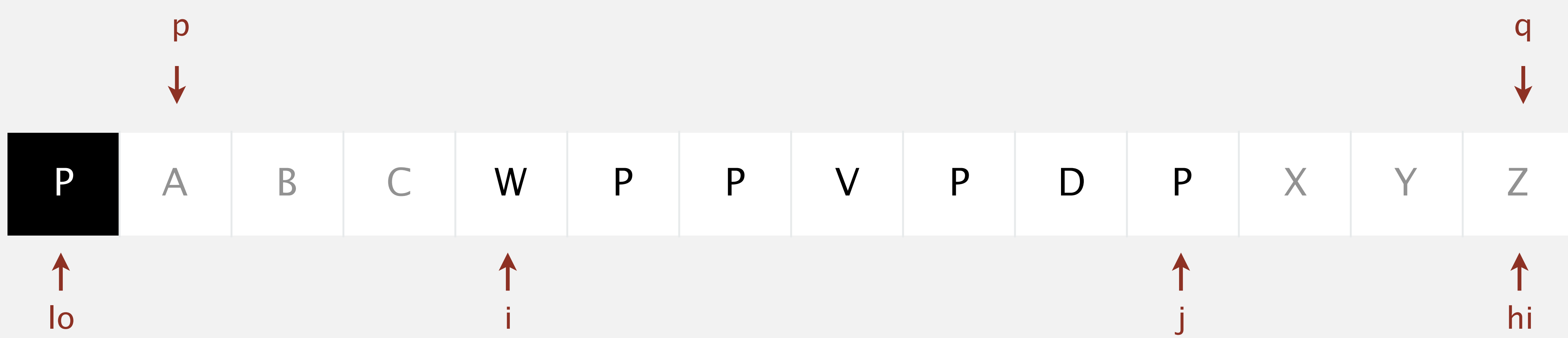

exchange `a[i]` with `a[j]`

Bentley–McIlroy 3-way partitioning demo

Phase I. Repeat until `i` and `j` pointers cross.

- Scan `i` from left to right so long as `(a[i] < a[lo])`.
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- If `(a[i] == a[lo])`, exchange `a[i]` with `a[p]` and increment `p`.
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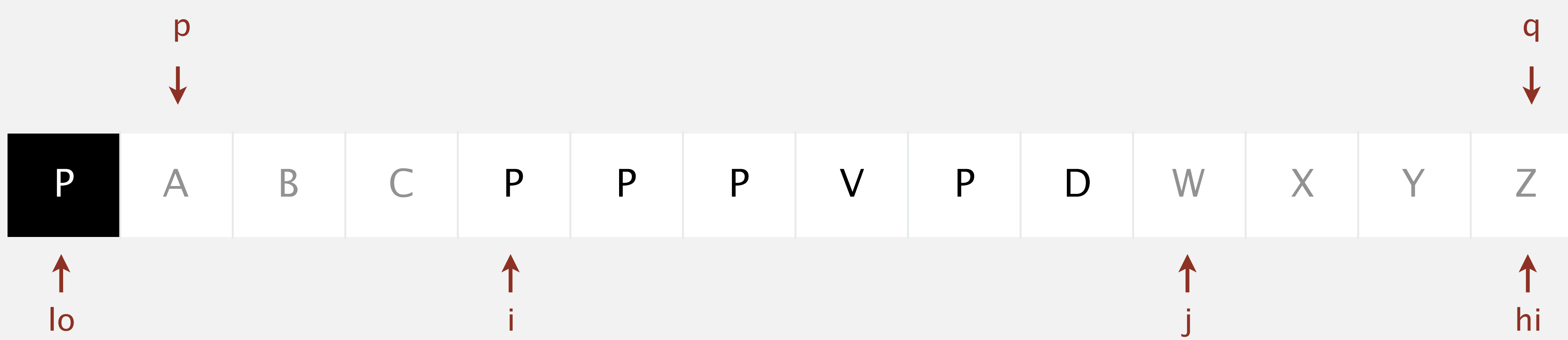

exchange `a[i]` with `a[p]` and increment `p`

Bentley–McIlroy 3-way partitioning demo

Phase I. Repeat until `i` and `j` pointers cross.

- Scan `i` from left to right so long as `(a[i] < a[lo])`.
- Scan `j` from right to left so long as `(a[j] > a[lo])`.
- Exchange `a[i]` with `a[j]`.
- If `(a[i] == a[lo])`, exchange `a[i]` with `a[p]` and increment `p`.
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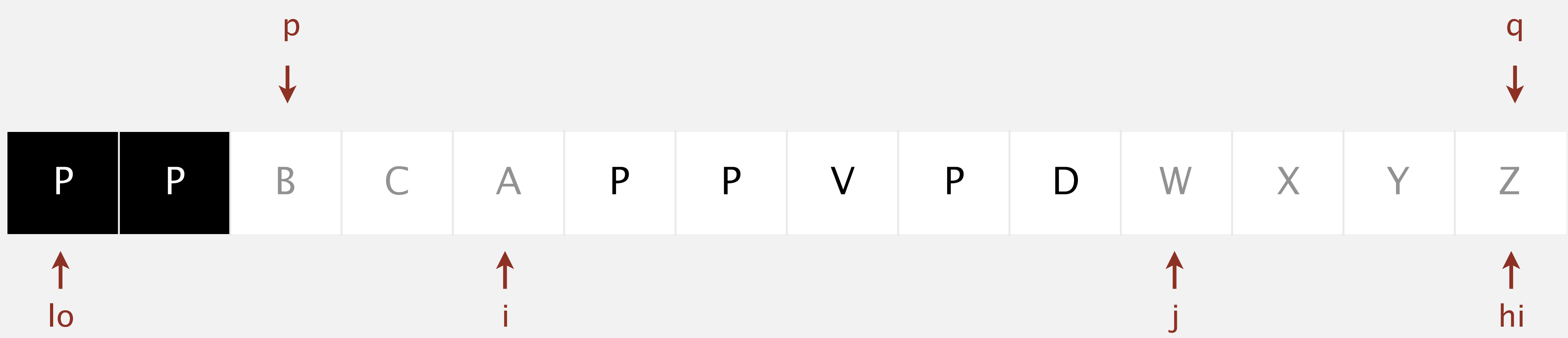

Bentley–McIlroy 3-way partitioning demo

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- Scan `i` from left to right so long as `(a[i] < a[lo])`.
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- Exchange `a[i]` with `a[j]`.
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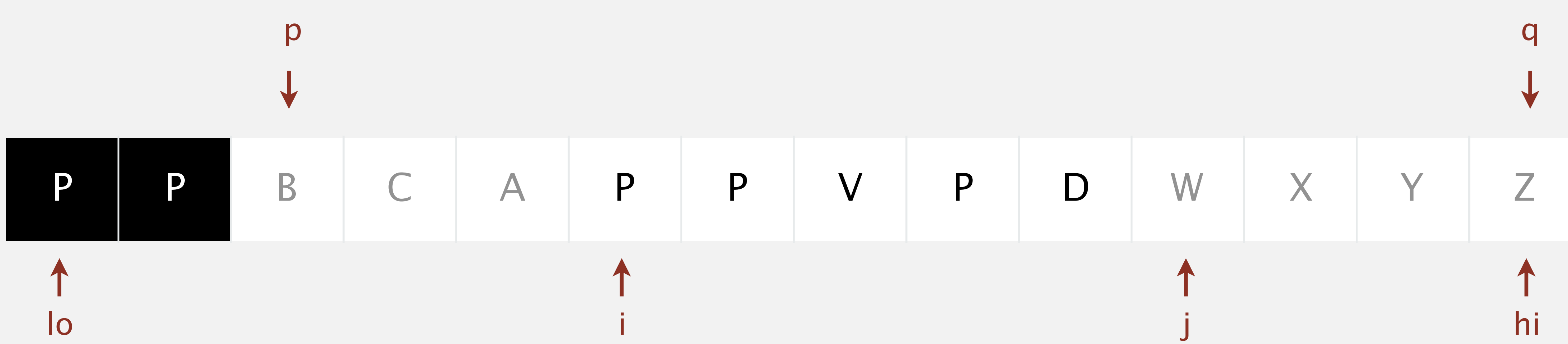

Bentley–McIlroy 3-way partitioning demo

Phase I. Repeat until `i` and `j` pointers cross.

- Scan `i` from left to right so long as `(a[i] < a[lo])`.
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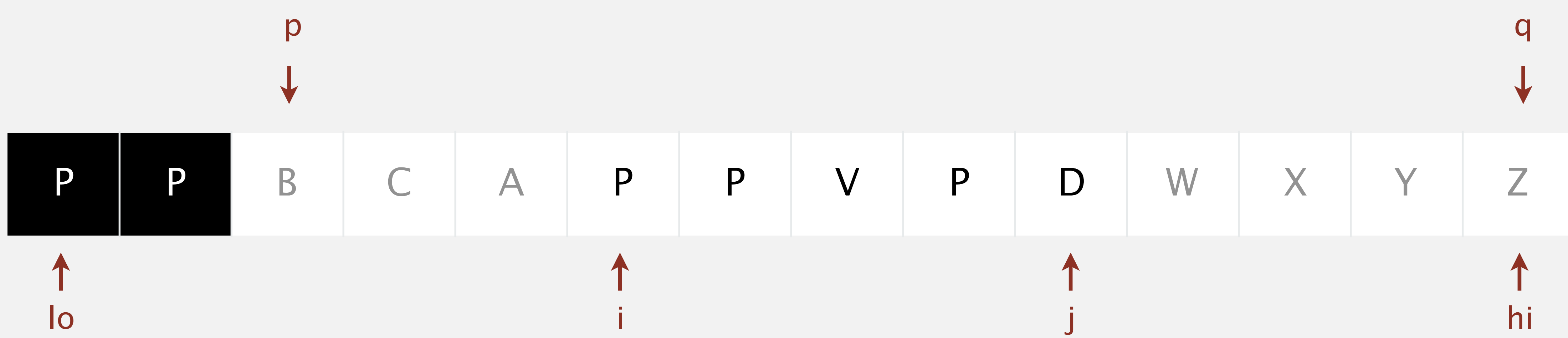

exchange `a[i]` with `a[j]`

Bentley–McIlroy 3-way partitioning demo

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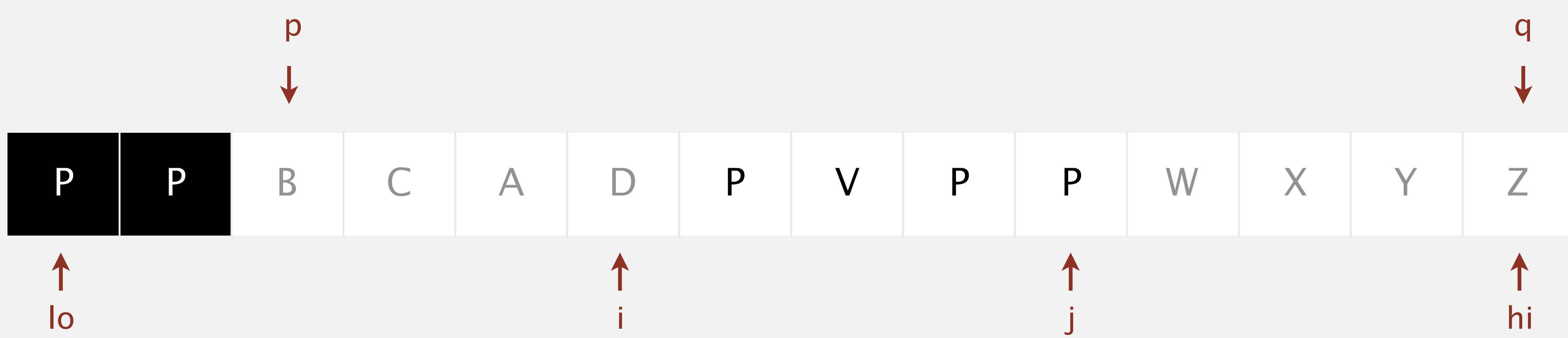

exchange `a[j]` with `a[q]` and decrement `q`

Bentley–McIlroy 3-way partitioning demo

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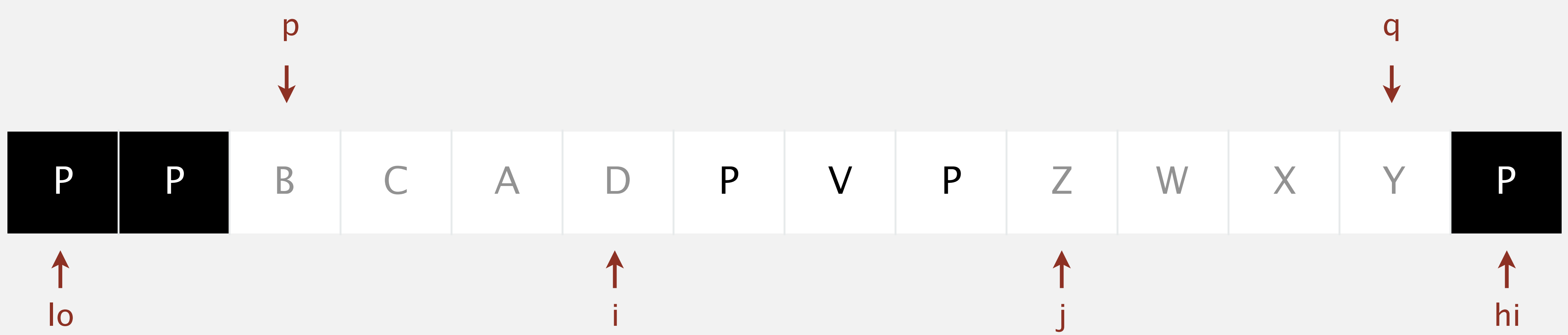

Bentley–McIlroy 3-way partitioning demo

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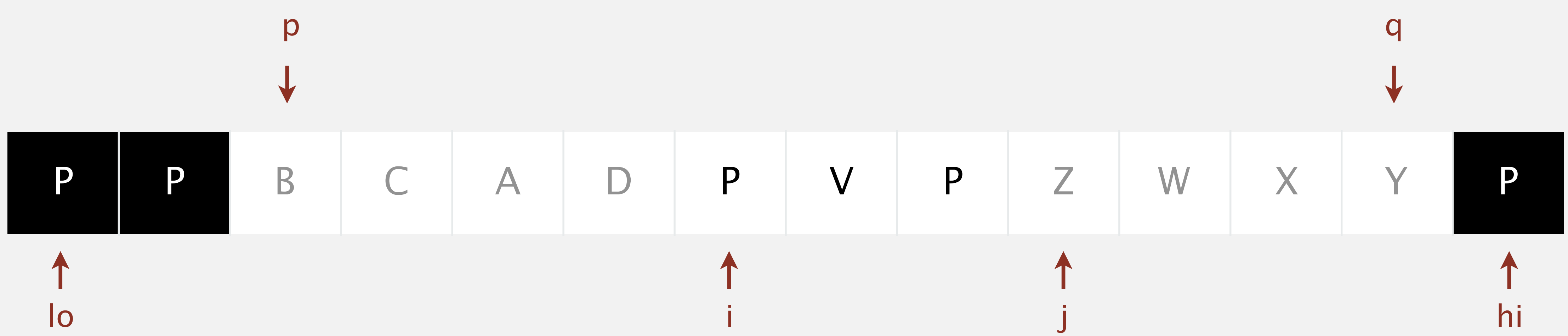

Bentley–McIlroy 3-way partitioning demo

Phase I. Repeat until `i` and `j` pointers cross.

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Bentley–McIlroy 3-way partitioning demo

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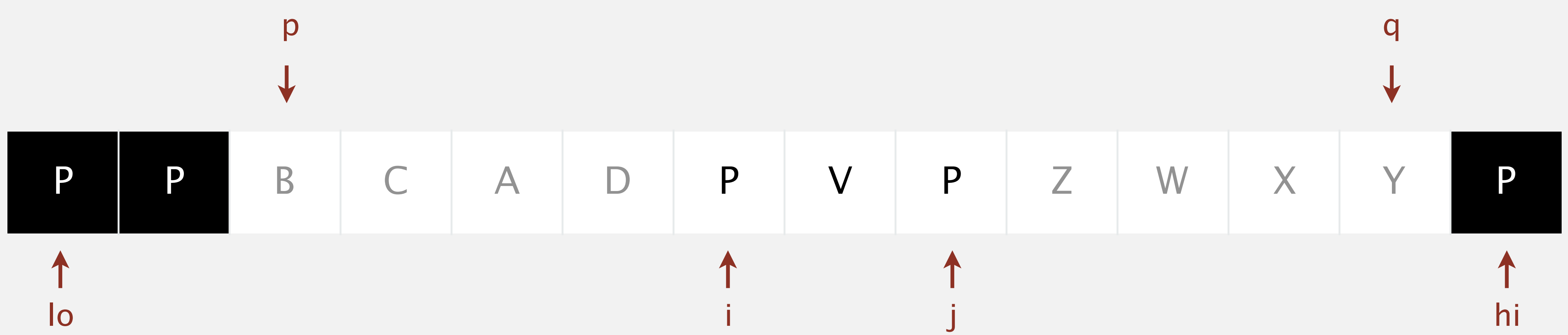

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Bentley–McIlroy 3-way partitioning demo

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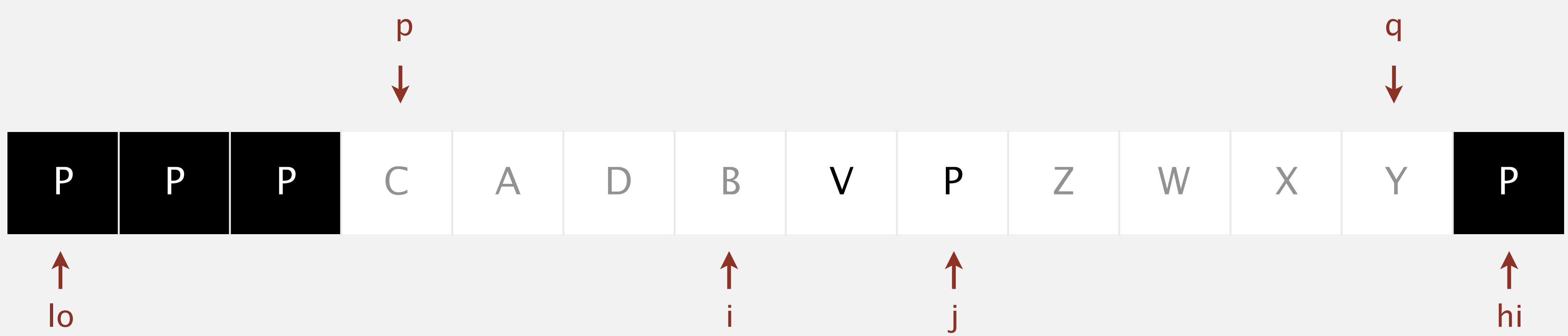

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Bentley–McIlroy 3-way partitioning demo

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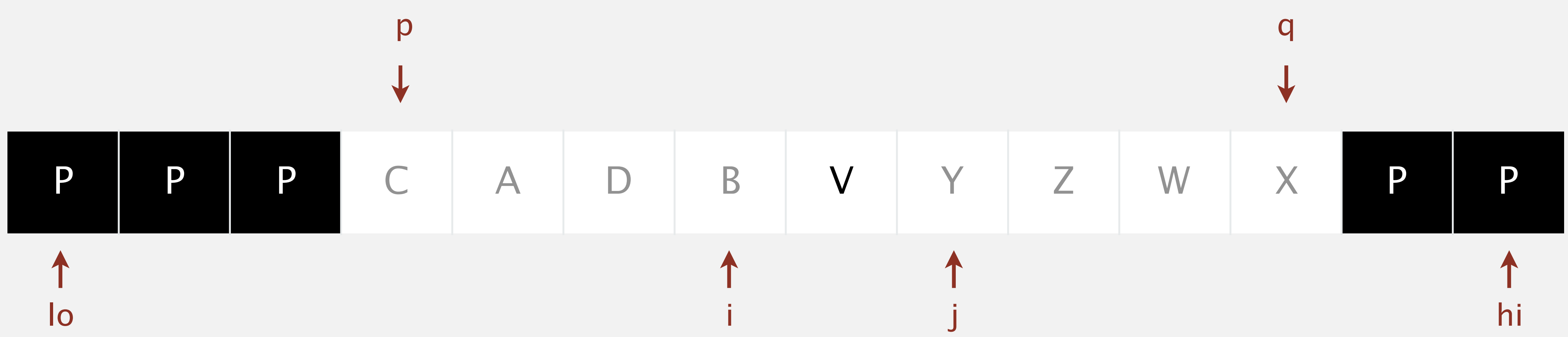

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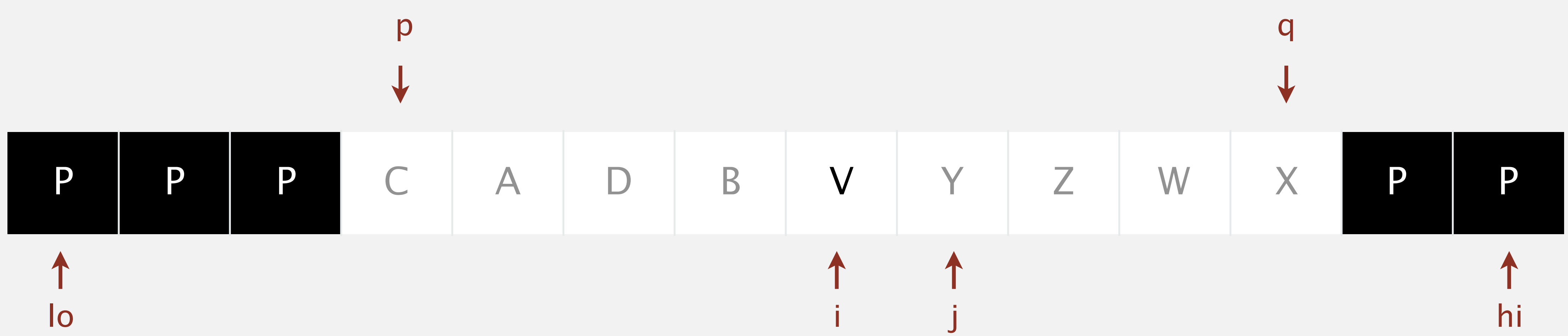

Bentley–McIlroy 3-way partitioning demo

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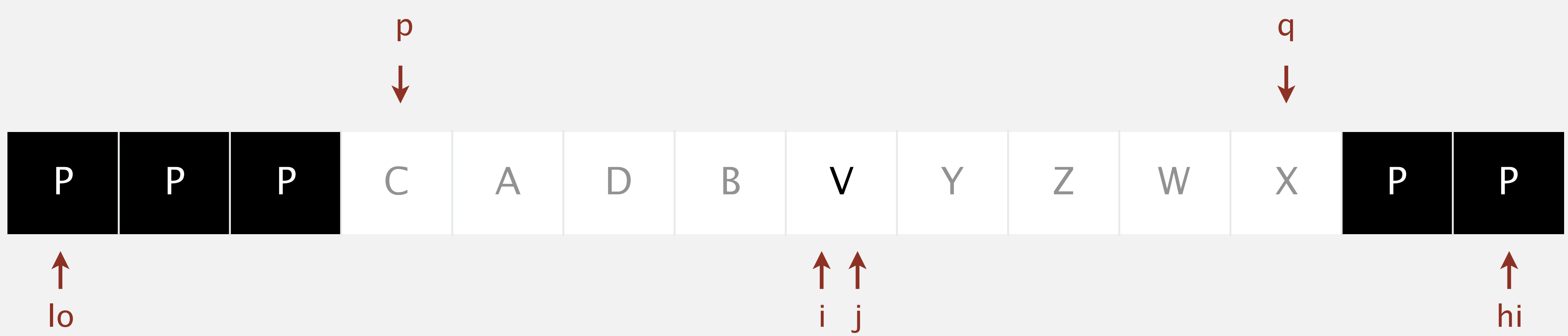

Bentley–McIlroy 3-way partitioning demo

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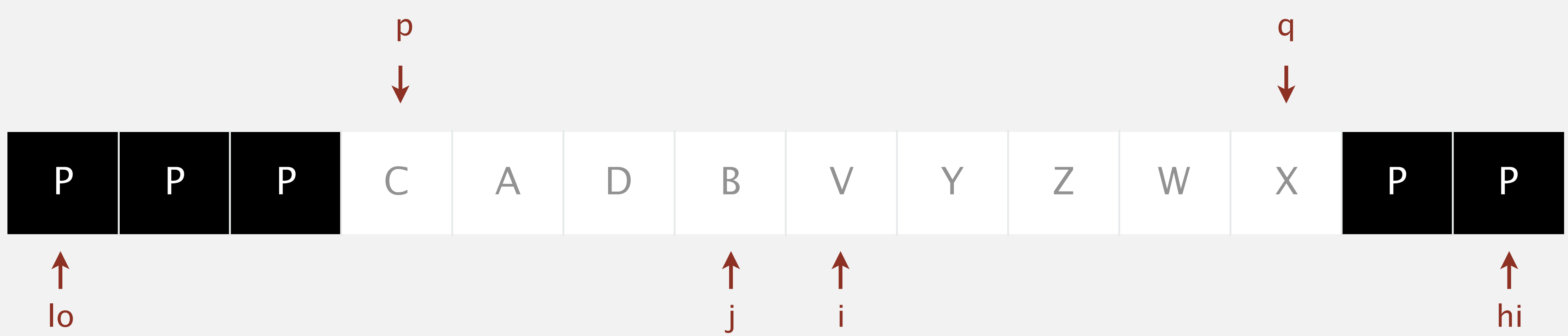
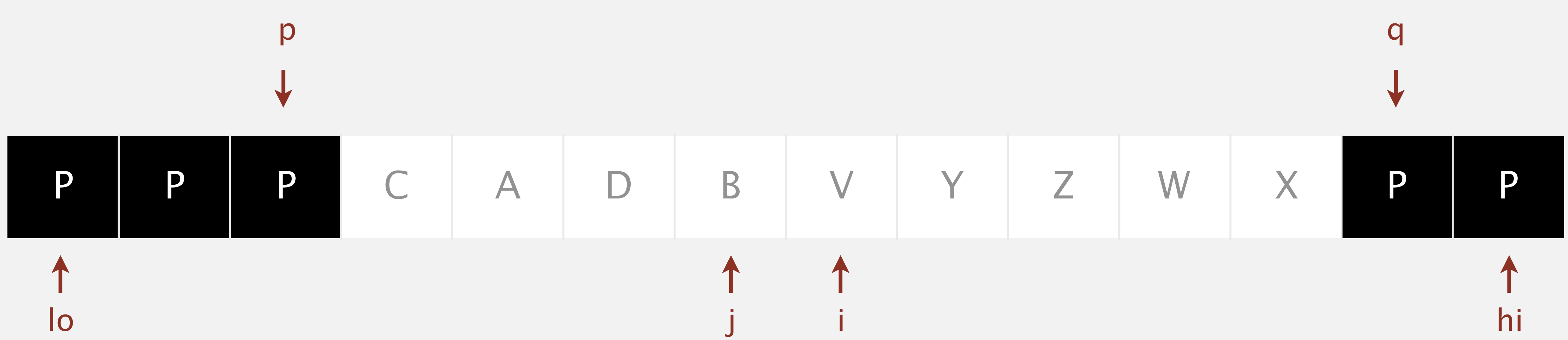


pointers cross

Bentley–McIlroy 3-way partitioning demo

Phase II. Swap equal keys to the center.

- Scan `j` and `p` from right to left and exchange `a[j]` with `a[p]`.
- Scan `i` and `q` from left to right and exchange `a[i]` with `a[q]`.



exchange `a[j]` with `a[p]`

Bentley–McIlroy 3-way partitioning demo

Phase II. Swap equal keys to the center.

- Scan `j` and `p` from right to left and exchange `a[j]` with `a[p]`.
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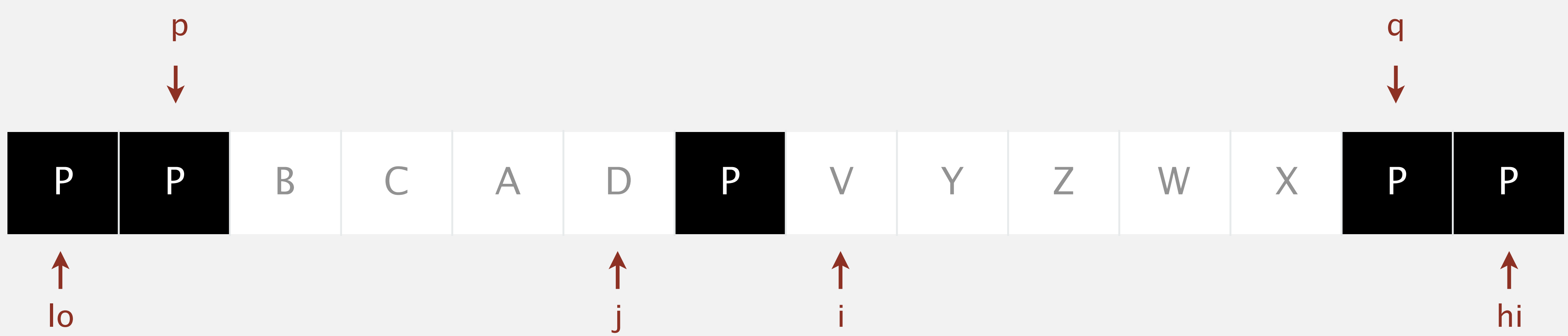

exchange `a[j]` with `a[p]`

Bentley–McIlroy 3-way partitioning demo

Phase II. Swap equal keys to the center.

- Scan `j` and `p` from right to left and exchange `a[j]` with `a[p]`.
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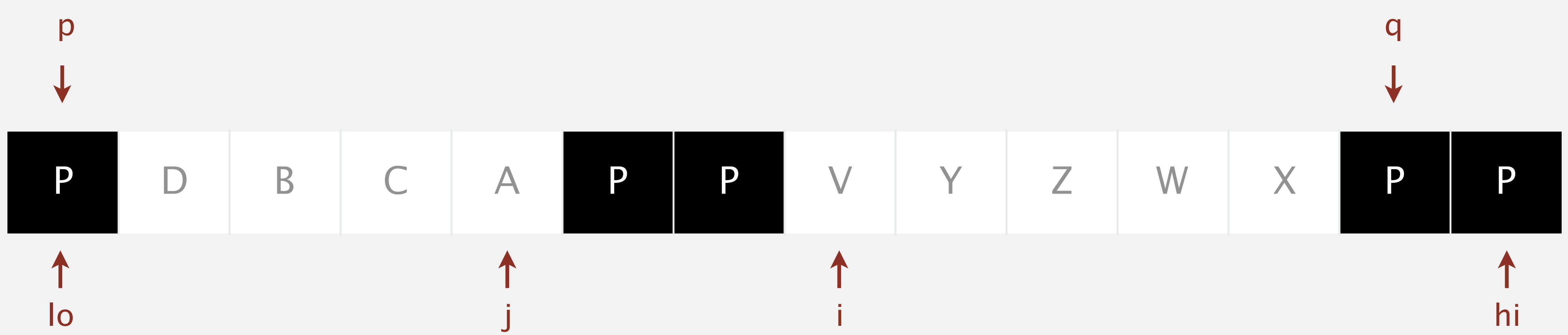

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Bentley–McIlroy 3-way partitioning demo

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Bentley–McIlroy 3-way partitioning demo

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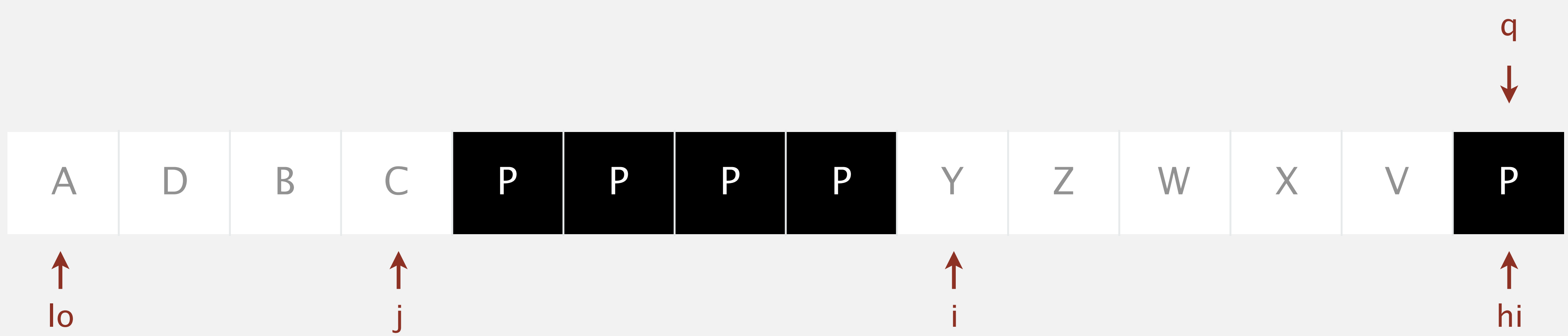

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Bentley–McIlroy 3-way partitioning demo

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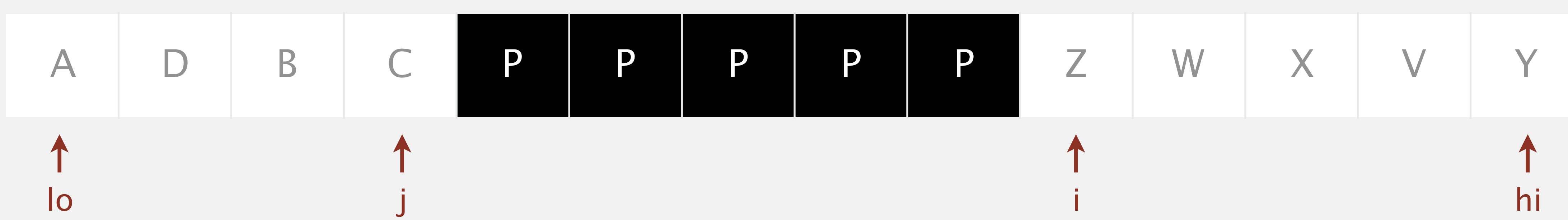


3-way partitioned

http://algs4.cs.princeton.edu

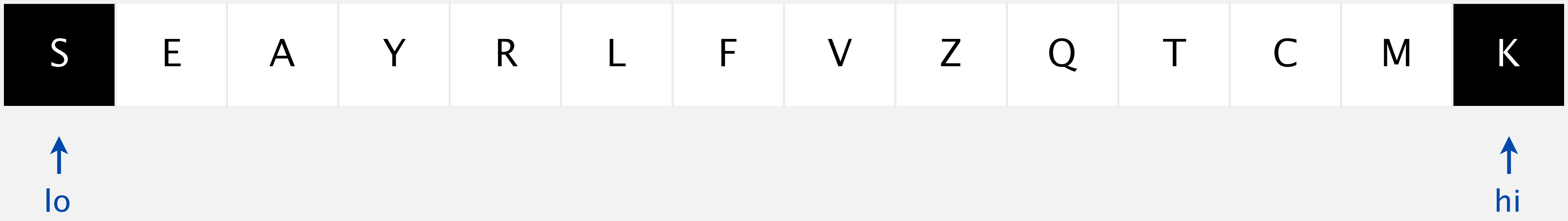
2.3 Partitioning Demos

- Sedgewick 2-way partitioning
- Dijkstra 3-way partitioning
- Bentley–McIlroy 3-way partitioning
- dual-pivot partitioning

Dual-pivot partitioning demo

Initialization.

- Choose `a[lo]` and `a[hi]` as partitioning items.
- Exchange if necessary to ensure `a[lo]` ≤ `a[hi]`.

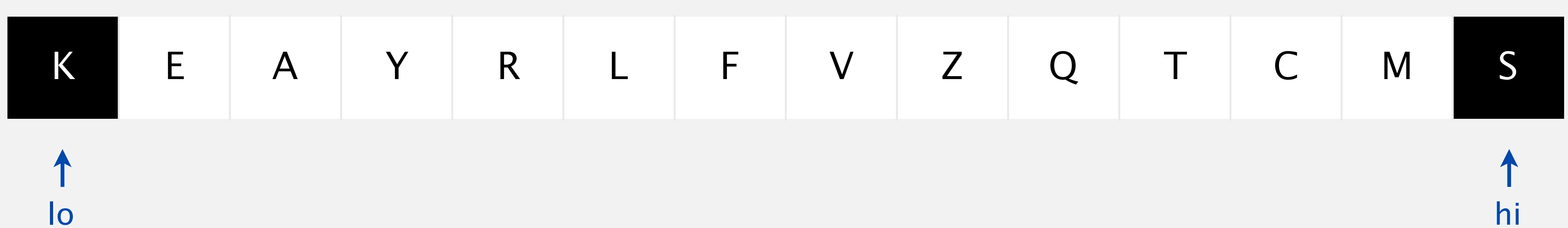


exchange a[lo] and a[hi]

Dual-pivot partitioning demo

Initialization.

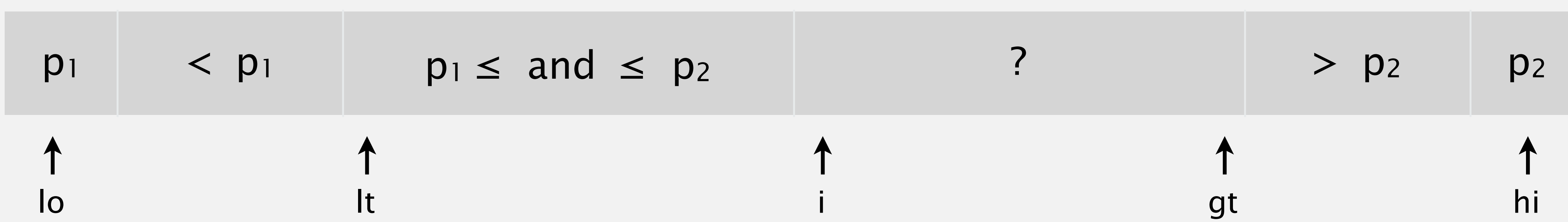
- Choose `a[lo]` and `a[hi]` as partitioning items.
- Exchange if necessary to ensure `a[lo]` $\leq$ `a[hi]`.

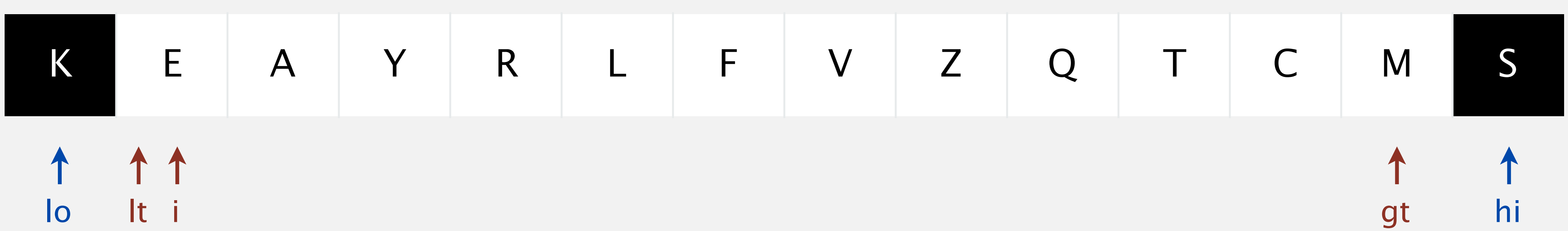


Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

- If `(a[i] < a[lo])`, exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
- Else if `(a[i] > a[hi])`, exchange `a[i]` with `a[gt]` and decrement `gt`.
- Else, increment `i`.





exchange a[i] and a[lt]; increment lt and i

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

- If `(a[i] < a[lo])`, exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
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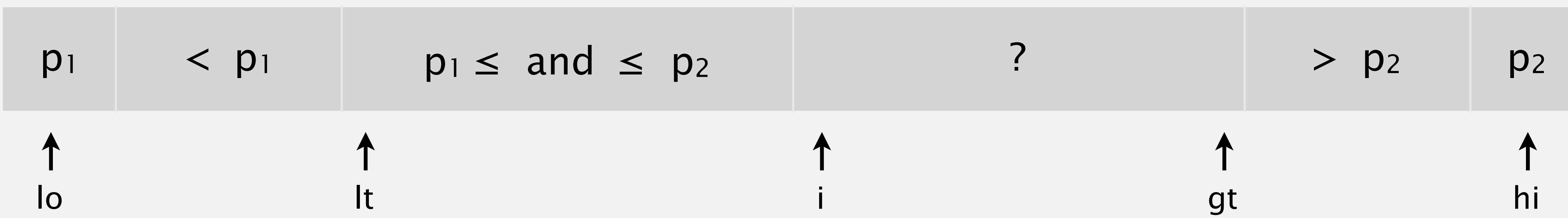
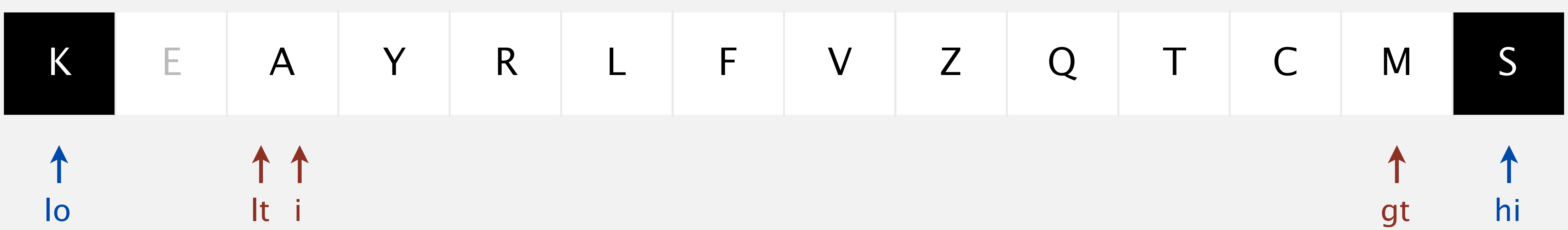
- Else, increment `i`.





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Dual-pivot partitioning demo

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- If `(a[i] < a[lo])`, exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
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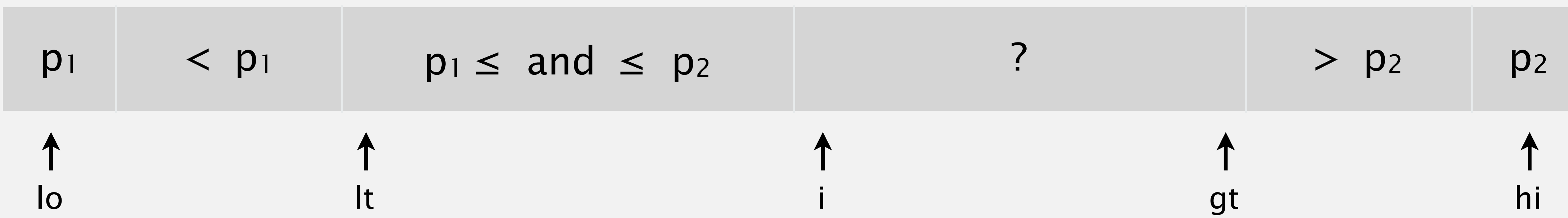
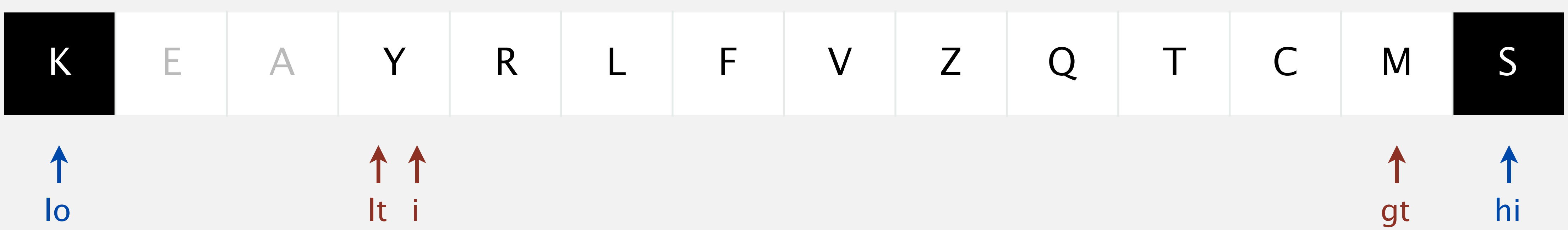
- Else, increment `i`.





exchange a[i] and a[gt]; decrement gt

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

- If (`a[i] < a[lo]`), exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
- Else if (`a[i] > a[hi]`), exchange `a[i]` with `a[gt]` and decrement `gt`.
- Else, increment `i`.

p_1	$< p_1$	$p_1 \leq$ and $\leq p_2$	?	$> p_2$	p_2
lo		lt	i	gt	hi

K	E	A	M	R	L	F	V	Z	Q	T	C	Y	S
lo			lt i								gt		hi

increment i

Dual-pivot partitioning demo

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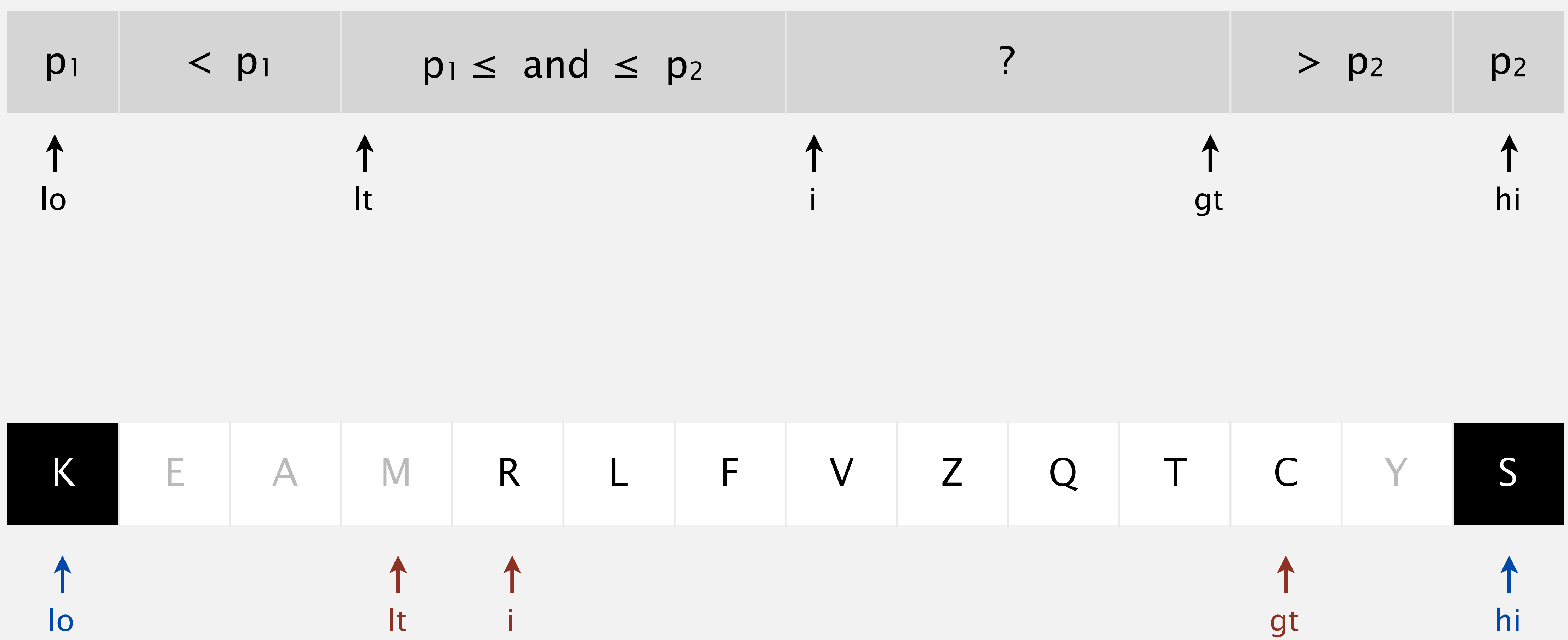

increment i

Dual-pivot partitioning demo

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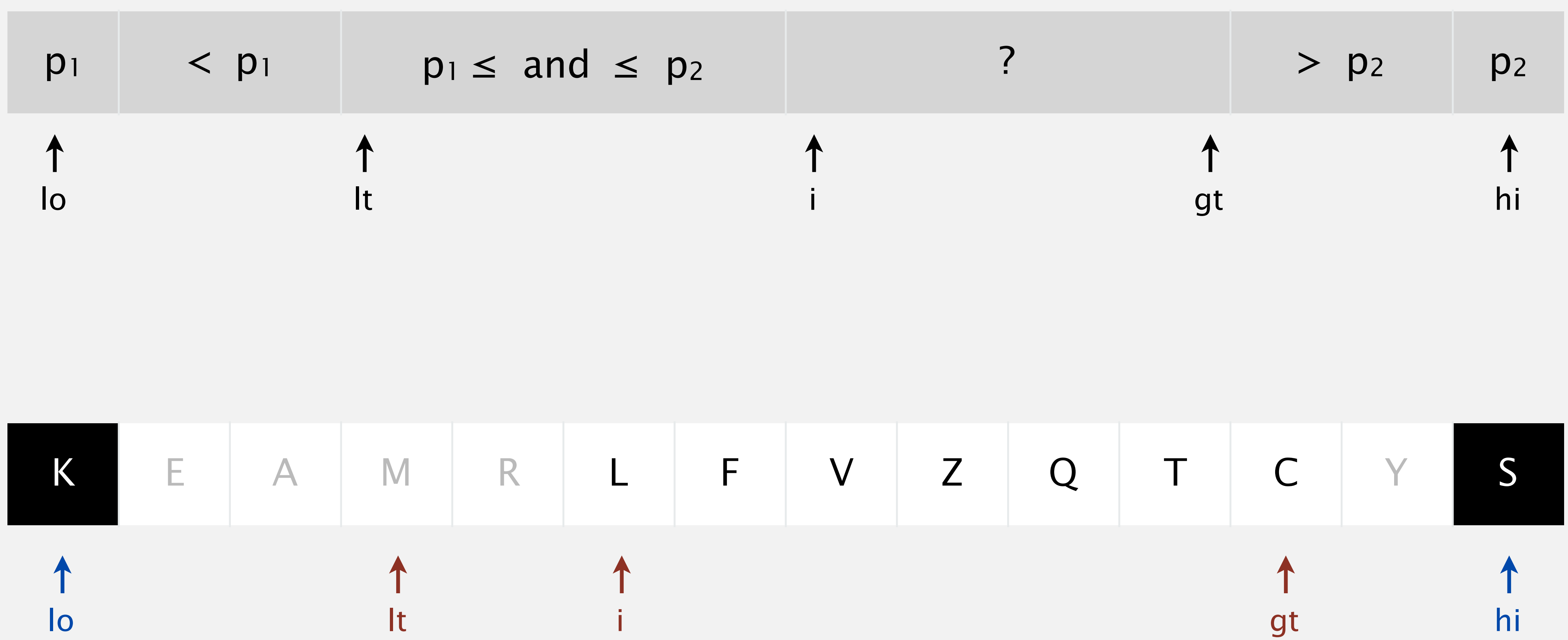

increment i

Dual-pivot partitioning demo

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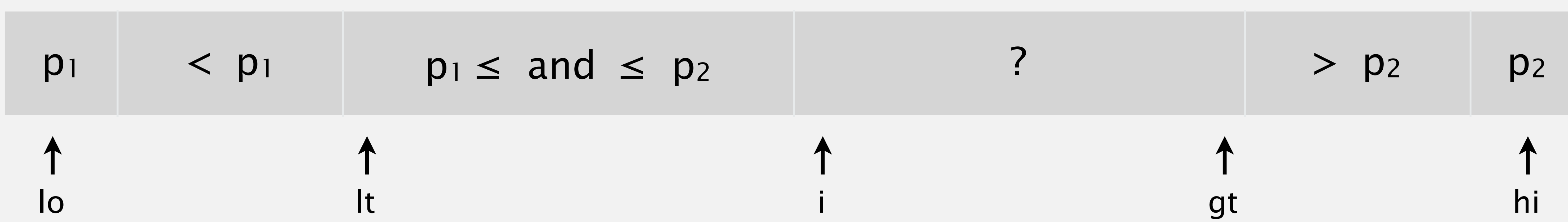
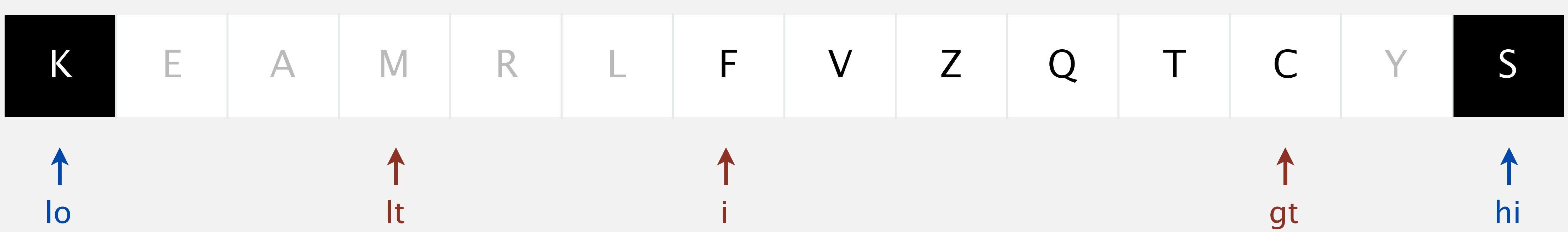


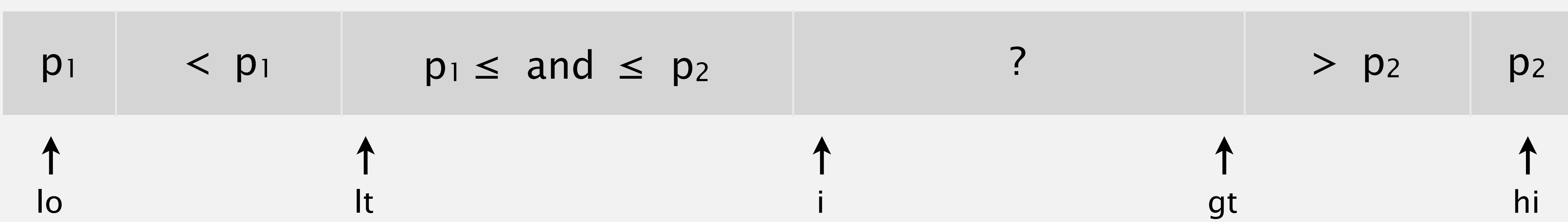


exchange a[i] and a[lt]; increment lt and i

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

- If (`a[i] < a[lo]`), exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
- Else if (`a[i] > a[hi]`), exchange `a[i]` with `a[gt]` and decrement `gt`.
- Else, increment `i`.



exchange a[i] and a[gt]; decrement gt

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

- If `(a[i] < a[lo])`, exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
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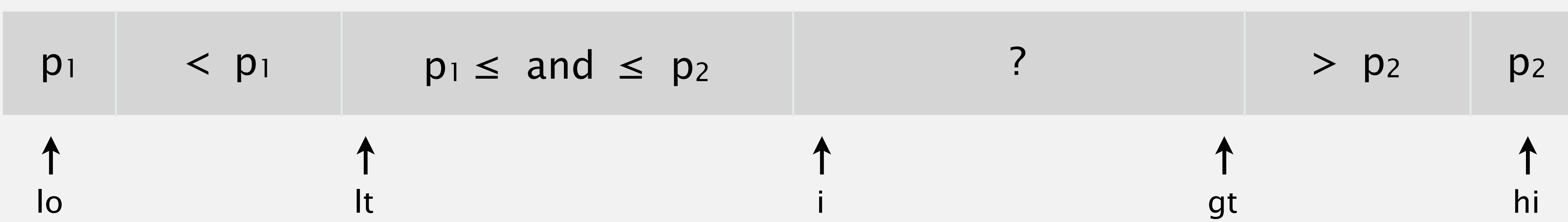
- Else, increment `i`.



exchange a[i] and a[lt]; increment lt and i

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

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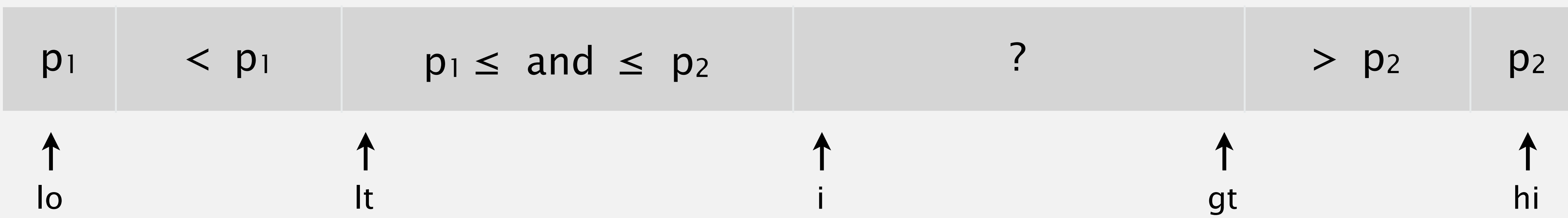

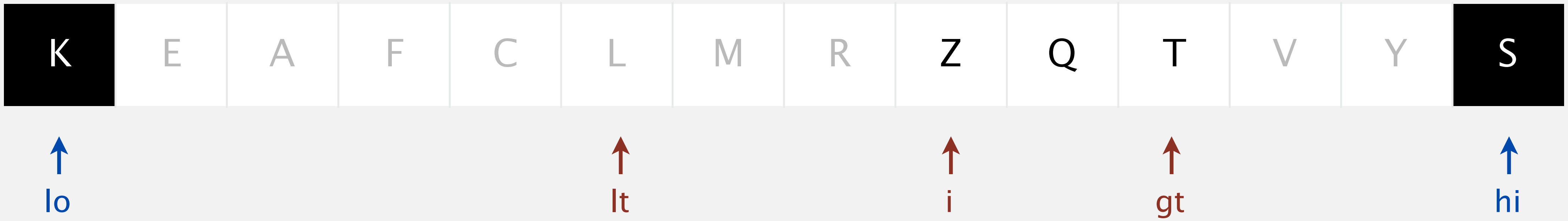

exchange a[i] and a[gt]; decrement gt

Dual-pivot partitioning demo

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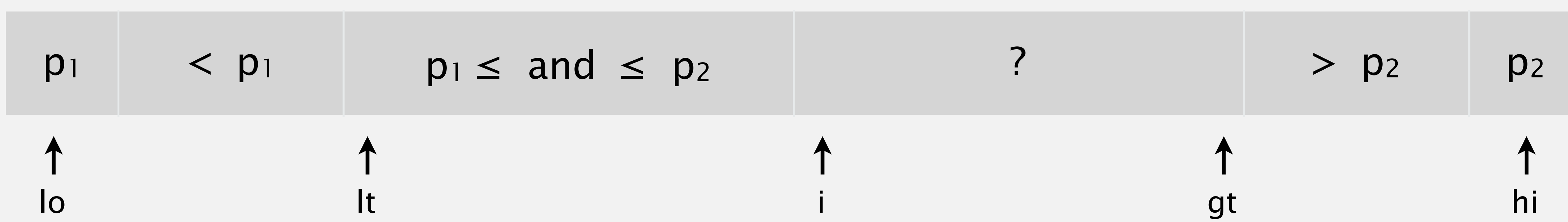

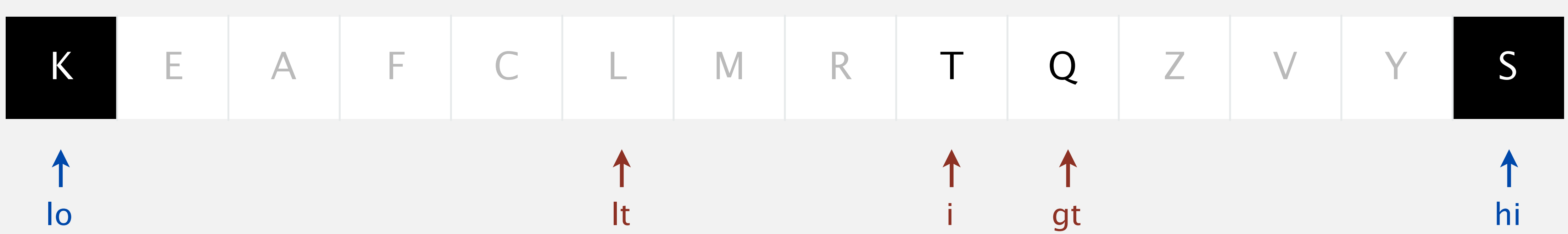

exchange a[i] and a[gt]; decrement gt

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

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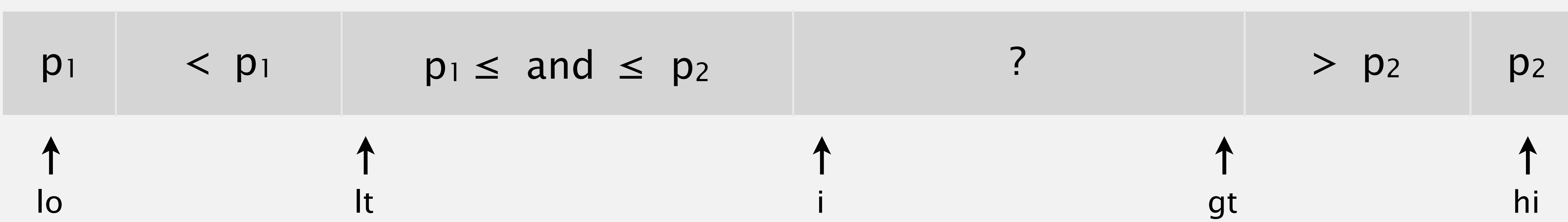

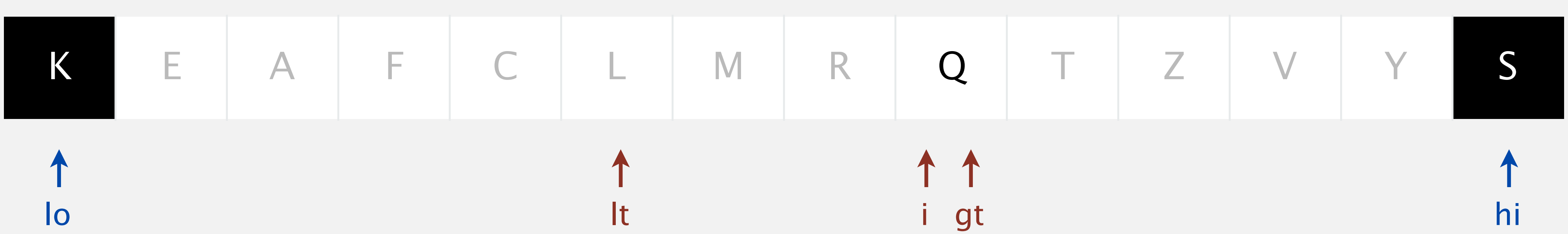

increment i

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

- If (`a[i] < a[lo]`), exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
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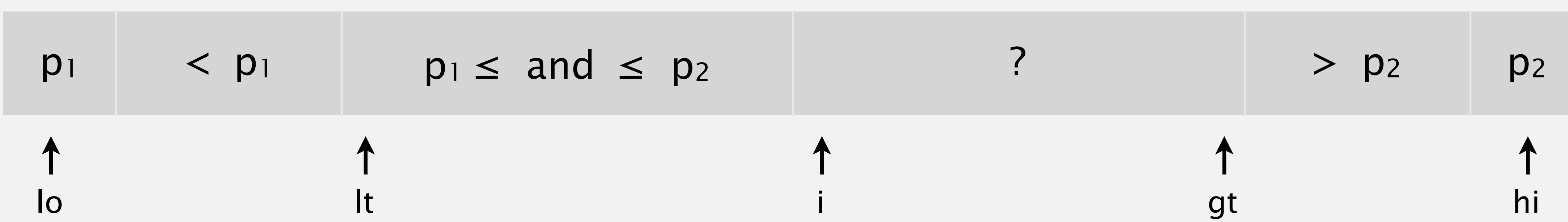
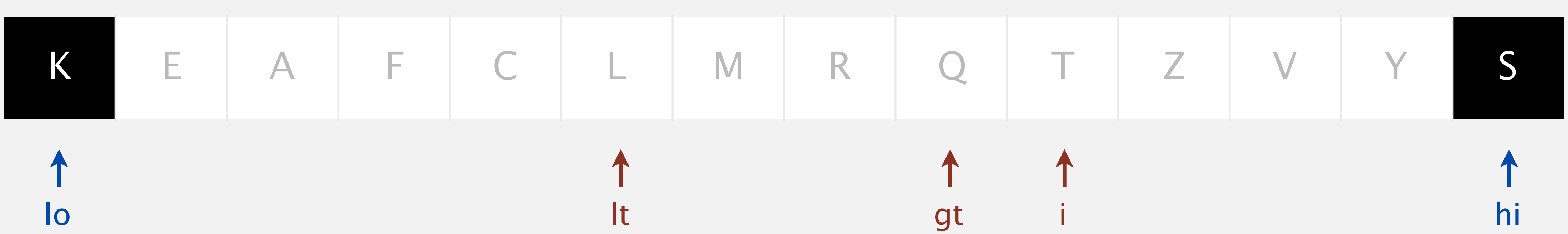


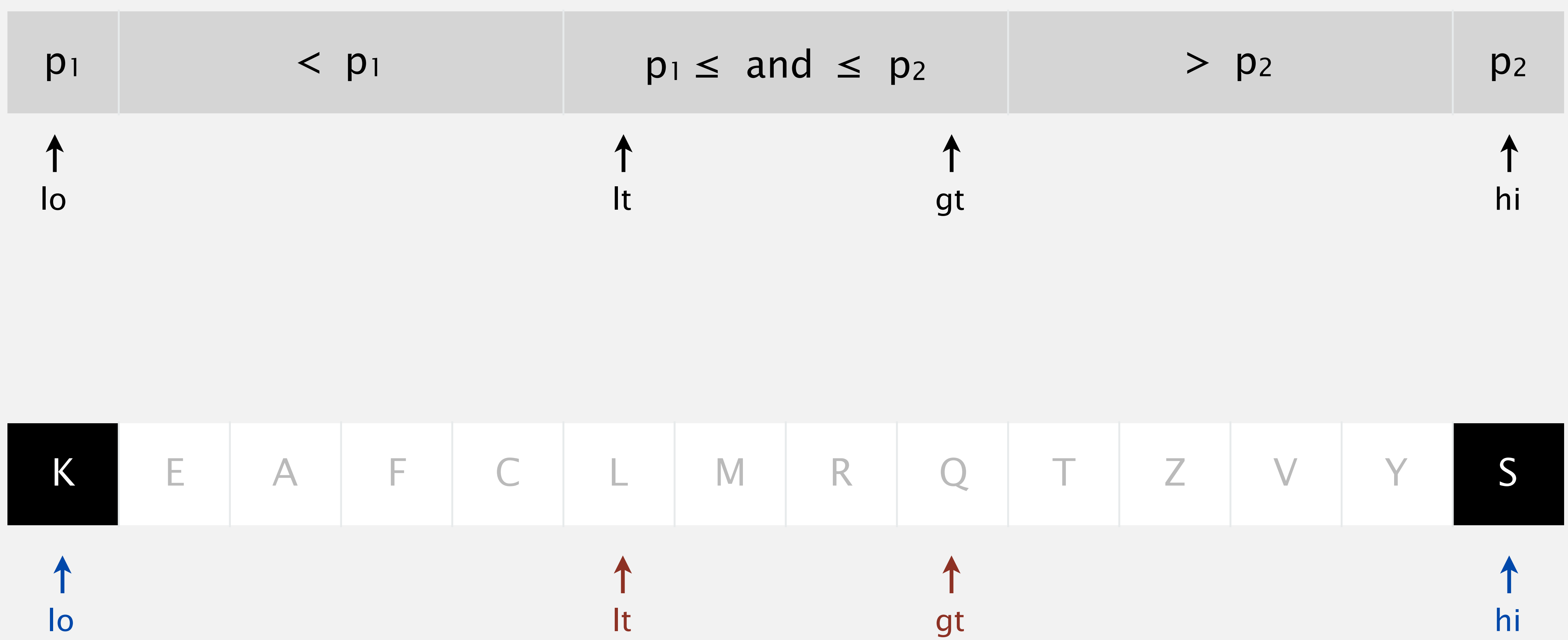


stop when pointers cross

Dual-pivot partitioning demo

Finalize.

- Exchange `a[lo]` with `a[--lt]`.
- Exchange `a[hi]` with `a[++gt]`.



Dual-pivot partitioning demo

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- Exchange `a[lo]` with `a[--lt]`.
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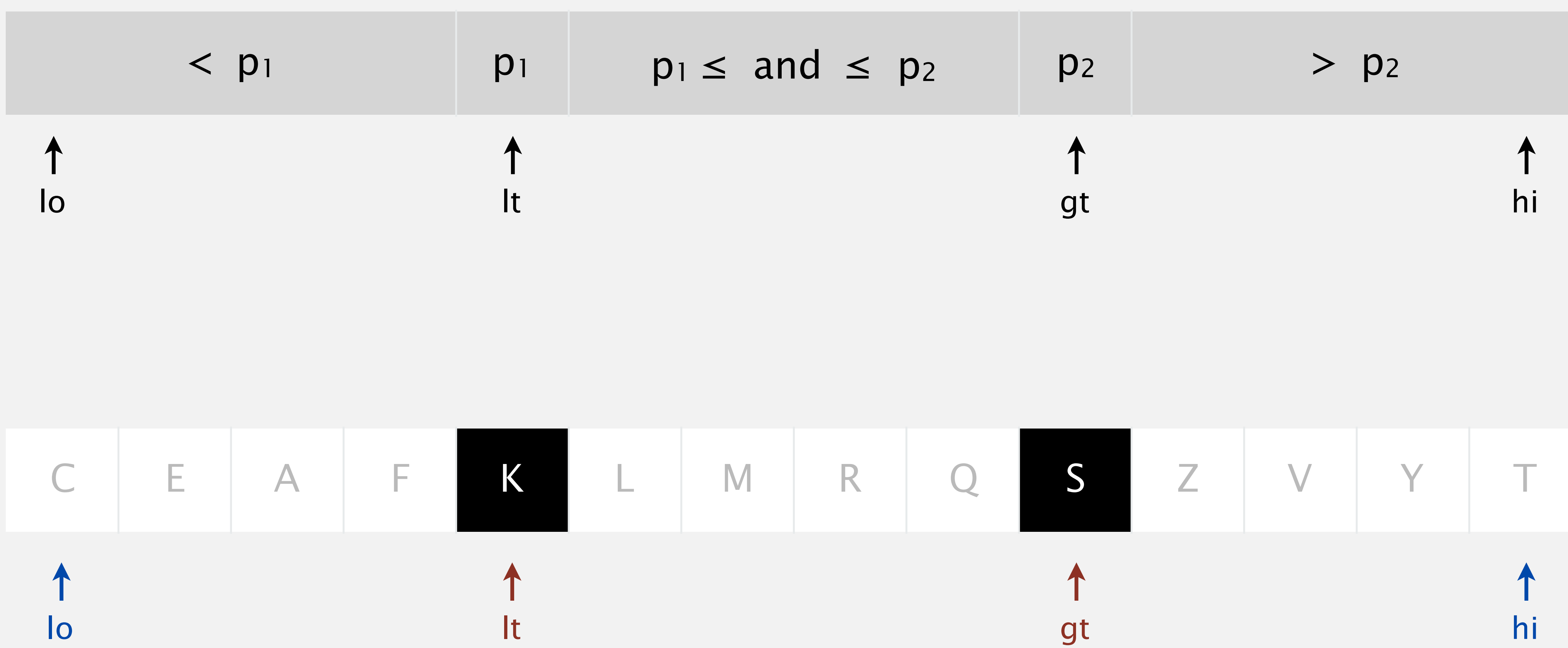

3-way partitioned