COS402- Artificial Intelligence
Fall 2015

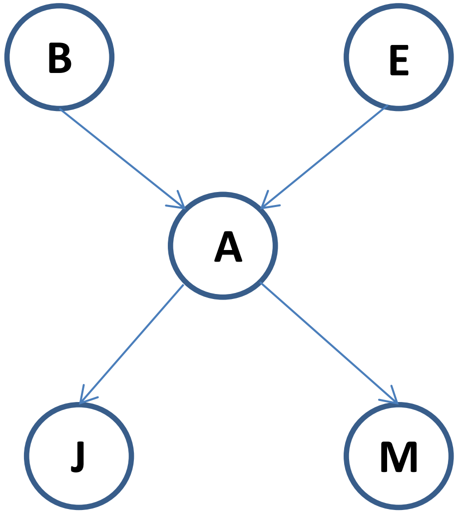
Lecture 15: Decision Theory: Utility and Expected values

Outline

- **Brief review on approximate inference in Bayes Nets**
- **Brief review on HMM**
- **Brief review on Kalman filters**
- **Brief review on particle filtering**
- **Decision Theory: Utility and expected value**

Approximate inference in BN

- **Direct sampling**
  - **Prior sample algorithm: for joint probability**
  - **Rejection sampling: for conditional probability**
  - **Likelihood sampling: for conditional probability**
- **How to sample the variables?**
- **P(J=t, M=t) = ?**
- **P(J=t, M=t|B=t) = ?**
- **P(J=t|E=t)= ?**



Approximate inference in BN

- **MCMC**
  - **A state in MCMC specifies a value for every variable in the BN.**
  - **Initialize the state with random values for all the non-evidence variable, and copy the evidence for the evidence variables**
  - **Repeat N times (long enough to assume convergence: stationary distribution.)**
    - **Randomly choose a non-evidence variable z, set the value of z by sampling from P(z|mb(z))**
  - **Estimate P(X|e)**

Review questions: true/false

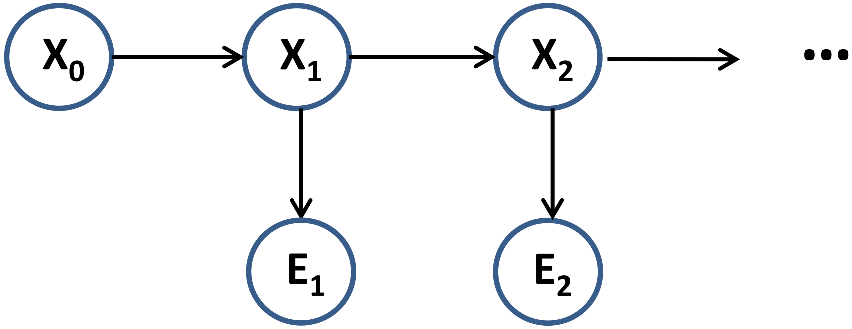
1. **Approximate inference methods are usually used in large multiply connected networks because exact inference is intractable in these networks.**
2. **When using direct sampling methods in a Bayesian network, variables can be sampled in any order.**
3. **The weakness of rejection sampling is that it could take a long time if the evidence rarely happens.**
4. **Likelihood weighting samples all variables, but does not reject samples that are not consistent with the evidence.**

Review questions: true/false (cont'd)

5. **Unlike direct sampling methods which generate each sample from scratch, MCMC generates each sample by making a random change to the preceding sample.**

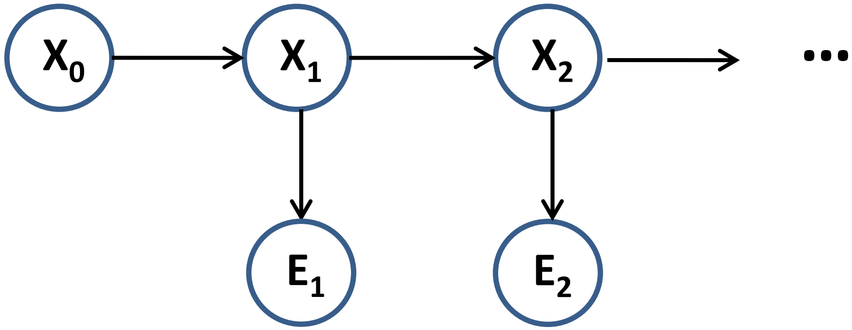
6. **Gibbs Samplings is a particular form of MCMC. It starts with an arbitrary state by setting a random value for each of the variables in the Bayesian network, and then generates the next state by randomly sampling a value for one of the variables.**

Hidden Markov Models



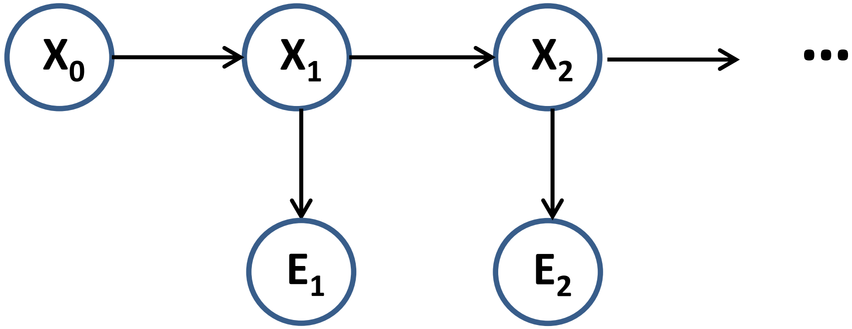
- X_t:
- E_t:

Hidden Markov Models



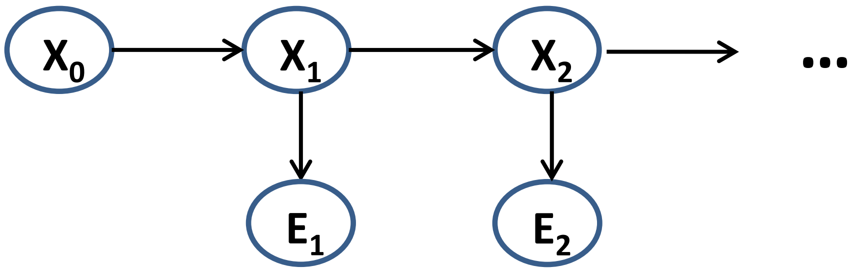
- X_t: random variable
  - State at time t
  - Discrete, finite number of values
  - Single variable representing a single state, can be decomposed into several variables
  - Hidden, invisible
- E_t: random variable, evidence at time t

Hidden Markov Models(parameters)



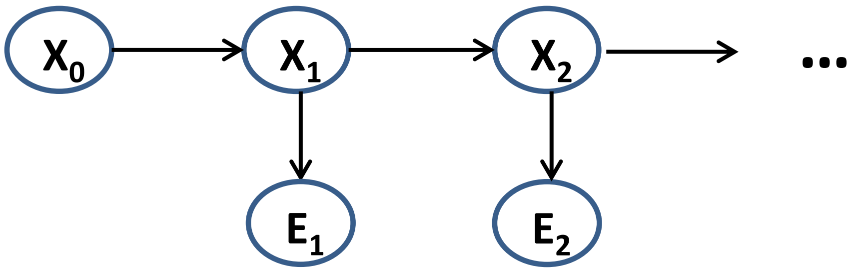
- **$P(X_0)$: the initial state model**
- **$P(X_t|X_{t-1})$: Transition model (usually assume stationary, same for all t)**
- **$P(E_t|X_t)$: sensor/observation model (usually assume stationary.)**

Hidden Markov Models (2 Markov assumptions)



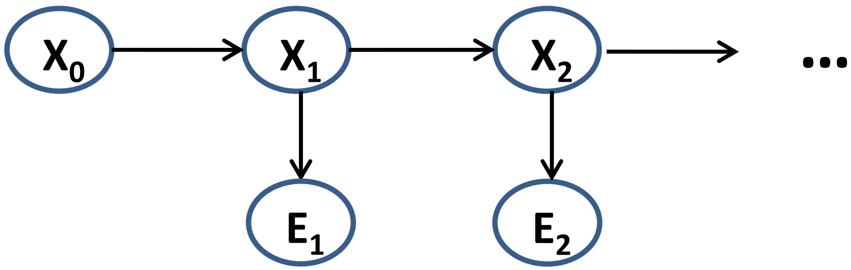
- $P(X_{t+1} | X_{0:t}) = P(X_{t+1} | X_t)$
  - The future is independent of the past given the present.
- $P(E_t | X_{0:t}, E_{1:t-1}) = P(E_t | X_t)$
  - Current evidence only depends on current state.
- **Note:**
  - given the 2 Markov assumptions, you can draw the Bayes Net; Given the Bayes net, the 2 Markov assumptions are implied.
  - HMMs are special cases of BNs, what is the full joint probability? $P(X_{0:t}, E_{1:t}) = ?$

Hidden Markov Models (4 basic tasks)



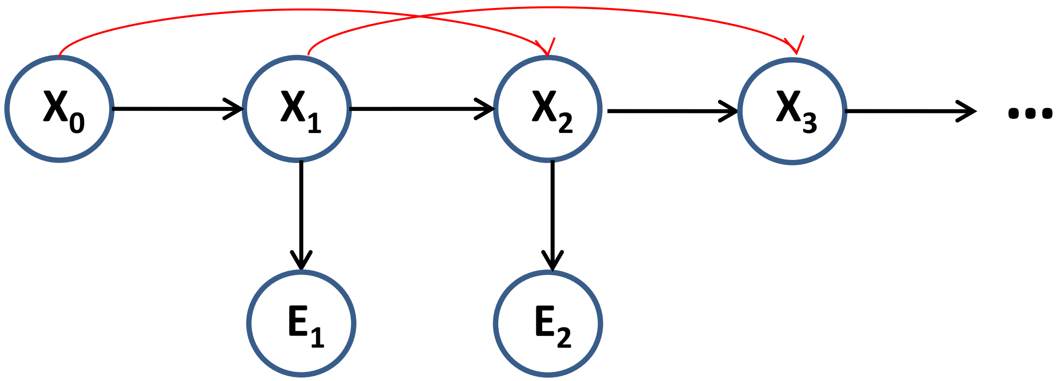
- **Filtering: Where am I now?**
  - **$P(X_{t+1} | e_{1:t+1}) = ?$**
- **Prediction : where will I be in k steps?**
  - **$P(X_{t+k} | e_{1:t}) = ?$**
- **Smoothing: Where was I in the past?**
  - **$P(X_k | e_{1:t}) = ?$ (k<t)**
- **Finding the most likely sequence**
  - **Max $P(X_{0:t} | e_{1:t}) = ?$**

Hidden Markov Models (4 basic tasks)



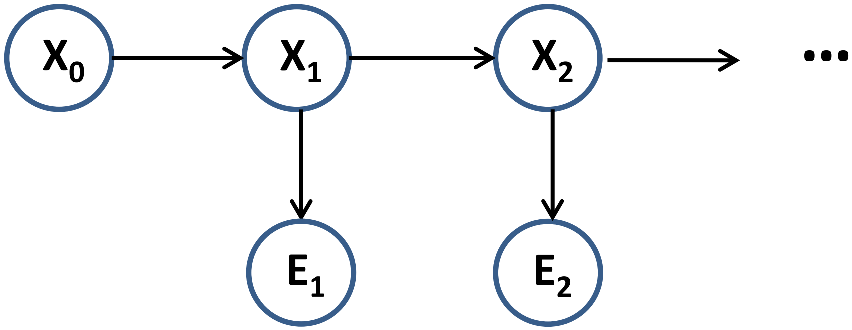
- **Filtering:** $P(X_{t+1}|e_{1:t+1}) = ?$
- **Prediction :** $P(X_{t+k}|e_{1:t}) = ?$
- **Smoothing:** $P(X_k|e_{1:t}) = ?$ $(k<t)$
- **Finding the most likely sequence:** $\text{Max } P(X_{0:t}|e_{1:t}) = ?$
- **Question?**
  - **Time complexity for the 4 basic tasks? $O(t \bullet (\#states)^2)$**
  - **Can we do other inference in HMM? $P(E_2|X_1,X_3) = ?$, time complexity?**

Kth order Hidden Markov Models



- **First order HMM**
  - $P(X_{t+1}|X_{0:t}) = P(X_{t+1}|X_t)$
- **Second order HMM**
  - $P(X_{t+1}|X_{0:t}) = P(X_{t+1}|X_t,X_{t-1})$
- **Kth order HMM?**
  - **The future is dependent on the last k states.**

Kalman Filters



- **$P(X_0)$: Gaussian distribution**
- **$P(X_{t+1}|X_t)$: Linear Gaussian distribution**
  - **The next state X_{t+1} is a linear function of the current state X_t, plus some Gaussian noise.**
- **$P(E_t|X_t)$: Linear Gaussian distribution**
- **Filtering: $P(X_{t+1}|e_{1:t+1})$ is also a Gaussian distribution.**

Particle Filtering—When to use it?

- **In DBNs where state variables are continuous, but both the initial state distribution and transitional model are not Gaussian.**
- **In DBNs where state variables are discrete, but the state space is huge.**
- **HMMs with huge state space.**

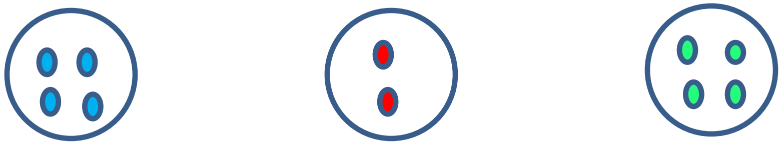
Particle Filtering—How does it work?

- **First, a population of N samples is created by sampling from the prior distribution $P(X_0)$.**
- **Repeat the update cycle for t= 0,1,…**
  - **1. each sample is propagated forward by sampling the next state value X_{t+1} based on the transitional model $P(X_{t+1} | x_t)$.**
  - **2. each sample is weighted by the likelihood it assigns to the new evidence. $P(e_{t+1} | x_{t+1})$**
  - **3. Resample to generate a new population of N samples: The probability that a sample is selected is proportional to its weight. The new samples are un-weighted.**

Particle Filtering—Example

$P(X_0)$=(0.4, 0.2, 0.4) , e={T,F}, , x={A,B,C}, N=10

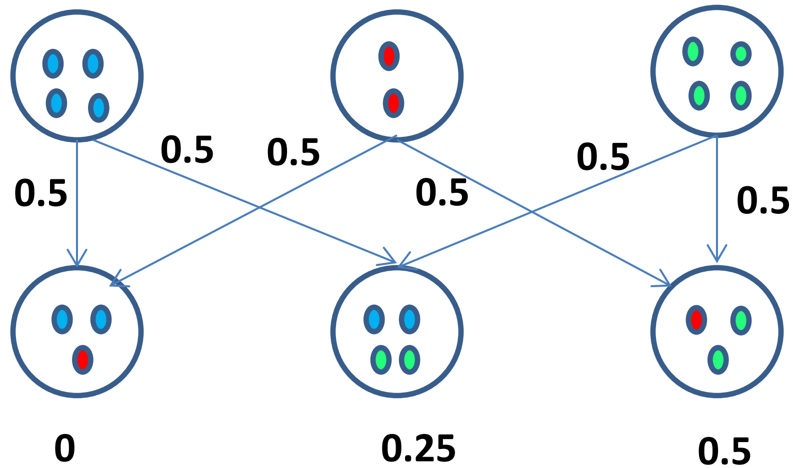
- **t=0, $P(X_0)$**



Particle Filtering—Example

$P(X_0)=(0.4, 0.2, 0.4)$, e={T,F}, , x={A,B,C}, N=10

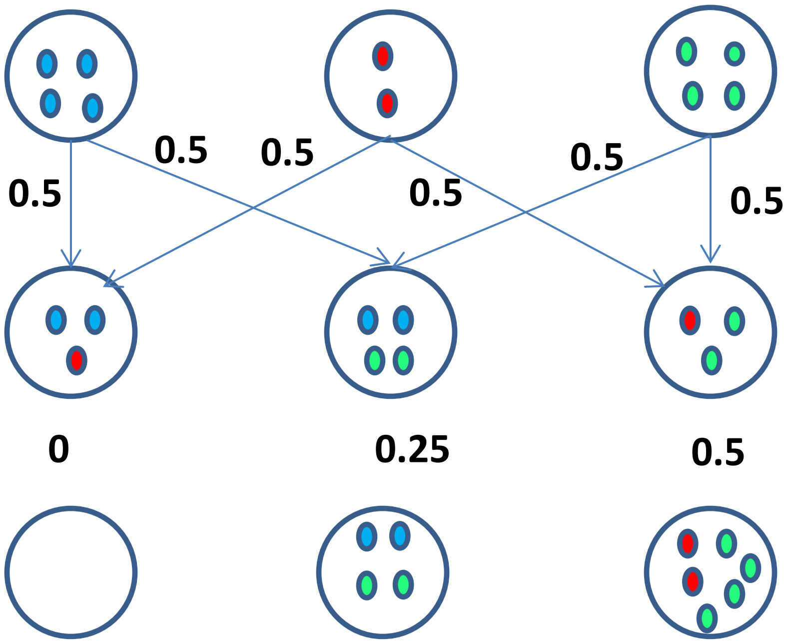
- **t=0, $P(X_0)$**
- **t=1,**
  - $P(X_1 | x_0)$
  - e_1= T
  - $P(e_1 | x_1)$



Particle Filtering—Example

$P(X_0)$=(0.4, 0.2, 0.4) , e={T,F}, , x={A,B,C}, N=10

- **t = 0**
  - $P(X_0)$
- **t = 1**
  - $P(X_1 | x_0)$
  - e_1= T
  - $P(e_1 | x_1)$
  - $P(X_1 | e_1)$=(0. 0.4, 0.6)
- **t = 2 …**



Particle Filtering—Demo?

– http://robots.stanford.edu/movies/sca80a0.avi

– **A robot is wandering around in some cluster of rooms**

– **Modeled as HMM**

- **States: locations**
- **Observations: sonar readings**
- **Task: Determining current state**
- **Particle filtering: the green dot is the robot's actual location; the little red dots are the particles(samples.)**

Review questions: true/false

1. **HMMS and Kalman Filters are special cases of Dynamic Bayesian Networks(DBNs).**
2. **HMMS, Kalman Filters, Particle filtering and Dynamic Bayesian Networks(DBNs) are all temporal models to model a world which changes over time.**
3. **Filtering, prediction, smoothing, and finding the most likely sequence are the four basic tasks in HMMS and they all can be done in time of O(t•(#states)^2).**
4. **The state variables in Kalman Filters are visible and continuous.**

Review questions: true/false (cont'd)

5. **In particle filtering, if more samples are used, the estimation of the belief state will be more accurate.**
6. **Particle filtering can be used in any Bayesian nets.**
7. **At resampling of particle filtering, the probability that a sample is selected is proportional to its weight.**
8. **As using more samples will increase the accuracy of the estimation, the number of samples of the new population will increase at each time step in particle filtering.**

Decision theory: Utility and expected value

- **Expected value (expectation) of a discrete random variable**
  - **Weighted average of all possible values**
  - **E[X] =** $\sum_x P(X = x) * x$
- **Expected value of a discrete random variable**
  - **Replace the sum with an integral and the probabilities with probability densities.**
- **Conditional expectation**
  - **E[X|y=a] =** $\sum_x P(X = x|y = a) * x$
- **Expectation of a real-valued function**
  - **E[f(x)] =** $\sum_x P(X = x) * f(x)$

Decision theory: Utility and expected value

- **Linearity of expectations**
  - **(1) $E[X + c] = E[X] + c$**
  - **(2) $E[c * X] = c * E[X]$**
  - **(3) $E[X + Y] = E[X] + E[Y]$**
  - **Note: X and Y do not need to be independent.**
- **Examples:**
  - **$E[X] = ?$ If X is the result of throwing a die.**
  - **$E[X] = ?$ If X is the number of heads when throw two fair coins.**

Decision theory: MDP

- **General principle:**
  - **Assign utilities to states**
  - **Take action that yields highest expected utility**
  - **Rational decision vs. human decision**
- **Simple decision vs complex decision**
  - **Simple decision: make a single decision, achieve short-term goal.**
  - **Complex decision: make a sequence of decisions, achieve long-term goal.**
    - **We will look at problems of making complex decisions**
    - **Markov assumption: The next state only depends on current state and action.**

Announcement & Reminder

- **W2 and Quiz 1 will be returned after class today**
- **W4 has been released and is due on Tuesday Nov. 24th**

  --- Turn in hard copies in class