Introduction to Shape Analysis

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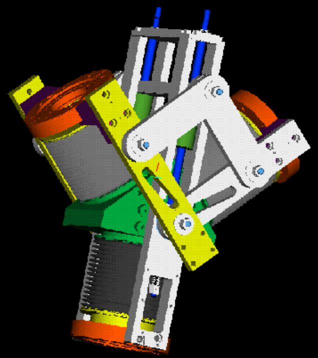
Princeton University

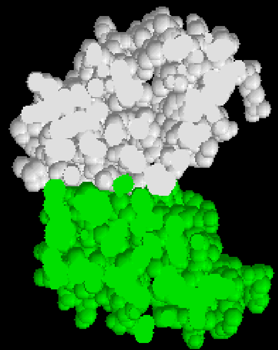
COS 526, Fall 2014

Motivation

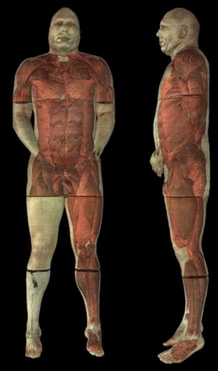
Large repositories of 3D data are becoming available

Computer Graphics





Molecular Biology



Medicine

Cultural Heritage

Computer Vision

Lecture Outline

Introduction

Applications

Problems

- Feature detection

Lecture Outline

Introduction

Applications ←

Problems

- Feature detection

Applications

Examples:

- Computer graphics
- Geometric modeling
- Archaeology
- Urban planning
- Paleontology
- Molecular bio
- Medicine
- Art

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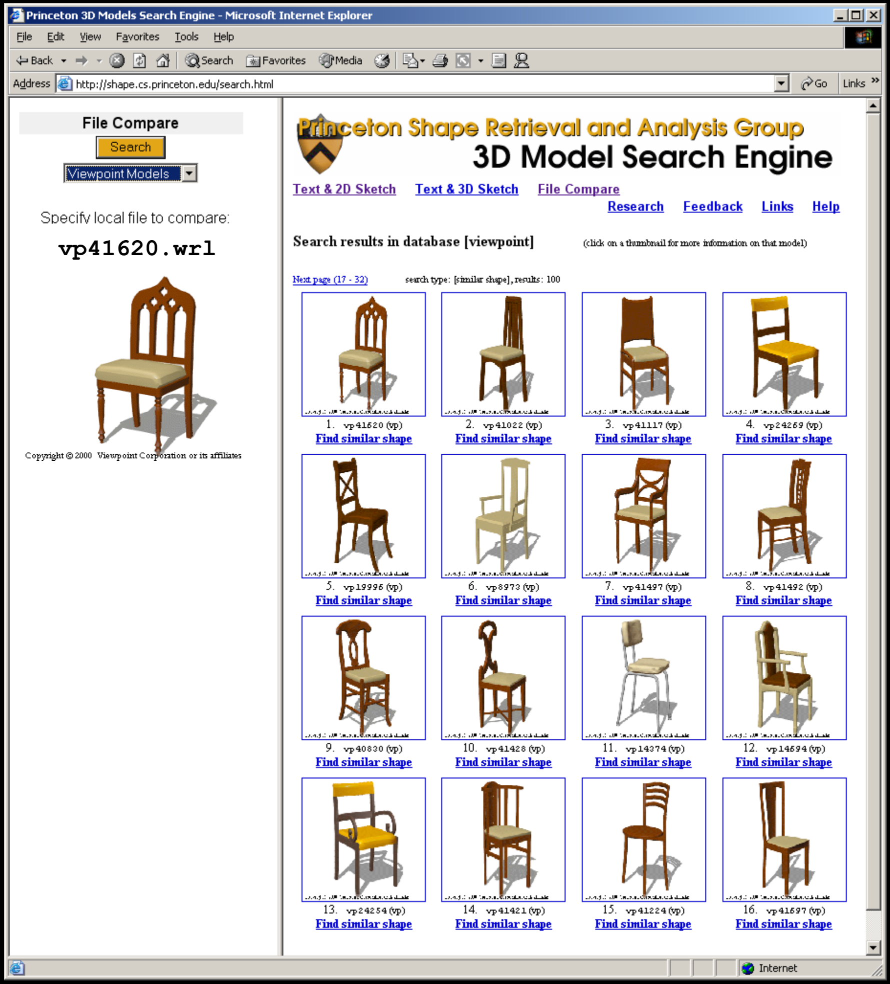

Applications

Examples:

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Applications

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Reconstructing Frescoes from Thera
(Weyrich, Brown, Rusinkiewicz, et al.)

Applications

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Applications

Images courtesy of
Delson & Freiss

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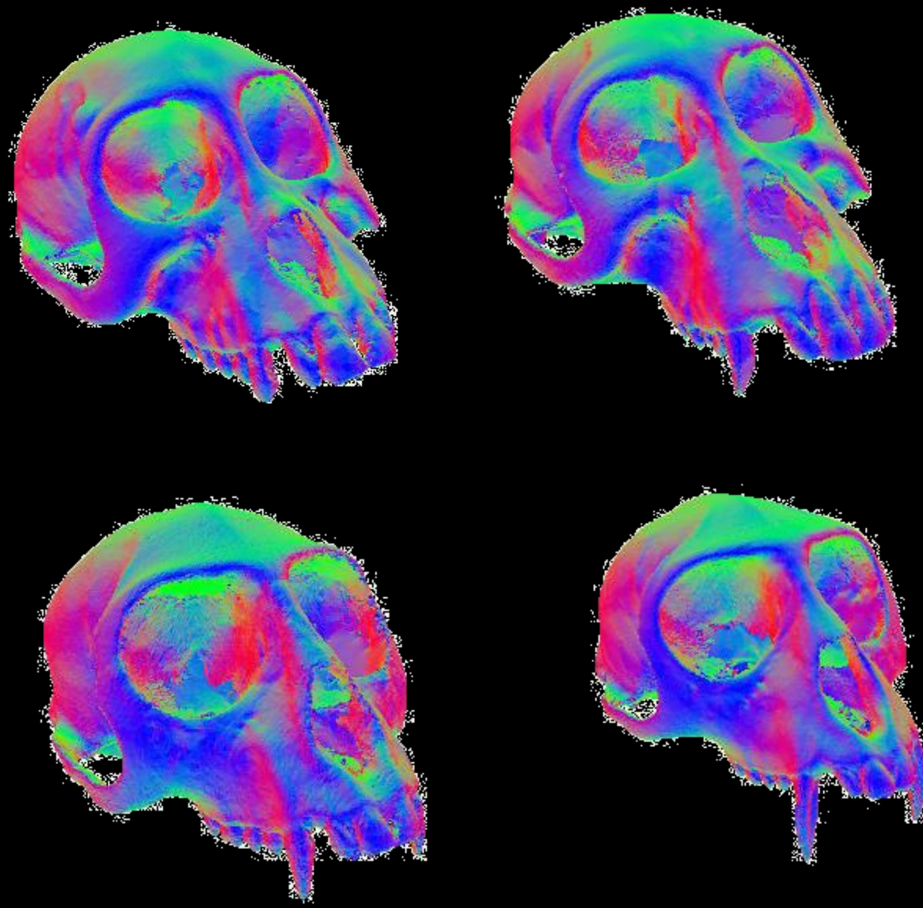

Applications

Image courtesy of
Ilya Vakser, GRAMM

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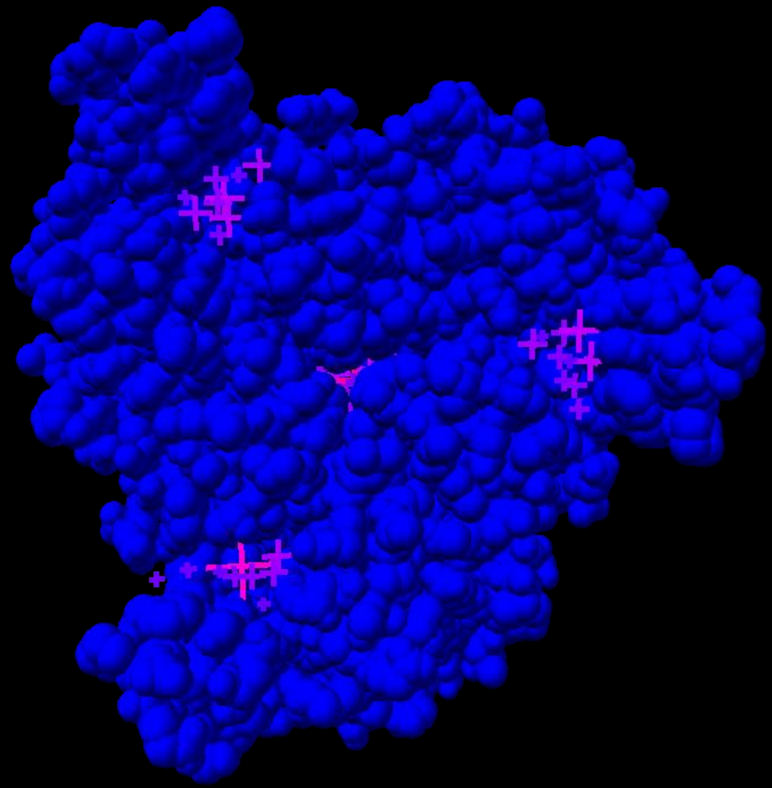

Applications

Image courtesy of
Polina Golland, MIT

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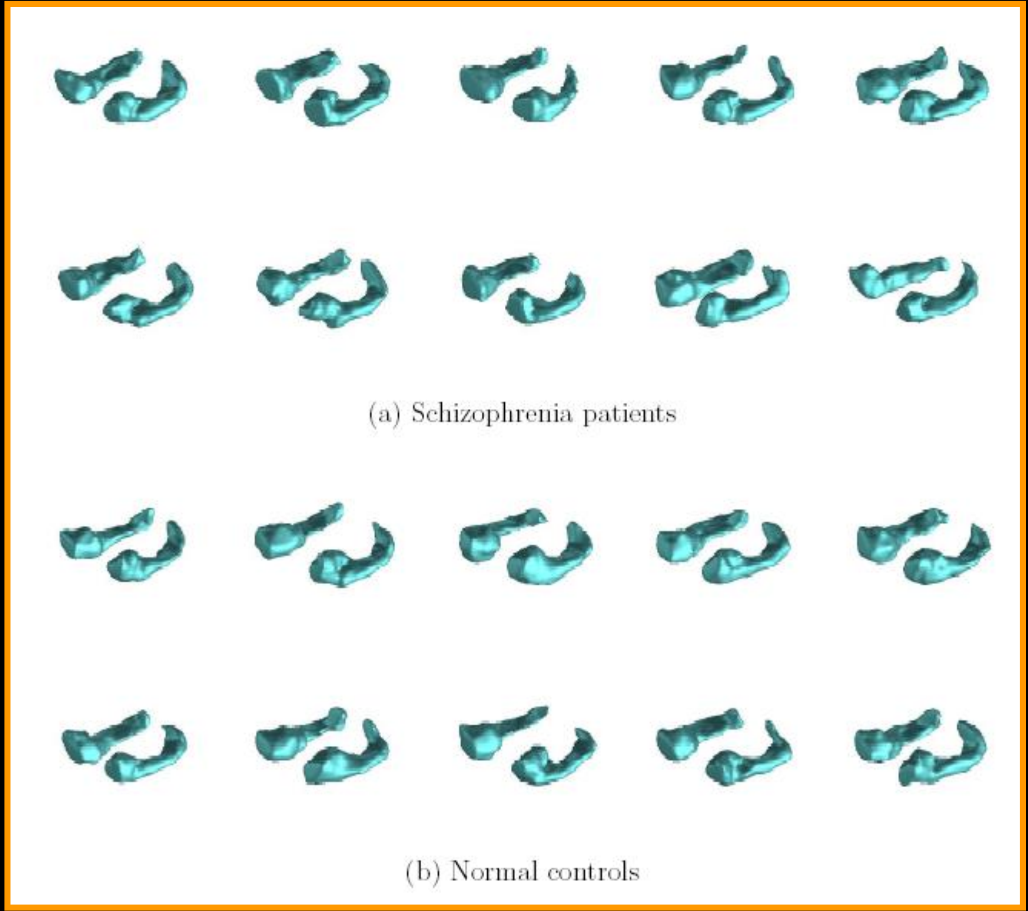

Hippocampus-amygdala study in schizophrenia

Applications

Images courtesy of Stanford University

Examples:

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Lecture Outline

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- Feature detection

Shape Analysis Problems

Examples:

- Feature detection
- Segmentation
- Labeling
- Registration
- Matching
- Recognition
- Classification
- Clustering
- Retrieval

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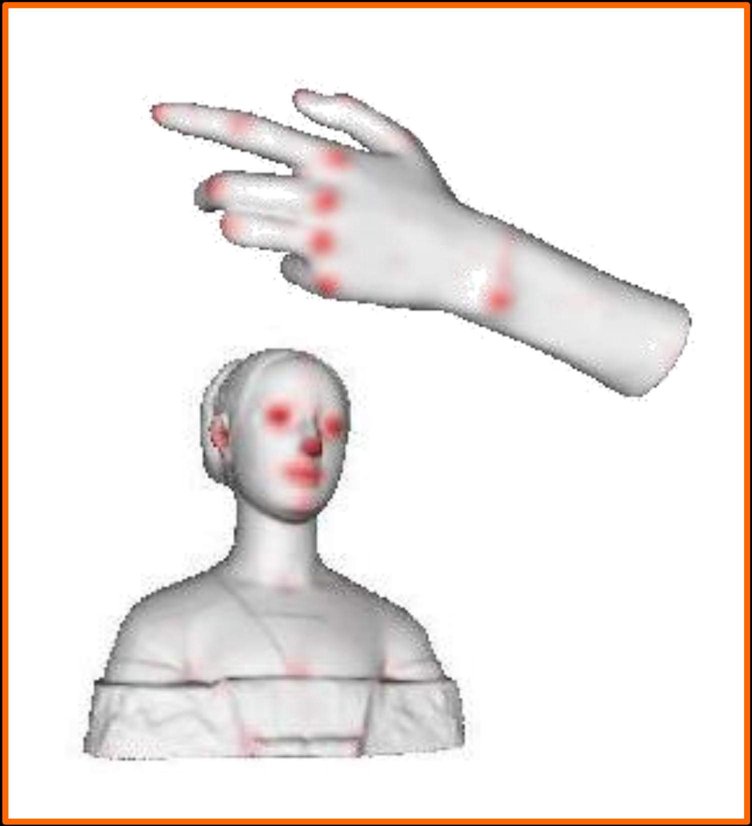

Schelling Points

"How can we find significant geometric features robustly?"

Shape Analysis Problems

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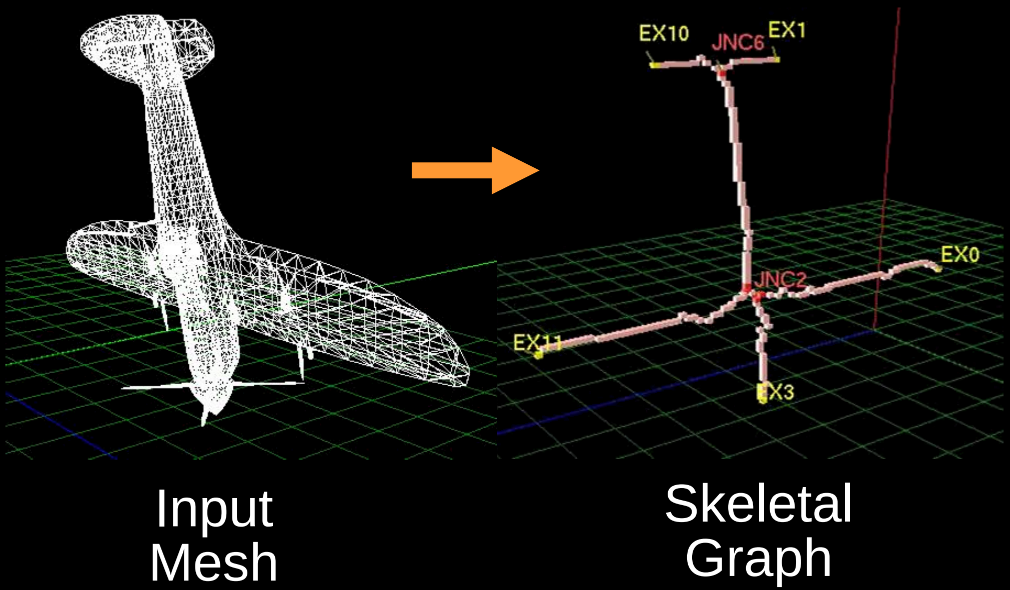

"How can we decompose a 3D model into its parts?"

Shape Analysis Problems

Images courtesy of
Ayellet Tal, Technion &
Princeton University

Examples:

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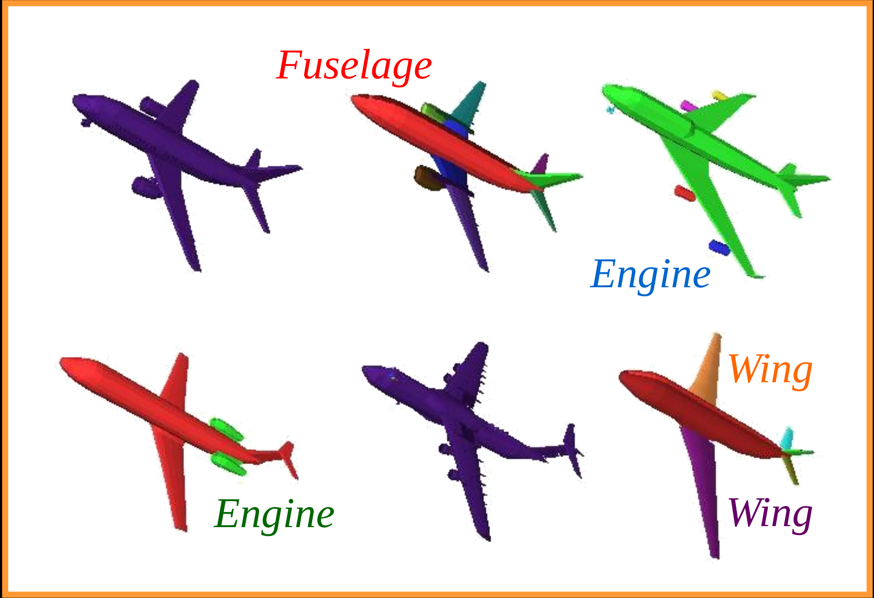

Semantic Labels

(Golovinskiy, Lee, et al.)

"How can we decompose a 3D model into its parts?"

Shape Analysis Problems

Images courtesy of
Emil Praun

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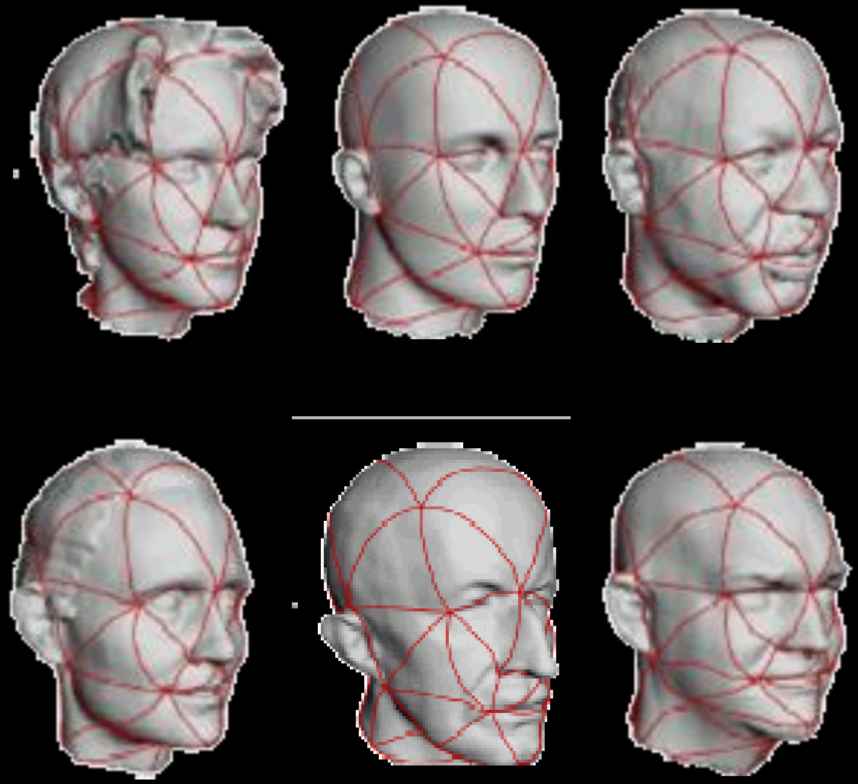

"How can we align features of 3D models?"

Shape Analysis Problems

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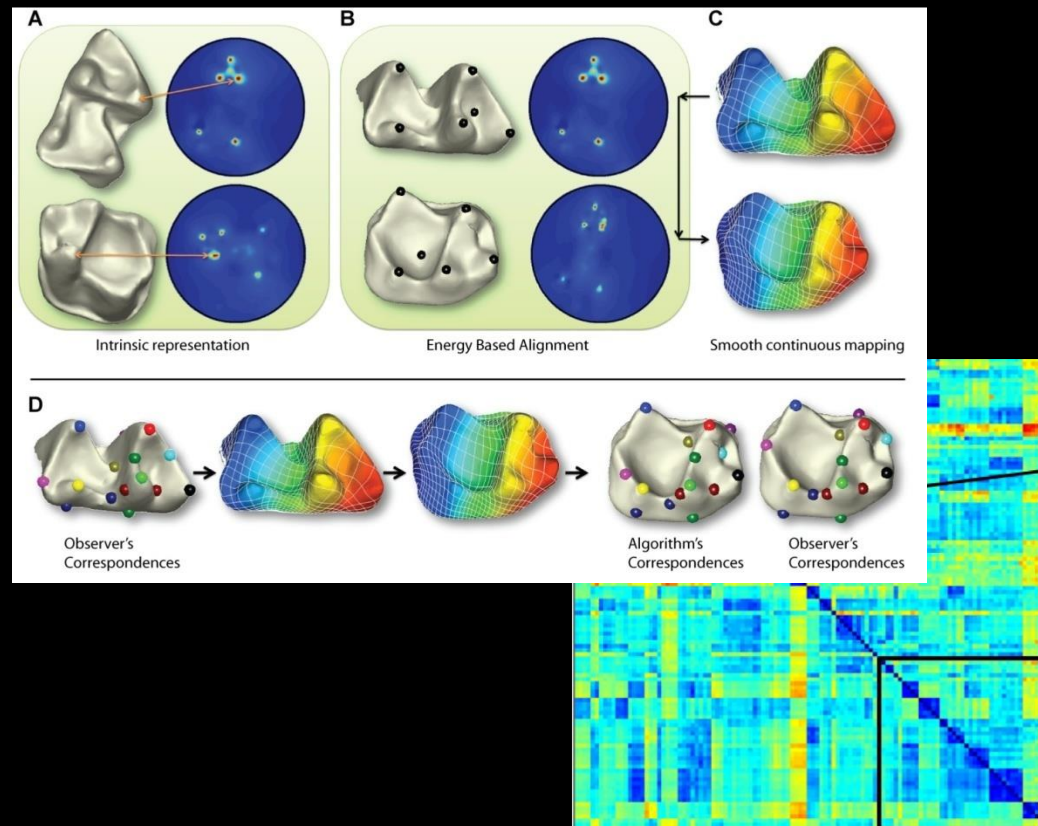

"How can we compute a measure of geometric similarity?"

Shape Analysis Problems

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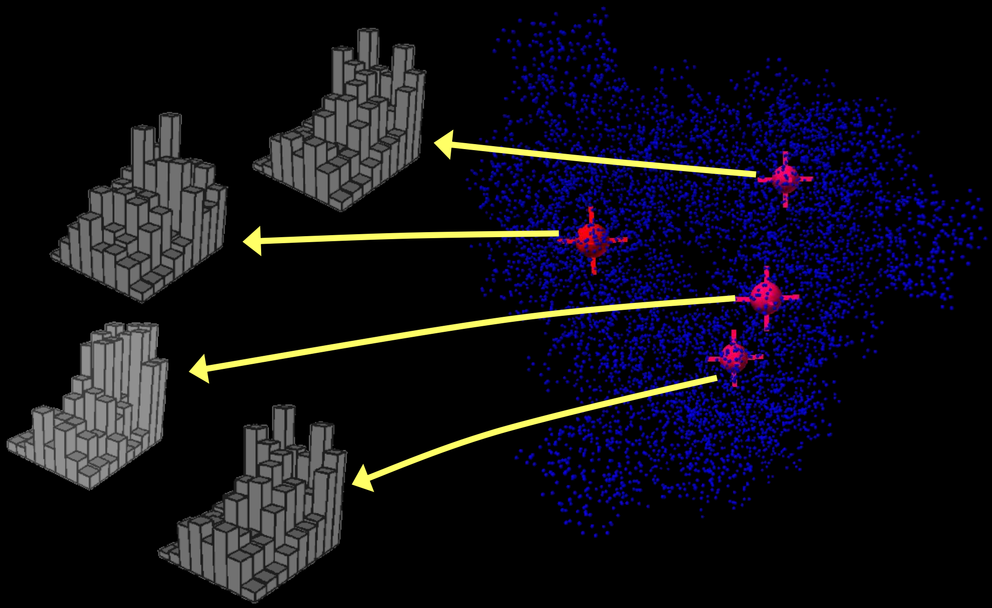

Harmonic Shape Descriptors

"How can we find similar 3D shapes in a database?"

Shape Analysis Problems

Images courtesy of
Florida State Univ.

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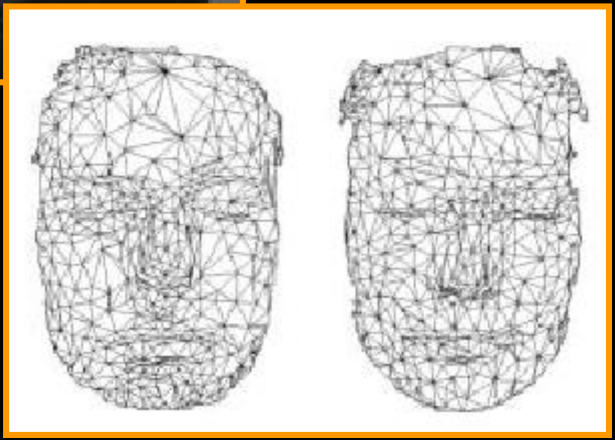

"How can we find a given 3D model in a large database?"

Shape Analysis Problems

Images courtesy of Darpa E3D Project

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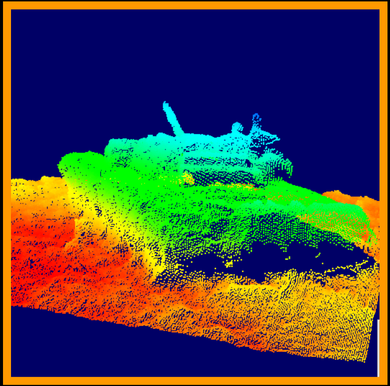
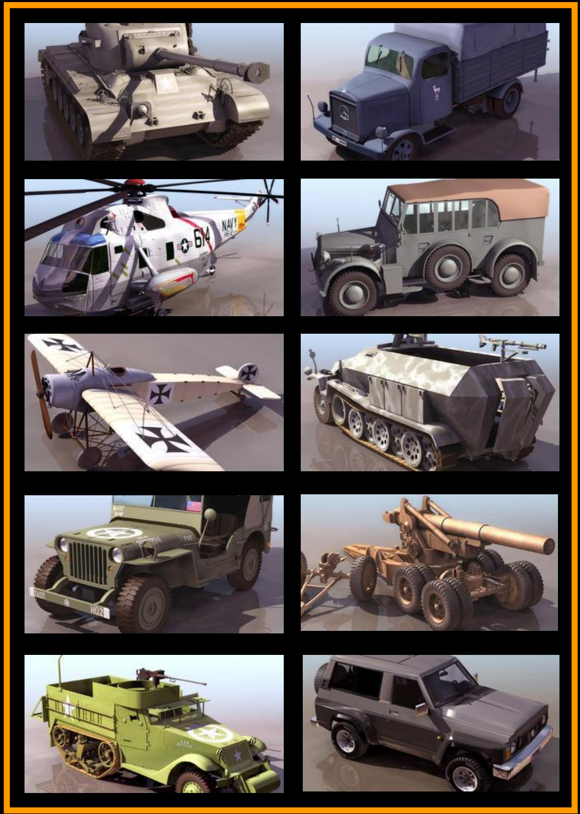


Query



Classes

"How can we determine the class of a 3D model?"

Shape Analysis Problems

Images courtesy of Viewpoint

Examples:

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- ➢ Clustering

"How can we learn classes of 3D models automatically?"

A Quick Diversion …

Images courtesy of
Georgia Tech and
www.dreamhorse.com

Which is harder to analyze?

3D Model

2D Image

Lecture Outline

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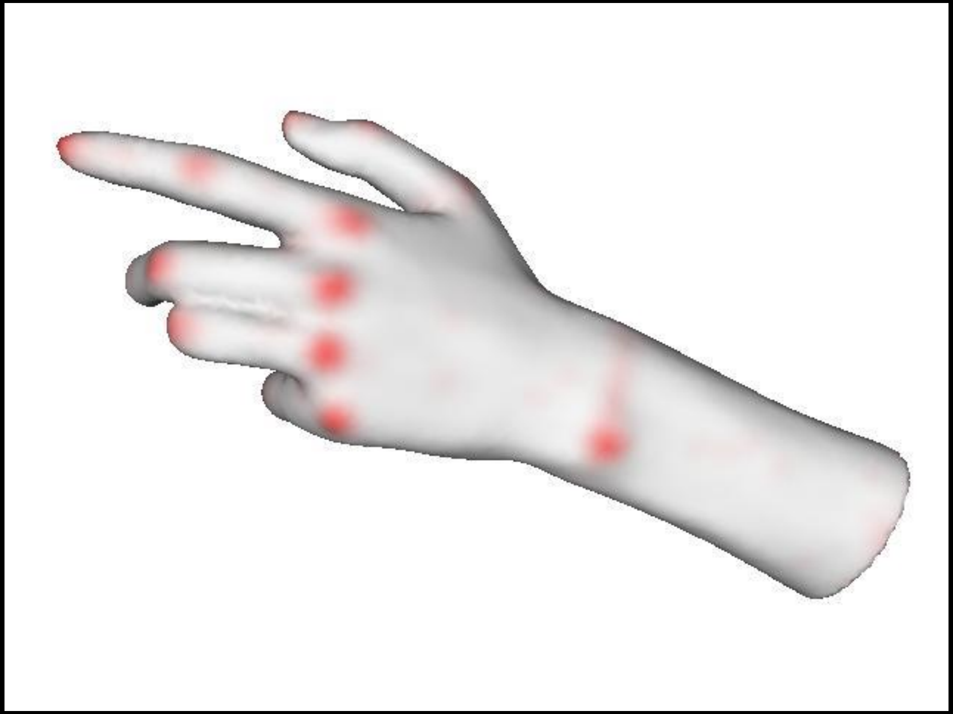
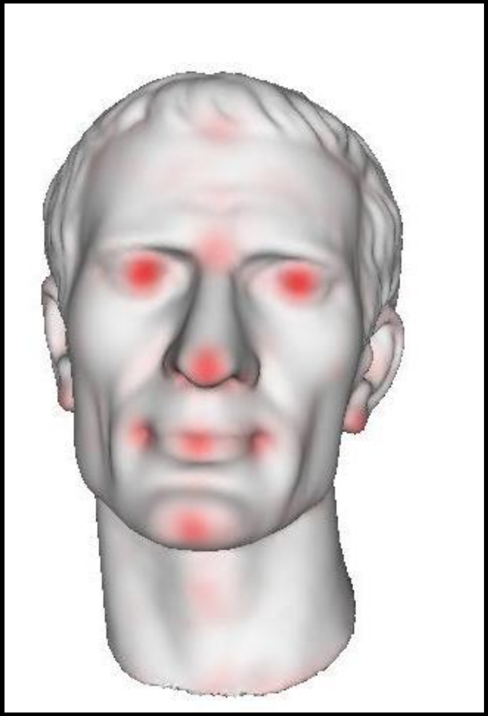
Features

Definition (Merriam-Webster)"

- "a prominent characteristic"

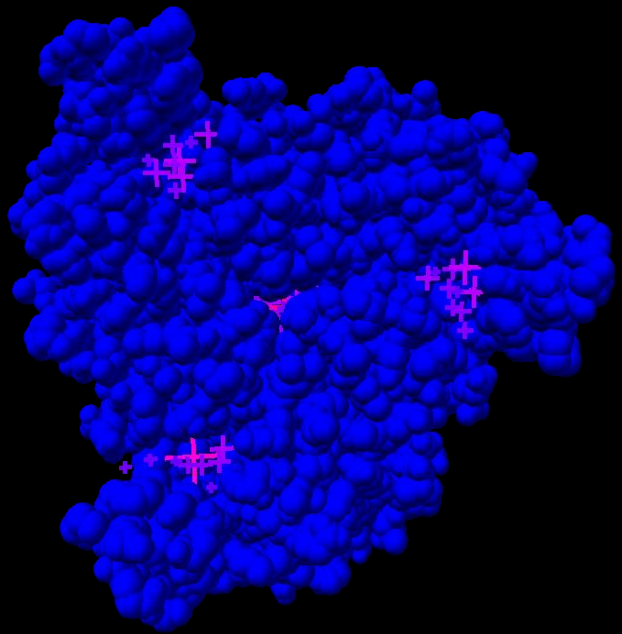




Point Features

Applications:

- Maintaining shape features as process mesh
- Matching shape features as align meshes
- Reasoning about part decomposition
- Visualization
- etc.



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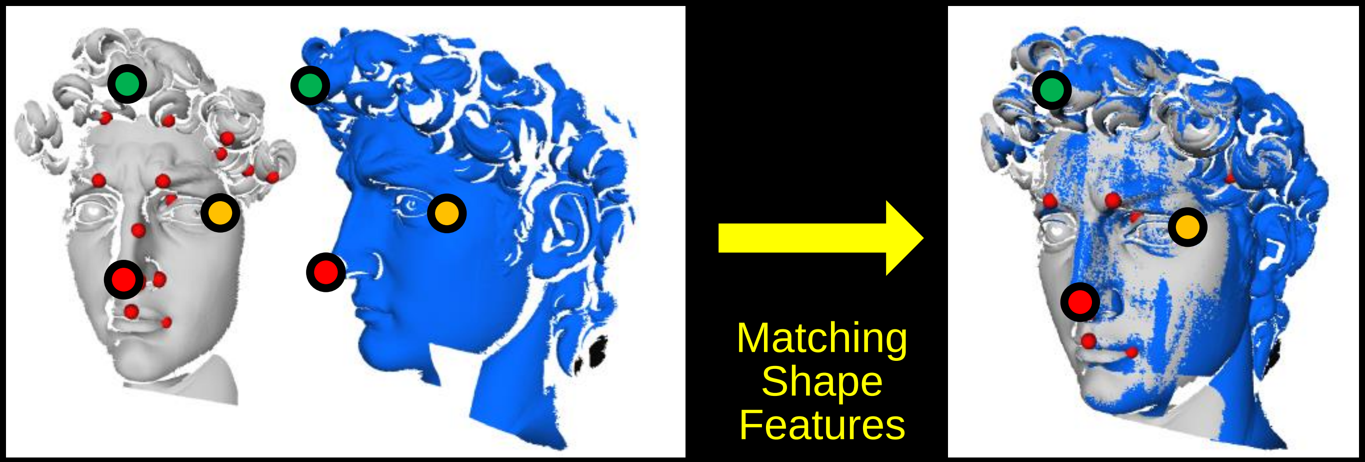
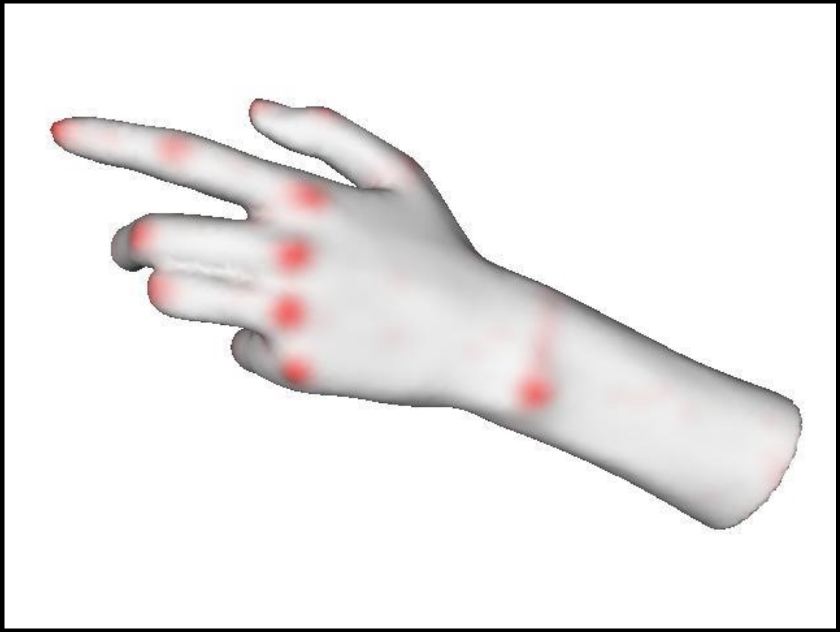


Scan A Scan B Best Alignment

Point Feature Detection

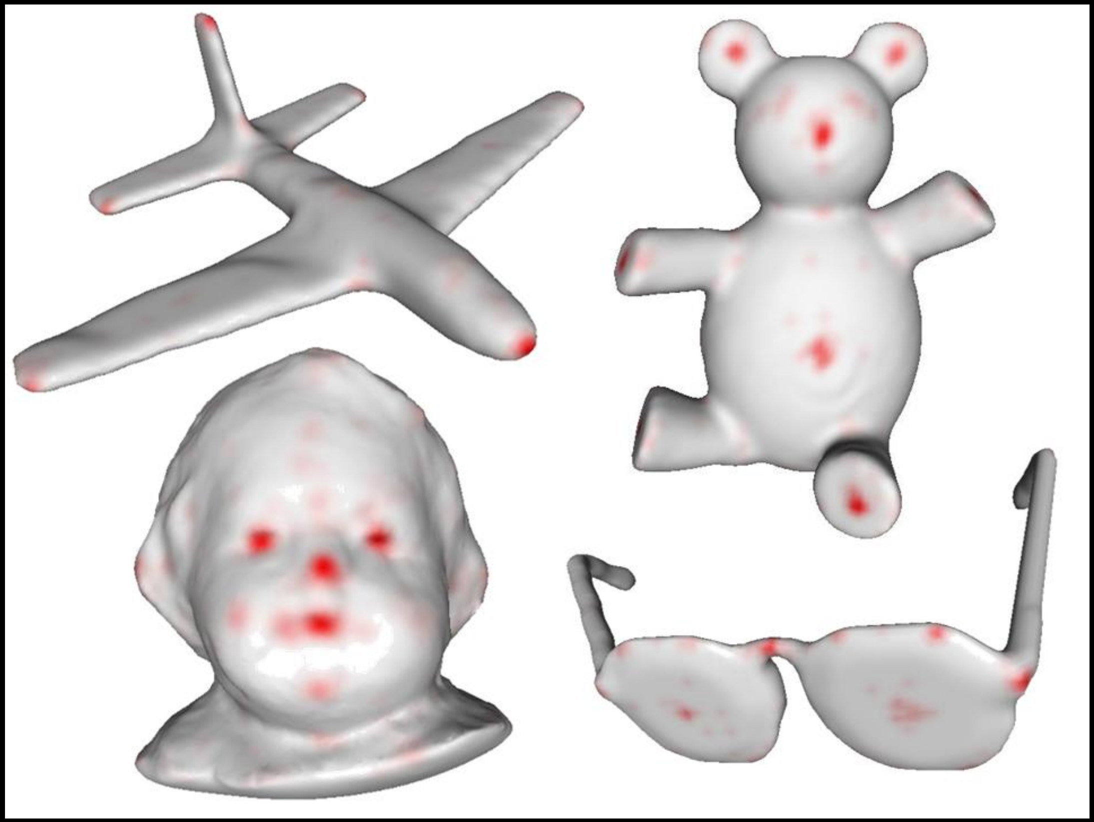
Goals:

- Invariant to transformations
- Robust to small surface deviations (holes, noise, etc.)
- Common across different surfaces in same class
- Semantic?



Point Feature Detection

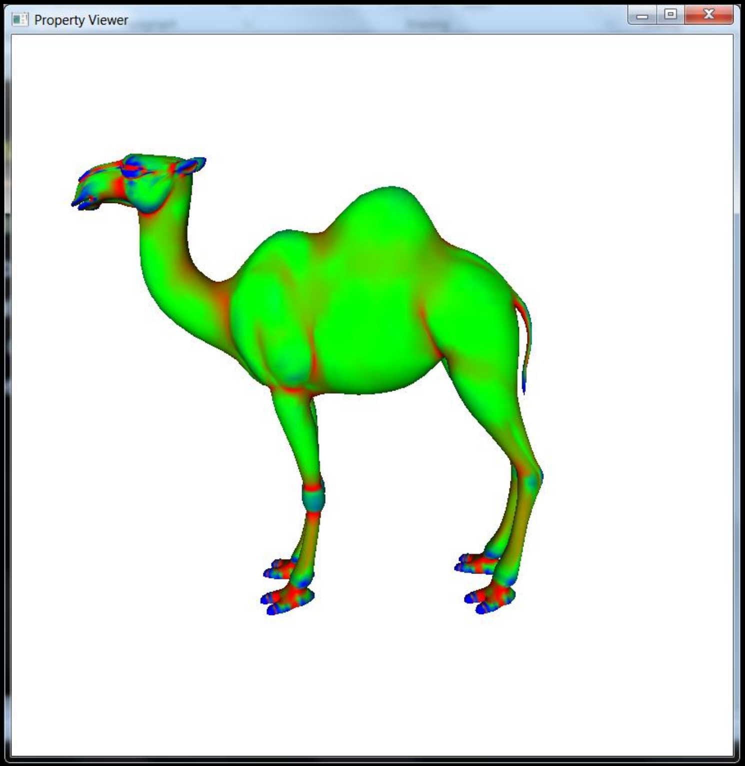
Algorithmic methods to detect feature points?



Point Feature Detection

Some relevant properties

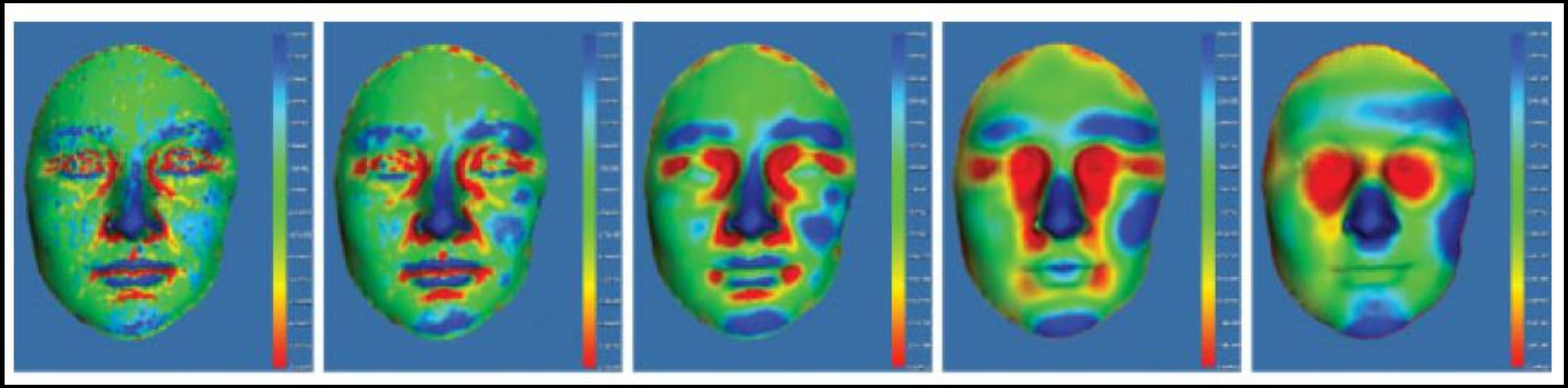
- Average geodesic distance
- Gauss curvature
- Differences of curvature
- Shape diameter function
- etc.



Point Feature Detection

Multiscale methods

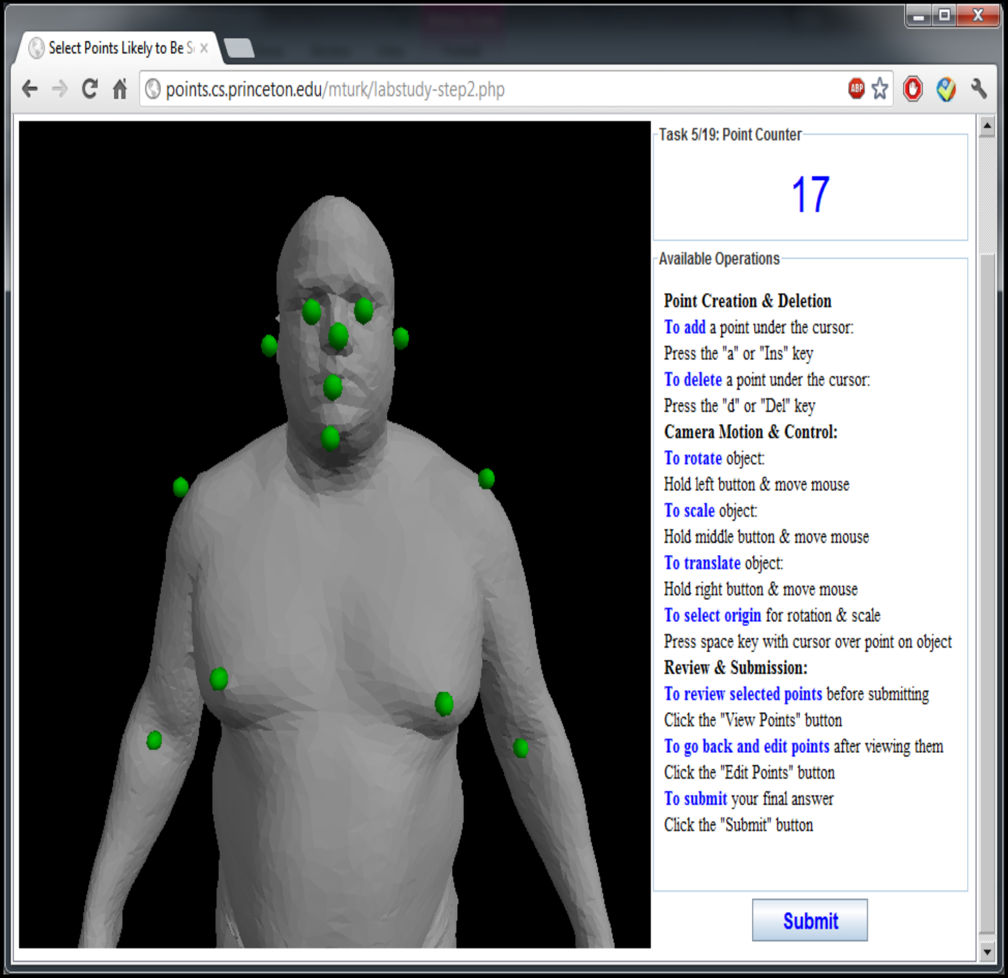
Many methods consider scale-space persistence



Zou08

Feature Point Study

Ask people on the Amazon Mechanical Turk



Key question

How should we ask people which points are salient?

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- "Please select salient points"

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- Please select a pattern of points from which another person can recognize the object's class by viewing only those points

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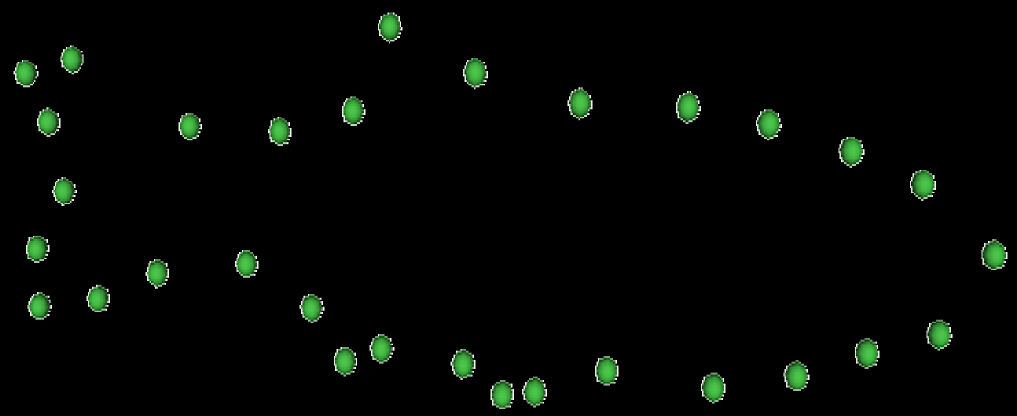


Schelling approach

We asked people to:

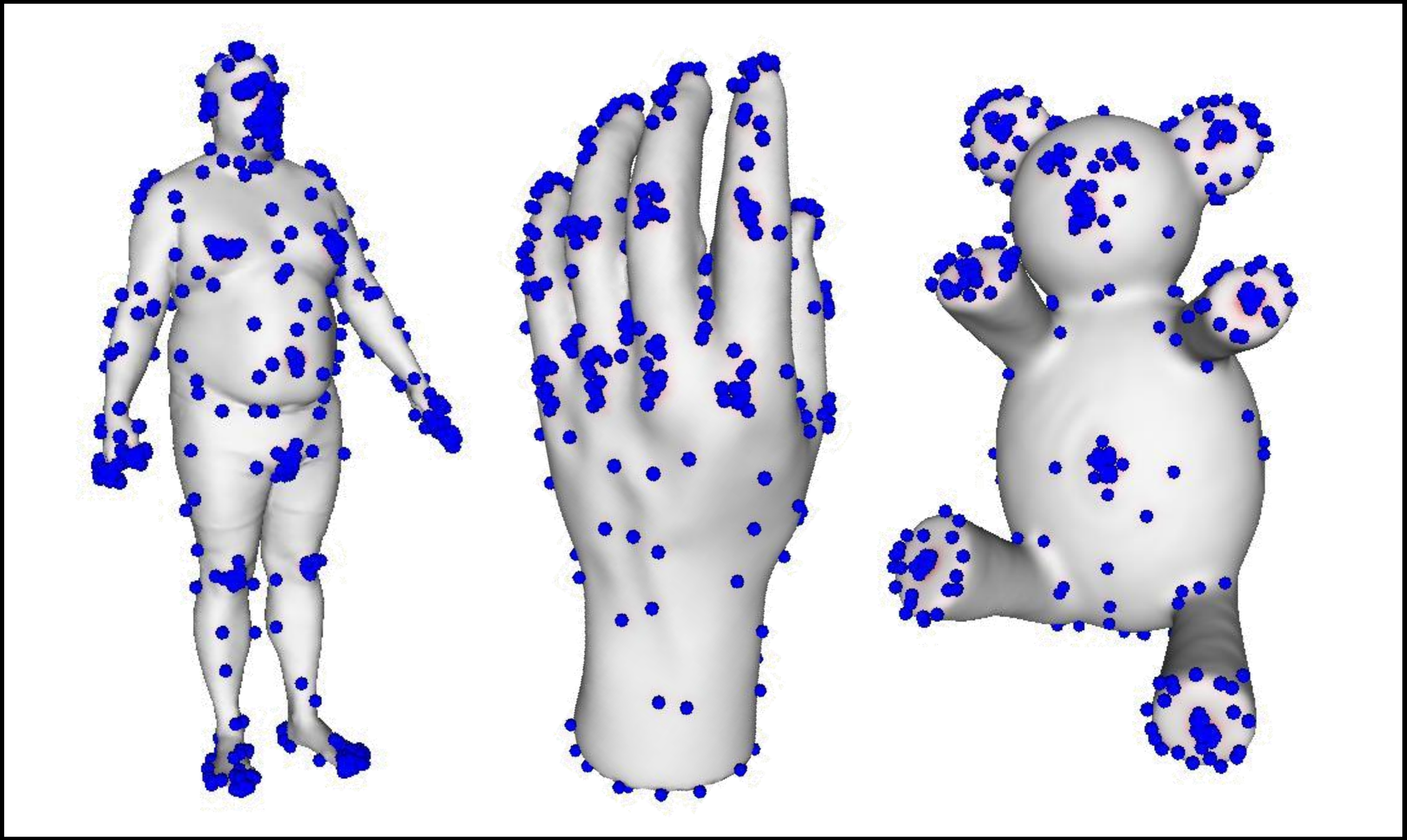
- Please select points that you think other people will select

Based on the "focal point" theory of [Schelling60]

- A solution that people tend to use in the absence of communication, because it seems natural, special or relevant to them

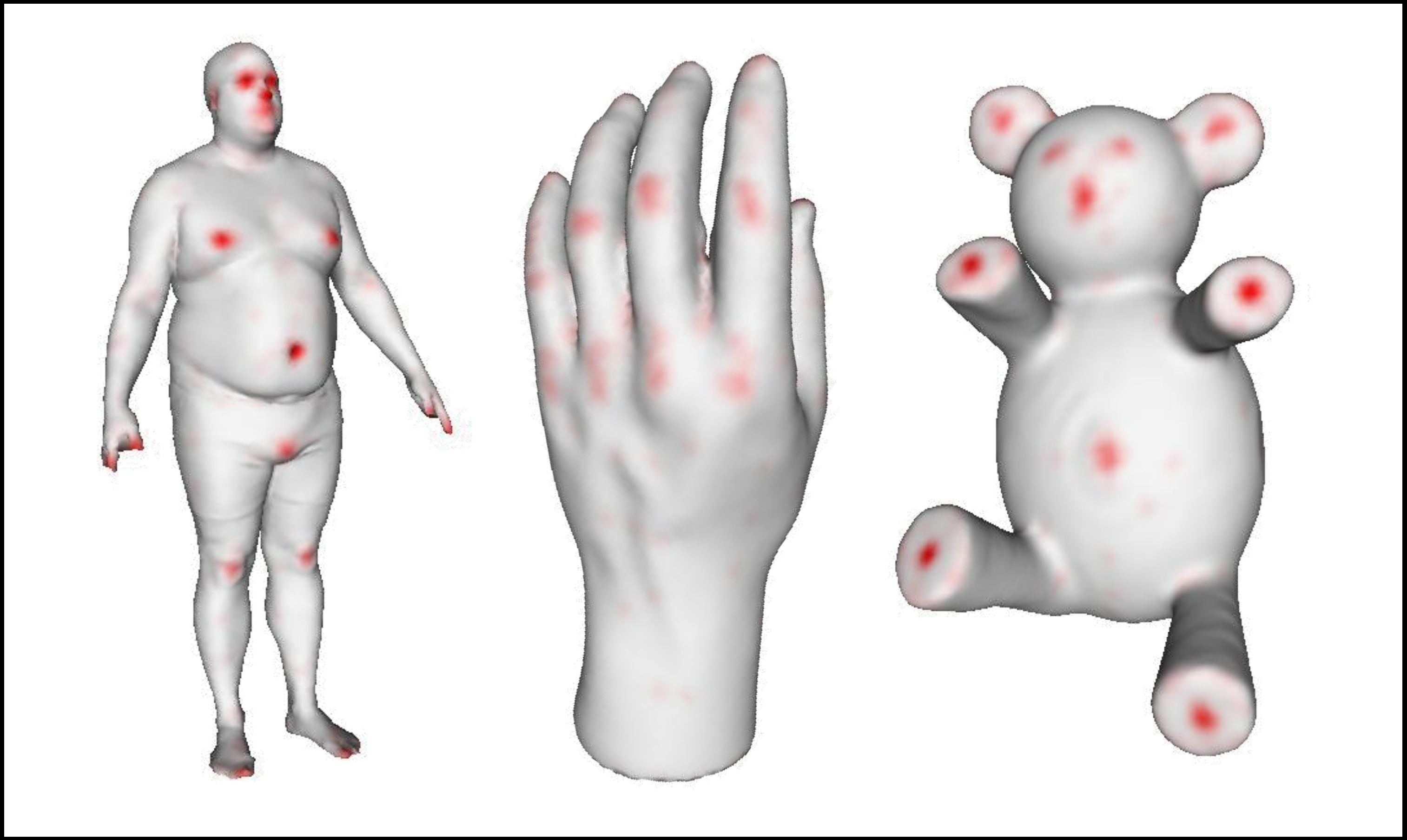
Schelling Feature Points

Schelling feature points



Schelling Feature Points

Schelling feature point distributions



Relation with geometric properties?

Local properties

- Curvatures
- Mesh Saliency
- HKS at small t

Global properties

- HKS at large t
- SDF [Shapira 08]
- Symmetry
- Segment Center
- AGD
- Etc.

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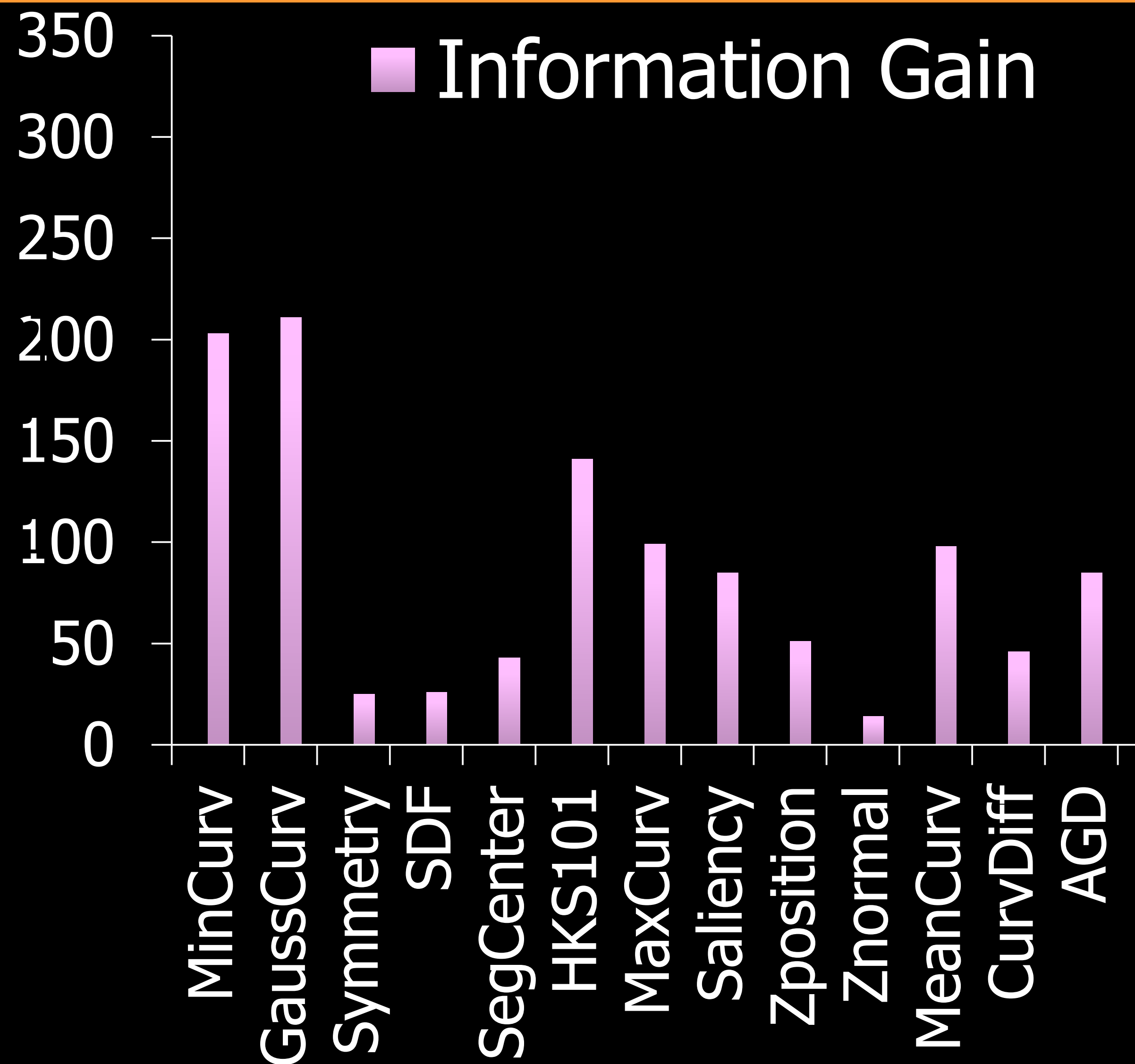

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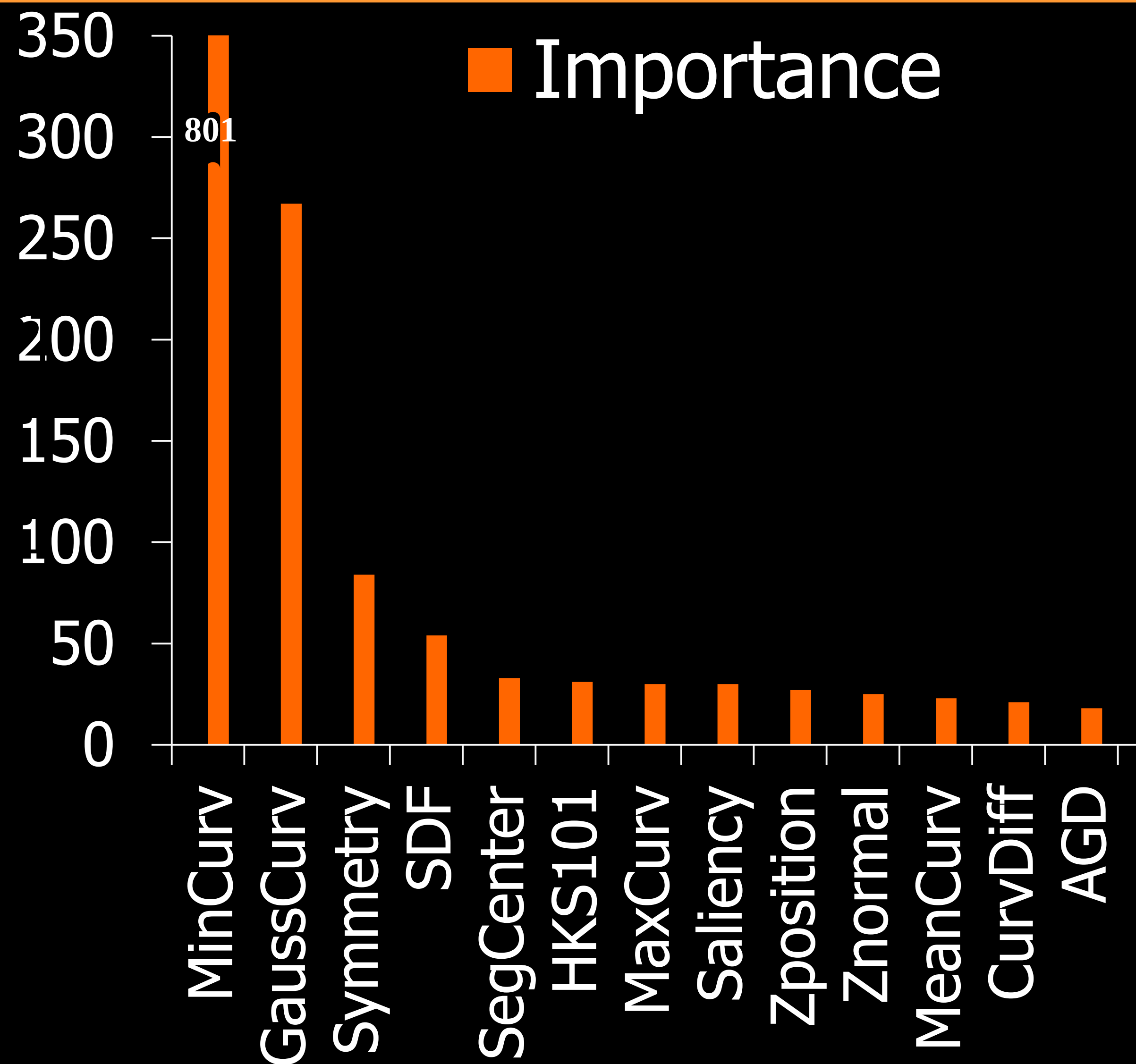
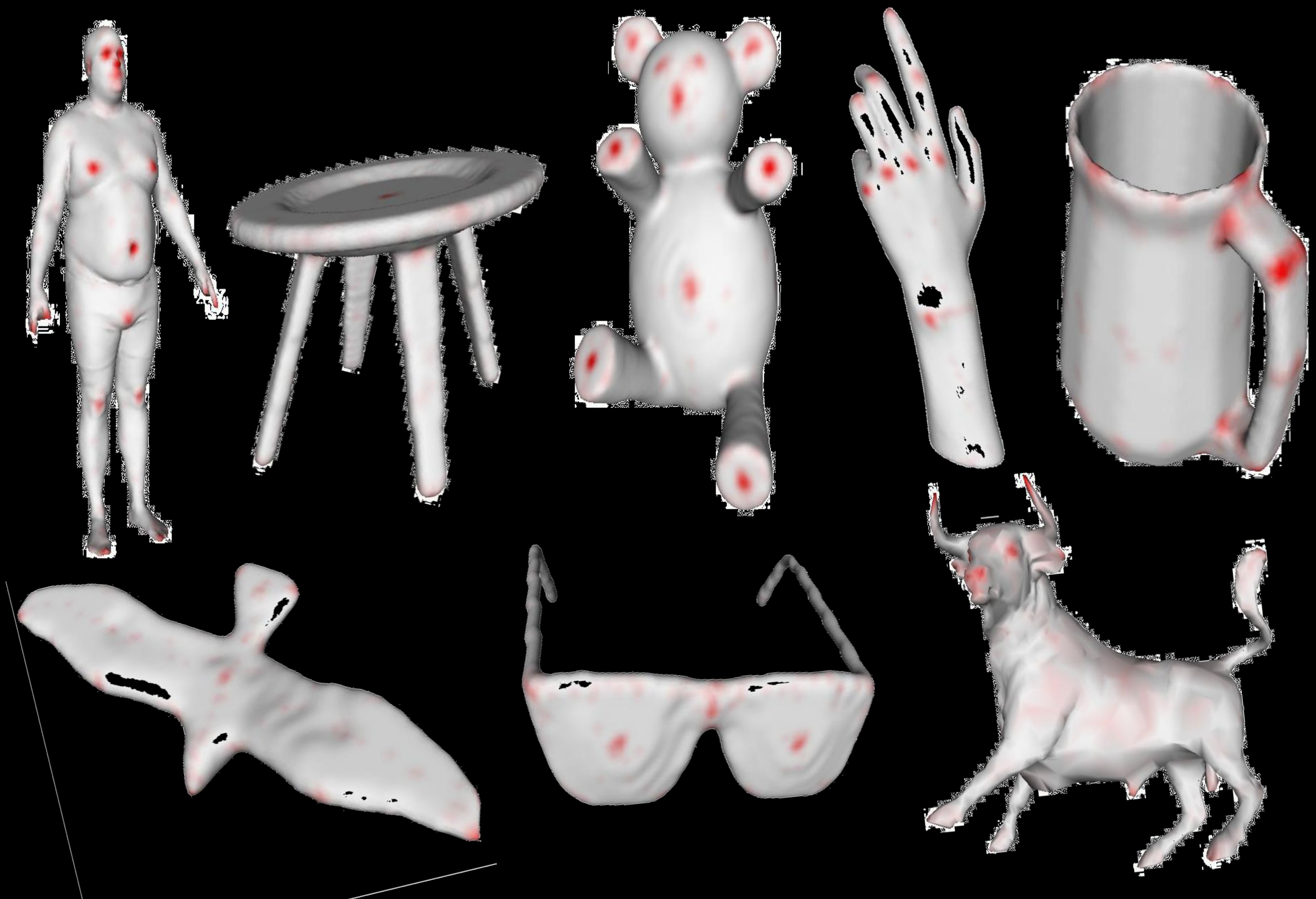


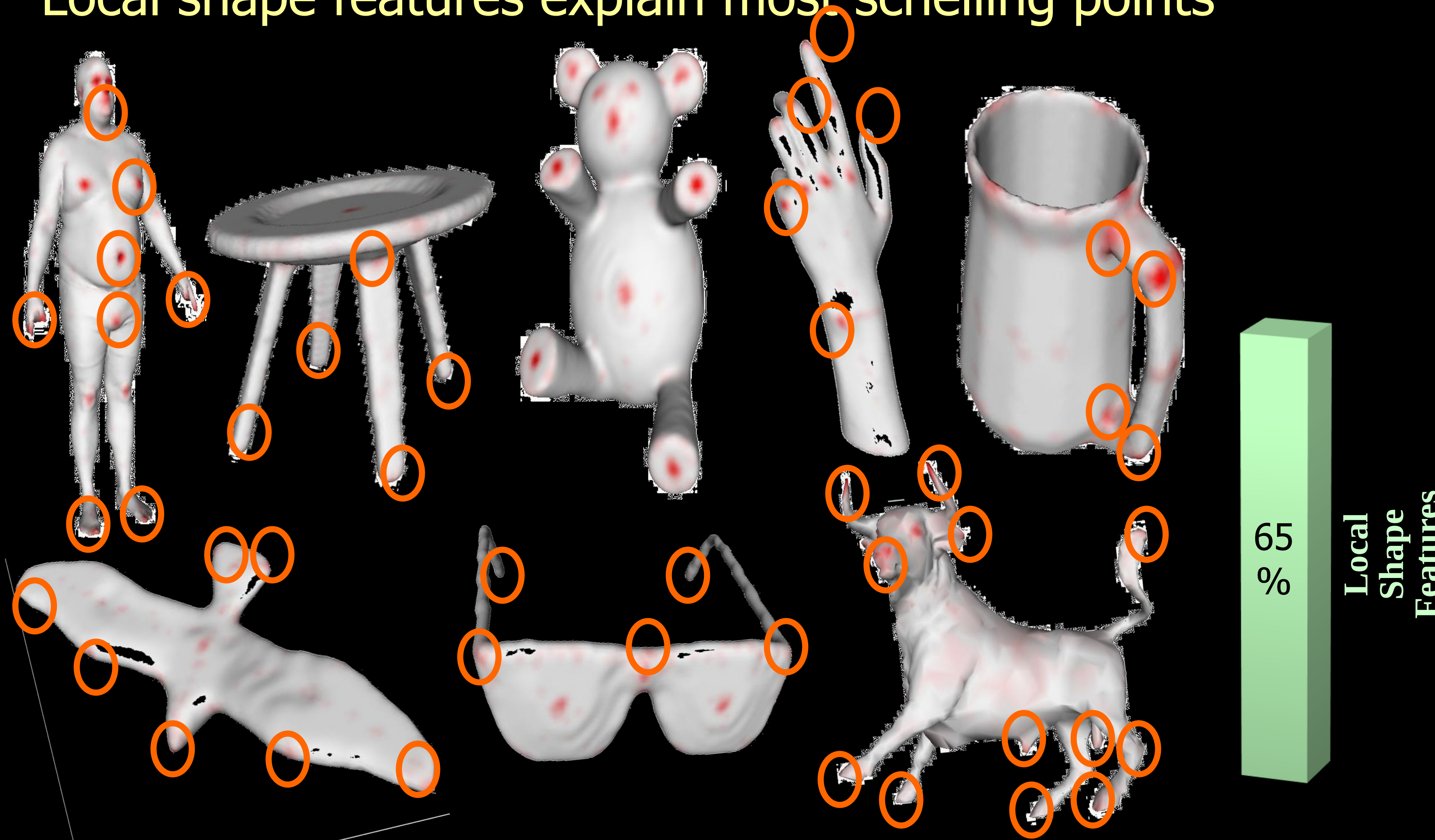
Relation with geometric properties?

Schelling point distribution



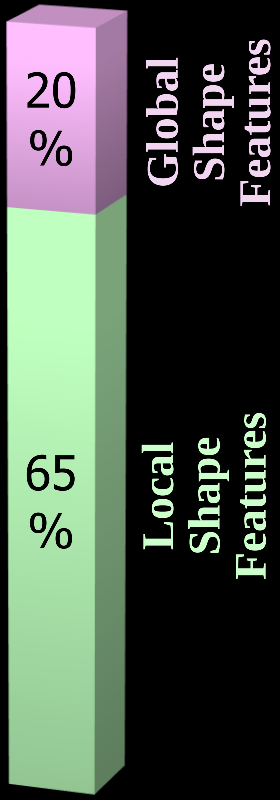
Relation with geometric properties?

Local shape features explain most schelling points



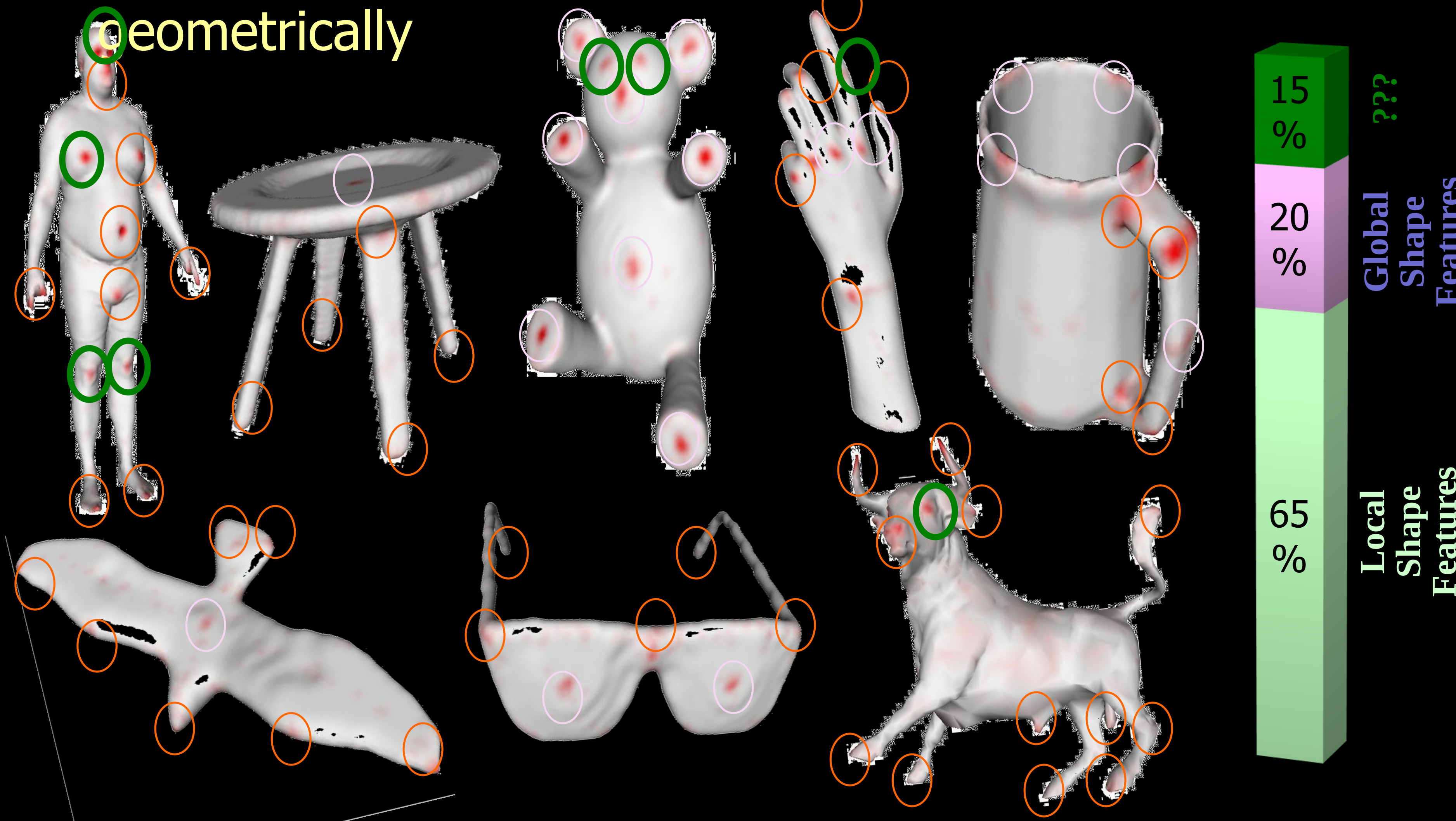
Relation with geometric properties?

Global shape features explain ~20% more



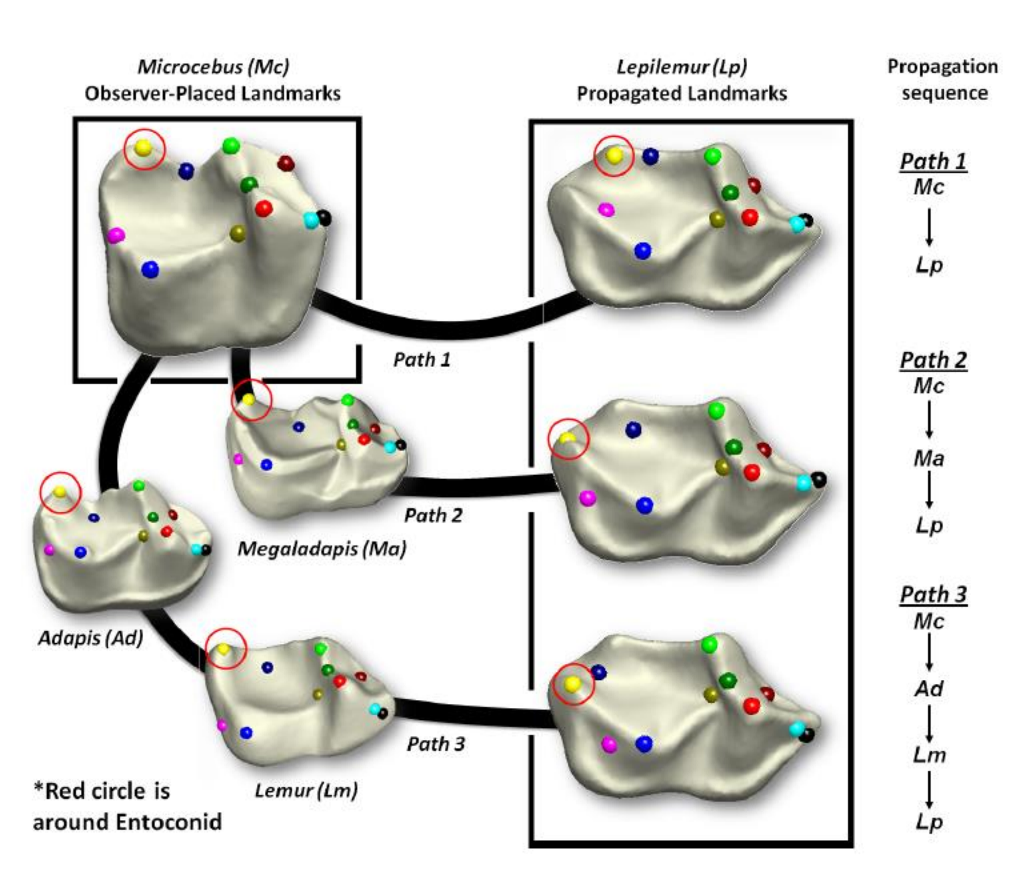
Relation with geometric properties?

~15% of schelling points are not explained geometrically



Summary

Geometric analysis can yield insights into features and relationships in 3D surface data

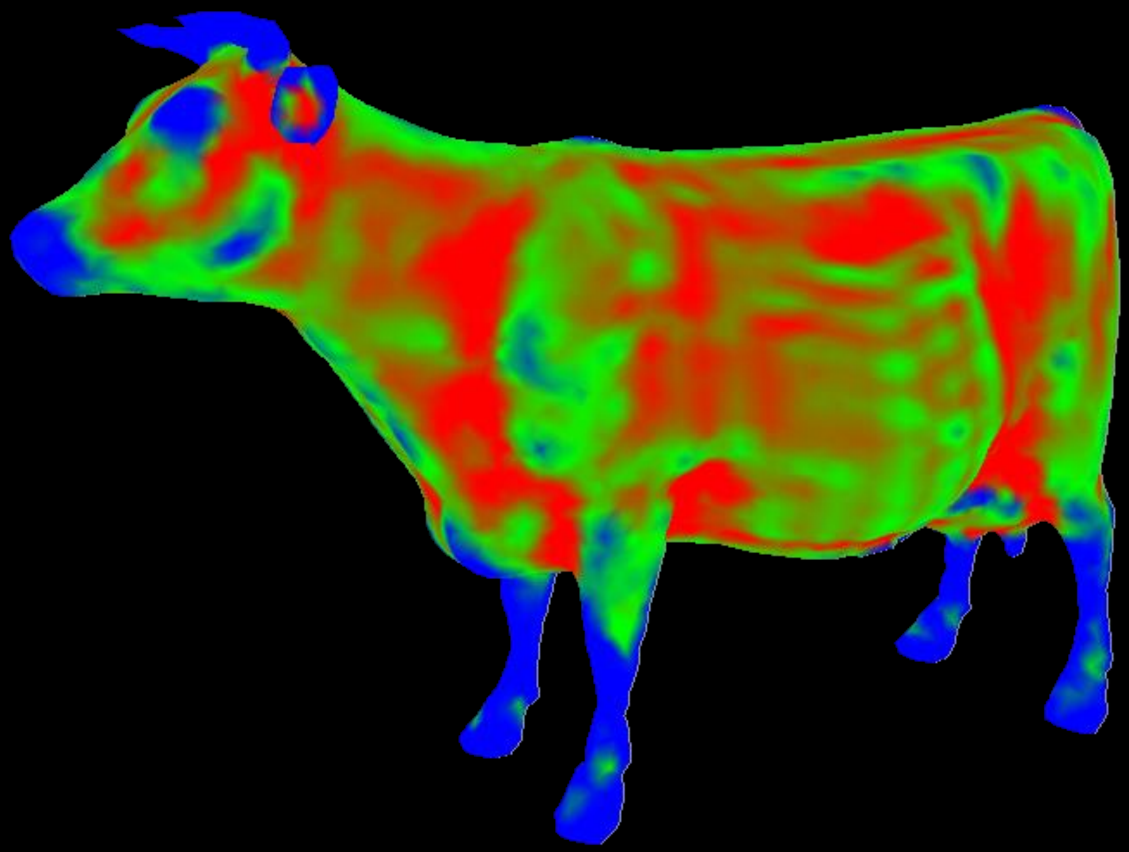


Boyer et al., 2011

Extra Slides

Curvature

Curvature

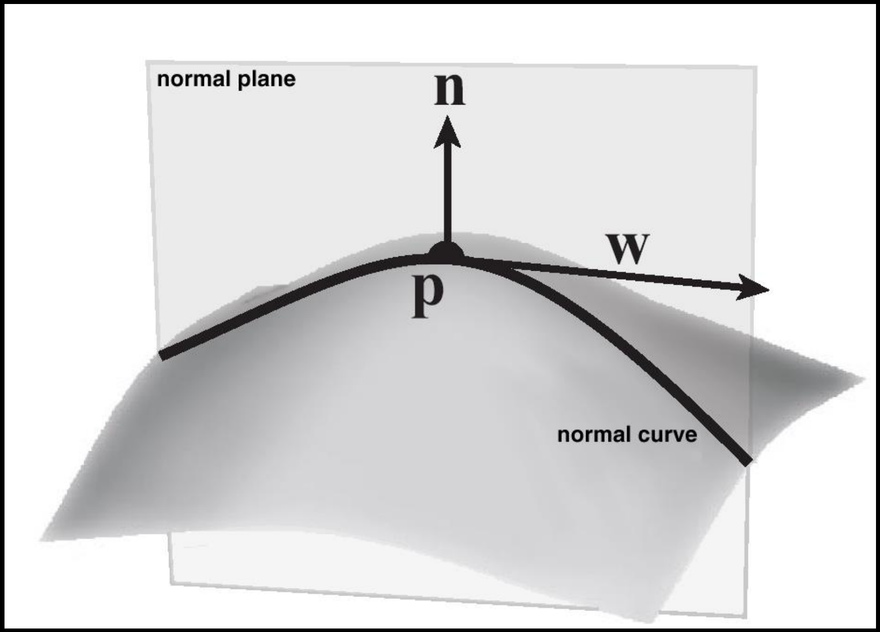


Curvature

Curvature κ of a curve is reciprocal of radius of circle that best approximates it

Defined at a point $\mathbf{p}$ in a direction $\mathbf{w}$

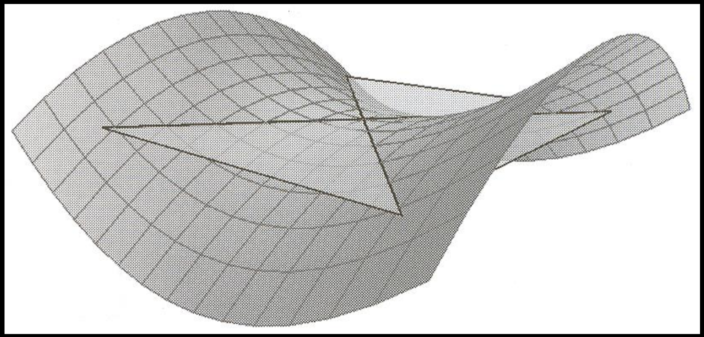
Line has $\kappa = 0$



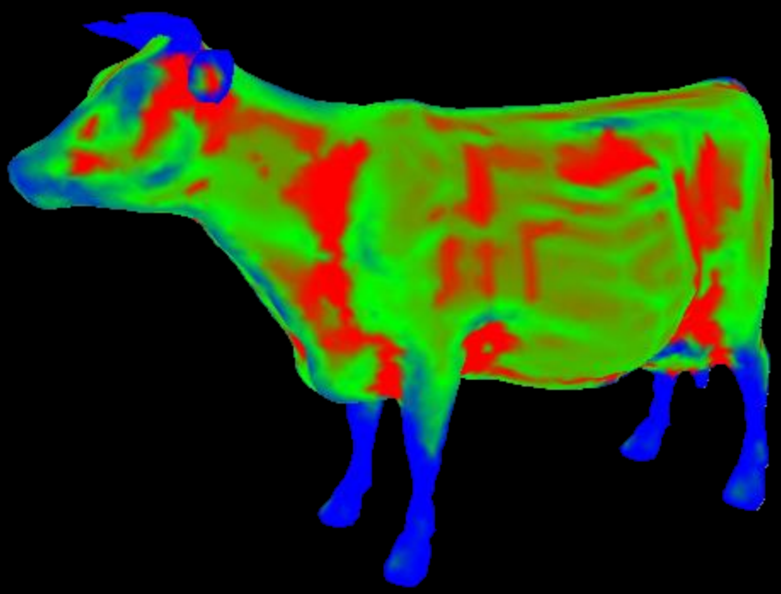
Principal Curvatures

The curvature at a point varies between some minimum and maximum – these are the *principal curvatures* κ_1 and κ_2

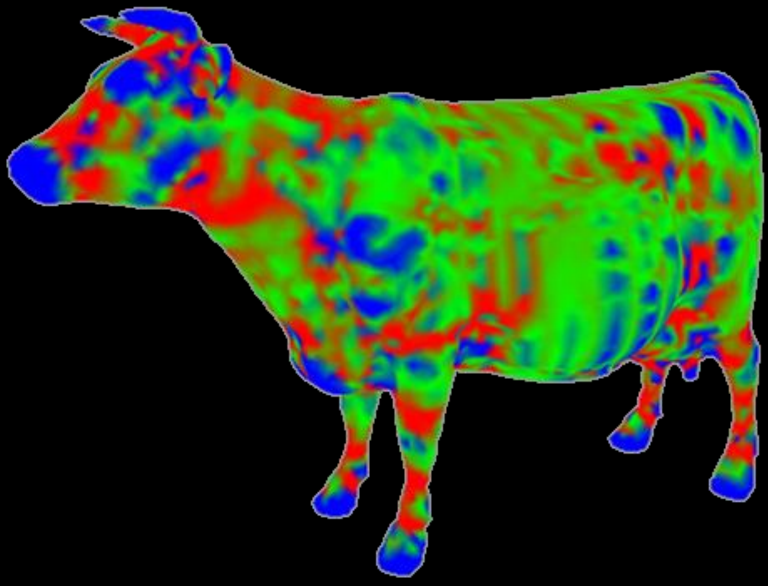
They occur in the *principal directions* d_1 and d_2 which are perpendicular to each other



Principal Curvatures

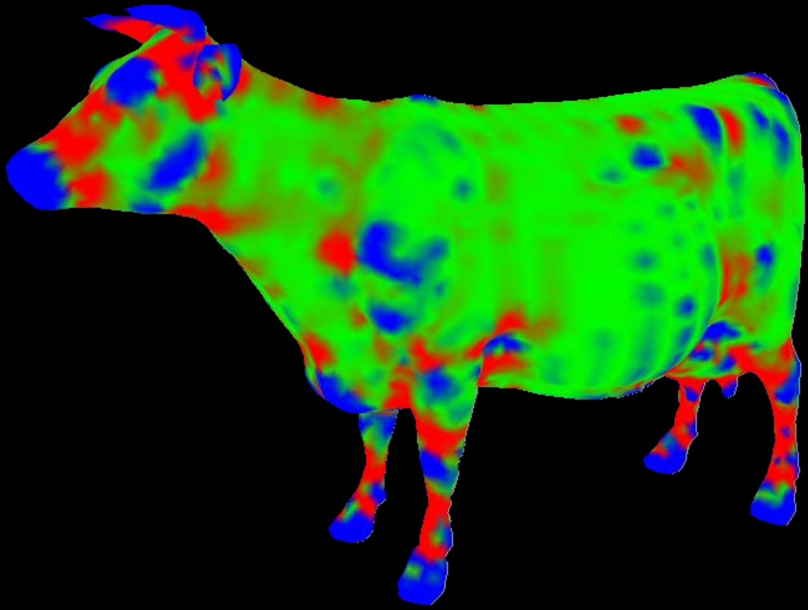


Minimum Curvature
κ_1

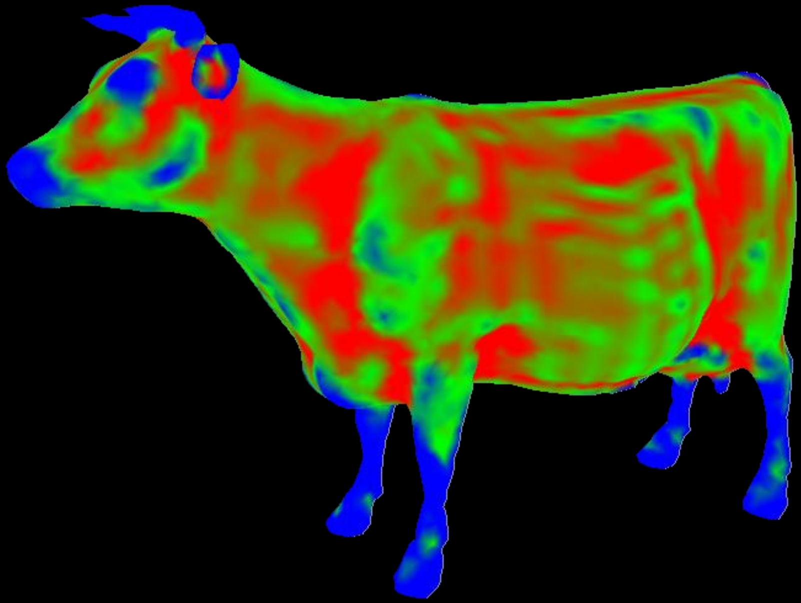


Maximum Curvature
κ_2

Gaussian and Mean Curvature



Gauss Curvature
$K=\kappa_1 \kappa_2$



Mean Curvature
$H=½ (\kappa_1 + \kappa_2)$

What Does Curvature Tell Us?

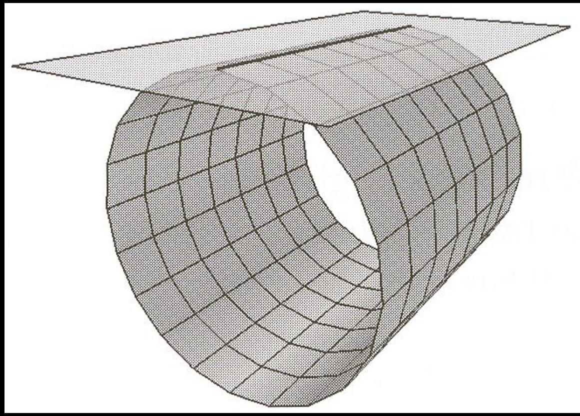
Planar points:

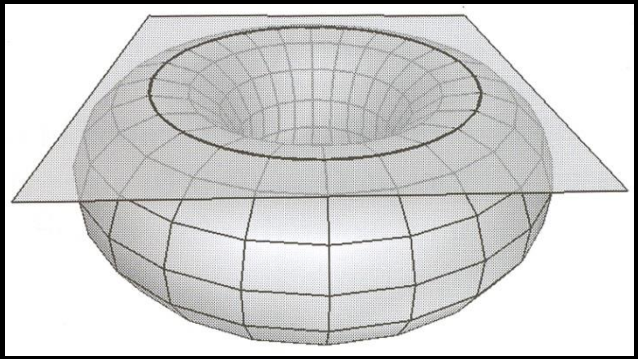
- Zero Gaussian curvature and zero mean curvature
- Tangent plane intersects surface at infinity points

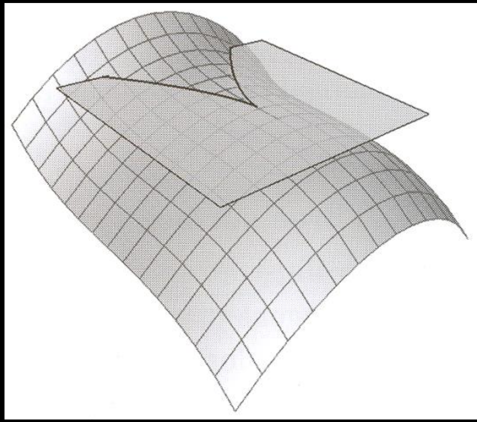
What Does Curvature Tell Us?

Parabolic points:

- Zero Gaussian curvature, non-zero mean curvature
- Tangent plane intersects surface along 1 curves



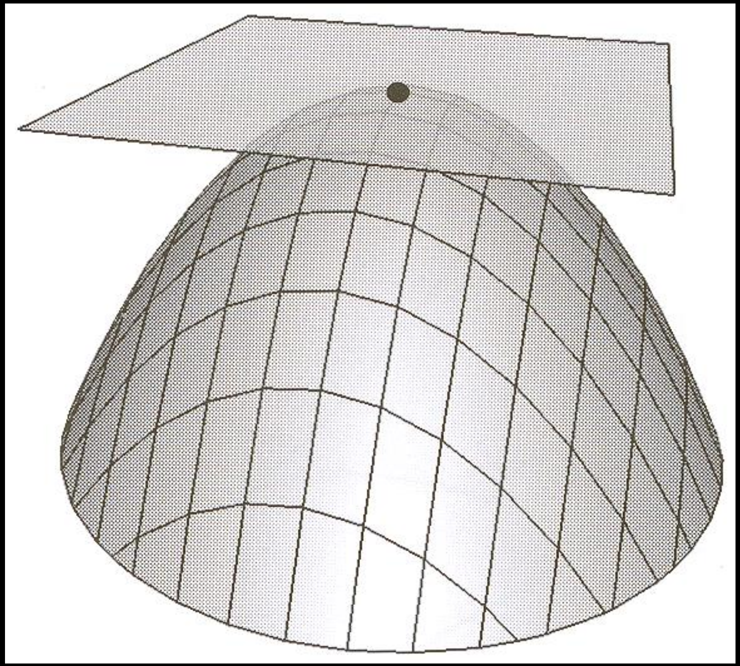




What Does Curvature Tell Us?

Elliptical points:

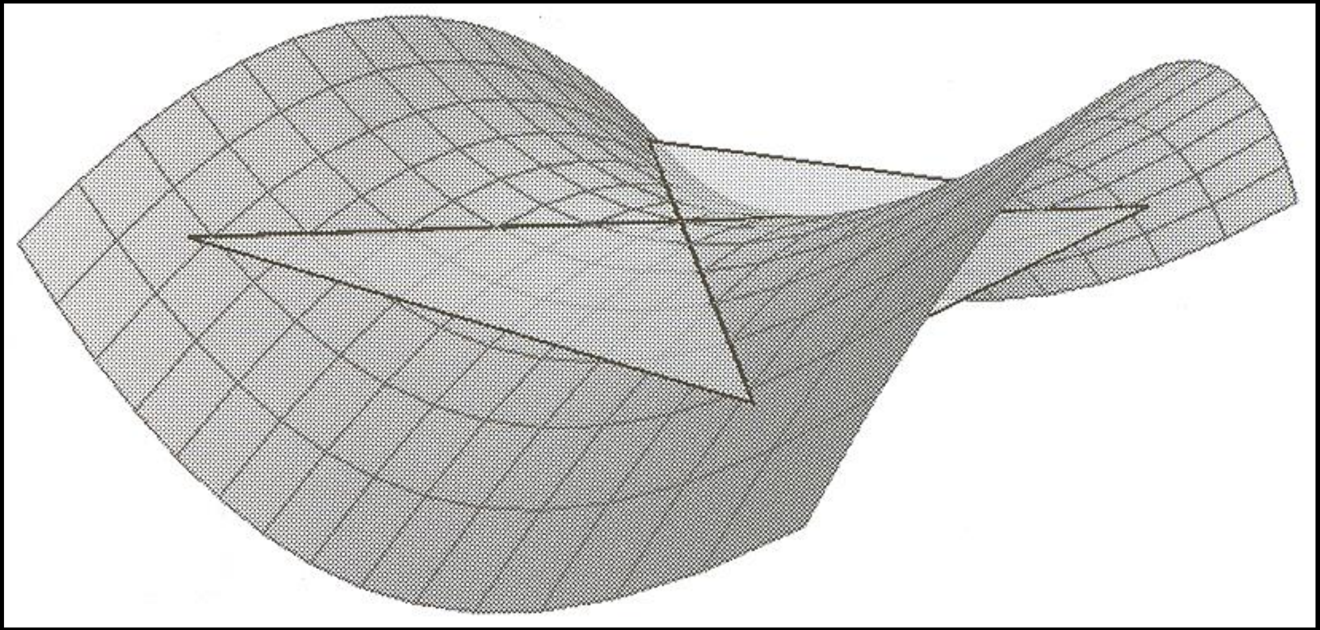
- Positive Gaussian curvature
- Convex/concave depending on sign of mean curvature
- Tangent plane intersects surface at 1 point



What Does Curvature Tell Us?

Hyperbolic points:

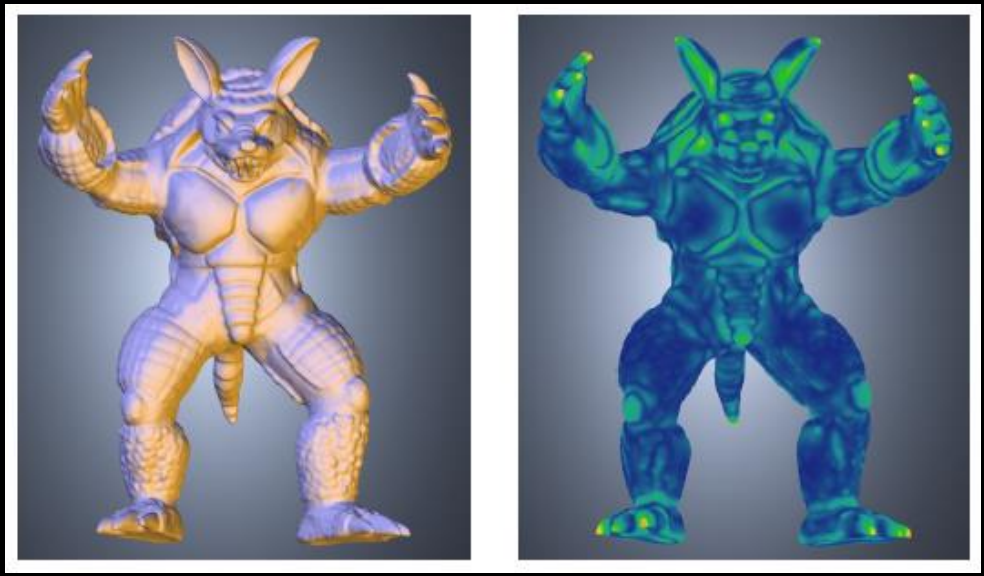
- Negative Gaussian curvature
- Tangent plane intersects surface along 2 curves



What Does Curvature Tell Us?

Mesh Saliency:

- Motivated by models of perceptual salience
- Difference between mean curvature blurred with σ and blurred with 2σ



Lee05

Principal Component Analysis (PCA)

Principal Component Analysis (PCA)

Tensor voting

- Extract points $\{q_i\}$ in neighborhood
- Compute covariance matrix M
- Analyze eigenvalues and eigenvectors of M (via SVD)
- Eigenvectors are Principal Axes

$$\mathbf{M} = \frac{1}{n}\sum_{i=1}^{n}\begin{bmatrix} q_i^x q_i^x & q_i^x q_i^y & q_i^x q_i^z \\ q_i^y q_i^x & q_i^y q_i^y & q_i^y q_i^z \\ q_i^z q_i^x & q_i^z q_i^y & q_i^z q_i^z \end{bmatrix}$$

Covariance Matrix

$$\mathbf{M} = \mathbf{U}\mathbf{S}\mathbf{U}^t$$

$$\mathbf{S} = \begin{bmatrix} \lambda_a & 0 & 0 \\ 0 & \lambda_b & 0 \\ 0 & 0 & \lambda_c \end{bmatrix} \quad \mathbf{U} = \begin{bmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{bmatrix}$$

Eigenvalues & Eigenvectors

Principal Component Analysis (PCA)

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- Extract points $\{q_i\}$ in neighborhood
- Compute covariance matrix M
- Analyze eigenvalues and eigenvectors of M (via SVD)

$$\mathbf{M} = \frac{1}{n}\sum_{i=1}^{n}\begin{bmatrix} q_i^x q_i^x & q_i^x q_i^y & q_i^x q_i^z \\ q_i^y q_i^x & q_i^y q_i^y & q_i^y q_i^z \\ q_i^z q_i^x & q_i^z q_i^y & q_i^z q_i^z \end{bmatrix}$$

Covariance Matrix

$$\mathbf{M} = \mathbf{U}\mathbf{S}\mathbf{U}^t$$

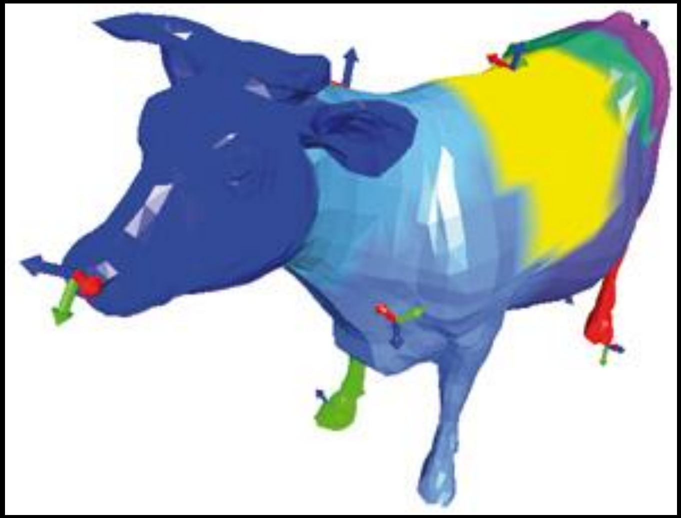
$$\mathbf{S} = \begin{bmatrix} \lambda_a & 0 & 0 \\ 0 & \lambda_b & 0 \\ 0 & 0 & \lambda_c \end{bmatrix} \quad \mathbf{U} = \begin{bmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{bmatrix}$$

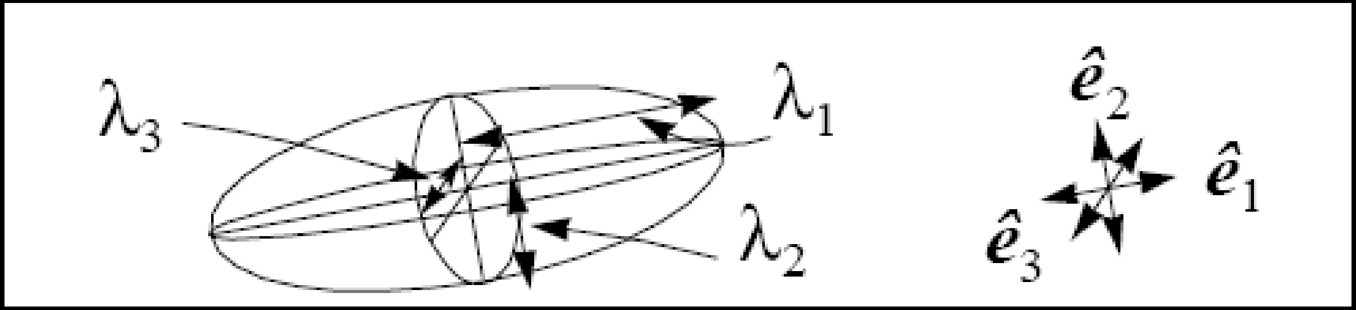
Eigenvalues & Eigenvectors

Principal Component Analysis (PCA)

Eigenvectors are
"Principal Axes of Inertia"

Eigenvalues are variances of the point distribution in those directions

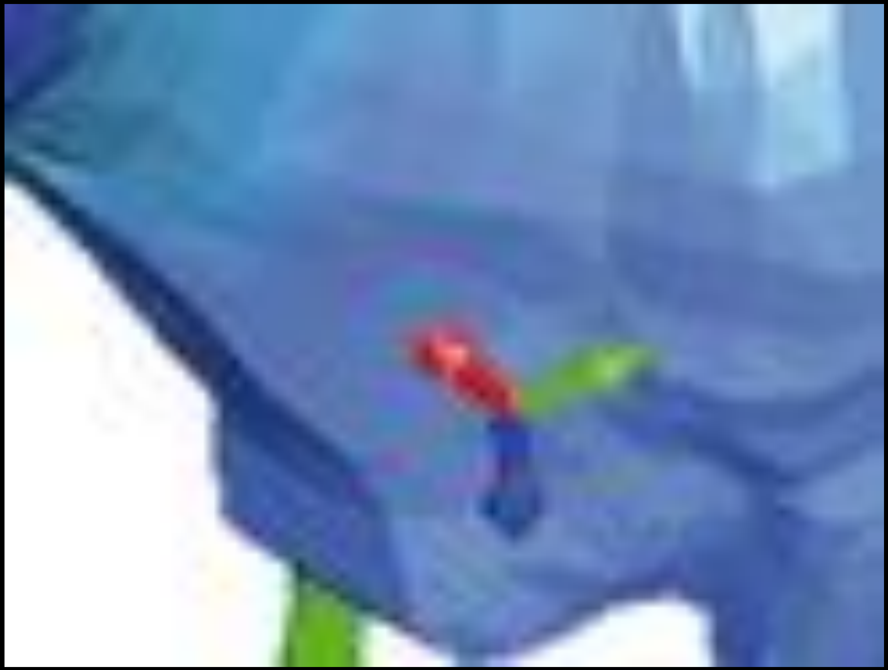




What Does PCA Tell Us?

Provides estimate of normal direction

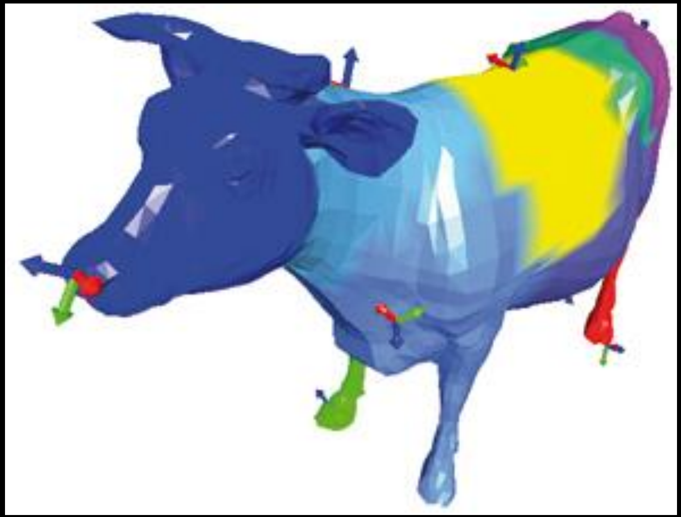
- Eigenvector (principal axis) associated with smallest eigenvalue

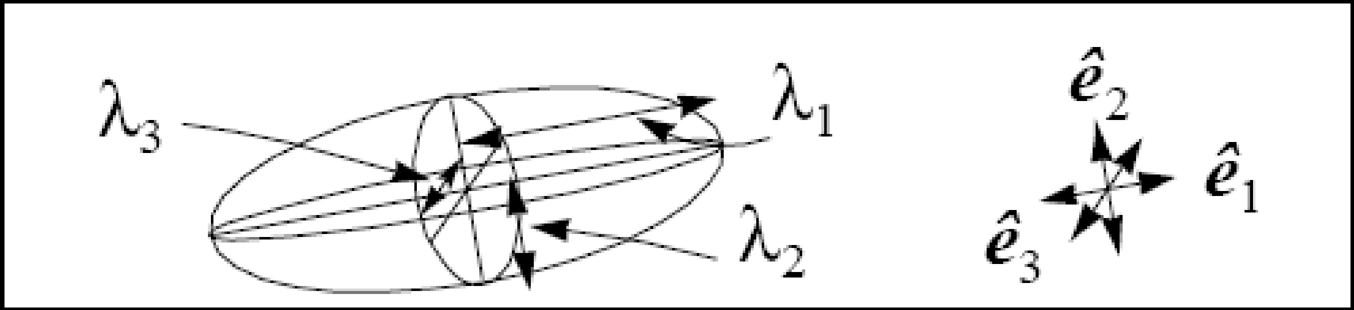


What Does PCA Tell Us?

Helps us construct a local coordinate frame for every point

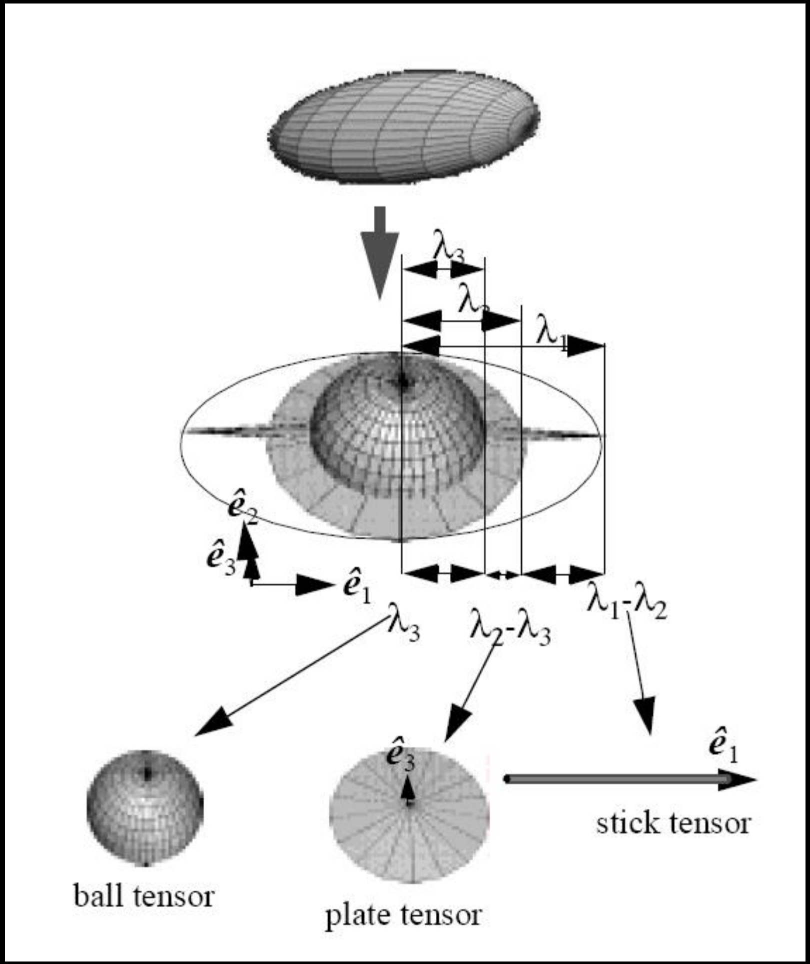
- Map $\hat{e}_1$ to X axis
- Map $\hat{e}_2$ to Y axis
- Map $\hat{e}_3$ to Z axis





What Does PCA Tell Us?

Helps differentiate
nearly plane-like,
from stick-like,
from sphere-like,
etc.



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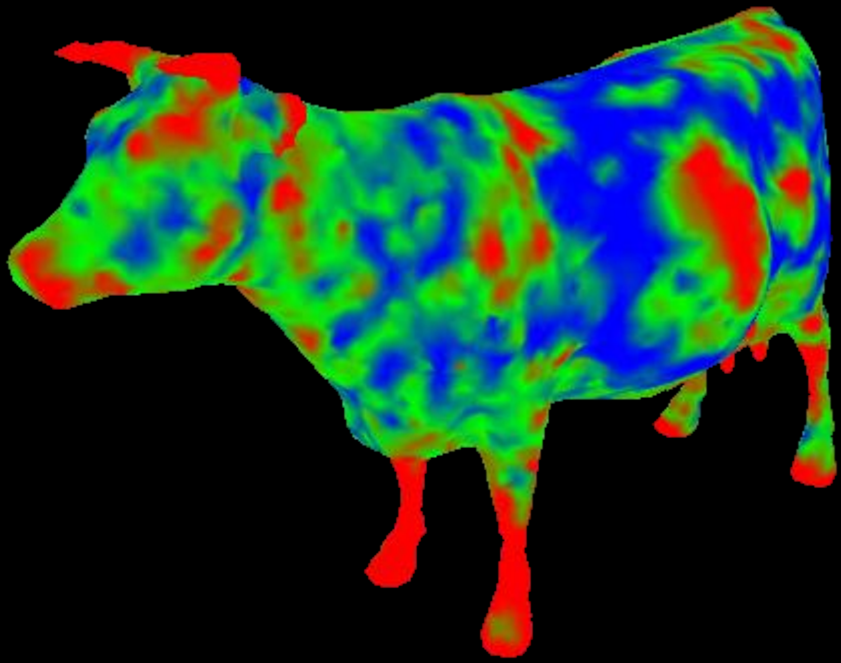

$\lambda_2 / (\lambda_1+\lambda_2+\lambda_3)$