COS 109 – Problem Set 5

1.a. Chips designed in 1981 had feature size of 6 micrometers and chips designed today have feature size of 14 nanometers. A transistor is \((\text{feature size} \times \text{feature size})\). Chips in 1981, could fit 10,000 transistors.

Hence, number of transistors that would fit in today’s chips would be:

\[
\frac{(6 \times 10^{-6} \text{ m})^2}{(14 \times 10^{-9} \text{ m})^2} \times 10,000 = 1,800,000,000 \text{ transistors}
\]

1b. i. Laptop Size: 13.3 inch (diagonally)
   Desktop Size: 27 inch (diagonally)

   \[
   \frac{\text{Bigger Screen Diagonal}}{\text{Smaller Screen Diagonal}} = \frac{\text{Bigger Screen Length}}{\text{Smaller Screen Length}} = \frac{\text{Bigger Screen Width}}{\text{Smaller Screen Width}}
   \]

   Since, diagonal has increased almost twice, length and width also would increase twice, since the ratio remains the same, and the area would increase four times (length \(*\) width).

   Hence, the bigger screen would have four times the number of pixels as the smaller screen, which is 4 M pixels.

   ii. If the larger display uses standard RGB representation of colors, each pixel is represented in 3 bytes. Hence the whole image would be of size \((4 \text{ M pixels} \times 3 \text{ bytes/pixel})\) 12 M bytes.

1c. i. Each day there are 45 students in class and about half of them use laptop (around 23). I assume that average amount of RAM in each laptop is around 8GB, and average disk space in each laptop is around 256GB.

   Hence, the total amount of RAM in each lecture = 23 students \(\times\) (8GB RAM / student) = 184GB of RAM
   Total amount of disk space in each lecture = 23 students \(\times\) (256GB disk space / student) = 5888GB of disk space.

   ii. An hour of HD video requires 4.5GB of disk space. In 5888GB of disk space, we can store:

   \[
   \frac{1 \text{ hour}}{4.5\text{GB}} \times \frac{1 \text{ day}}{24 \text{ hours}} \times 5888\text{GB} = 55 \text{ days of HD video}
   \]

   iii. Princeton network runs at about 100Mb/second.

   \[
   \frac{100,000,000 \text{ bits}}{1 \text{ second}} \times \frac{1 \text{ byte}}{8 \text{ bits}} = 0.0125 \text{ GB per second}
   \]
All of the videos, occupy the size 5888GB, and this can be transmitted through the Princeton network in:

\[
\frac{5888 GB \times \frac{1 \text{ day}}{(60 \times 60 \times 24) \text{ seconds}} \times \frac{1 \text{ second}}{0.0125 GB}}{5.45185 \text{ days}}
\]

2a. i. It will likely take 250 seconds \((5^2 \times 10)\) to process 500 data items.

ii. It appears to be growing as a function of \(N^2\).

2b. i. In the year 1990, an algorithm took 10 milliseconds to process \(N\) items. For the same algorithm to process same \(N\) items in 10 nanoseconds, it would need approximately 20 doublings as shown below.

\[
\log_2 \frac{10 \times 10^{-3}}{10 \times 10^{-6}} = \log_2 (1000000) = 19.93
\]

According to Moore’s Law, each doubling period is 1.5 years, and hence for 20 doublings it would need \((1.5 \times 20)\) 30 years. 30 years since 1990 would be 2020.

Hence, by 2020, the same algorithm would process the same \(N\) items in 10 nanosecond time.

ii. For the same algorithm to process same \(N\) items in 10 picoseconds, it would need approximately 30 doublings as shown below.

\[
\log_2 \frac{10 \times 10^{-3}}{10 \times 10^{-12}} = \log_2 (1000000000) = 29.89
\]

According to Moore’s Law, each doubling period is 1.5 years, and hence for 30 doublings it would need \((1.5 \times 30)\) 45 years. 45 years since 1990 would be 2035.

Hence, by 2035, the same algorithm would process the same \(N\) items in 10 picosecond time.

3a. Number of bits needed to store the number 3685 billion would be: \(\log_2 (3,685 \text{ billion}) = 41.7\), or 42 bits. 42 bits can be represented by 6 bytes \((42/8 = 5.25)\)

3b. Number of bits needed to store the number 18.69 trillion would be: \(\log_2 (18.69 \text{ trillion}) = 44.09\), or 45 bits. 45 bits can be represented by 6 bytes \((45/8 = 5.625)\)

3c.i. Each tweet is 140 bytes, and there are 500 million tweets per day currently. Amount of memory needed to store all of the tweets from one year at the current rate of tweeting would be: \(500 \text{ million tweets per day} \times 365 \text{ days per year} \times 140 \text{ bytes per tweet} = 2.56 \times 10^{13} \text{ bytes.}\)
ii. Starting from 5000 tweets per day in 2007, to get to 500 million tweets per day it would need 17 doublings, as shown below

\[
\log_2 \frac{500 \text{ million}}{5000} = \log_2 (100000) = 16.6
\]

According to Moore’s Law, each doubling period is 1.5 years, and hence for 17 doublings it would need \((1.5 \times 17)\) around 25.5 years.

iii. Time period between 2007 and 2013 is 6 years. During this 6 years (which is \(6 \times 12 = 72\) months), the number of tweets doubled 17 times. Hence it doubled every \(72 \text{ months} / 17 \text{ doublings} = 4.235 \text{ months} \)

3d. Cecilia is 21 in Base 13.
In decimal it is: \(2(13^1) + 1(13^0) = 27\) years.
In Binary it is 11011.
In Hexadecimal it is 1B.