

# Mesh Segmentation

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COS 526, Fall 2014

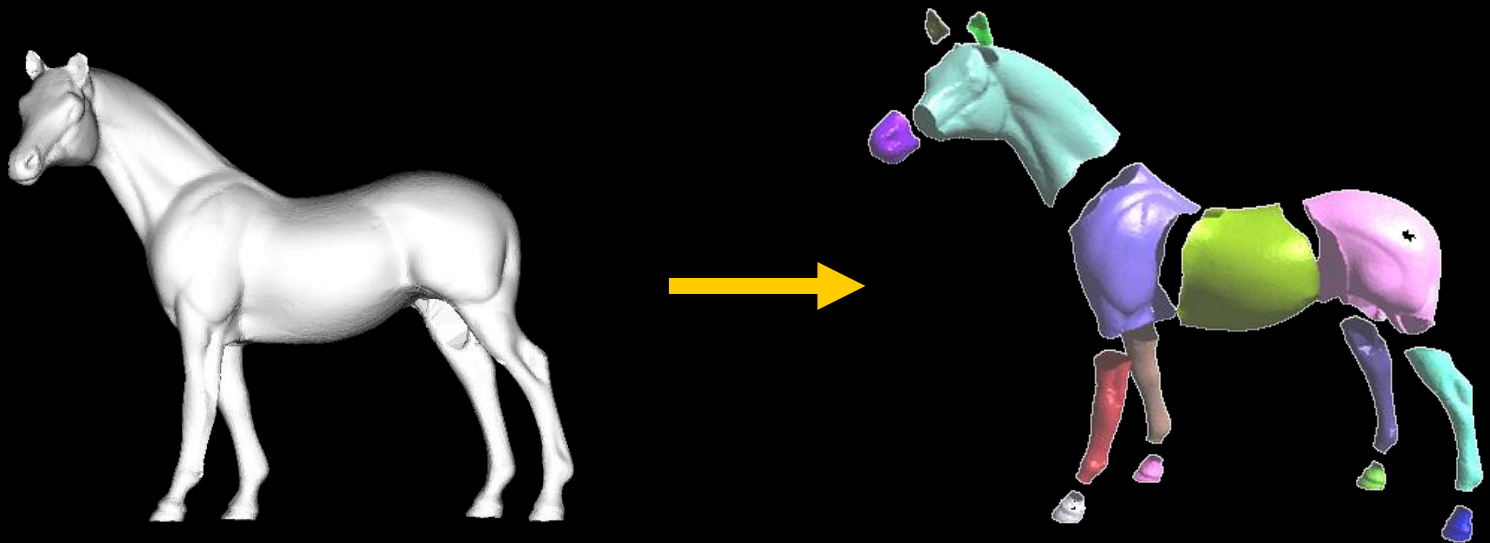
(most slides by Arik Shamir)



# Introduction

## Goal:

- Given: a mesh  $M = \{V, E, F\}$
- Create: a set  $S$  of submeshes  $M_i$  that partition the faces of  $M$  into disjoint subsets.



# Motivation



## Applications:

- Analysis
- Representation
- Recognition
- Collision detection
- Animation
- Modeling
- etc.

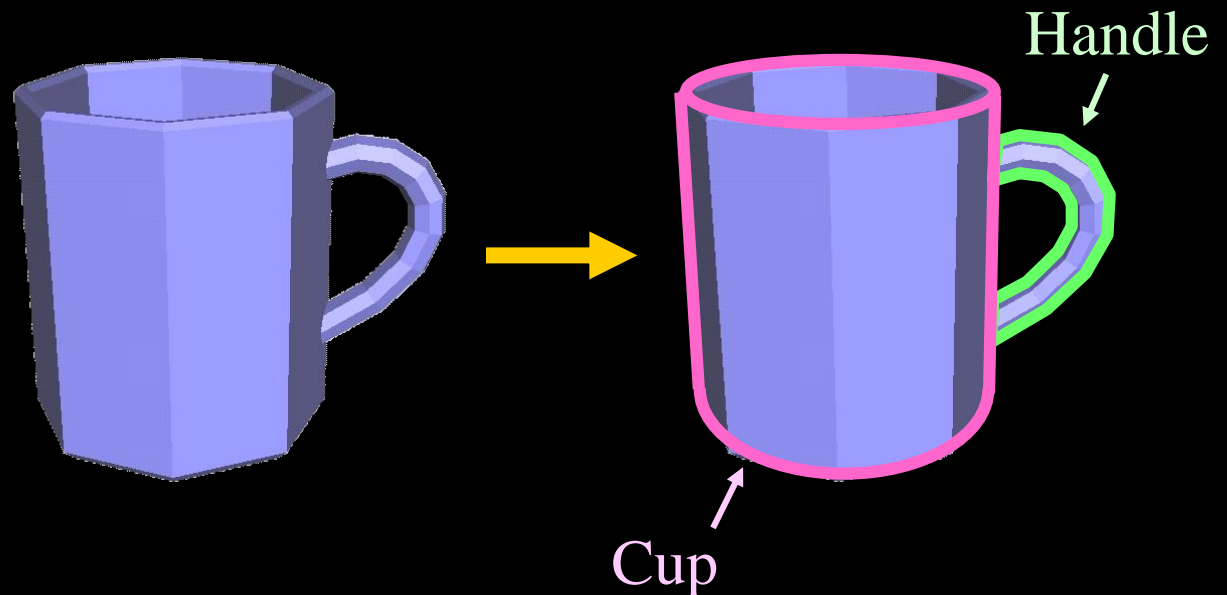


# Motivation

## Applications:

### ➤ Analysis

- Representation
- Recognition
- Collision detection
- Animation
- Modeling
- etc.



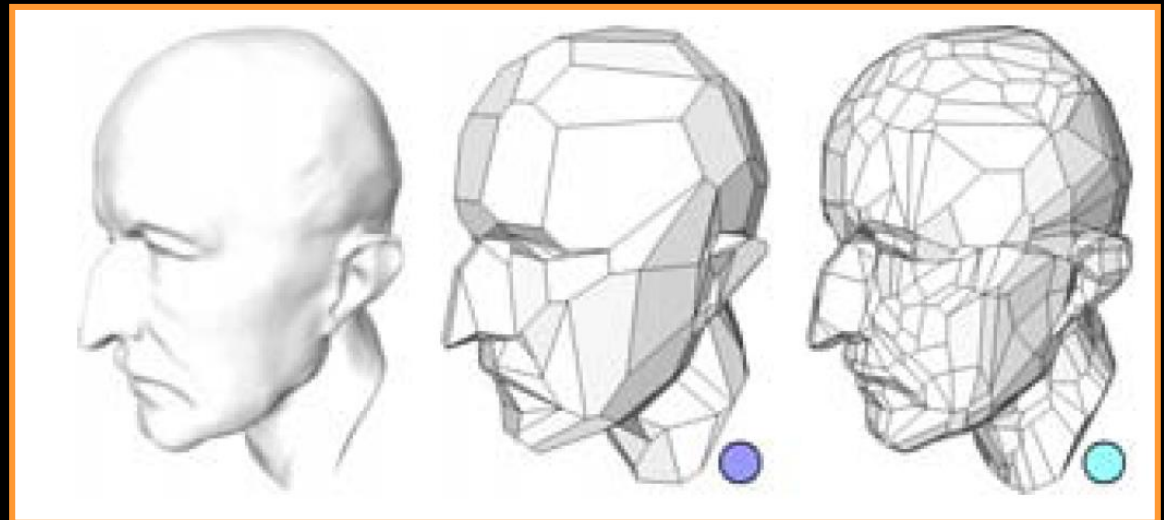
# Motivation



## Applications:

- Analysis
- **Representation**
- Recognition
- Collision detection
- Animation
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- etc.

*Cohen-Steiner et al.*

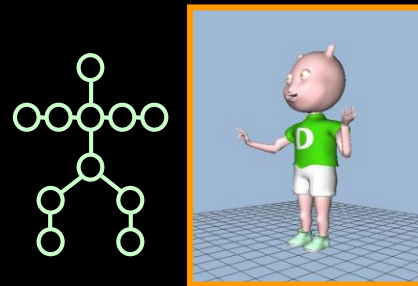




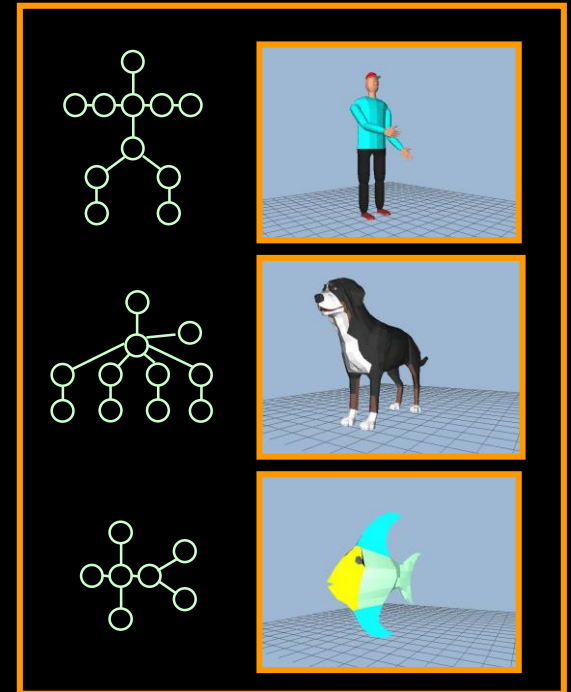
# Motivation

## Applications:

- Analysis
- Representation
- **Recognition**
- Collision detection
- Animation
- Modeling
- etc.



Query



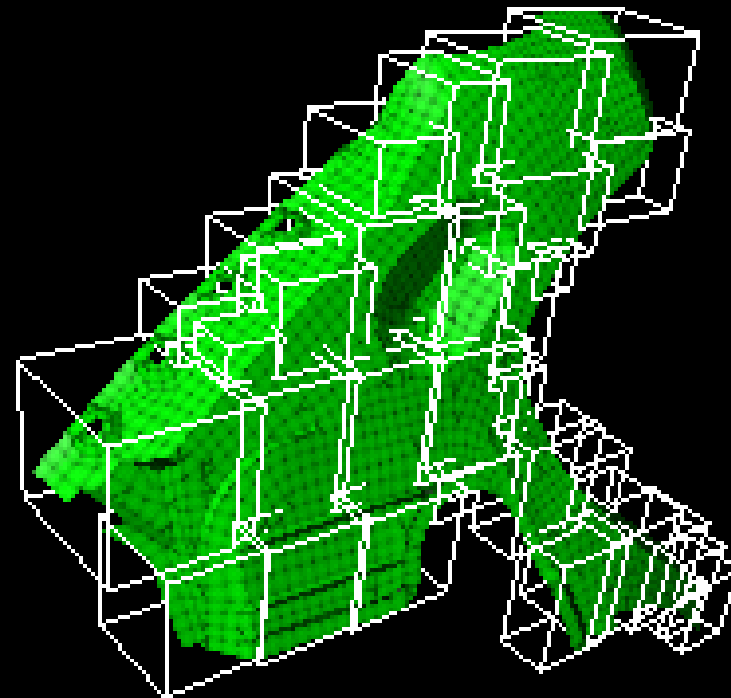
Database

# Motivation



## Applications:

- Analysis
- Representation
- Recognition
- **Collision detection**
- Animation
- Modeling
- etc.

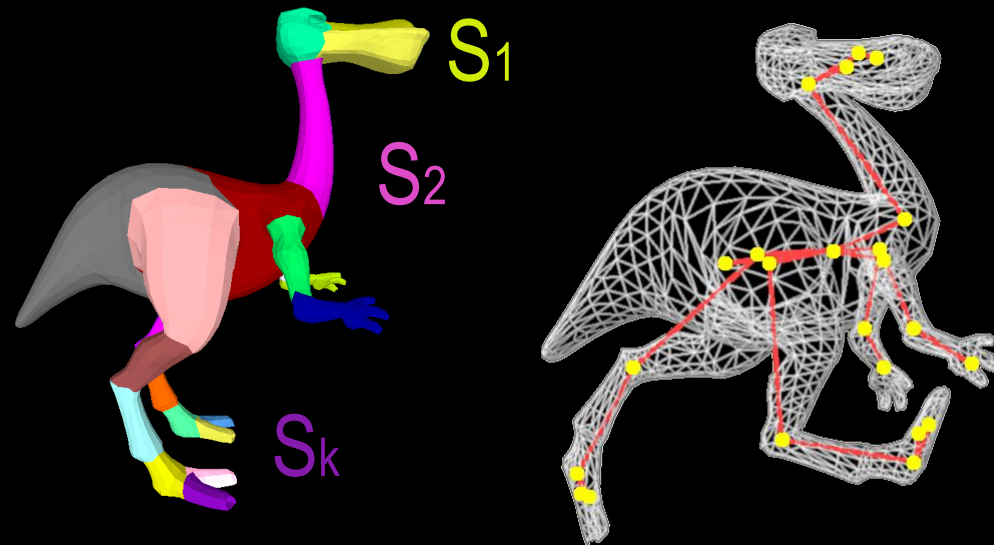
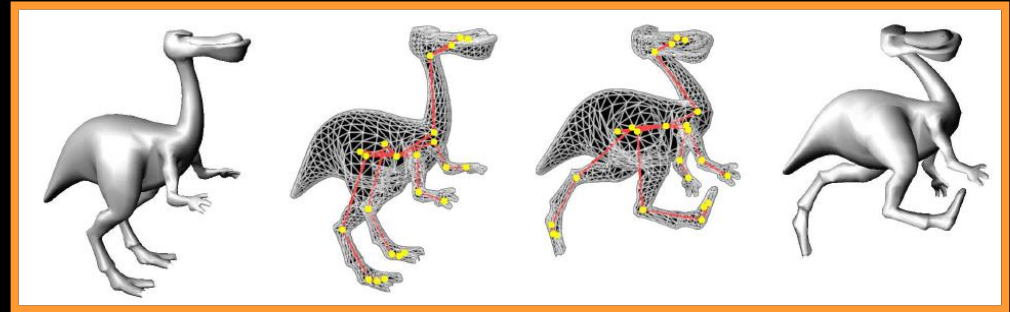


# Motivation



## Applications:

- Analysis
- Representation
- Recognition
- Collision detection
- Animation
- Modeling
- etc.





# Motivation



## Applications:

- Analysis
- Representation
- Recognition
- Collision detection
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- **Modeling**
- etc.

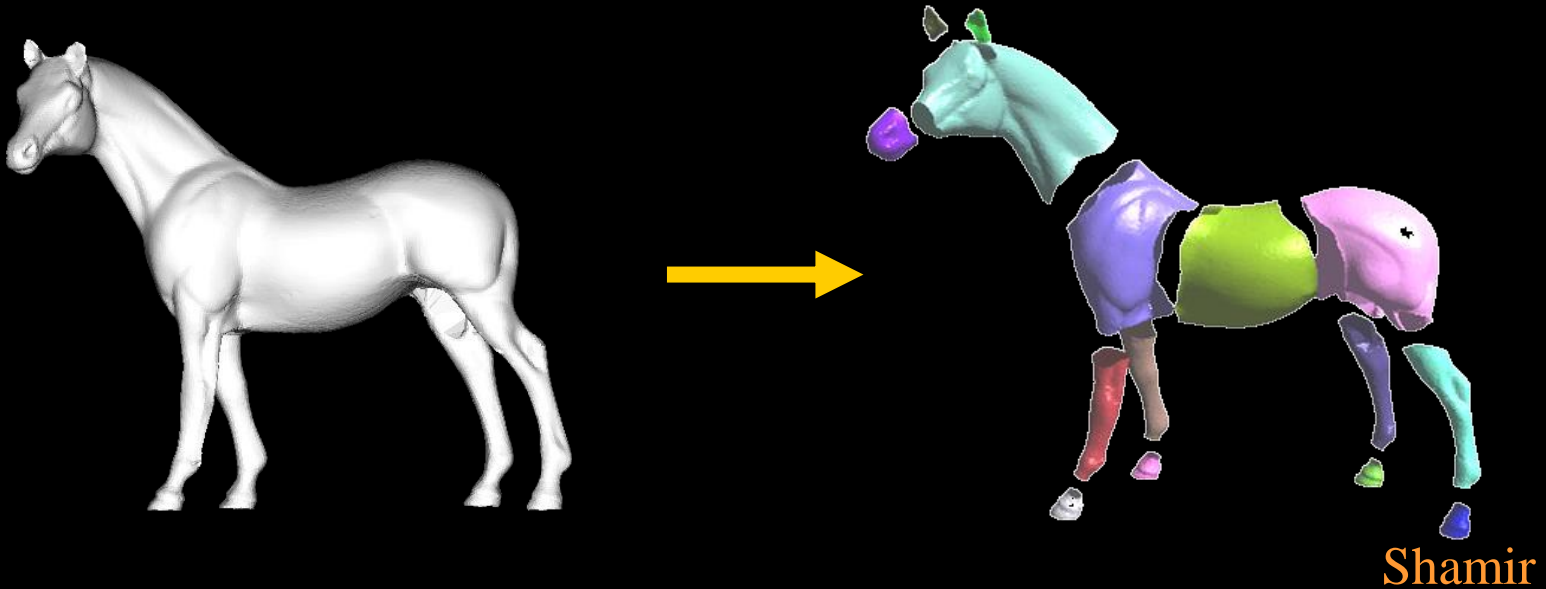




# Problem Statement

Optimization formulation:

- Given: a mesh  $M = \{V, E, F\}$
- Create: a set  $S$  of submeshes  $M_i$  that partition the faces of  $M$  into disjoint subsets that minimize an objective function  $J$  under a set of constraints  $C$



# Outline



Constraints

Objective function

Algorithmic strategies

Evaluation

# Constraints



## Cardinality

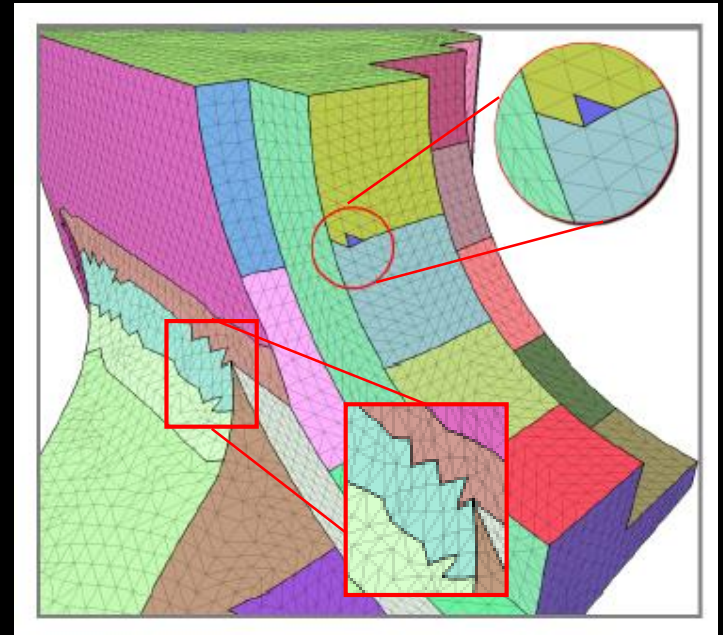
- Not too small and not too large or a given number (of segment or elements)
- Overall balanced partition

## Geometry

- Size: area, diameter, radius
- Convexity, Roundness
- Boundary smoothness

## Topology

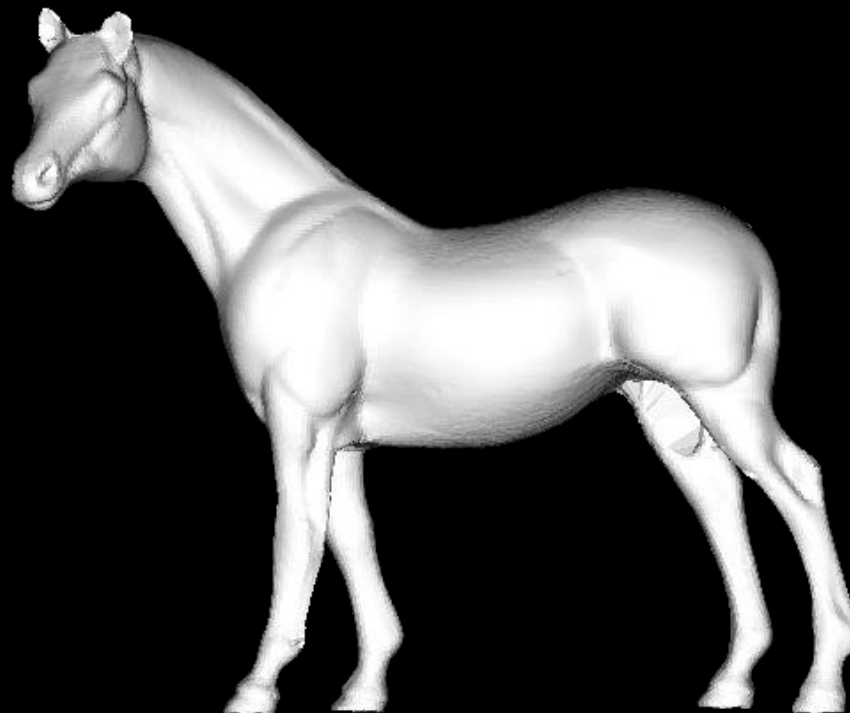
- Connectivity (single component)
- Disk topology



# Objective Function



Object function  $J$  says how "good" a segmentation is ...



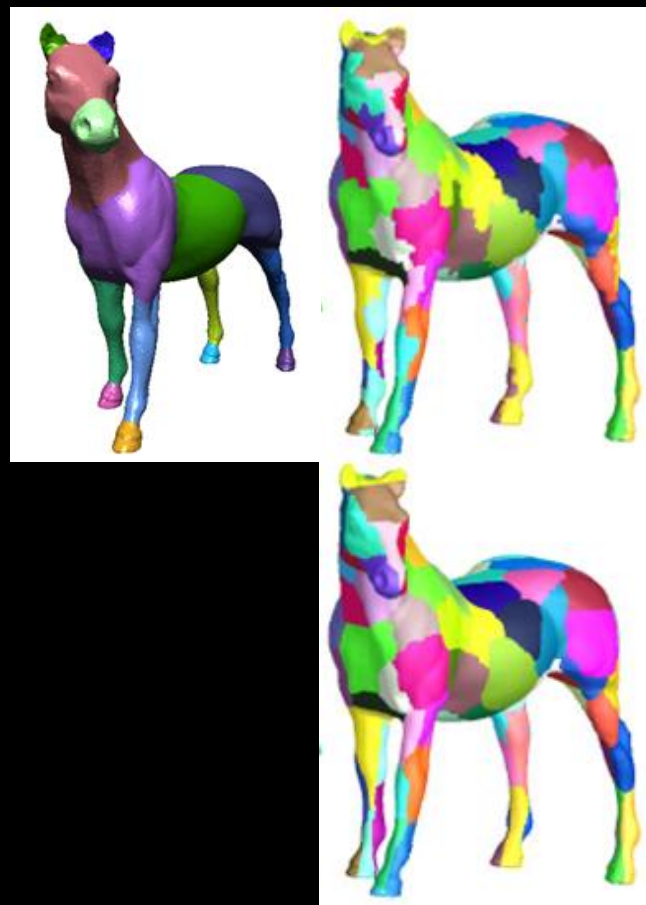
What properties define a good segmentation of this horse?

# Objective Function



Object function  $J$  says how "good" a segmentation is ...

- Number of segments?
- Surface properties?
- Boundary properties?
- Global shape properties?
- Match examples?
- Semantics?
- etc.

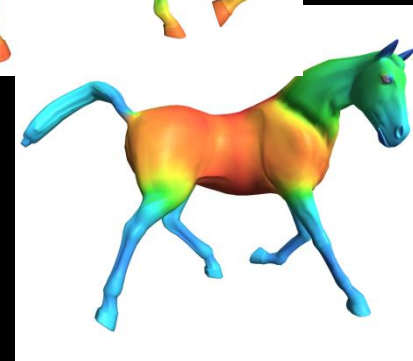
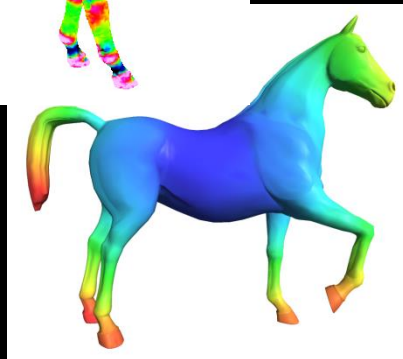
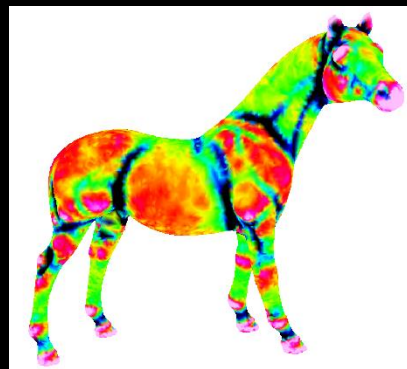


# Objective Function



Mesh attributes to consider:

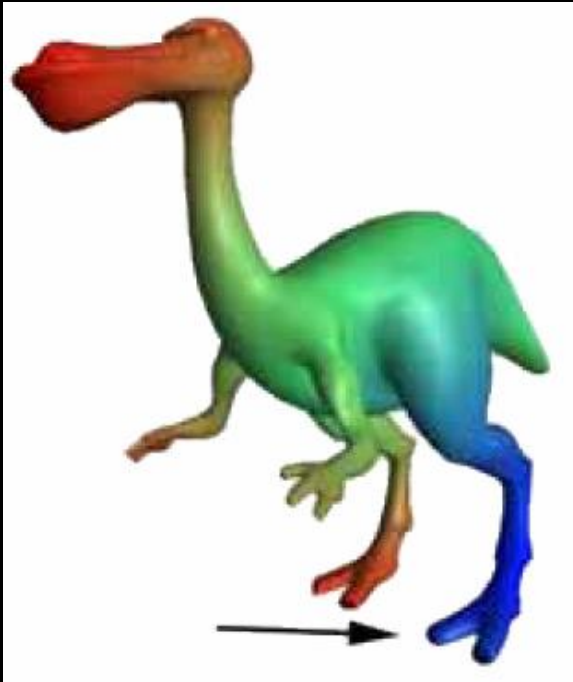
- Distances
- Normal directions
- Smoothness, curvature
- Shape diameter
- Distance to proxies
- Convexity
- Symmetry
- etc.



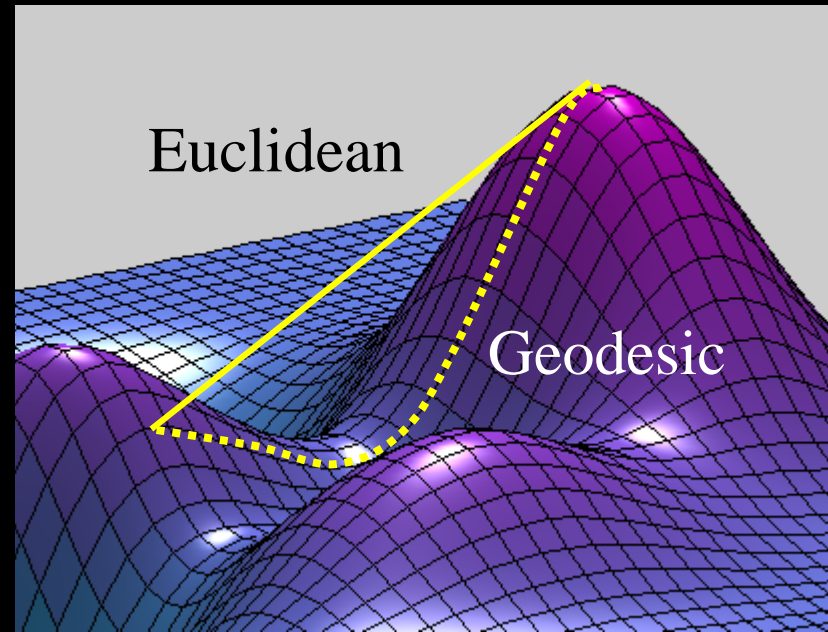


# Distances

Triangles in same segment ought to be close



Geodesic distance to point



Geodesic vs. Euclidean distance

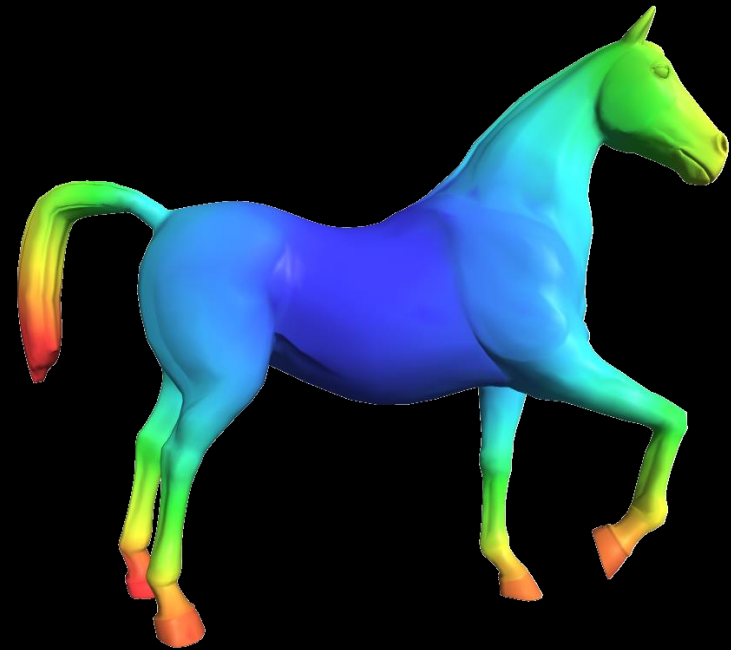




# Distances

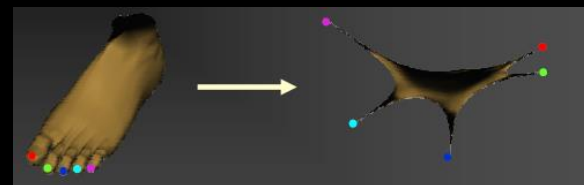
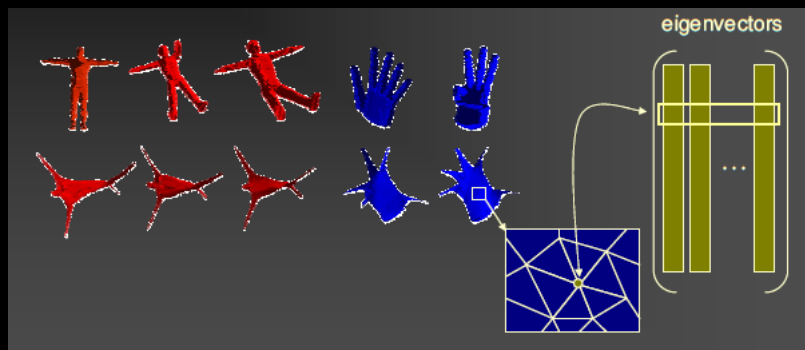
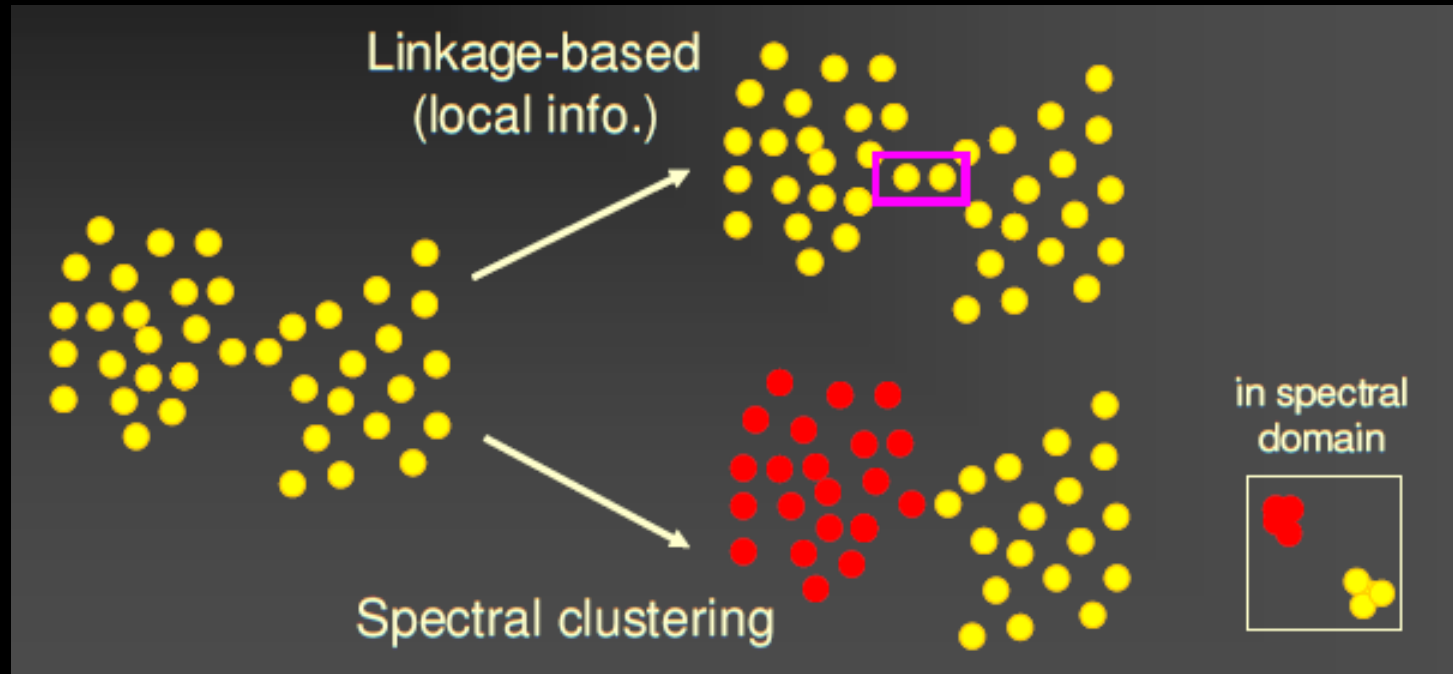
Triangles in same segment ought to be close

Discontinuities in functions of distance  
indicate possible boundaries



Average geodesic distance to other points

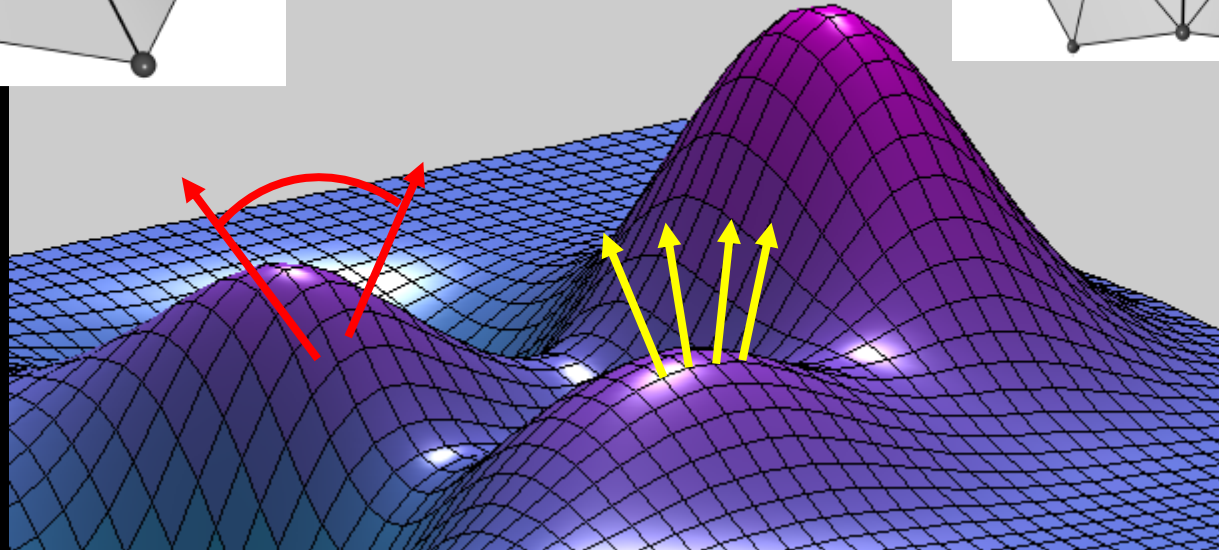
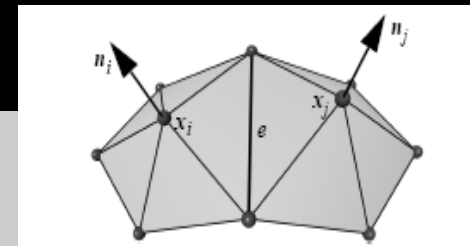
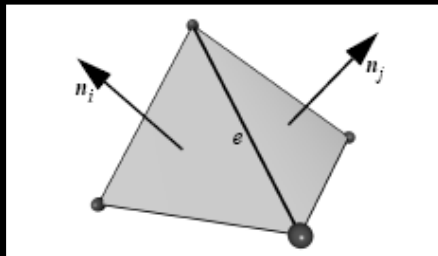
# Distances with Spectral Embedding





# Normal direction, Dihedral Angles

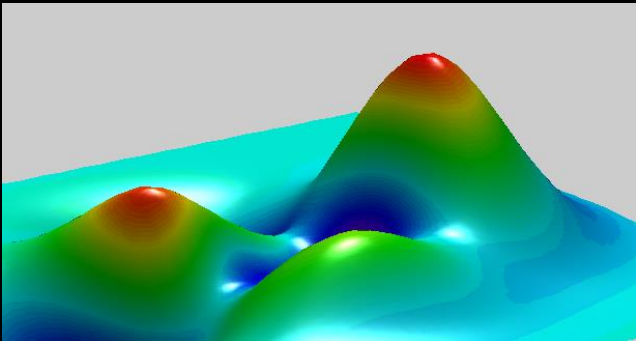
Triangles in same segment ought to have normals that are: similar (planar)?, continuous (no creases)?





# Smoothness, Curvature

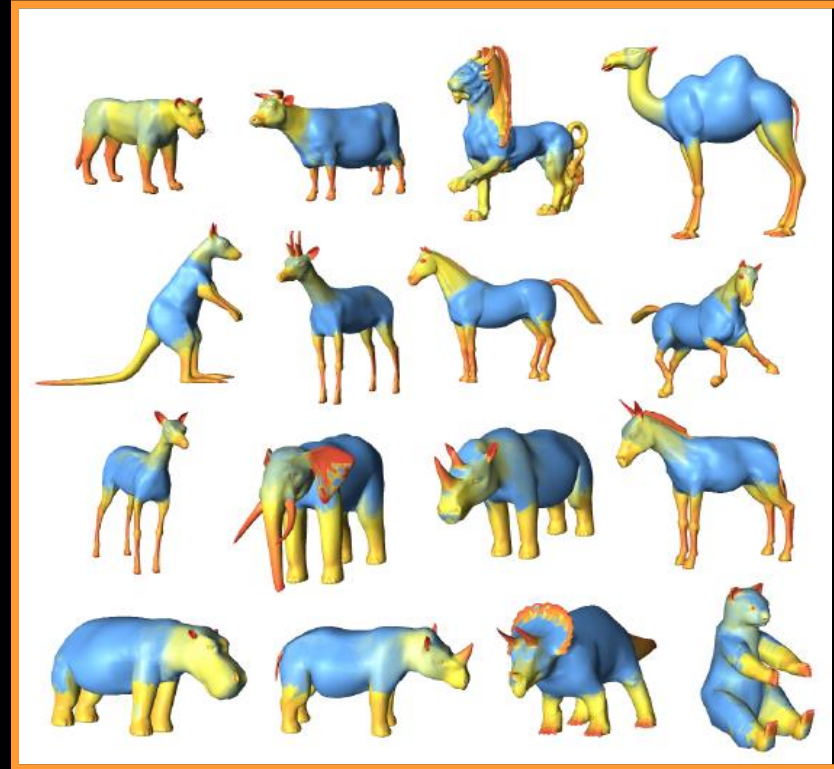
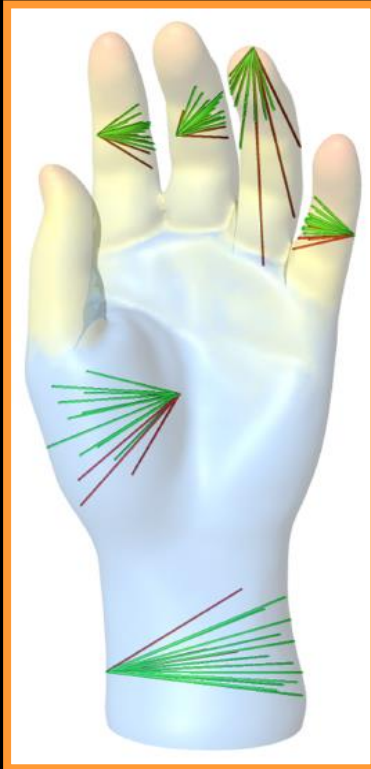
Concave creases indicate good segmentation boundaries





# Diameter

Distinguish between thin and thick parts in a model

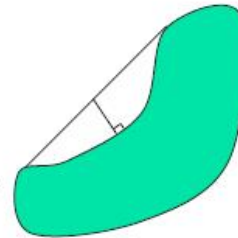




# Convexity

Parts generally should be convex and compact

$$\text{Convexity} = \frac{\sum_{t \in P} \text{dist}(t, C(P)) \cdot \text{area}(t)}{\sum_{t \in P} \text{area}(t)},$$



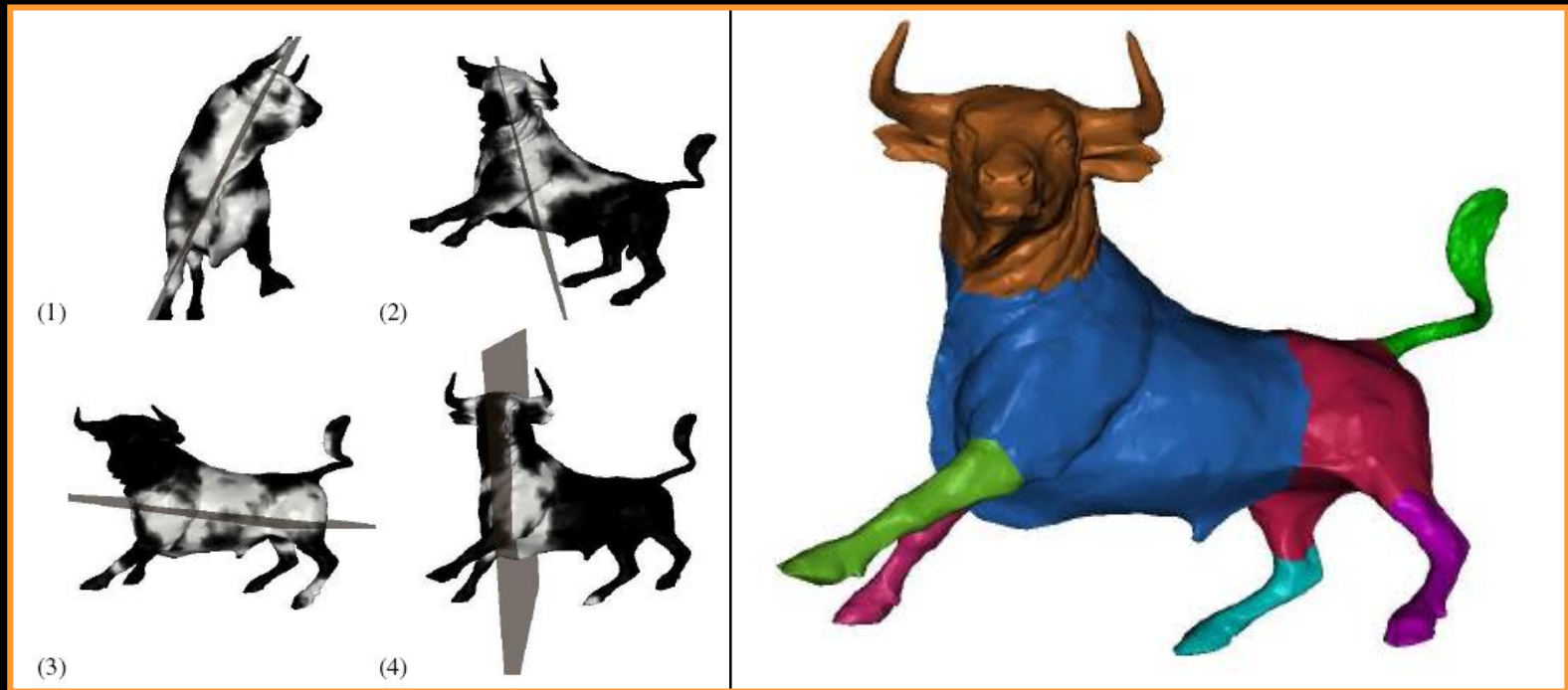
$$\text{Compactness} = \frac{\text{area}(C)}{\text{volume}(C)^{2/3}}$$



# Symmetry



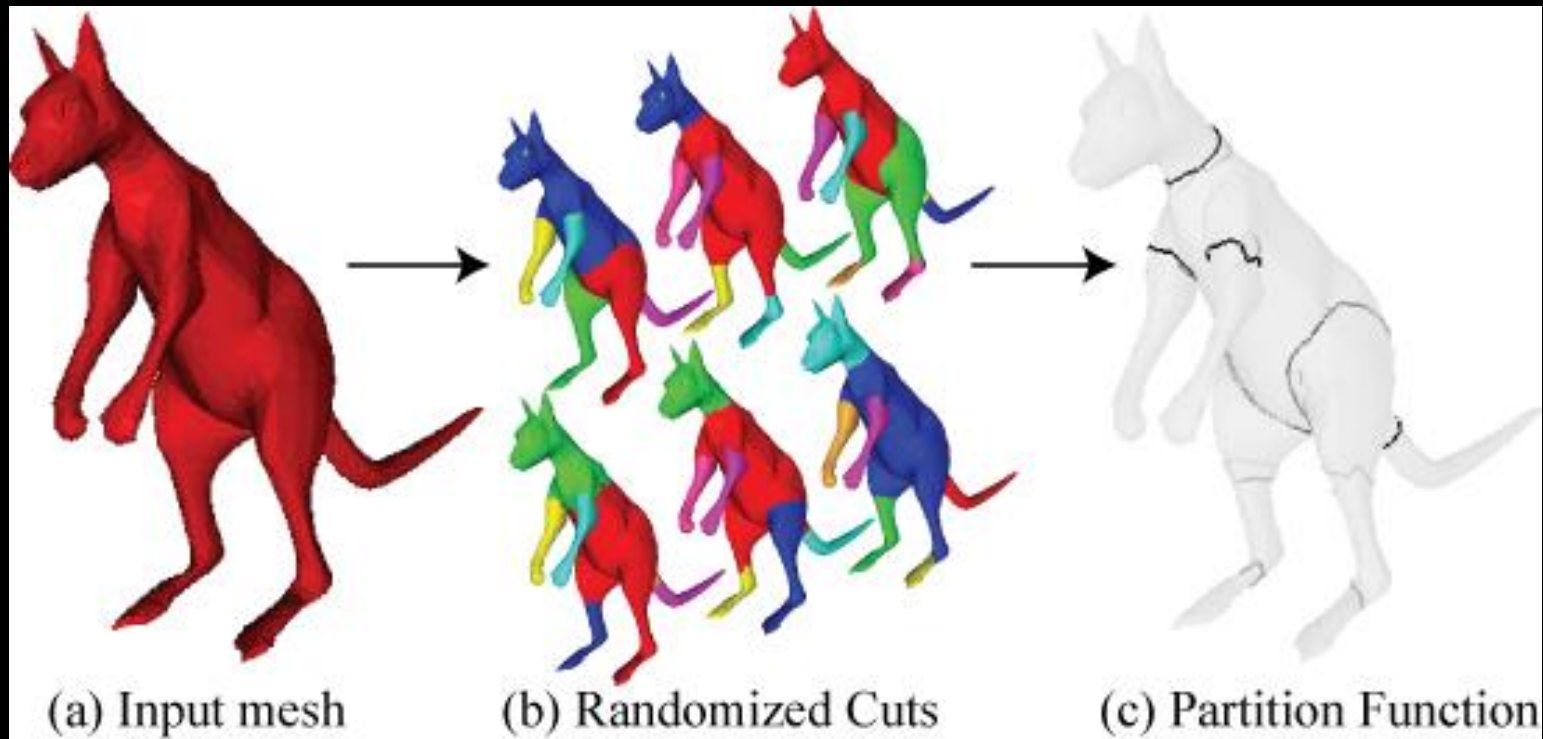
Segments should be locally symmetric



# Combining many properties



## Randomized cuts





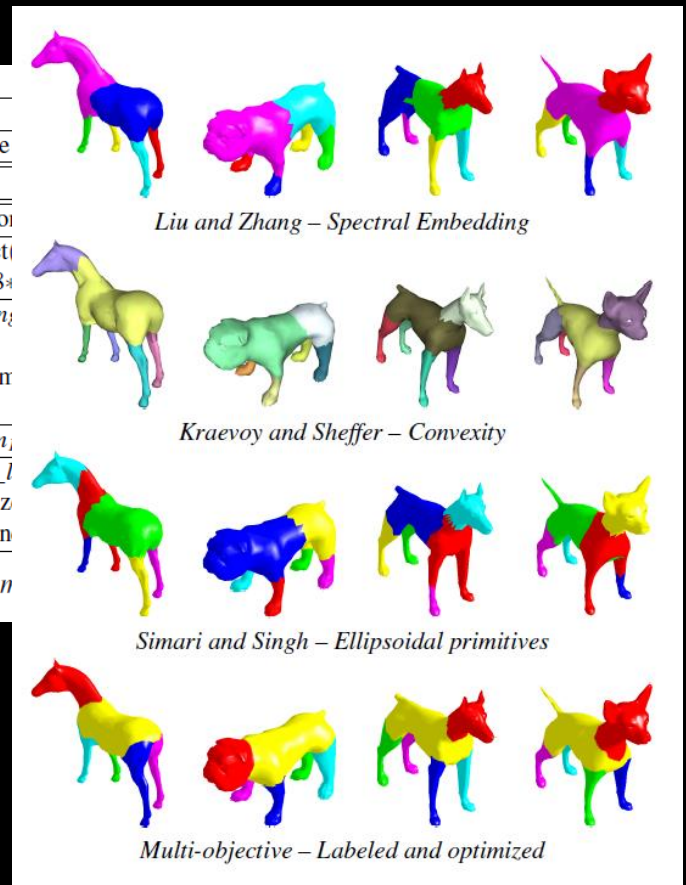
# Segmenting and Labeling



## Multi-objective mesh segmentation

Summary of objectives used		
narrow( $\cdot$ ), flat( $\cdot$ ), planarsymmetric( $\cdot$ ), ellipsoidal( $\cdot$ ), perpendicular( $\cdot$ , $\cdot$ ), similarsize		
Model	Labels	Segmentation objectives
Hammer	$handle, head$	$5 * narrow(handle)$ , perpendicular( $handle, head$ ), compact
Quadruped	$head, body, leg_1, \dots, leg_4$	$\forall_i narrow(leg_i)$ , similarsize( $leg_1, \dots, leg_4$ ), compact In dogs, compactness of head is emphasized with $8 * compact(head)$
Bird	$body, wing_1, wing_2, tail$	narrow( $body$ ), $\forall_i flat(wing_i)$ , similarsize( $wing_1, wing_2$ ), compact( $tail$ ), $10 * convexparts(Seg)$ Constraints: body and tail lie on plane of global symmetry are reflected from $wing_1$ .
Octopus	$head, arm_1, \dots, arm_8$	ellipsoidal( $head$ ), $\forall_i narrow(arm_i)$ , similarsize( $arm_1, \dots, arm_8$ )
Humanoid	$head, torso, arm\_left, arm\_right, leg\_left, leg\_right$	narrow( $arm\_left$ ), narrow( $arm\_right$ ), narrow( $leg\_left$ ), narrow( $leg\_right$ ), compact( $head$ ), similarsize( $arm_1, arm_2$ ), similarsize( $leg_1, leg_2$ ) Subparts (upper arm, forearm, hand, etc.) are obtained

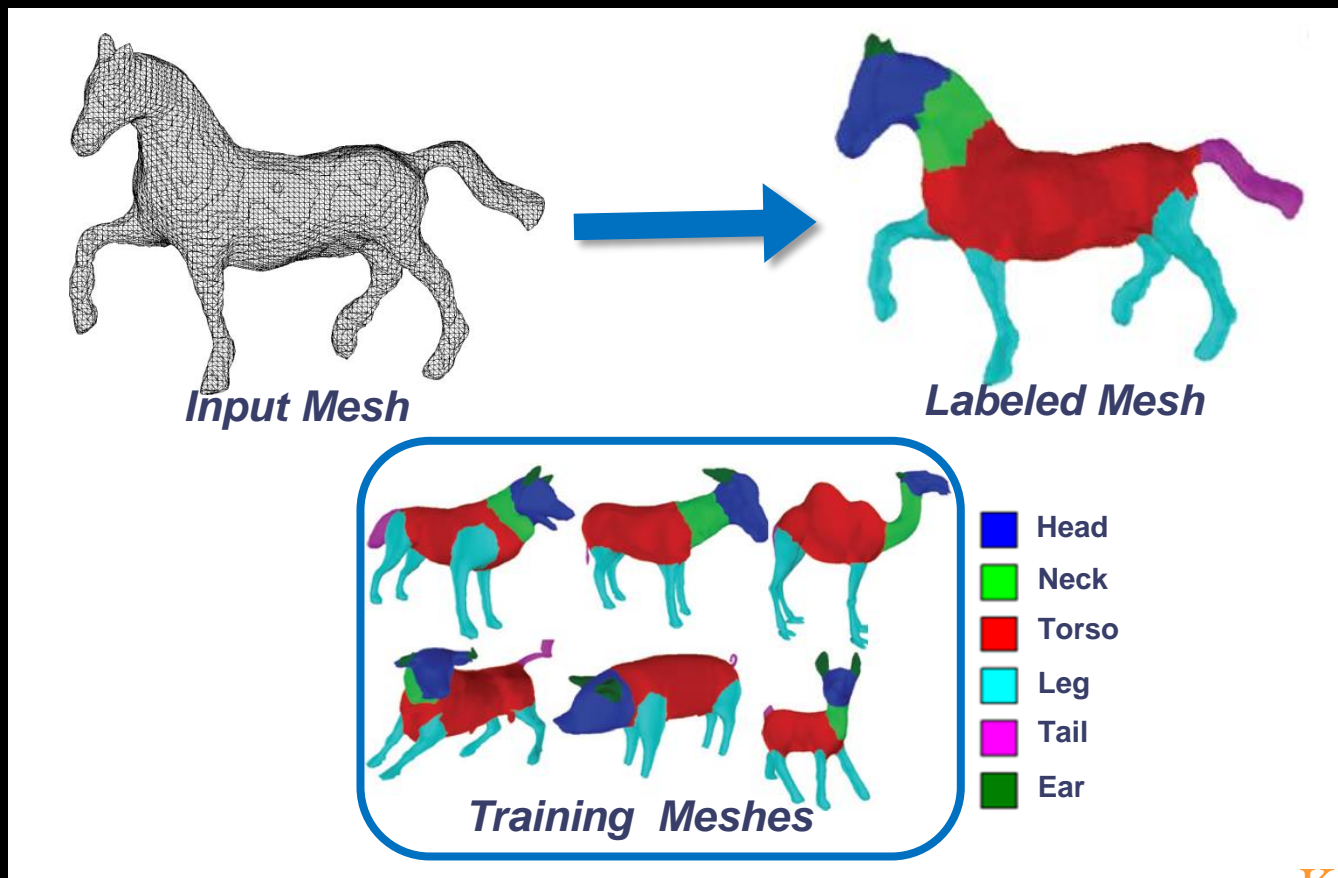
Figure 13: Objectives used to obtain segmentations of each model





# Segmenting and Labeling

Use conditional random field to learn segments and labels based on examples





# Outline

Constraints

Objective function

**Algorithmic strategies** ←

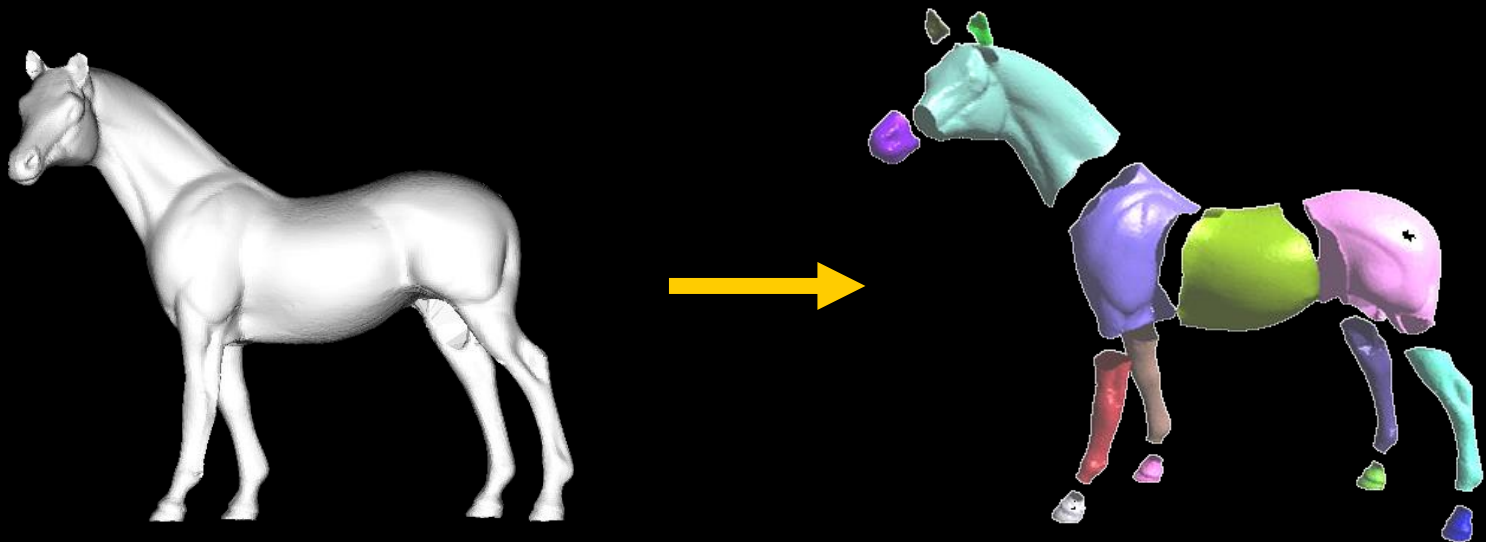
Evaluation

# Algorithmic Strategies



Segmentation problem:

- Given: a mesh  $M = \{V, E, F\}$
- Create: a set  $S$  of submeshes  $M_i$  that partition the faces of  $M$  into disjoint subsets.





# Algorithmic Strategies

If  $|M| = n$  and  $|S| = k$ , then the search space of possible mesh decompositions is of order  $k^n$ .

- NP-complete
- Must revert to approximation algorithm



# Segmentation as Clustering

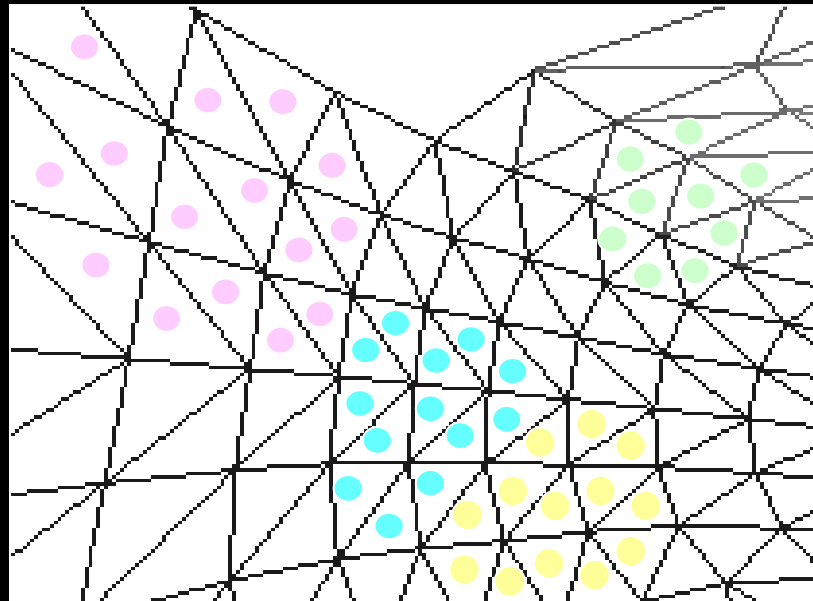
The basic segmentation problems can be viewed as assigning primitive mesh elements to sub meshes

- Clustering problem
- Well-studied in machine learning

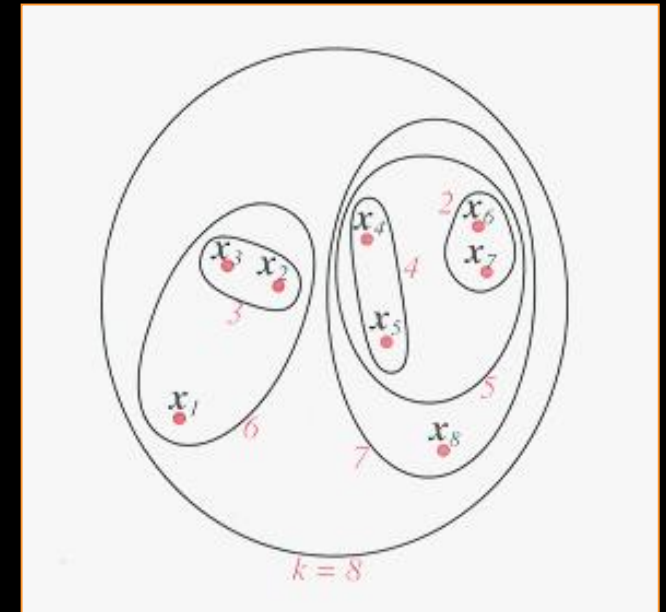
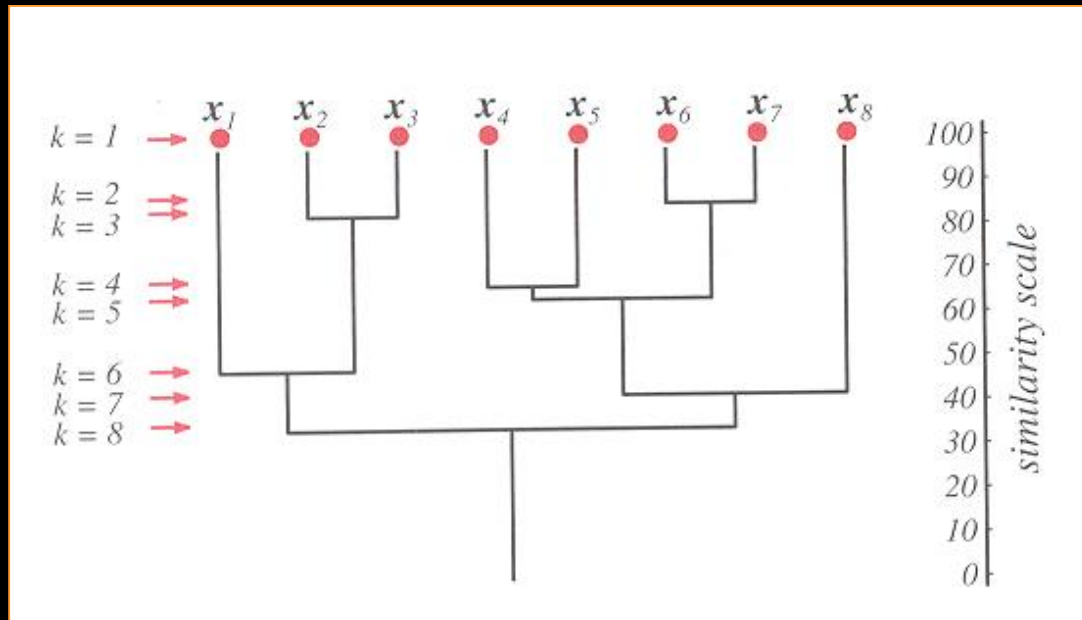
Most segmentation strategies have their basis in classic clustering algorithms:

- Region growing (local greedy)
- Primitive fitting (model-based)
- Hierarchical clustering (global greedy)
- K-means (iterative)
- Graph Cut

# Region Growing

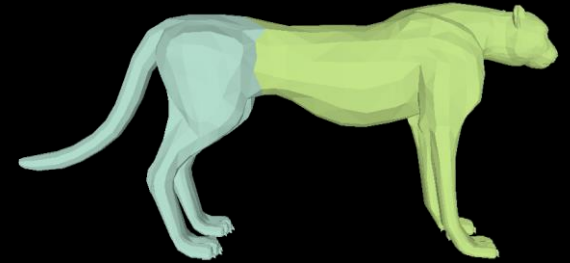
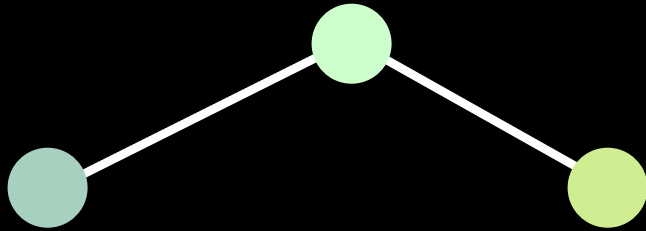


# Hierarchical Clustering

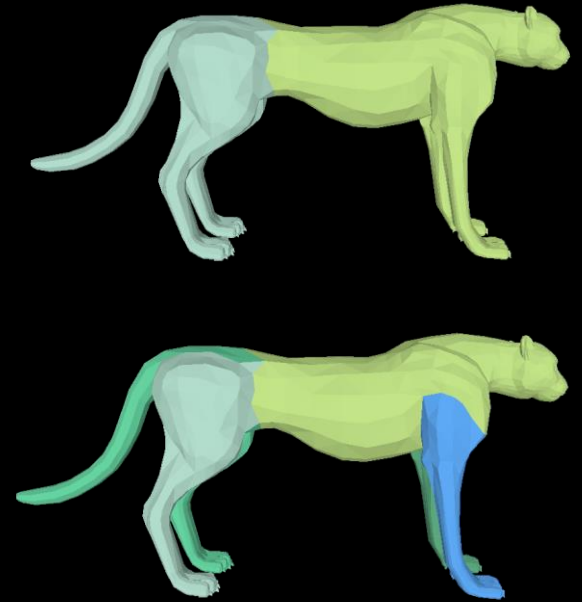
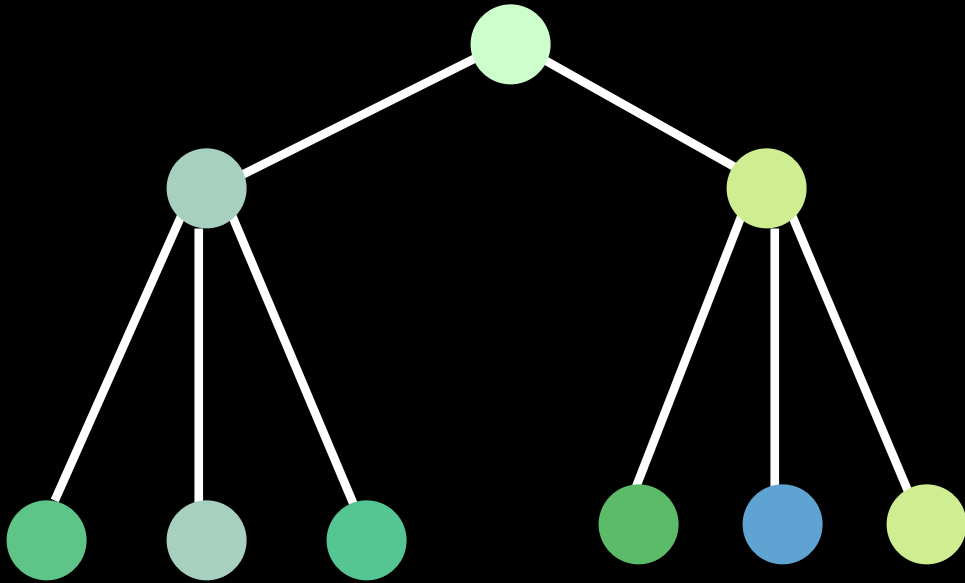




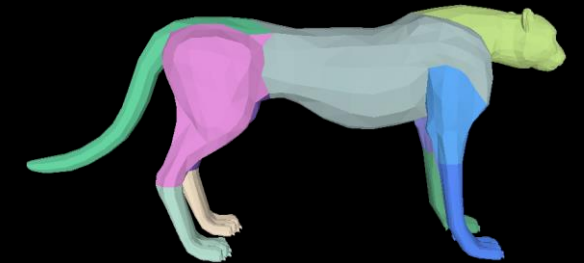
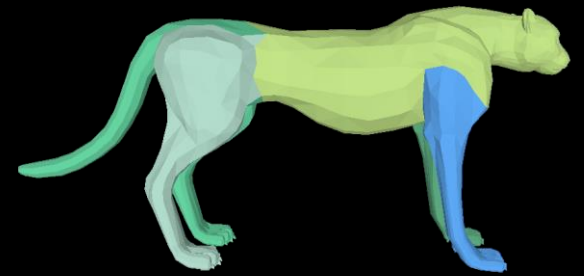
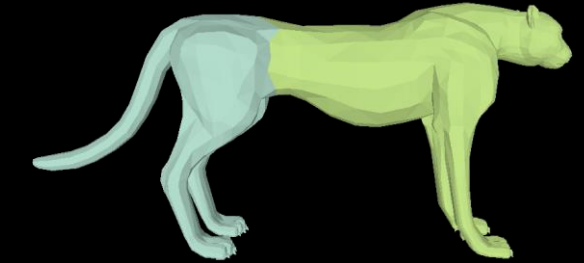
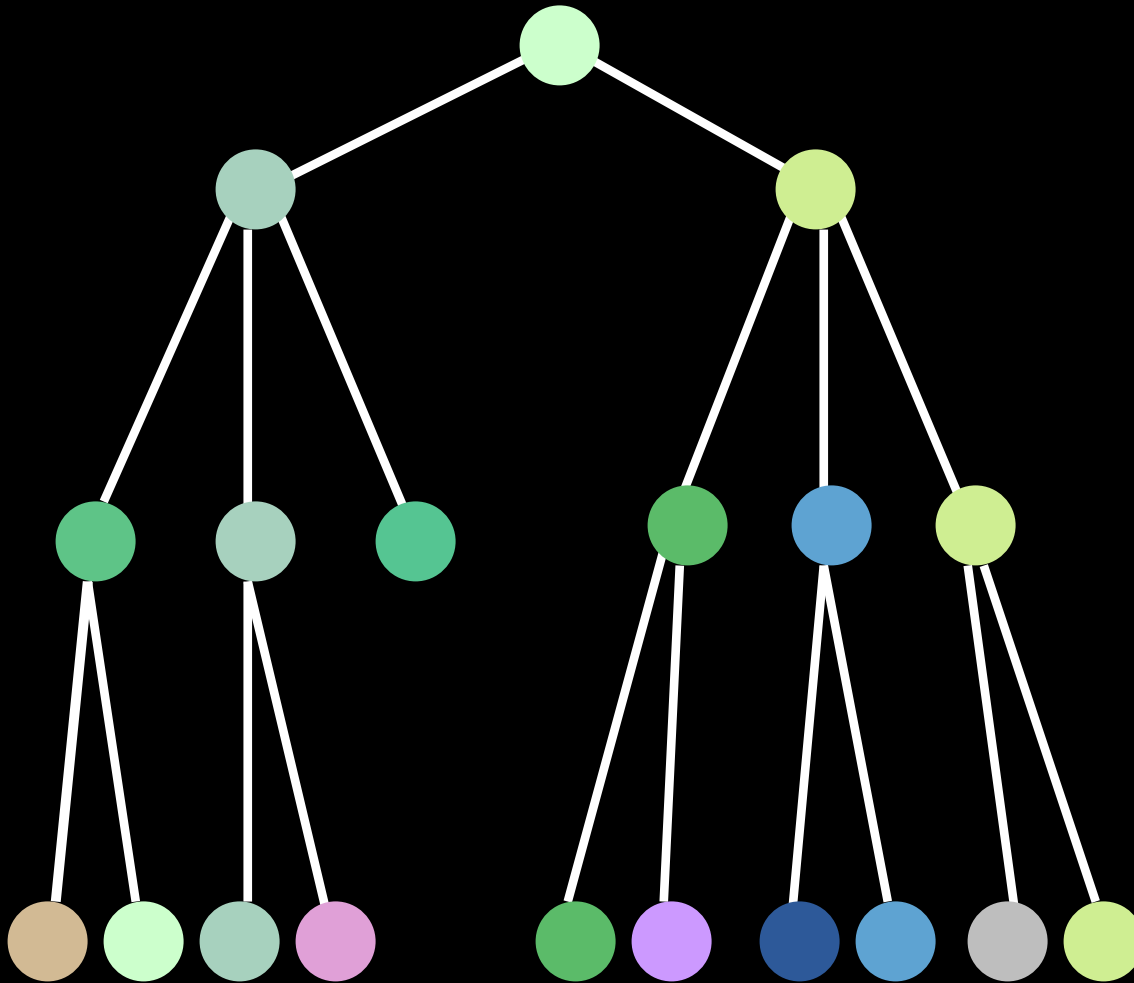
# Hierarchical Clustering



# Hierarchical Clustering



# Hierarchical Clustering



Katz

# Lloyd (k-means)



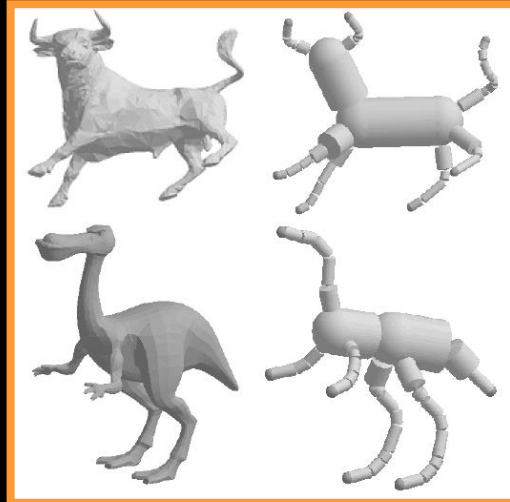


# Primitive Fitting

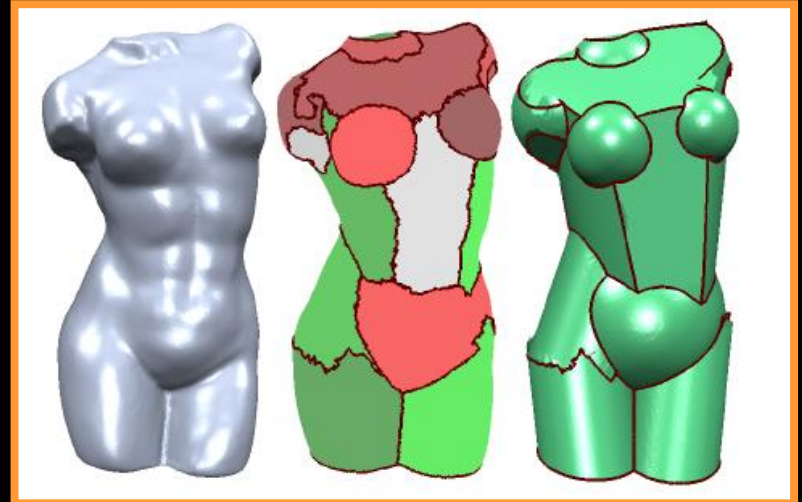
Find set of primitives that best approximates shape and map triangles to primitives



Planes



Cylinders

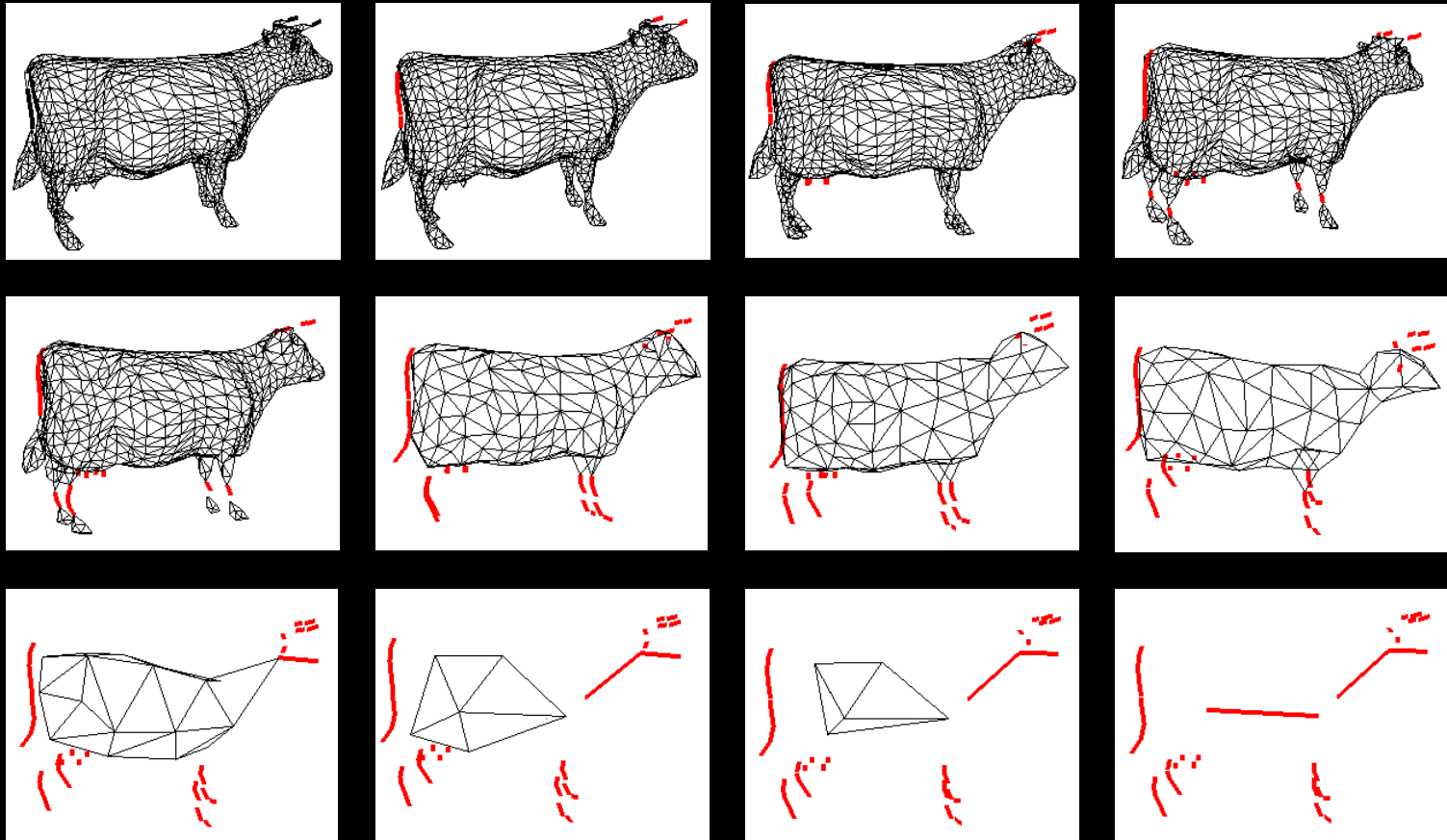


Spheres, cylinders, & rolling ball surfaces

# Simplification



## Iterative edge collapses

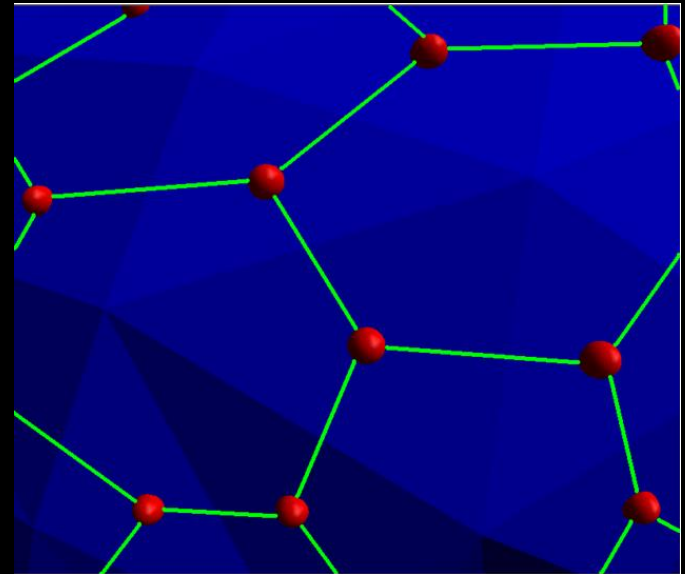




# Graph Cuts

Define a graph where each node is an element and the edges hold weights according to the distances between the elements.

Example: dual graph and the weight is the dihedral angle.

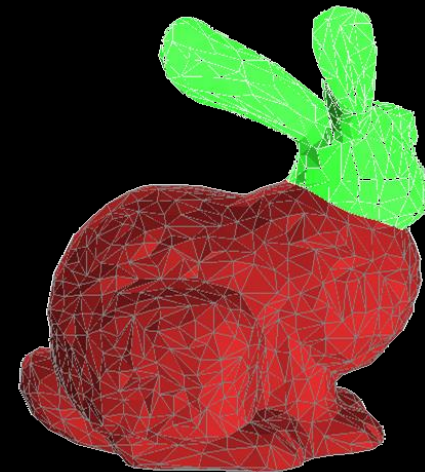
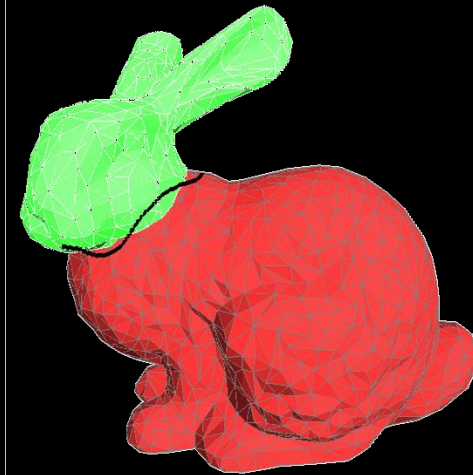
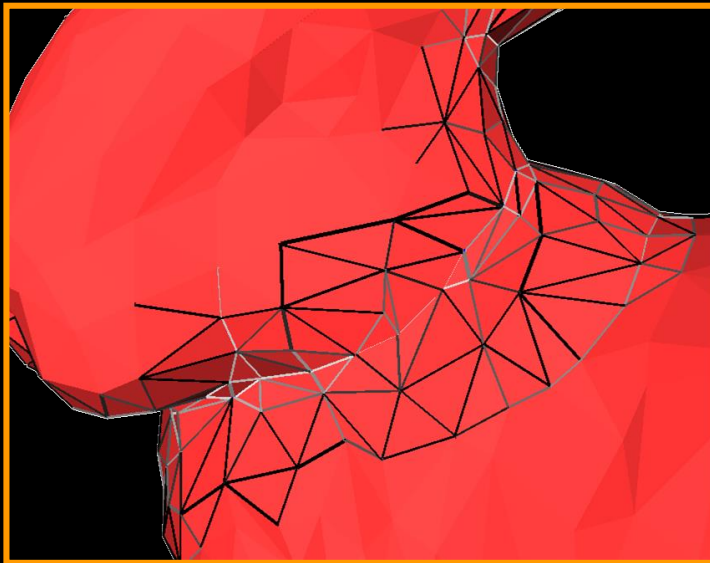




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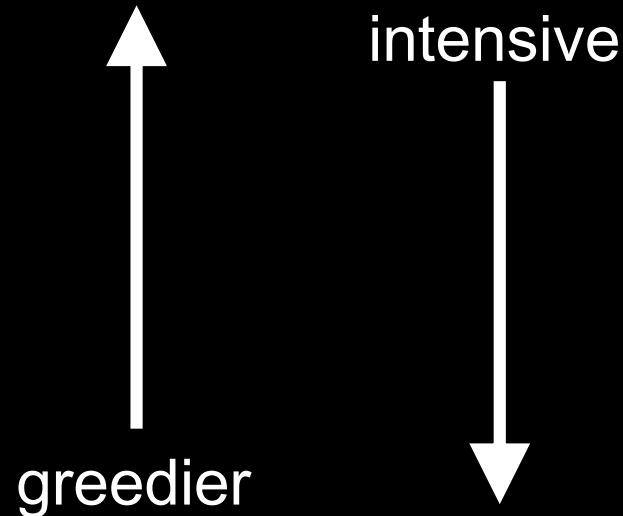


# Comparison of Strategies



## Strategies

- Region growing
- Hierarchical
- Iterative
- Graph cut

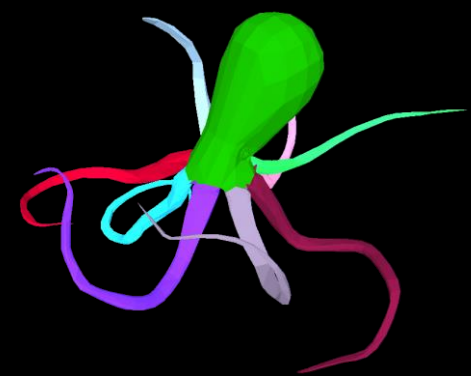


Other considerations: local control, hierarchy, convergence, parametric vs. non parametric...

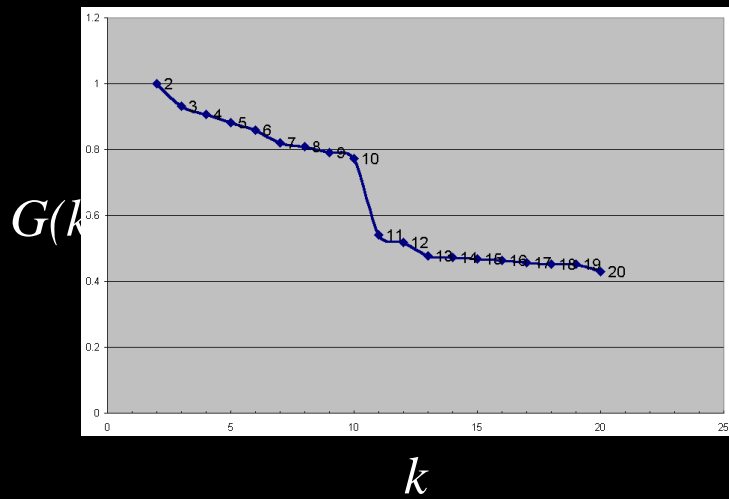
# Choosing the Number of Segments



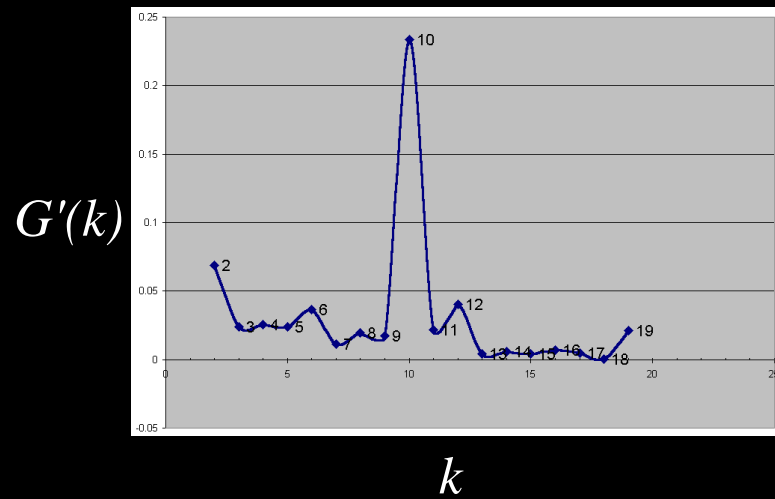
$$G(k) = \min_{i < k} (Dist(REP_k, REP_i))$$



G



G'





# Outline

Constraints

Objective function

Algorithmic strategies

**Evaluation** ←

# Benchmark for Mesh Segmentation

