Reflectance

COS 526, Fall 2016
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(slides from Szymon Rusinkiewicz)

Surface Reflectance – BRDF

- Bidirectional Reflectance Distribution Function

$$f_r(\omega_i \to \omega_o) = \frac{L_o(\omega_o)}{E_i(\omega_i)}$$

- 4-dimensional function: also written as

$$f_r(\theta_i, \varphi_i, \theta_o, \varphi_o) = \frac{L_o(\theta_o, \varphi_o)}{E_i(\theta_i, \varphi_i)}$$

(the symbol ρ is also used sometimes)

Surface Reflectance – BRDF

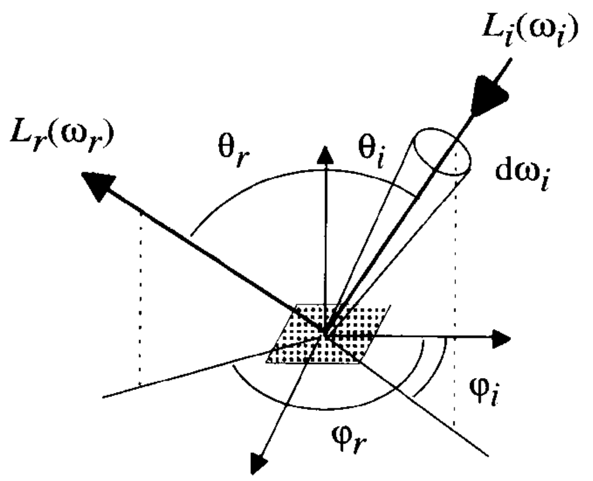


Figure 2.9: *Bidirectional reflection distribution function.*

$$f_r(\vec{\omega}_i \rightarrow \vec{\omega}_r) \equiv \frac{L_r(\vec{\omega}_r)}{L_i(\vec{\omega}_i)\cos\theta_i d\omega_i}$$

Properties of the BRDF

- Positivity:

$$f_r(\omega_i \rightarrow \omega_o) \geq 0$$

Properties of the BRDF

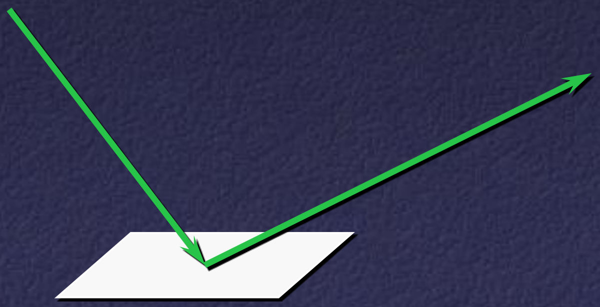
- Energy conservation:

$$\int_{\Omega} f_r(\theta_i, \varphi_i, \theta_o, \varphi_o) \cos\theta_o \, d\omega_o \leq 1$$

Properties of the BRDF

- Helmholtz reciprocity:

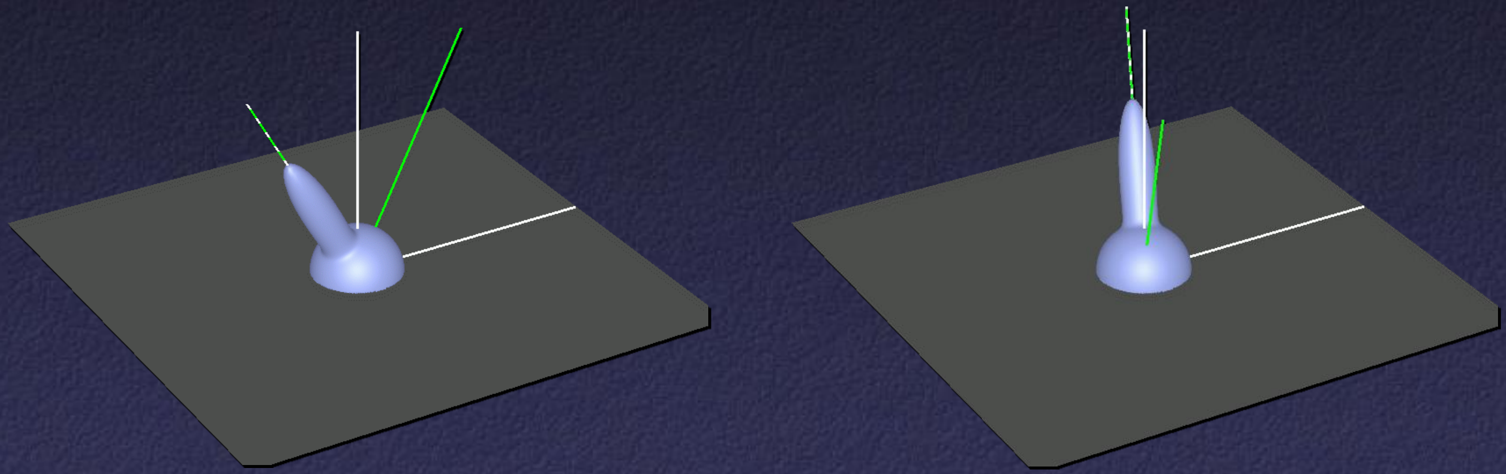
$$f_r(\omega_i \to \omega_o) = f_r(\omega_o \to \omega_i)$$



(not always obeyed by “BRDFs” used in graphics)

Isotropy

- A BRDF is isotropic if it stays the same when surface is rotated around normal

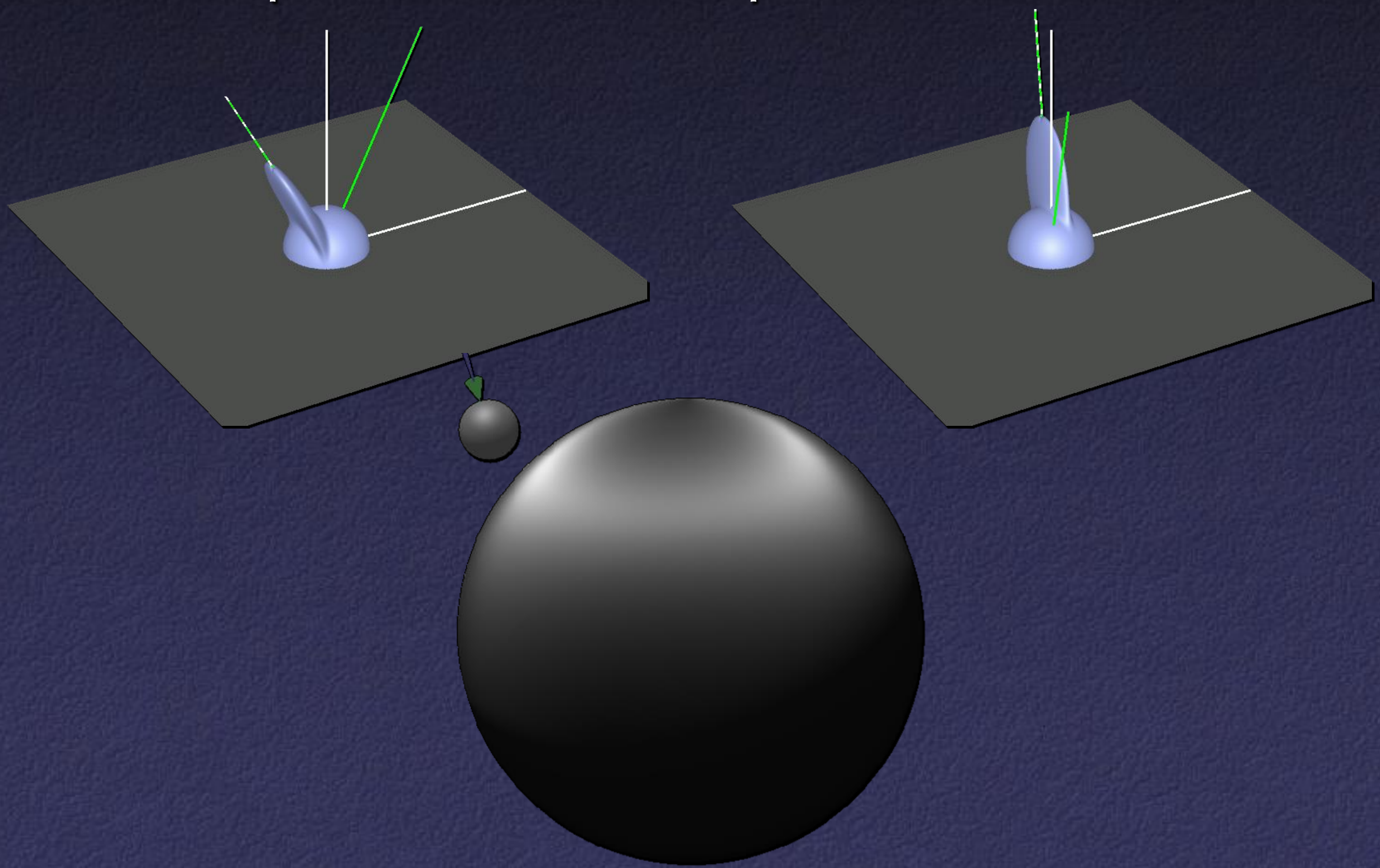


- Isotropic BRDFs are 3-dimesional functions:

$$f_r(\theta_i, \theta_o, \varphi_i - \varphi_o)$$

Anisotropy

- Anisotropic BRDFs **do** depend on surface rotation



BRDF Representations

- Physically-based vs. phenomenological models
- Measured data
- Desired characteristics:
  - Fast to evaluate
  - Maintain reciprocity, energy conservation
  - For global illumination: easy to importance sample

Diffuse

- The simplest BRDF is "ideal diffuse" or *Lambertian*: just a constant

$$f_r(\omega_i \to \omega_o) = k_d$$

- Note: does *not* include $\cos(\theta_i)$
  - Remember definition of irradiance

Diffuse BRDF

- Assume BRDF reflects a fraction ρ of light

$$\int_{\Omega} f_{r,Lambertian}(\omega_i \to \omega_o)\cos\theta_o \, d\omega_o = \rho$$

$$\int_{\substack{\theta\in[0..\pi/2] \\ \varphi\in[0..2\pi]}} k_d \cos\theta_o \sin\theta_o \, d\theta_o d\varphi_o = \rho$$

$$2\pi k_d \int_{\theta\in[0..\pi/2]} \sin\theta_o \cos\theta_o \, d\theta_o = \rho$$

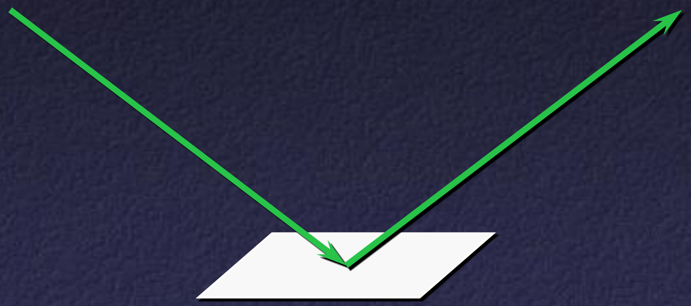
$$\pi k_d = \rho$$

$$\therefore f_{r,Lambertian} = \frac{\rho}{\pi}$$

- The quantity ρ is called the albedo

Ideal Mirror

- All light incident from one direction is reflected into another



- BRDF is zero everywhere except where

$$\theta_o = \theta_i$$

$$\varphi_o = \varphi_i + \pi$$

Ideal Mirror

- To conserve energy,

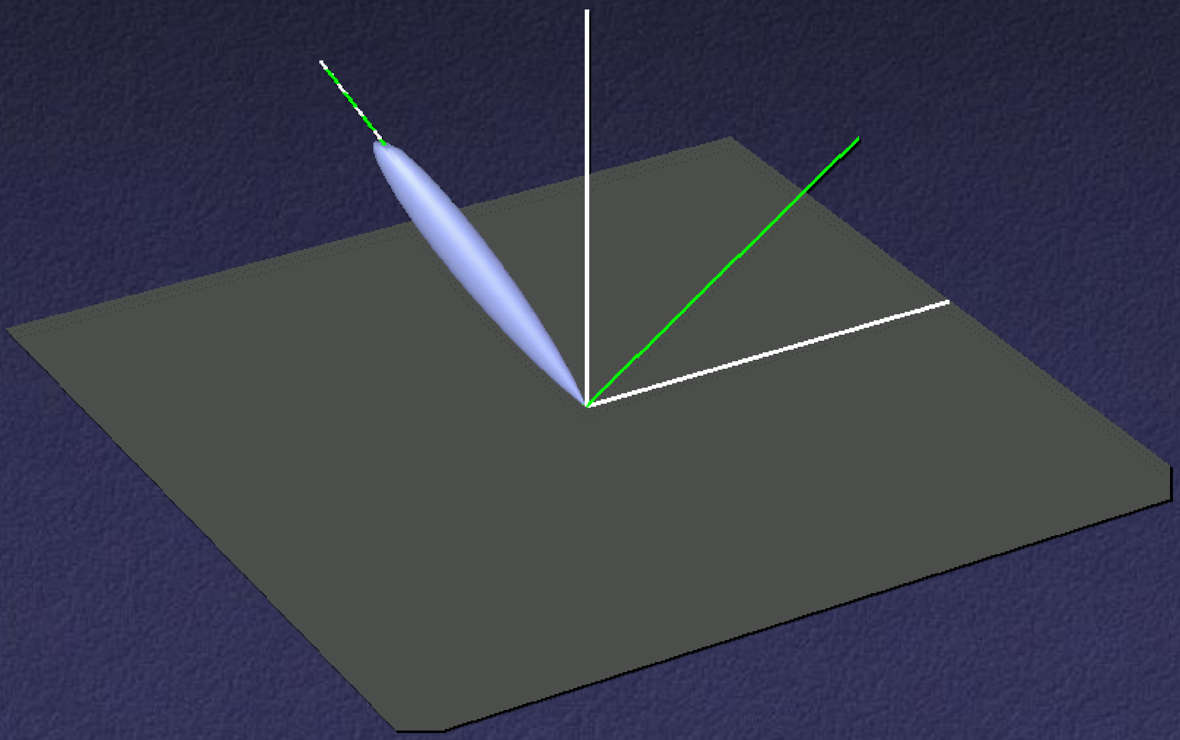
$$\int_{\Omega} f_{r,Mirror}(\omega_i \rightarrow \omega_o)\cos\theta_o \, d\omega_o = \rho$$

- So, BRDF is a delta function at direction of ideal mirror reflection

$$f_{r,Mirror} = \frac{\delta(\theta_i - \theta_o)\delta(\varphi_i + \pi - \varphi_o)}{\cos(\theta_i)}$$

Glossy Reflection

- Non-ideal specular reflection
- Most light reflected *near* ideal mirror direction



Phong BRDF

- Phenomenological model for glossy reflection

$$f_{r,Phong} = k_s(\hat{l} \cdot \hat{r})^n$$

l is a vector to the light source
r is the direction of mirror reflection

- Exponent n determines width of specular lobe
- Constant k_s determines size of lobe

Torrance-Sparrow BRDF

- Physically-based BRDF model
  - Originally used in the physics community
  - Adapted by Cook & Torrance and Blinn for graphics

$$f_{r,T-S} = \frac{DGF}{\pi \cos\theta_i \cos\theta_o}$$

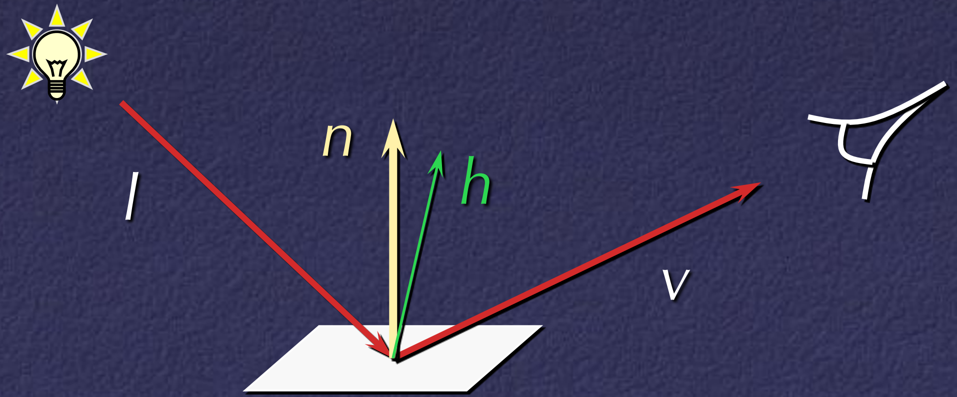
- Assume surface consists of tiny "microfacets" with mirror reflection off each

Torrance-Sparrow BRDF

- *D* term is distribution of microfacets (i.e., how many are pointing in each direction)
- Beckmann distribution

$$D = \frac{e^{-[(\tan\beta)/m]^2}}{4m^2\cos^4\beta}$$

β is angle between *n* and *h*
h is halfway between *l* and *v*
m is "roughness" parameter



Torrance-Sparrow BRDF

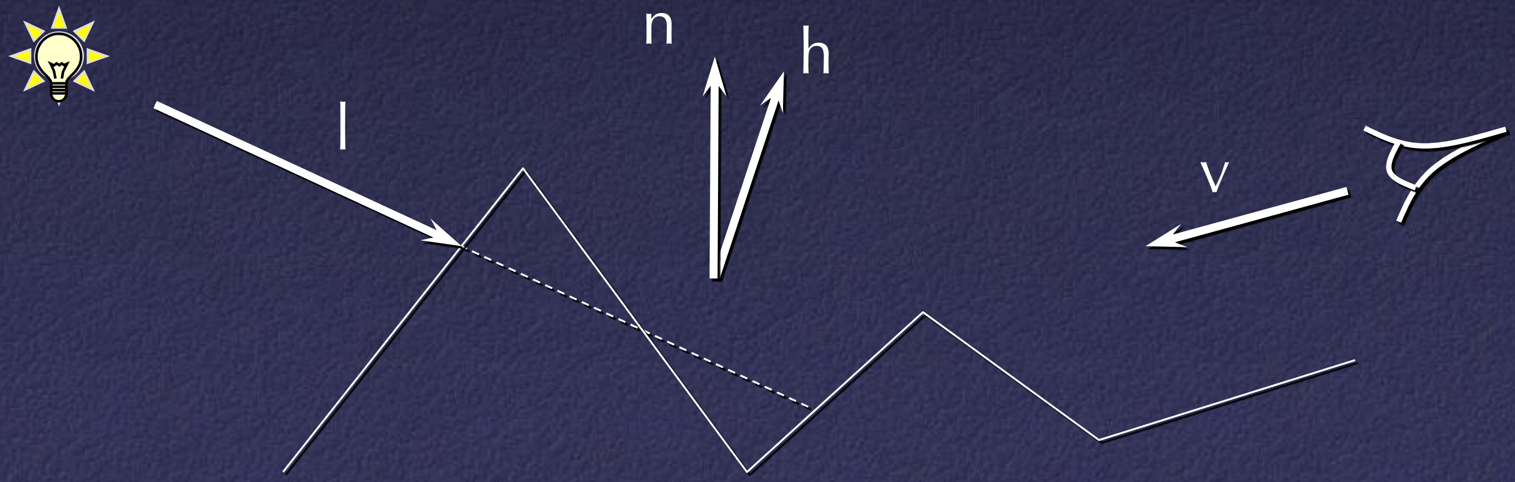
- Effect on D of increasing “m” (roughness):

D: From left to right roughness is 0.1, 0.3, 0.6, 0.8, 1.0

Torrance-Sparrow BRDF

- *G* term accounts for self-shadowing

$$G = \min\left\{1, \frac{2(n \cdot h)(n \cdot v)}{(v \cdot h)}, \frac{2(n \cdot h)(n \cdot l)}{(v \cdot h)}\right\}$$



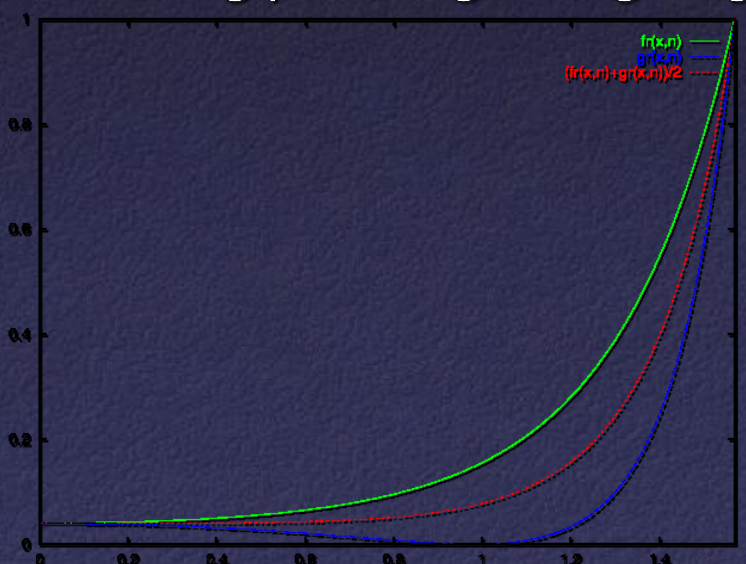
Torrance-Sparrow BRDF

- Effect on G of increasing “m” (roughness):

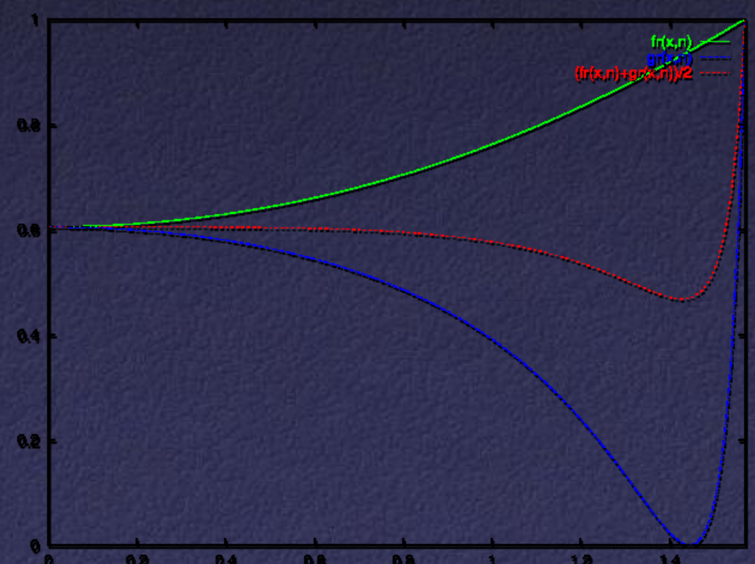
G: From left to right roughness is 0.0, 0.2, 0.5, 0.8, 1.0

Torrance-Sparrow BRDF

- *F* term is Fresnel term – reflection from an ideal smooth surface (solution of Maxwell's equations)
- Consequence: most surfaces reflect (much) more strongly near grazing angles



Dielectric

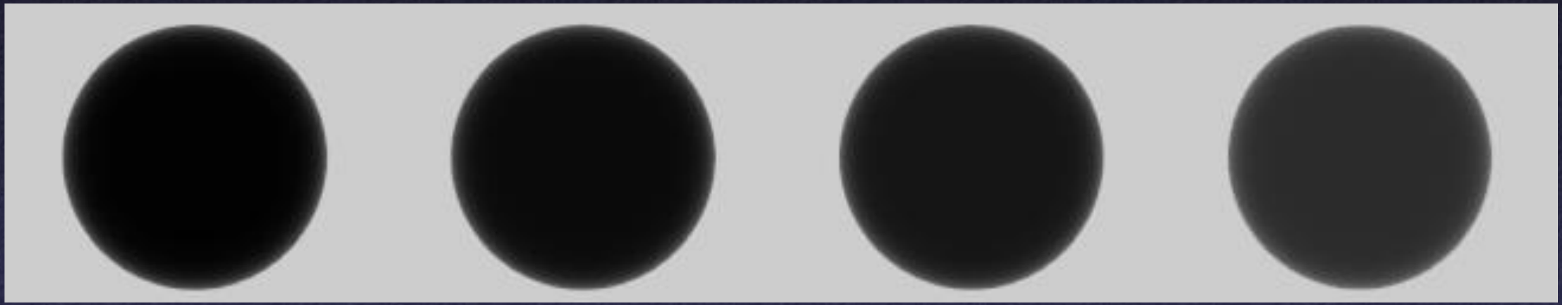


Metal

(note behavior at Brewster's angle)

Torrance-Sparrow BRDF

- Effect on F of increasing "index of refraction":



Dielectric F: From left to right the index of refraction is 1.2, 1.5, 1.8, 2.4

Other BRDF Models

- Ward – specular microfacet model
- Oren-Nayar – diffuse microfacet model
- Ashikhmin-Shirley – diffuse substrate, anisotropic glossy
- Lafortune – multiple specular lobes
- Lebedev – analytical grid approximation
- He, Torrance, Sillion, Greenberg – physically based
- etc.

Beyond BRDFs

- So far, have assumed 4D BRDF
- Function of wavelength: 5D
- Fluorescence (absorb at one wavelength, emit at another): 6D
- Phosphorescence (absorb now, emit later): 7D
- Temporal dependence: 8D
- Spatial dependence: 10D
- Subsurface scattering: 12D
- Polarization
- Wave optics effects (diffraction, interference)
- …

"Cross product" of two plenoptic functions