COS 226	Algorithms and Data Structures	Fall 2015

Midterm Exam

This exam has 8 questions worth a total of 100 points. You have 80 minutes. The exam is closed book, except that you are allowed to use one page of notes (8.5-by-11, one side, in your own handwriting). No calculators or other electronic devices are permitted. Give your answers and show your work in the space provided. You may use the back of each page for scratch space, or to continue long answers.

Name:

NetID:

Room:

Precept:

P01	9:00	Andy Guna
P02	10:00	Andy Guna
P02A	10:00	Elena Sizikova
P03	11:00	Maia Ginsburg
P03A	11:00	Nora Coler
P04	12:30	Maia Ginsburg
P04A	12:30	Miles Carlsten
P05	1:30	Tom Wu

Write and sign: "*I pledge my honor that I have not violated the Honor Code during this examination.*"

Problem	Score
0	
1	
2	
3	
4	
Sub 1	

Problem	Score
5	
6	
7	
8	
Sub 2	

Total:

0. Constructor. (1 point)

In the space provided on the front of the exam, write your name and Princeton netID; write the name of the room in which you are taking the exam; mark your precept number; and write and sign the honor code.

1. The Usual COS226 Sorting Question. (16 points)

The column on the left is an array of strings to be sorted or shuffled. The column on the right is in sorted order. The other columns are the contents of the array at some intermediate step during one of the algorithms below. Write the number of each algorithm under the corresponding column. Use each number exactly once.

```
mars   care   dart   barn   barn   barn   yard   bark   lard   bark
part   gary   lard   fare   care   care   warm   barn   care   barn
care   mars   care   rare   fare   dart   vary   card   gary   card
gary   part   gary   jars   gary   fare   part   care   barn   care
barn   barn   barn   harp   harp   gary   tart   dart   card   dart
park   fare   card   mars   jars   harp   park   earn   farm   earn
rare   park   farm   gary   mars   jars   rare   fare   fare   fare
fare   rare   fare   warm   park   mars   gary   farm   harm   farm
warm   harp   harm   care   part   park   mars   gary   earn   gary
tarp   jars   earn   tarp   rare   part   tarp   harm   jars   harm
jars   tarp   jars   part   tarp   rare   oars   harp   harp   harp
harp   warm   harp   park   warm   tarp   nary   jars   bark   jars
vary   bark   bark   vary   bark   vary   care   vary   dart   lard
dart   dart   mars   dart   dart   warm   dart   part   mars   mars
bark   vary   vary   bark   earn   bark   bark   mars   yard   nary
yard   yard   yard   yard   harm   yard   fare   yard   vary   oars
earn   earn   tarp   earn   vary   earn   earn   park   tarp   park
harm   farm   warm   harm   yard   harm   harm   tarp   warm   part
farm   harm   rare   farm   card   farm   farm   rare   tart   rare
tart   tart   tart   tart   farm   tart   barn   tart   rare   tarp
card   card   park   card   lard   card   card   warm   park   tart
lard   lard   part   lard   nary   lard   lard   lard   oars   vary
oars   nary   oars   oars   oars   oars   jars   oars   nary   warm
nary   oars   nary   nary   tart   nary   harp   nary   part   yard
----   ----   ----   ----   ----   ----   ----   ----   ----   ----
 0                                                             9
```

(0) Original input

(1) Knuth shuffle

(2) Selection sort

(3) Insertion sort

(4) Mergesort *(top-down)*

(5) Mergesort *(bottom-up)*

(6) Heapsort

(7) Quicksort *(no shuffle)*

(8) 3-way Quicksort *(no shuffle)*

(9) Sorted

2. Playing Cards. (16 points)

We would like to sort playing cards from a deck. Associated with each card is a denomination (1 to 13) and a suit (CLUBS < DIAMONDS < HEARTS < SPADES).

A card c_1 is considered less than a card c_2 if either of the following is true:

- the suit of c_1 is less than the suit of c_2, or
- c_1 and c_2 are of the same suit, but the denomination of c_1 is less than the denomination of c_2.

(a) Let us first consider sorting cards of the same suit, based purely on their denominations. Specifically, consider using ***2-way quicksort*** to sort the 3, 4, 5, 6, 7, 8 and 9 of hearts. After a random shuffle, we have the following sequence of denominations: 5, 6, 8, 3, 9, 4, 7. Show the result of the first call to `partition()` by giving contents of the array ***after each exchange***. Please write only the two elements that were exchanged.

5	6	8	3	9	4	7

Now write the entire contents of the partitioned array, and ***draw a box*** around each of the left and right subarrays on which recursive calls will be executed.

(b) The `Card` class is implemented in Java as follows. Complete the `compareTo()` function, assuming that the argument is not null.

```
public class Card implements Comparable<Card> {
    // Comparators by suit and by denomination
    public static final Comparator<Card> SUIT_ORDER = new SuitOrder();
    public static final Comparator<Card> DENOM_ORDER = new DenomOrder();

    // Suit of the card (CLUBS = 1, DIAMONDS = 2, HEARTS = 3, SPADES = 4)
    private final int suit;

    // Denomination of the card
    private final int denom;

    public Card(int suit, int denom) {
        if (suit < 1 || suit > 4)
            throw new IllegalArgumentException("Invalid suit");
        if (denom < 1 || denom > 13)
            throw new IllegalArgumentException("Invalid denomination");
        this.suit = suit;
        this.denom = denom;
    }

    // COMPLETE THE FOLLOWING FUNCTION
    public int compareTo(Card that) {











    }

    // Compare cards according to the suit only
    private static class SuitOrder implements Comparator<Card> {
        // Implementation not shown
    }

    // Compare cards according to the denomination only
    private static class DenomOrder implements Comparator<Card> {
        // Implementation not shown
    }
}
```

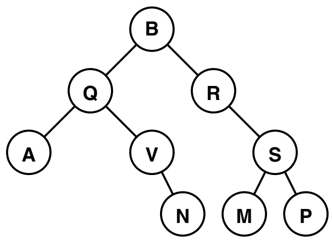
(c) Suppose that the variable `cards` is an array of cards. We could sort it, using your `compareTo` function, with a call to `MergeX.sort(cards)`. Write a Java code fragment that produces the same result, using one or more of the static comparators defined in `Card` instead. You may use a variant of the `MergeX.sort` function for mergesort.

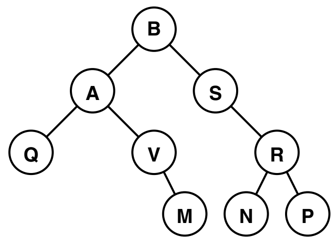
(d) Instead of using Mergesort, could we have gotten the same result as in (c) using multiple calls to Quicksort (using the same static comparators)? If not, why not?

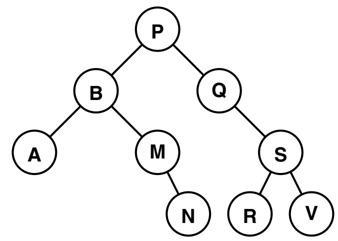
3. Traversing Trees. (10 points)

(a) Circle the correct ***binary tree*** (not necessarily a BST) that would produce both of the following traversals:

In-order: AQVNBRMSP
Pre-order: BQAVNRSMP

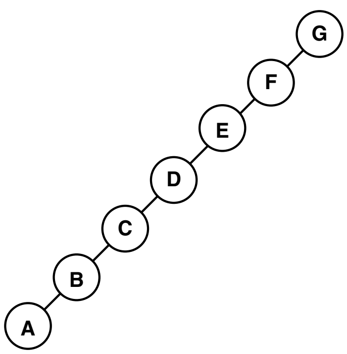


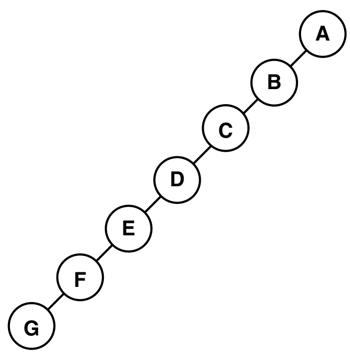


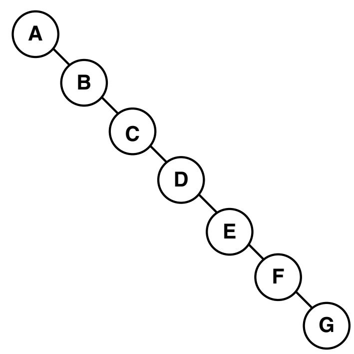


(b) Circle the correct ***Binary Search Tree*** that would produce the following traversal:

Post-order: ABCDEFG







(c) If you know that a tree is a BST, which of the following ***is*** or ***is not*** always sufficient to reconstruct it? For each one, write ***yes*** if it is enough to reconstruct the tree, or ***no*** if it is not.

Pre-order traversal:

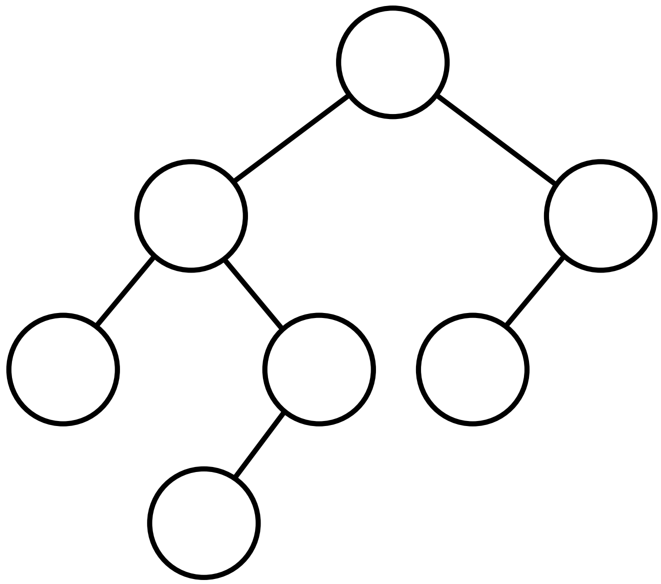
In-order traversal:

Post-order traversal:

Level-order traversal:

4. BSTs, LLRB and otherwise. (12 points)

(a) Label each node in the following binary tree with numbers from the set $\{2, 26, 10, 27, 20, 15, 42\}$ so that it is a legal Binary Search Tree. (Hint: use the back of the page as scratch space, and only write down the answer once you have it.)



(b) Now label each *edge* in the figure with `r` or `b`, denoting RED and BLACK, so that the tree is a legal Left-Leaning Red-Black Tree.

(c) Is the red/black labeling you created in (b) unique? That is, is it possible to create a *different* legal LLRB tree using the same nodes and edges (just with different red/black labels)?

(d) If the answer to (c) is ***yes***, draw and label the second tree. If the answer is ***no***, how do you know that this is not possible?

5. Heaps. (10 points)

Starting from the following max-heap (using the array representation presented in lecture), give the resulting array after each operation:

X	10	7	4	5	6	2	3	0	1	

(a) After `insert(9)`

X										

(b) After `delMax()`, starting from the original heap (i.e., assuming that (a) has ***not*** been performed)

X										

(c) For implementing a max-priority queue, which of the following are advantages of a resizing-array implementation of a heap ***over*** a sorted linked list? Circle ***all*** that apply.

expected time for `insert` is lower

`insert` has lower worst-case order of growth

expected time for `delMax` is lower

`delMax` has lower worst-case order of growth

expected storage cost is lower

`max` has lower worst-case order of growth

6. FortyTwoPQ. (15 points)

You have been hired by Deep Thought Enterprises to implement a priority-queue-like data structure supporting the following operations:

- `insert()` an item in $O(\log N)$ time.
- `fortytwo()` — return the 42^{nd} smallest item in constant time.
- `delFortyTwo()` — delete the 42^{nd} smallest item in $O(\log N)$ time.

Explain how you would implement the required functionality, using one or more data structures that we have seen in class. Write pseudocode for each of the three operations listed above. You may assume that $N > 42$, and omit all checks for smaller N.

For full credit, your implementation should support finding the k^{th} smallest item with an order-of-growth running time ***independent*** of k. That is, it should be possible to change 42 to some other constant without changing the running time.

If you need more space, use the back of the sheet.

7. Divide and Conquer. (12 points)

Consider the following three algorithms:

- **Algorithm 1** solves problems of size N by recursively dividing them into 2 sub-problems of size $N/2$ and combining the results in time c (where c is some constant).
- **Algorithm 2** solves problems of size N by solving one sub-problem of size $N/2$ and peforming some processing taking some constant time c.
- **Algorithm 3** solves problems of size N by solving two sub-problems of size $N/2$ and performing a *linear* amount (i.e., cN where c is some constant) of extra work.

(a) For each algorithm, write down a *recurrence relation* showing how $T(N)$, the running time on an instance of size N, depends on the running time of a smaller instance.

Algorithm 1: $T(N) =$

Algorithm 2: $T(N) =$

Algorithm 3: $T(N) =$

(b) For each recurrence relation, pick the solution for $T(N)$ from the following list. Just write the letter corresponding to the correct running time.

Algorithm 1:

Algorithm 2:

Algorithm 3:

A:	$T(N)$	$\sim$	c
B:	$T(N)$	$\sim$	$c \log N$
C:	$T(N)$	$\sim$	cN
D:	$T(N)$	$\sim$	$cN \log N$
E:	$T(N)$	$\sim$	cN^2

(c) For each of the following algorithms, pick which of the above classes of algorithms (1, 2, or 3) applies to that algorithm:

Mergesort:

Binary search in a sorted array:

Quicksort (if partitioning always divides the array in half):

8. You didn't think we forgot about the assignments, did you? (8 points)

(a) Suppose we wanted to simulate percolation in a *cube* with N sites on a side, with each site connected to its neighbors up, down, left, right, forward, and back. If we used `WeightedQuickUnionUF`, what would be the order of growth of the expected running time, as a function of N?

a. N^2
b. $N^2 \log N$
c. N^3
d. $N^3 \log N$
e. N^4
f. $N^4 \log N$
g. None of the above.

(b) If you run your `BinarySearchDeluxe` on a sorted array with N items but only 3 distinct keys, what is the order of growth of the expected running time for a call to `firstIndexOf()`?

a. constant
b. $\log N$
c. $\log_N 3$
d. N
e. $N \log N$
f. None of the above.

(c) True or False: The amount of memory necessary to solve `8puzzle` is equal to some constant times the size of the game board.

(d) True or False: `8puzzle` will still work without implementing the critical optimization, but it may take much more memory and running time to find the answer.

(e) True or False: it is always legal to call `equals` on two objects that do not have the same type.

(f) True or False: a `KdTreeST` always has a lower order-of-growth running time than the brute-force `PointST` for the `contains()` operation, for all possible query points.

(g) True or False: a `KdTreeST` always has a lower order-of-growth running time than the brute-force `PointST` for the `range()` operation, for all possible query rectangles.

(h) True or False: a `KdTreeST` always has a lower order-of-growth running time than the brute-force `PointST` for the `nearest()` operation, for all possible query points.