COS402- Artificial Intelligence
Fall 2015

Lecture 10: Bayesian Networks & Exact Inference

Outline

- **Logical inference and probabilistic inference**
- **Independence and conditional independence**
- **Bayes Nets**
  - **Semantics of Bayes Nets**
  - **How to construct a Bayes net**
  - **Conditional Independence in Bayes nets**
- **Variable elimination algorithm**
- **Naïve Bayes**

Logical inference vs. probabilistic inference

- **Problem: KB |= α ?**
- **Model checking can determine entailment**
-

P1	P2	P3	KB	α
T	T	T		
T	T	F	T	?
…	…			
F	F	T	T	?
F	F	F	T	?

- **Is M(KB) a subset of M(α)?**
- **# of models: 2^n, n=3 here.**

- **Problem: P(X,Y)=? Or P(X|Y)=?**
- **Full joint probability distribution can be used to answer any query.**

X	Y	Z	P(X,Y,Z)
x_1	y_1	z_1	0.3
x_1	x_1	Z_2	0.25
…	…		
X_h	y_m	z_{k-1}	0.1
x_h	y_m	Z_k	0.05

- **# of parameters: hmk > 2^n**
- **How to answer the query?**

Inference given full joint probability distribution

- **Joint probability**
  - **P(x,y) = $\sum_z P(x, y, z)$ (Marginalization)**
- **Conditional probability**
  - **P(x|y) =$\frac{P(x,y\}}{P(y)} = \frac{\sum_z P(x,y,z)}{\sum_{x,z} P(x,y,z)}$ (definition + marginalization)**
  - **Or P(x|y) = α$\sum_z P(x, y, z)$ (normalization)**
    - **α = $\frac{1}{\sum_{x,z} P(x,y,z)}$**
- **Time and space: O(2^n)**

Independence and conditional independence

- **Independence of two events**
  - **Events a and b are independent if knowing b tells us nothing about a**
  - **$P(a|b) = P(a)$ or $P(a|b) = P(a)P(b)$**
- **Independence of two random variables**
  - **Random variable X and Y are independent if for all x,y, $P(X=x, Y=y) = P(X=x)P(Y=y)$**
  - **Shorthand: $P(X,Y)=P(X)P(Y)$**
- **Conditional independence**
  - **X and Y are conditionally independent given Z if $P(X,Y|Z)=P(X|Z)P(Y|Z)$**

Bayesian Network/Bayes Net (1)

- **Semantics**
  - Nodes are random variables
  - Edges are directed. Edge X --> Y indicates x has a direct influence on Y
  - There is no cycles
  - Each node is associated with a conditional probability distribution: P(x|Parents(x))
- **How to construct a Bayes Net?**
  - Topology comes from human expert
  - Conditional probabilities: learned from data

Bayesian Network/Bayes Net(2)

- **Conditional independence in Bayes Nets**
  - **A node is conditionally independent of non-descendants given its parents.**
  - **A node conditionally independent of all other nodes given its Markov blanket.**
    - **A Markov blanked of a node is composed of its parents, its children, and its children's other parents.**

Bayesian Network/Bayes Net(3)

- **Bayes nets represent the full joint probability**

$$\mathbf{P}(X_1, X_2, \ldots, X_n) = \prod_{i}^{n} \boldsymbol{P}(X_i | \boldsymbol{Parents}(X_i))$$

- **Exact inference (P(b|j,m) = ? example in the textbook)**

$$\mathbf{P}(\mathbf{b},|\mathbf{j},\mathbf{m}) = \boldsymbol{\alpha}\, \mathbf{P}(\mathbf{b}, \mathbf{j}, \mathbf{m}) = \boldsymbol{\alpha} \sum_{e,a} \boldsymbol{P}(\boldsymbol{b}, \boldsymbol{j}, \boldsymbol{m}, \boldsymbol{e}, \boldsymbol{a})$$

$$= \boldsymbol{\alpha} \sum_{e,a} \boldsymbol{P}(\boldsymbol{b})\boldsymbol{P}(\boldsymbol{e})\boldsymbol{P}(\boldsymbol{a}|\boldsymbol{e}, \boldsymbol{b})\boldsymbol{P}(\boldsymbol{j}|\boldsymbol{a})\boldsymbol{P}(\boldsymbol{m}|\boldsymbol{a})$$

$$= \boldsymbol{\alpha}\, \mathbf{P}(\mathbf{b}) \sum_{e} \boldsymbol{P}(\boldsymbol{e}) \sum_{a} \boldsymbol{P}(\boldsymbol{a}|\boldsymbol{e}.\boldsymbol{b})\boldsymbol{P}\,(\boldsymbol{j}|\boldsymbol{a})\boldsymbol{P}(\boldsymbol{m}|\boldsymbol{a})$$

Variable Elimination Algorithm (1)

- **Variable elimination algorithm**
  - **P(b,|j,m) =** $\boldsymbol{\alpha}\, \mathbf{P(b)} \sum_e P(e) \sum_a P(a|e.b)P\,(j|a)P(m|a)$
    - **g_1(e,b) =** $\sum_a P(a|e.b)P\,(j|a)P(m|a)$
    - **g_2(b) =** $\sum_e P(e)$**g_1(e,b)**
    - **g_3(b) = P(b) g_2(b)**
  - **Define and evaluate function for each summation from right to left.**
  - **Evaluate once and store the values to be used later.**
  - **Normalize.**

Variable elimination algorithm (2)

- **Time and space:**
  - **linear in terms of the size of Bayes net for singly connected networks.**
  - **Exponential for multiply connected networks.**
- **Singly-connected networks vs Multiply-connected networks**
  - **In singly-connected networks, also called polytrees, there is at most one undirected path between any two nodes.**
  - **In mutliply-connected networks, there could be 2 or more undirected paths between 2 nodes.**

Naïve Bayes

- **Naïve Bayes:**
  - A special case of Bayes net: one parent node and the rest are its children.
  - Random variables: One cause and multiple effects.
  - Assume that all effects are conditionally independent given the cause.
  - Very tractable.
  - Can be used for classification: Naïve Bayes classifier.

Review questions: true/false

1. **Given the full joint probability distribution, we can answer most, but not all, inference queries.**
2. **A Bayes net completely and implicitly defines the full joint probability distribution of all random variables in a probabilistic model.**
3. **In a Bayes net, a node is conditionally independent of all other nodes given its parents.**
4. **In a Bayes net, a node is conditionally independent of all other nodes given its Markov blanket.**
5. **The Markov blanket of a node consists of its parents and its children.**

Review questions: true/false (cont'd)

6. **Variable elimination algorithm can be used to do exact inference in any Bayes net.**
7. **For any Bayes net, variable elimination algorithm takes linear time and space in terms of the size of the Bayes net.**
8. **In singly connect networks, also called polytrees, there is at most one indirect path between any two nodes.**
9. **Bayes rule is very useful because it provides a way to calculate the conditional probability of a hidden variable given some evidence, which is usually hard to estimate directly.**

Announcement & Reminder

- **W2 is due today**

  --- Turn in hard copies in class

- **W1 will be returned after class today**

- **W3 has been released and is due on Tuesday Nov. 10th**

  --- Turn in hard copies in class