Some Mesh Surface Properties

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Curvature
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Curvature $\kappa$ of a curve is reciprocal of radius of circle that best approximates it.

Defined at a point $p$ in a direction $w$

*Line has $\kappa = 0*$
Principal Curvatures

The curvature at a point varies between some minimum and maximum – these are the *principal curvatures* $\kappa_1$ and $\kappa_2$.

They occur in the *principal directions* $d_1$ and $d_2$ which are perpendicular to each other.
Principal Curvatures

Minimum Curvature $\kappa_1$

Maximum Curvature $\kappa_2$
Gaussian and Mean Curvature

Gauss Curvature
\[ K = \kappa_1 \kappa_2 \]

Mean Curvature
\[ H = \frac{1}{2} (\kappa_1 + \kappa_2) \]
What Does Curvature Tell Us?

Planar points:

- Zero Gaussian curvature and zero mean curvature
- Tangent plane intersects surface at infinity points
What Does Curvature Tell Us?

Parabolic points:

- Zero Gaussian curvature, non-zero mean curvature
- Tangent plane intersects surface along 1 curves
What Does Curvature Tell Us?

Elliptical points:

- Positive Gaussian curvature
- Convex/concave depending on sign of mean curvature
- Tangent plane intersects surface at 1 point
What Does Curvature Tell Us?

Hyperbolic points:

- Negative Gaussian curvature
- Tangent plane intersects surface along 2 curves
What Does Curvature Tell Us?

Mesh Saliency:

- Motivated by models of perceptual salience
- Difference between mean curvature blurred with $\sigma$ and blurred with $2\sigma$
Principal Component Analysis (PCA)
Principal Component Analysis (PCA)

Tensor voting

- Extract points \( \{ q_i \} \) in neighborhood
- Compute covariance matrix \( M \)
- Analyze eigenvalues and eigenvectors of \( M \) (via SVD)
- Eigenvectors are Principal Axes

\[
M = \frac{1}{n} \sum_{i=1}^{n} \begin{bmatrix}
q_i^x q_i^x & q_i^x q_i^y & q_i^x q_i^z \\
q_i^y q_i^x & q_i^y q_i^y & q_i^y q_i^z \\
q_i^z q_i^x & q_i^z q_i^y & q_i^z q_i^z
\end{bmatrix}
\]

Covariance Matrix

\[
M = \text{USU}^t
\]

Eigenvalues & Eigenvectors
Principal Component Analysis (PCA)

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q_i^z q_i^x & q_i^z q_i^y & q_i^z q_i^z 
\end{bmatrix}
\]

Eigenvalues & Eigenvectors

\[
M = USU^T
\]

\[
S = \begin{bmatrix}
\lambda_a & 0 & 0 \\
0 & \lambda_b & 0 \\
0 & 0 & \lambda_c
\end{bmatrix}, \quad U = \begin{bmatrix}
A_x & A_y & A_z \\
B_x & B_y & B_z \\
C_x & C_y & C_z
\end{bmatrix}
\]
Principal Component Analysis (PCA)

Eigenvectors are “Principal Axes of Inertia”

Eigenvalues are variances of the point distribution in those directions
What Does PCA Tell Us?

Provides estimate of normal direction

- Eigenvector (principal axis) associated with smallest eigenvalue
What Does PCA Tell Us?

Helps us construct a local coordinate frame for every point

- Map $\hat{e}_1$ to X axis
- Map $\hat{e}_2$ to Y axis
- Map $\hat{e}_3$ to Z axis
What Does PCA Tell Us?

Helps differentiate nearly plane-like, from stick-like, from sphere-like, etc.
What Does PCA Tell Us?

Helps differentiate nearly plane-like, from stick-like, from sphere-like, etc.

\[ \frac{\lambda_2}{(\lambda_1 + \lambda_2 + \lambda_3)} \]
Statistics of Distances
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Distances can be along surface (geodesic) or as a crow flies (Euclidean)

Geodesic distance to point

Geodesic vs. Euclidean distance
Statistics of Distances

Average geodesic distance to other points on surface
What Do Statistics of Distance Tell Us?

Histograms of geodesic distances

- Small distances relate to curvature
- Long distances relate to centeredness
Shape Diameter Function
Shape Diameter Function

Median distance along sampling of rays through interior
Shape Diameter Function

Distinguish between thin and thick parts in a model
Sharp changes often correlate with part boundaries
Mesh Surface Properties in Assignment 2