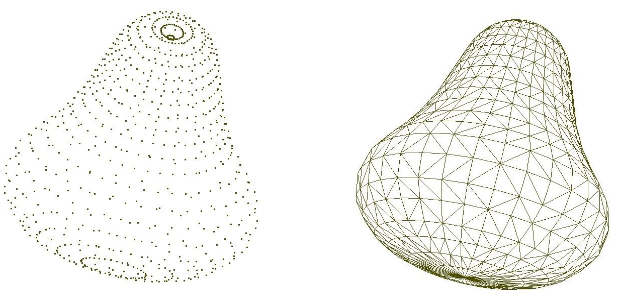
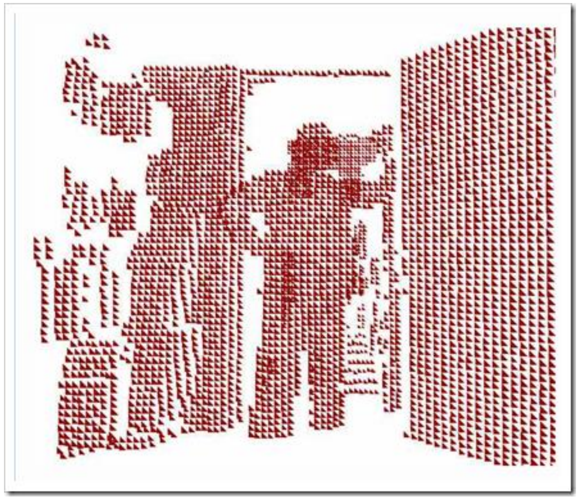
# Surface Reconstruction From Unorganized Point Sets

COS 526, Fall 2014

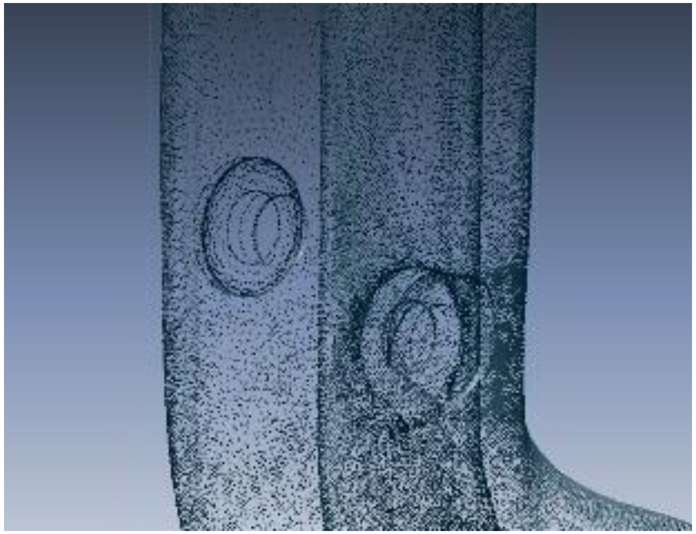


*Slides from Misha Kazhdan, Fisher Yu, Szymon Rusinkiewicz, Ioannis Stamos, Hugues Hoppe, and Piyush Rai*

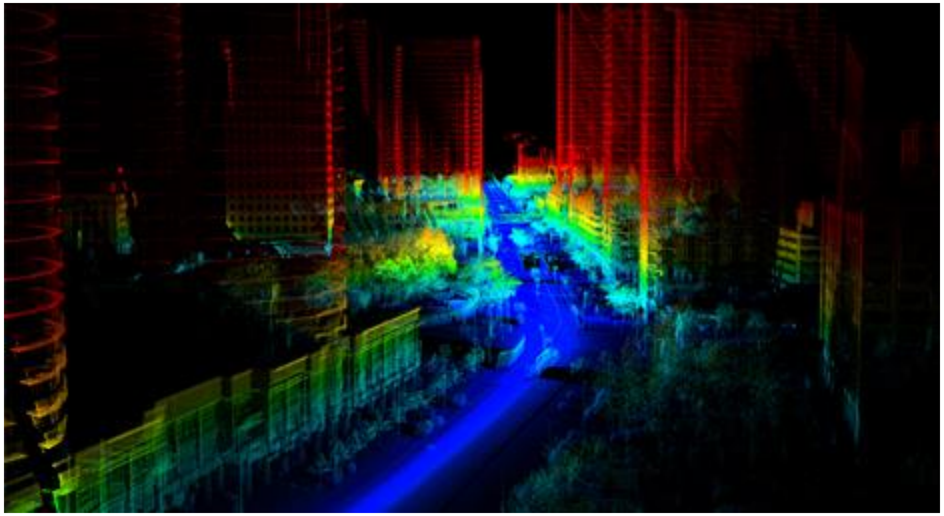
# Point Sets



Greg Duncan



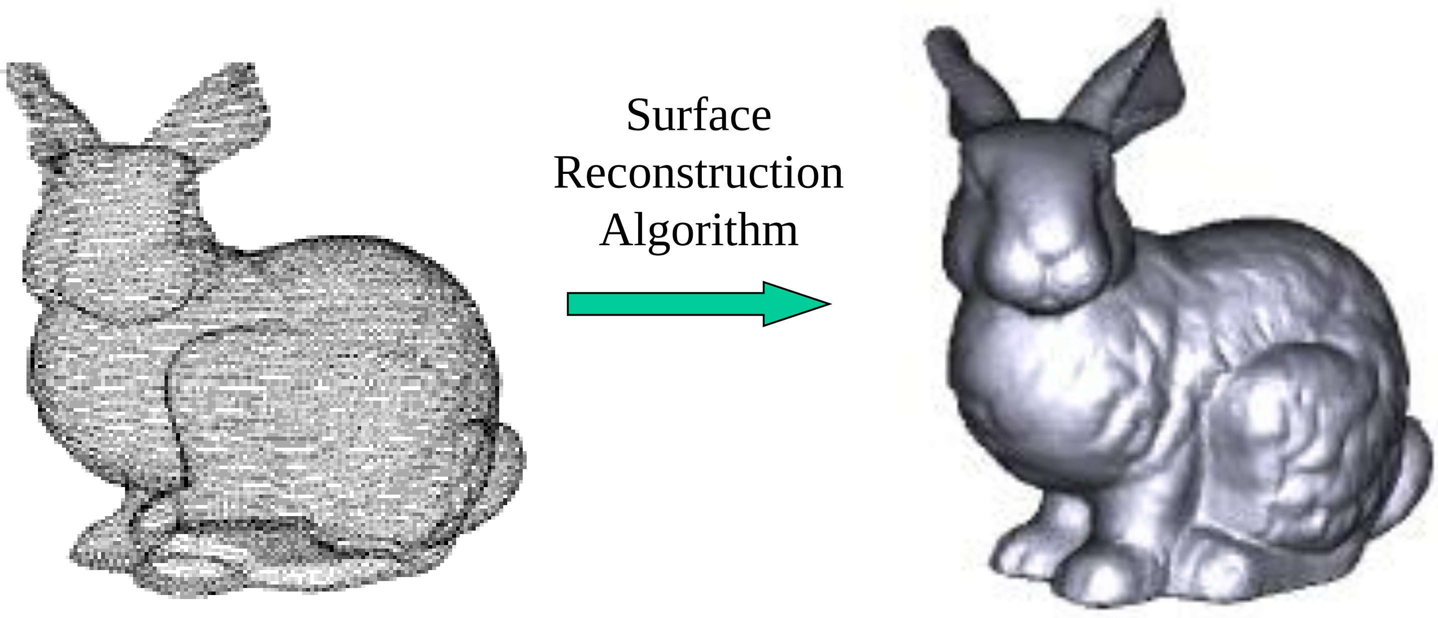
Absolute Geometries



OpTech

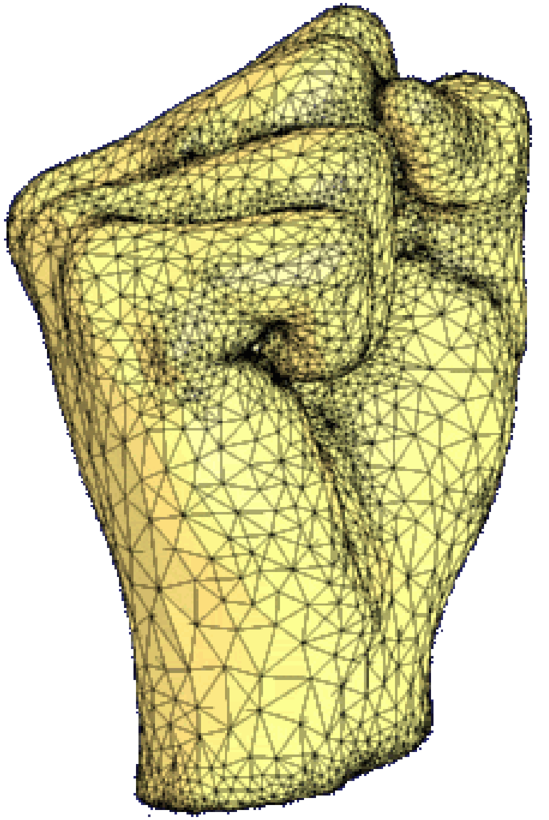
# Problem Statement

- Given a set of sample points in three dimensions produce a simplicial surface that captures the “most reasonable shape” the points were sampled from.



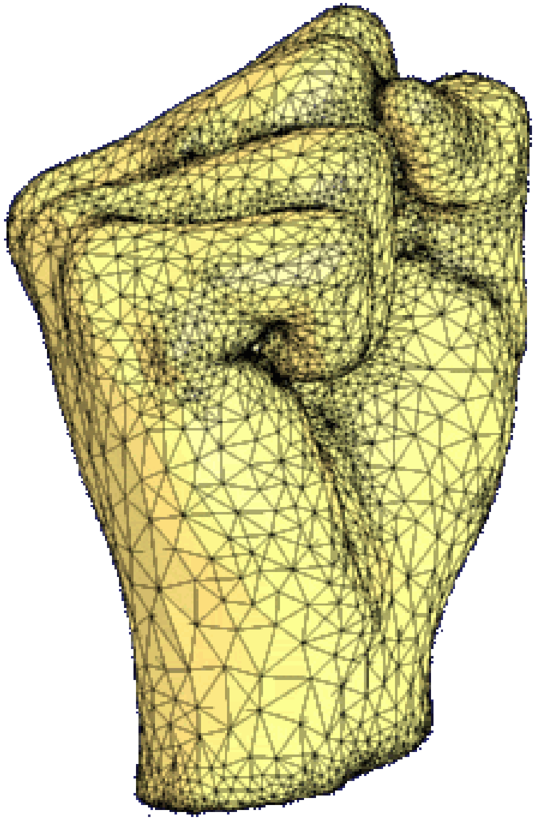
# Applications

- Computer graphics,
- Medical imaging
- Cartography
- Compression
- Reverse engineering
- Urban modeling
- etc.

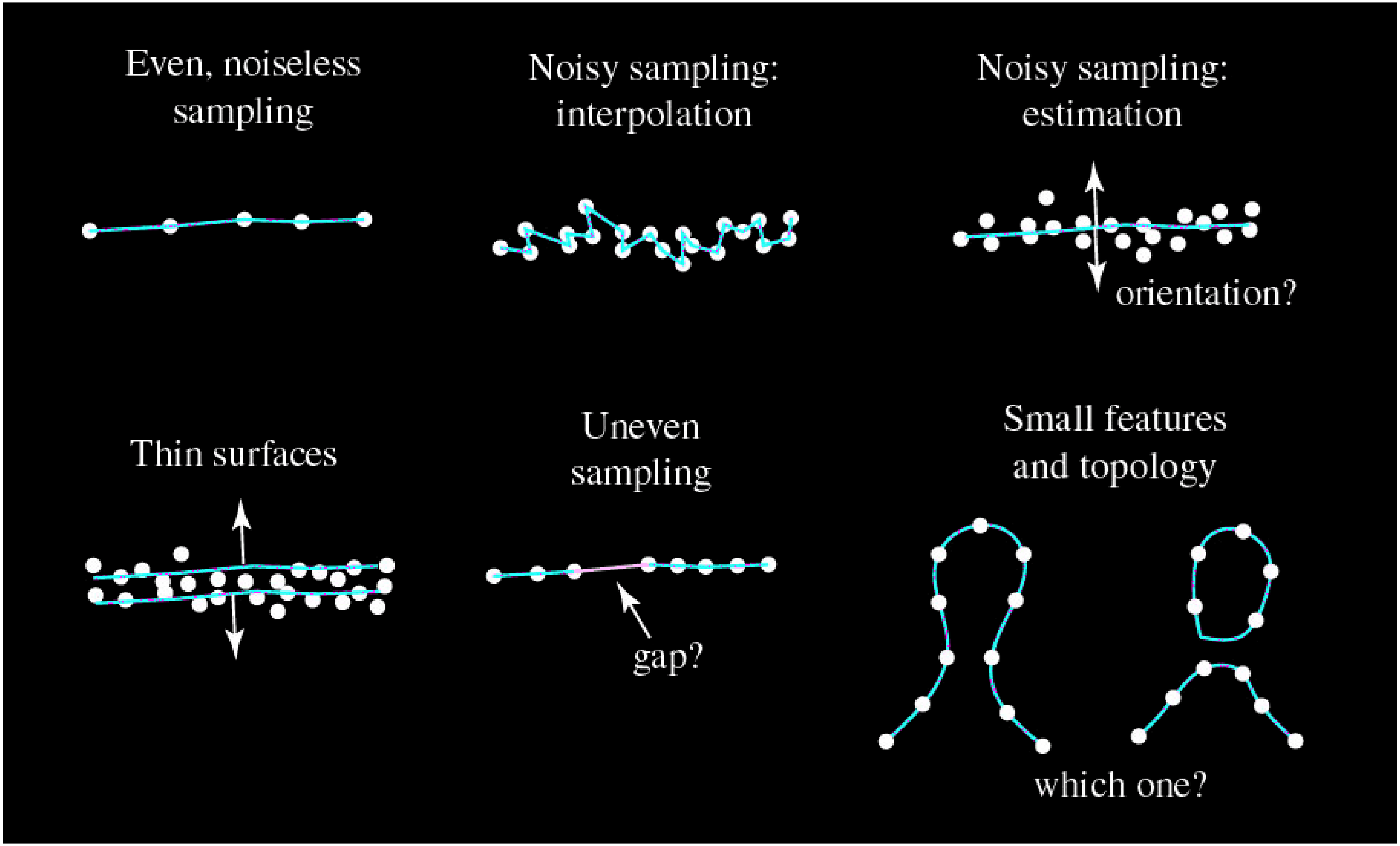


# Desirable Properties

- Interpolate input points
- Handle arbitrary genus
- Generally smooth
- Retain sharp features
- Watertight surface

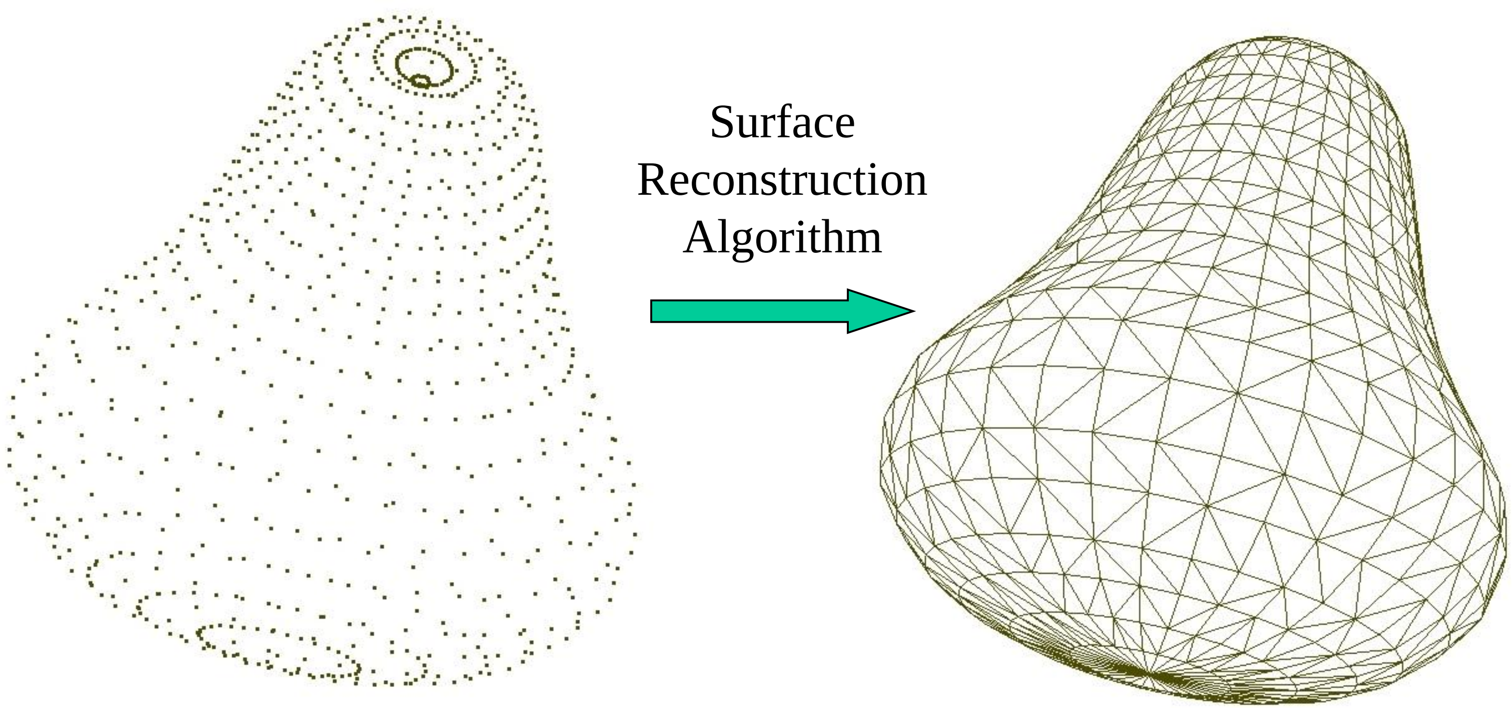


# Challenges



Even, noiseless sampling
Noisy sampling: interpolation
Noisy sampling: estimation
orientation?
Thin surfaces
Uneven sampling
gap?
Small features and topology
which one?

# Possible Approaches?



# Possible Approaches

- Explicit Meshing
  - Ball pivoting algorithm
  - Crust
  - etc.
- Implicit Reconstruction
  - Hoppe’s algorithm
  - Moving Least Squares (MLS)
  - Poisson surface reconstruction
  - etc.
- Surface fitting
  - Deformable templates
  - etc.

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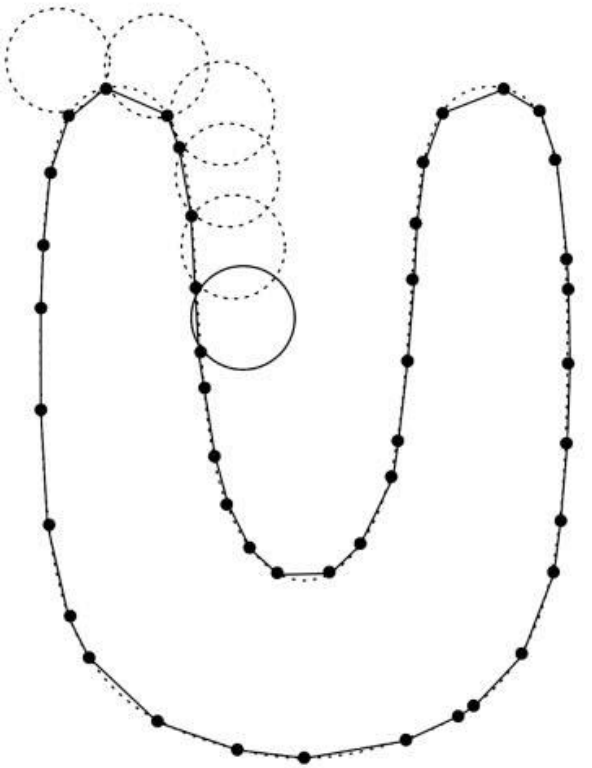
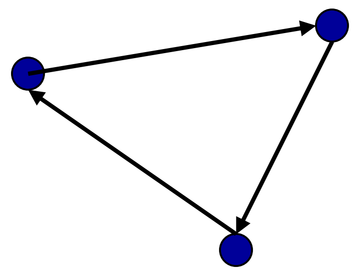
# The Ball Pivoting Algorithm

- Pick a ball radius, roll ball around surface, connect what it hits



# The Ball Pivoting Algorithm

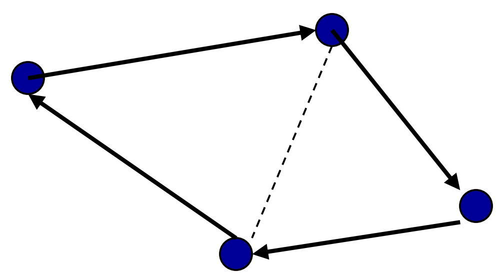


Initial seed triangle

Active edge

Point on front

# The Ball Pivoting Algorithm

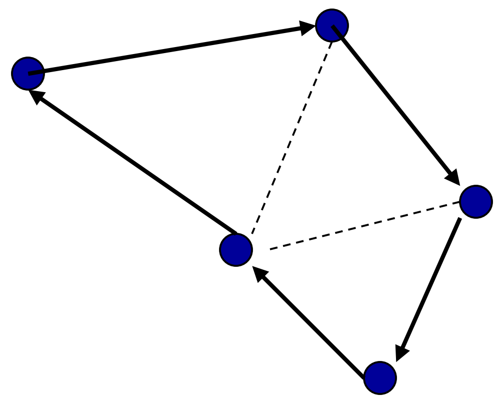


Ball pivoting around active edge

Active edge

Point on front

# The Ball Pivoting Algorithm



Ball pivoting around active edge

Active edge

Point on front

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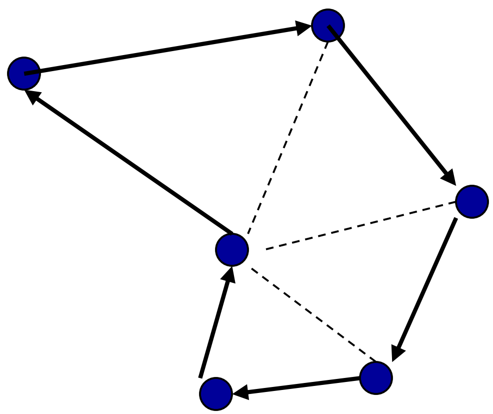
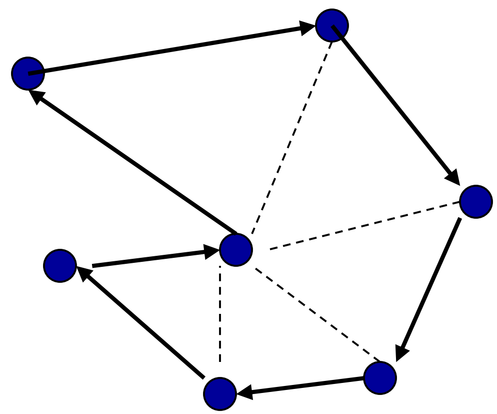


Ball pivoting around active edge

Active edge

Point on front

# The Ball Pivoting Algorithm



Ball pivoting around active edge

Active edge

Point on front

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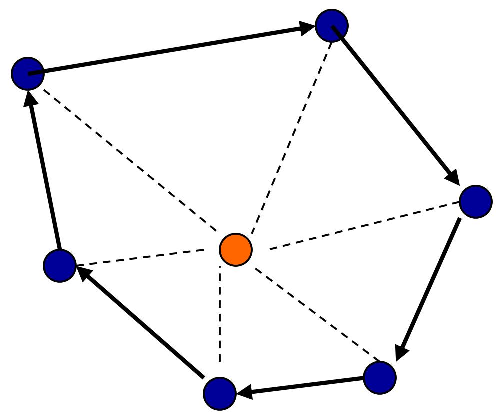


Ball pivoting around active edge

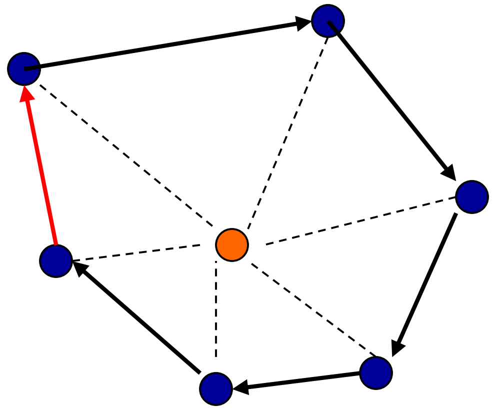
Active edge

Point on front

Internal point

# The Ball Pivoting Algorithm

Boundary edge



Ball pivoting around active edge

No pivot found

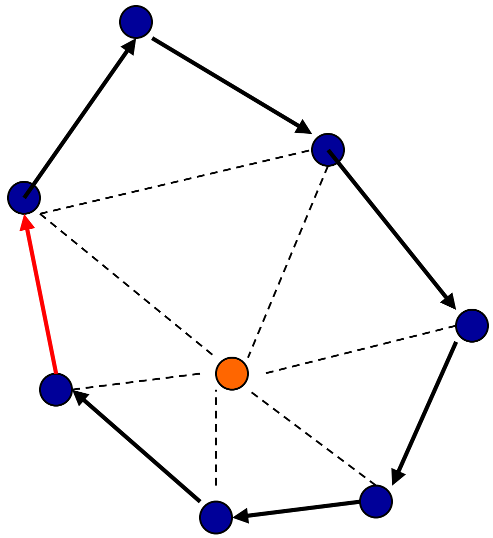
Active edge

Point on front

Internal point

# The Ball Pivoting Algorithm

Boundary edge



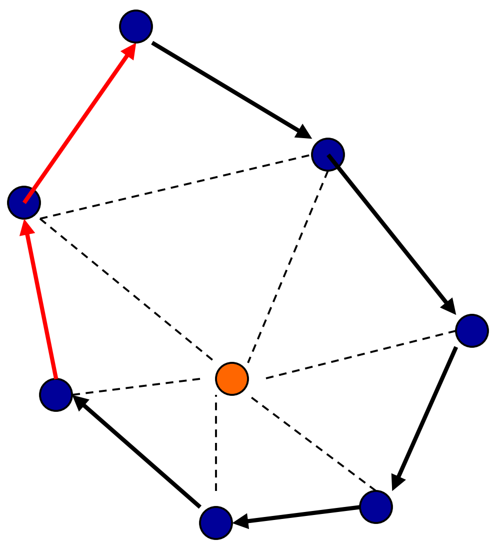
Ball pivoting around active edge

Active edge

- Point on front
- Internal point

# The Ball Pivoting Algorithm

Boundary edge



Ball pivoting around active edge

No pivot found

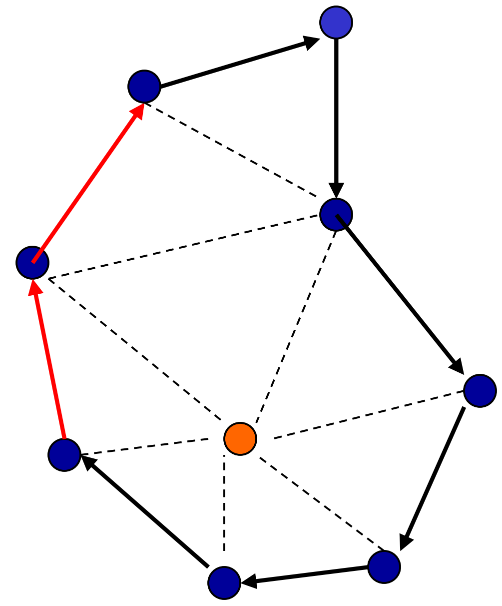
Active edge

Point on front

Internal point

# The Ball Pivoting Algorithm

Boundary edge



Ball pivoting around active edge

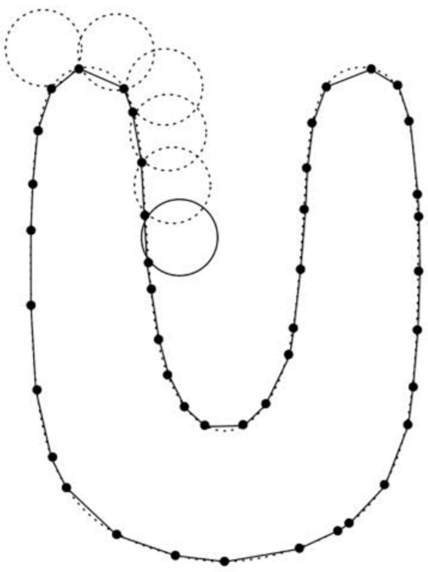
Active edge

Point on front

Internal point

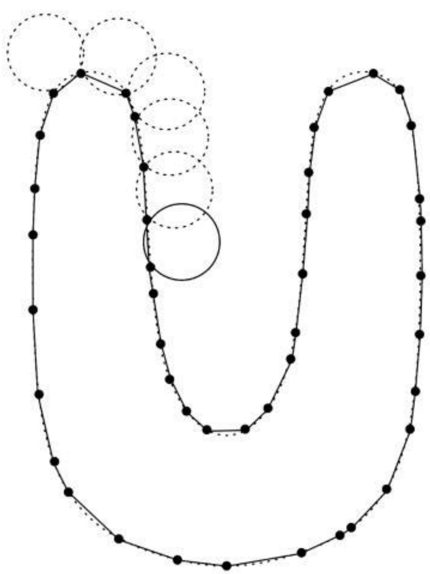
# The Ball Pivoting Algorithm

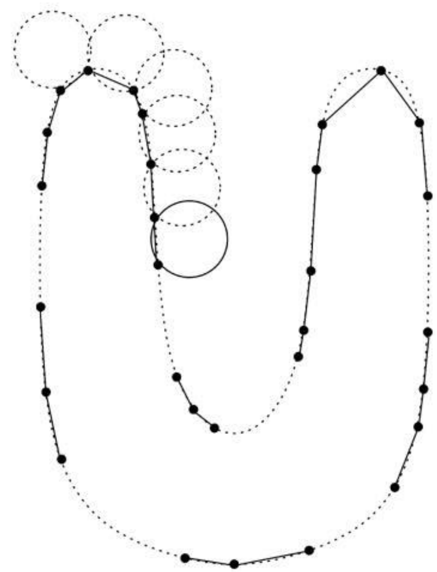
Possible problems?



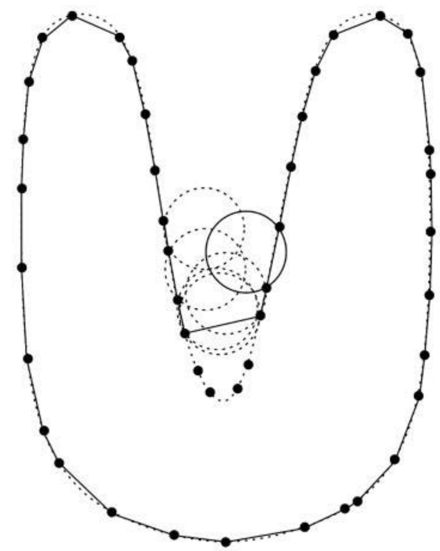
# The Ball Pivoting Algorithm

Possible problems?





Undersampling

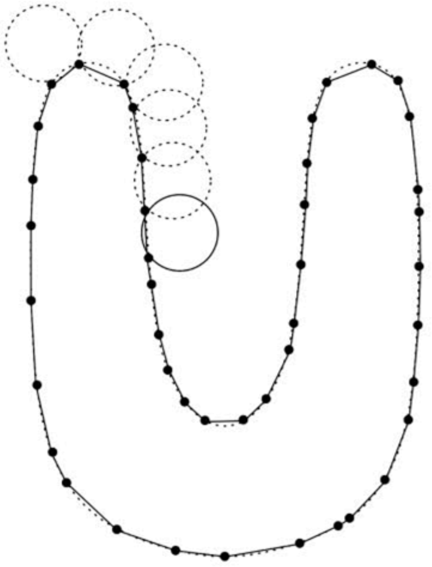


Small Concavities

# The Ball Pivoting Algorithm

Possible problems?

Self-intersection?   Watertight?



# Possible Approaches

- Explicit Meshing
  - Ball pivoting algorithm
  - Crust
  - etc.
- Implicit Reconstruction
  - Hoppe's algorithm
  - Moving Least Squares (MLS)
  - Poisson surface reconstruction
  - etc.
- Surface fitting
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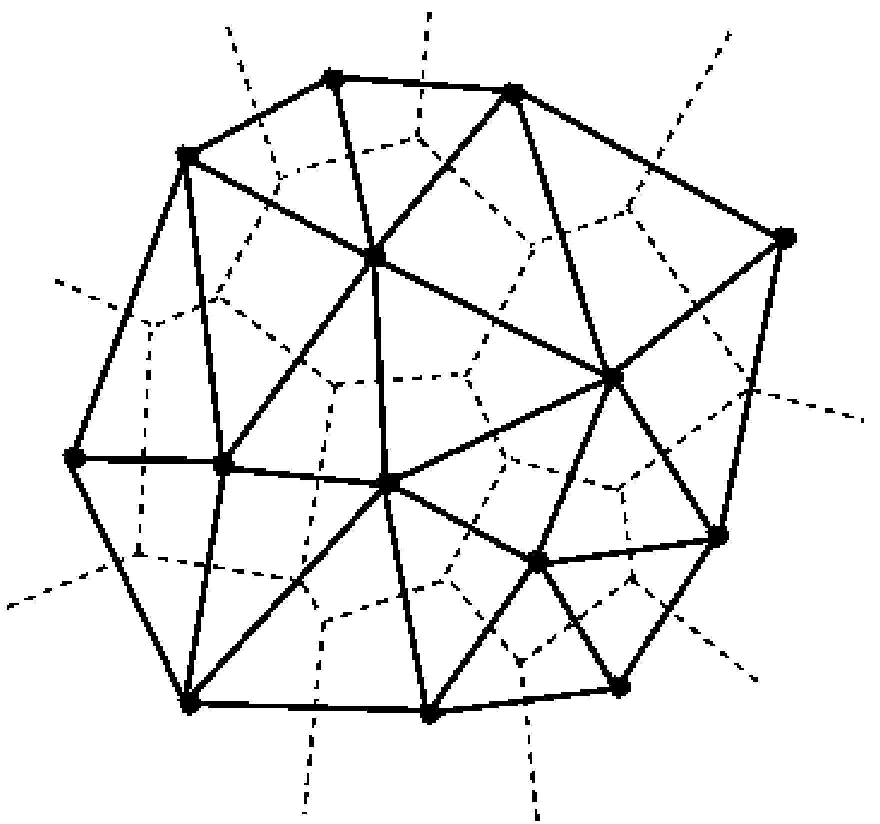
  - etc.

# Crust

Aims to find adjacent surface without a parameter specifying feature sizes

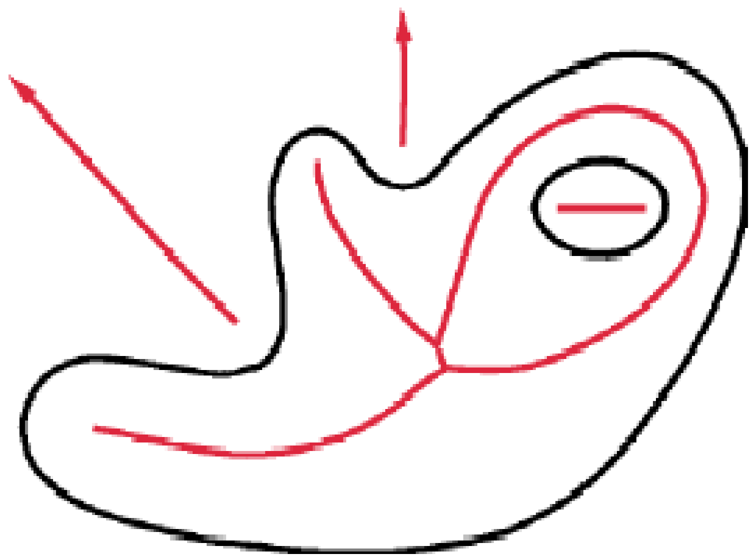
# Definitions

Delaunay Triangulation,  Voronoi Diagram



# Definitions

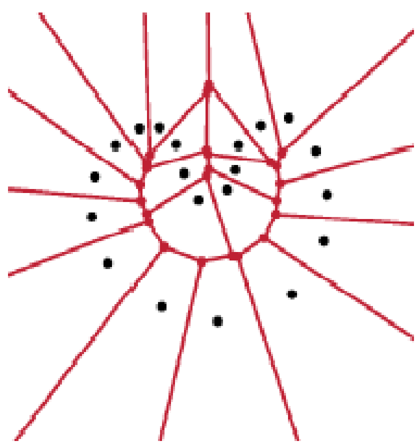
*Medial Axis*: of surface F is the closure of points that have more than one closest point in F.

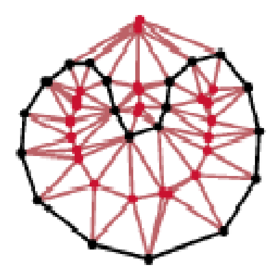


# The Intuition behind Crust

The Voronoi Cells of a dense sampling are thin and long.

The Medial Axis is the extension of Voronoi Diagram for continuous surfaces in the sense that the Voronoi Diagram of S Can be defined as the set of points with more than one closest point in S. (S = Sample Point Set)





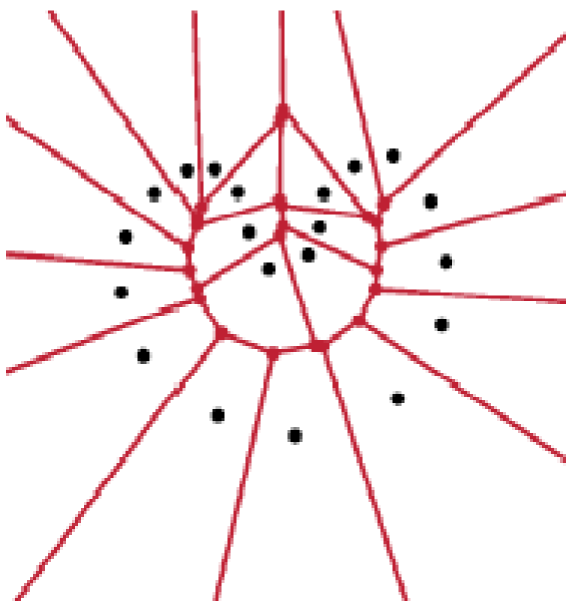
# Crust in 2D

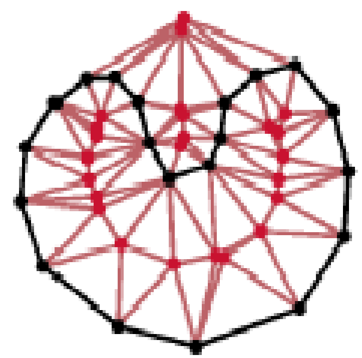
Input : P = Set of sample points in the plane
Output: E = Set of edges connecting points in P

The Algorithm

Compute the Voronoi vertices of P = V
Calculate the Delaunay of (P U V)
Pick the edges (p,q) where both p,q are in P

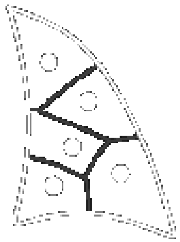
# Sample Output

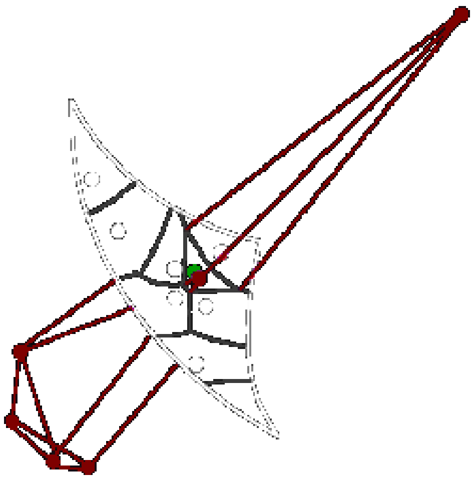


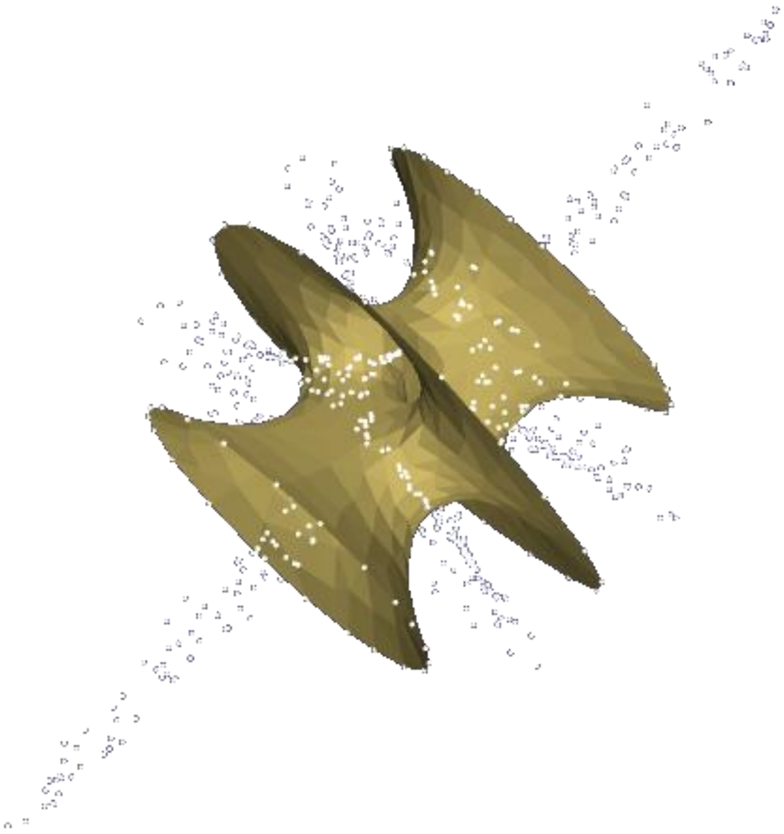


# Crust in 3D

- Some Voronoi vertices lie neither near the surface nor near the medial axis
- Keep the "poles"



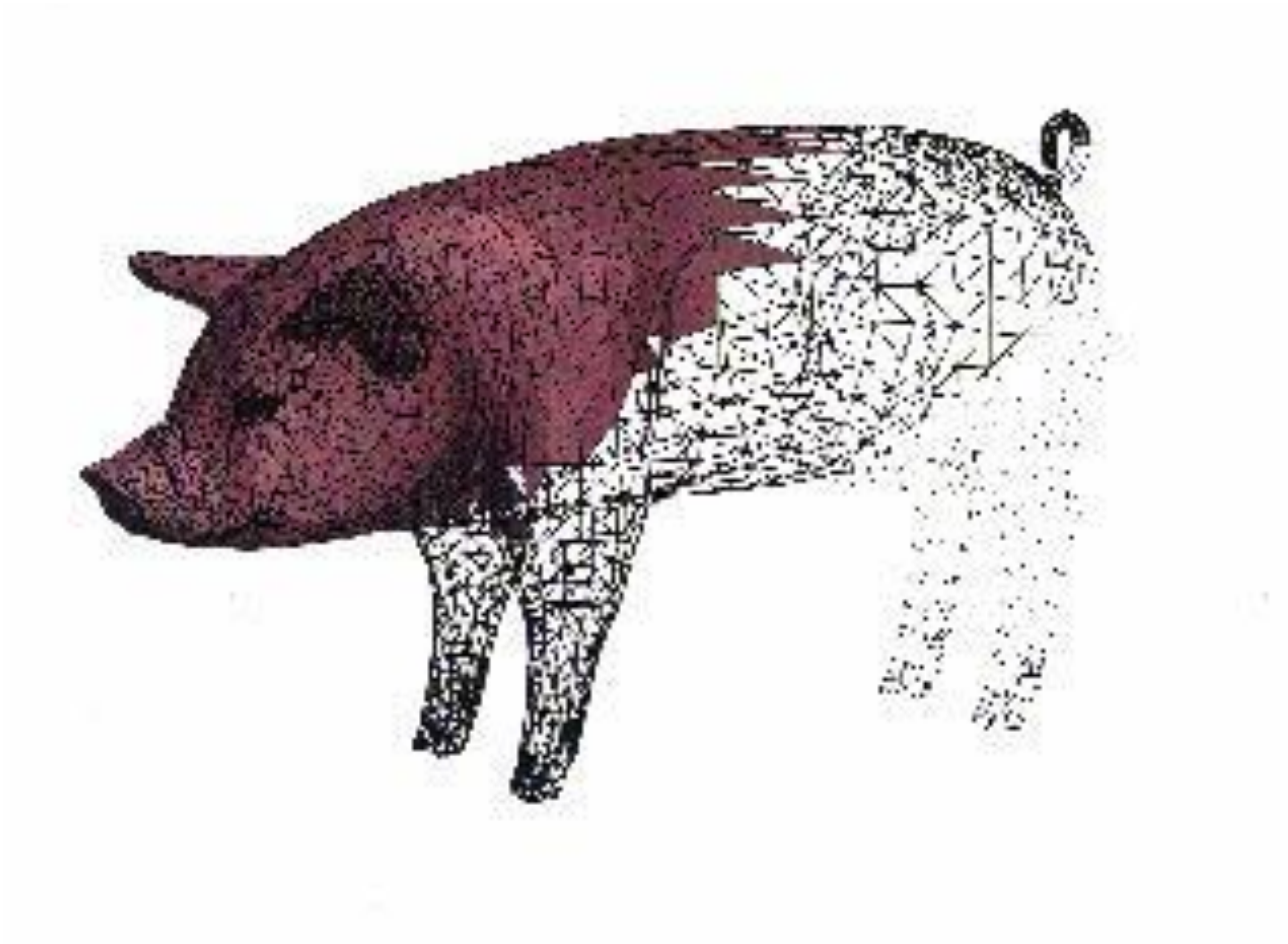




# Crust in 3D

- Compute the 3D Voronoi diagram of the sample points.
- For each sample point $s$, pick the farthest vertex $v$ of its Voronoi cell, and the farthest vertex $v'$ such that angle $vsv'$ exceeds 90 degrees.
- Compute the Voronoi diagram of the sample points and the "poles", the Voronoi vertices chosen in the second step.
- Add a triangle on each triple of sample points with neighboring cells in the second Voronoi diagram.

# Sample Output



# Possible Approaches

- Explicit Meshing
  - Ball pivoting algorithm
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  - etc.
- Implicit Reconstruction
  - Hoppe’s algorithm
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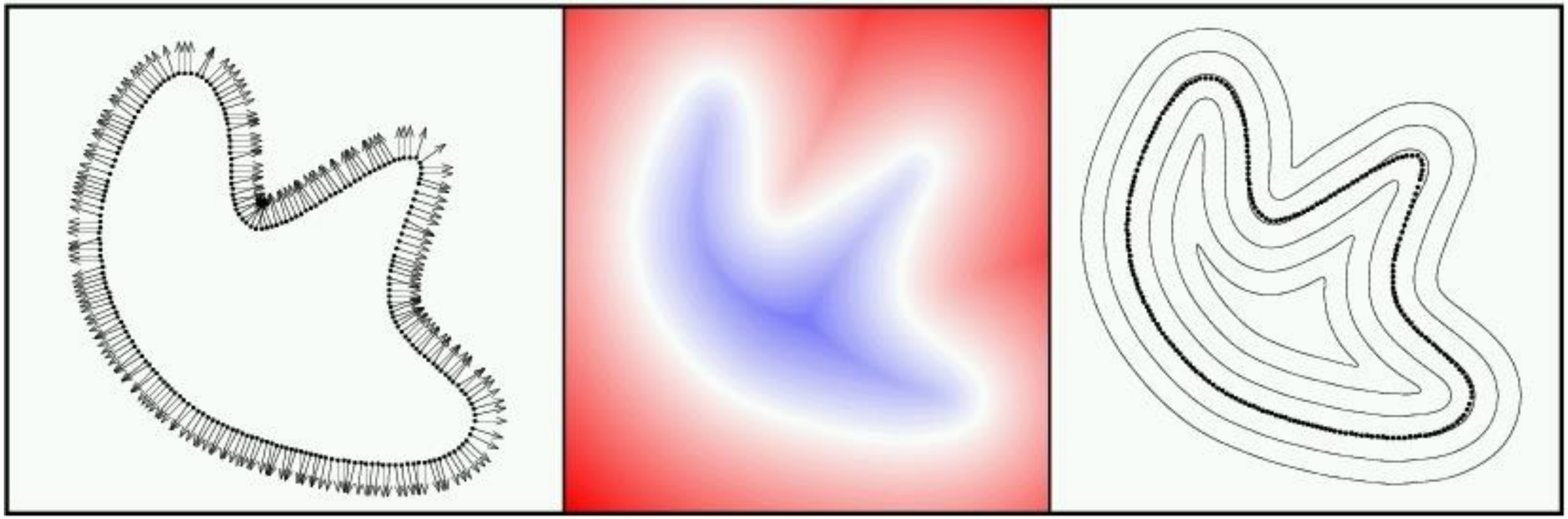
  - Poisson surface reconstruction
  - etc.
- Surface fitting
  - Deformable templates
  - etc.

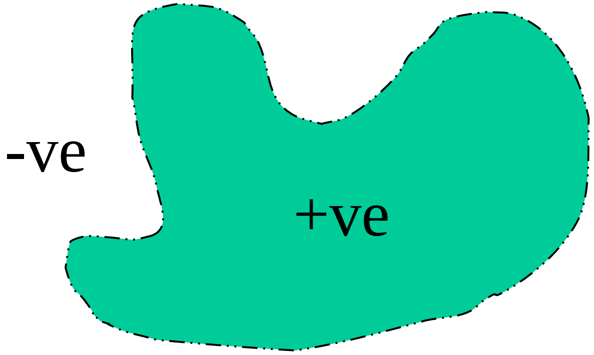
# Implicit Reconstruction

- Main idea:
  - Compute an implicit function f(p) (negative outside, positive inside)
  - Extract surface where f(p)=0



# Hoppe et al's Algorithm

1. Tangent Plane Estimation
2. Consistent tangent plane orientation
3. Signed distance function computation
4. Surface extraction



# Tangent Plane Estimation

- Principal Component Analysis (PCA)
  - Extract points $\{q_i\}$ in neighborhood
  - Compute covariance matrix M
  - Analyze eigenvalues and eigenvectors of M (via SVD)

$$\mathbf{M} = \frac{1}{n}\sum_{i=1}^{n}\begin{bmatrix} q_i^x q_i^x & q_i^x q_i^y & q_i^x q_i^z \\ q_i^y q_i^x & q_i^y q_i^y & q_i^y q_i^z \\ q_i^z q_i^x & q_i^z q_i^y & q_i^z q_i^z \end{bmatrix}$$

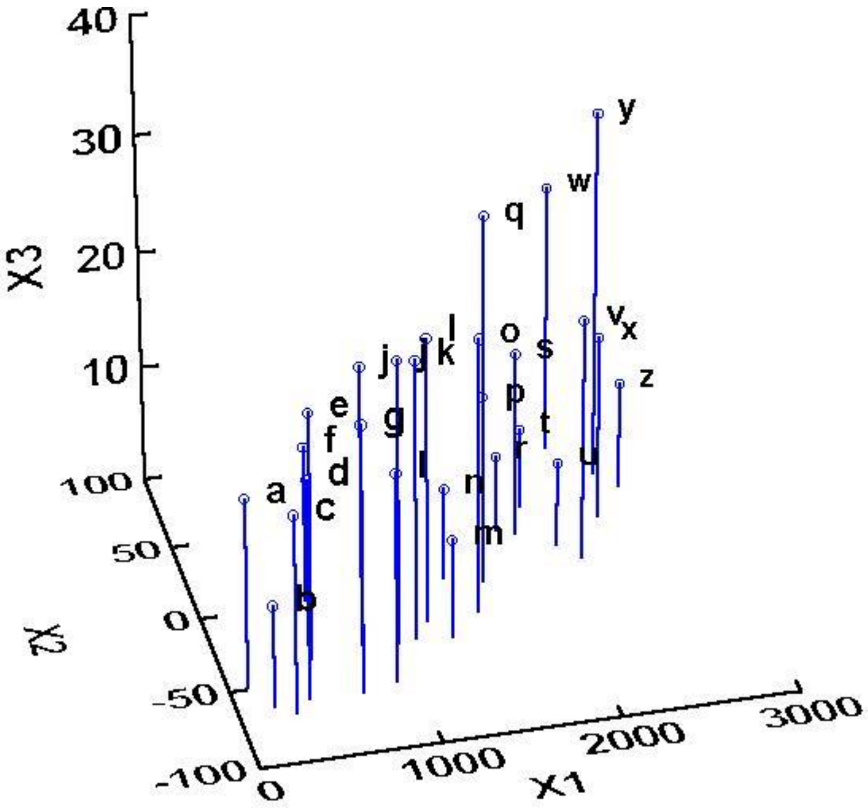
Covariance Matrix

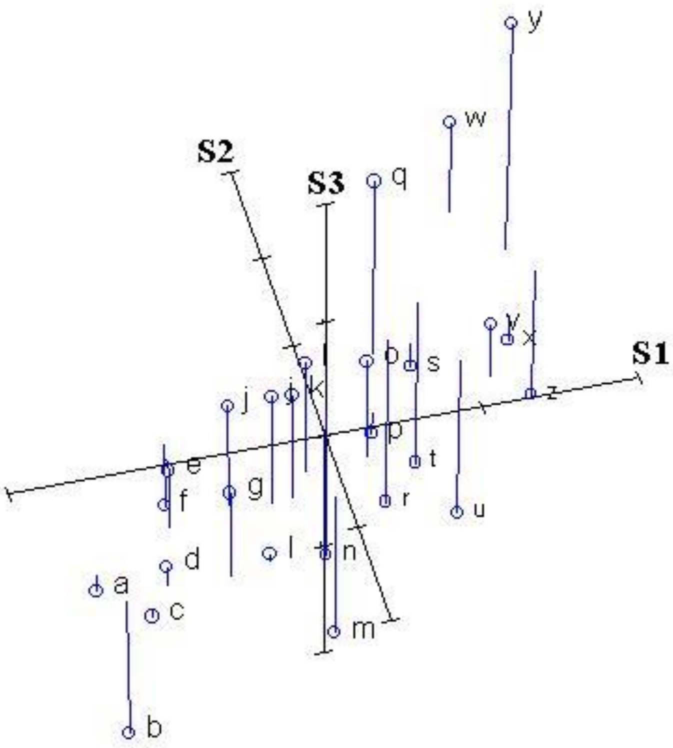
$$\mathbf{M} = \mathbf{U}\mathbf{S}\mathbf{U}^t$$

$$\mathbf{S} = \begin{bmatrix} \lambda_a & 0 & 0 \\ 0 & \lambda_b & 0 \\ 0 & 0 & \lambda_c \end{bmatrix} \quad \mathbf{U} = \begin{bmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{bmatrix}$$

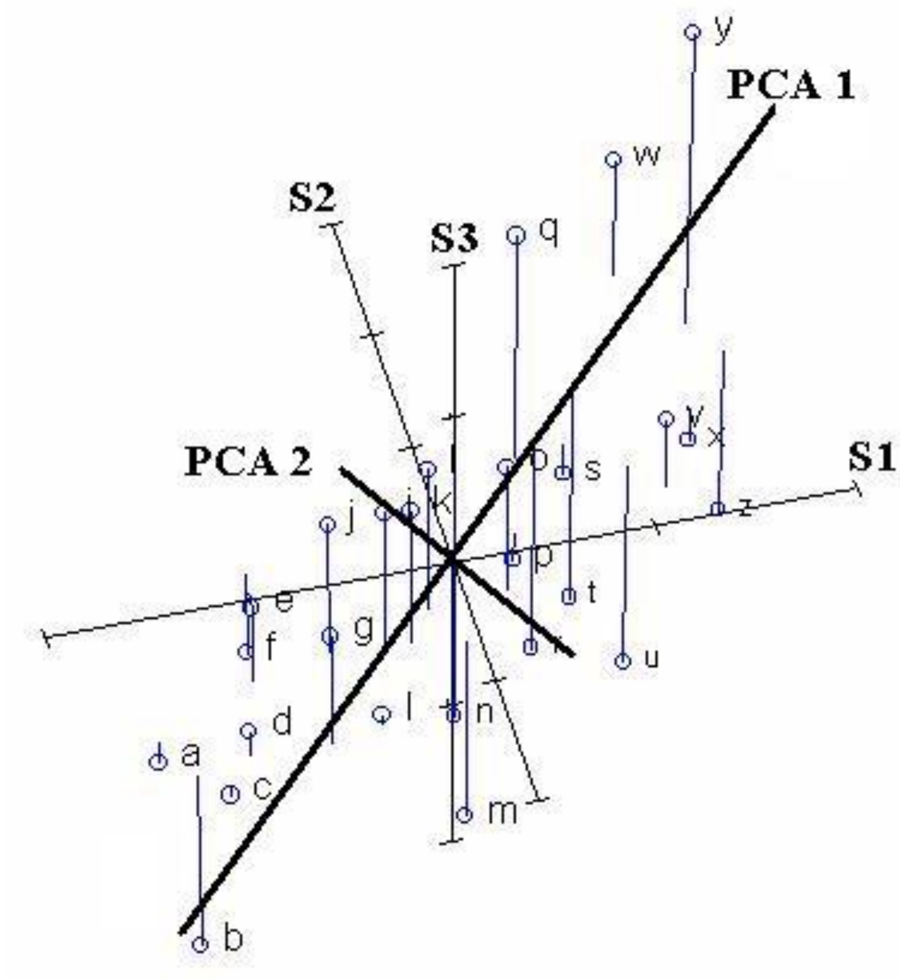
Eigenvalues & Eigenvectors

# Tangent Plane Estimation



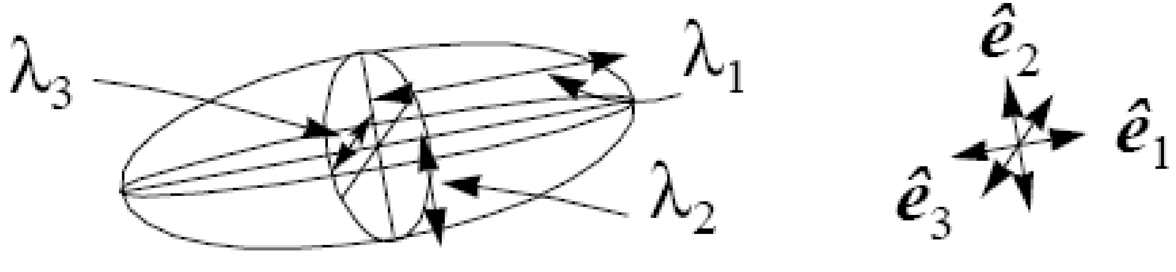



# Tangent Plane Estimation



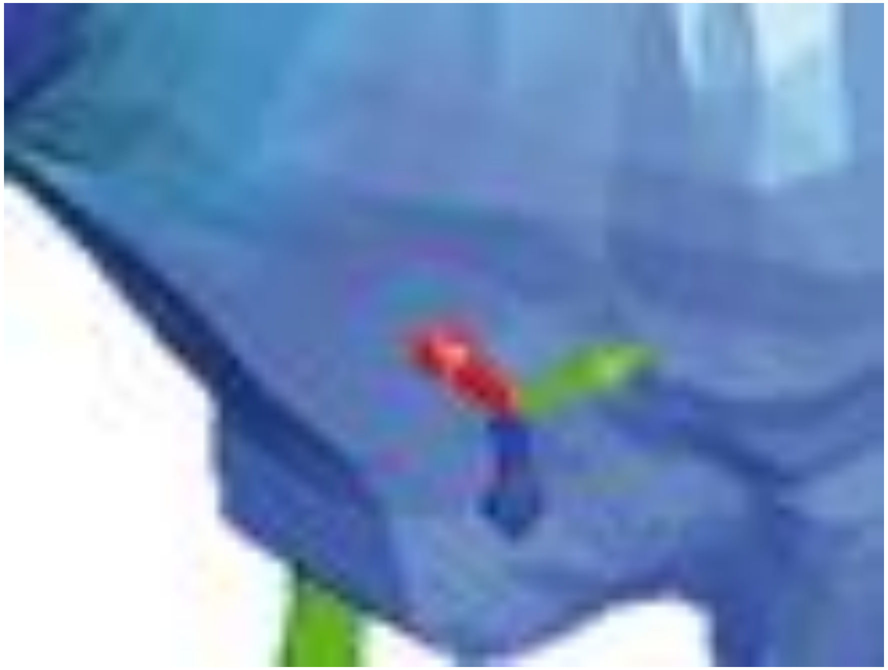
# Tangent Plane Estimation

- Eigenvectors are "Principal Axes of Inertia"
- Eigenvalues are variances of the point distribution in those directions



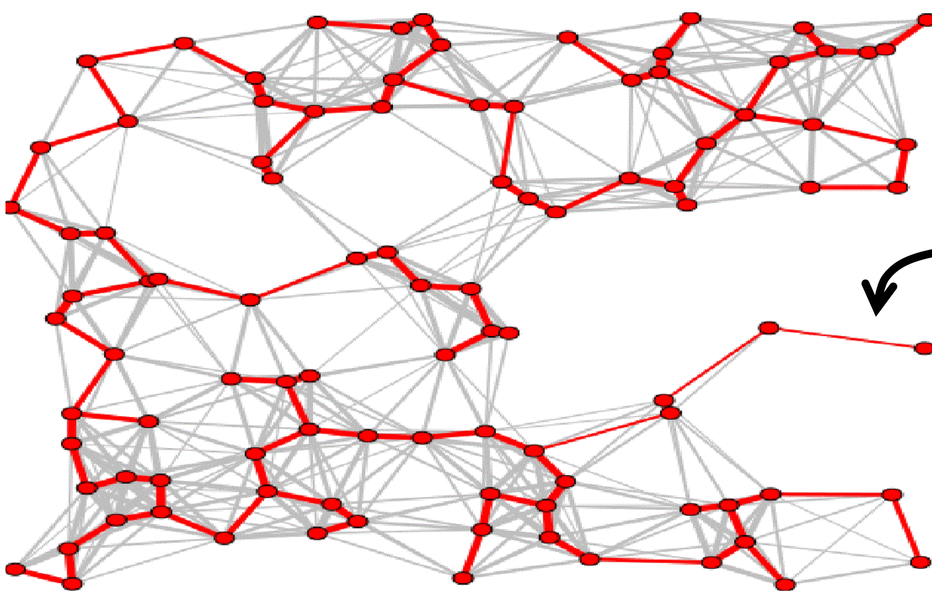
# Tangent Plane Estimation

- Surface normal is estimated by eigenvector (principal axis) associated with smallest eigenvalue



# Consistent Tangent Plane Orientation

- Traverse nearest neighbor graph flipping normals for consistency
  - Greedy propagation algorithm (minimum spanning tree of normal similarity)



$$1 - |\hat{\mathbf{n}}_i \cdot \hat{\mathbf{n}}_j|$$

# Signed Distance Function

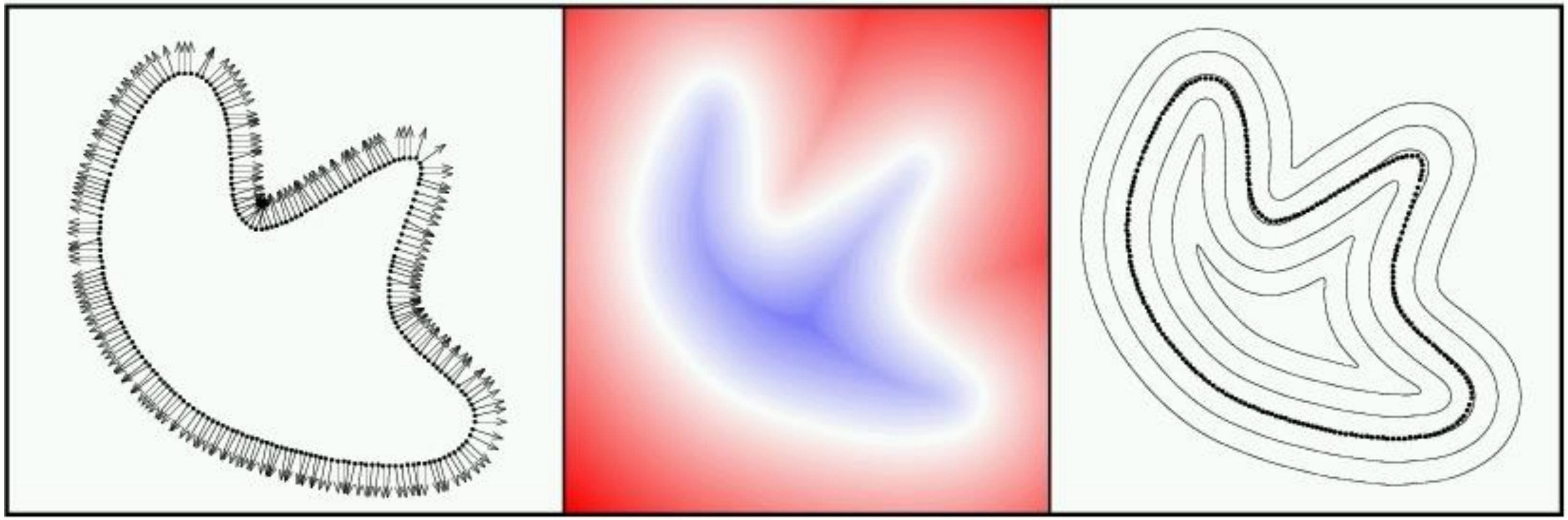
- f(p) is signed distance to tangent plane of closest point sample

$$\{ \textit{ Compute } \mathbf{z} \textit{ as the projection of } \mathbf{p} \textit{ onto } Tp(\mathbf{x}_i) \}$$
$$\mathbf{z} \leftarrow \mathbf{o}_i - ((\mathbf{p} - \mathbf{o}_i) \cdot \hat{\mathbf{n}}_i)\, \hat{\mathbf{n}}_i$$

$$\begin{array}{ll} \textbf{if} \quad d(\mathbf{z}, X) < \rho + \delta \quad \textbf{then} & \\ \qquad f(\mathbf{p}) \leftarrow (\mathbf{p} - \mathbf{o}_i) \cdot \hat{\mathbf{n}}_i & \qquad \{= \pm \|\mathbf{p} - \mathbf{z}\|\} \\ \textbf{else} & \\ \qquad f(\mathbf{p}) \leftarrow \textbf{undefined} & \\ \textbf{endif} & \end{array}$$

# Signed Distance Function

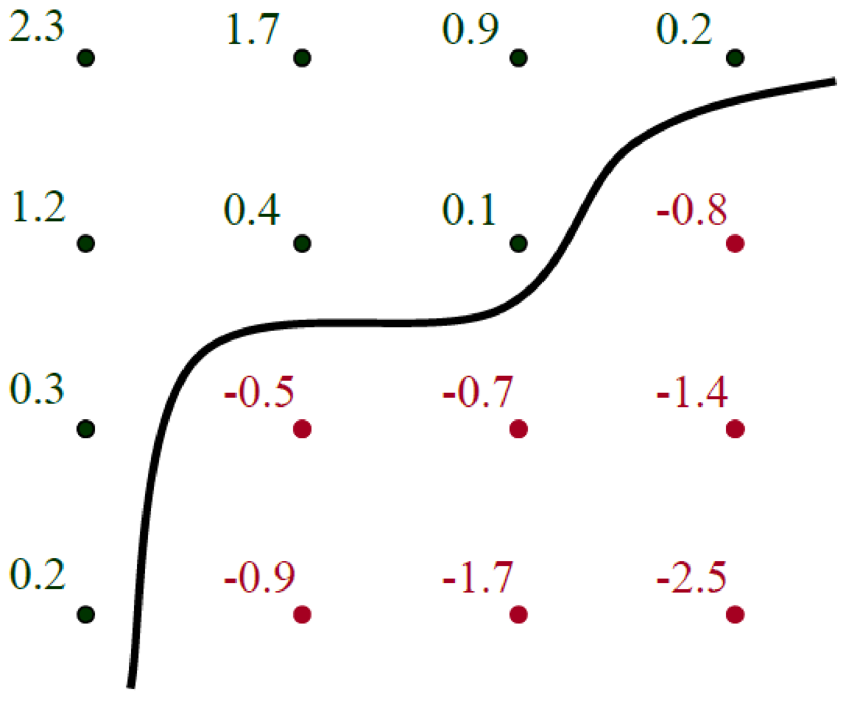
- f(p) is signed distance to tangent plane of closest point sample



Ravikrishna Bvs Kolluri

# Surface Extraction

- Extract triangulated surface where f(p)=0
  - e.g., Marching Cubes



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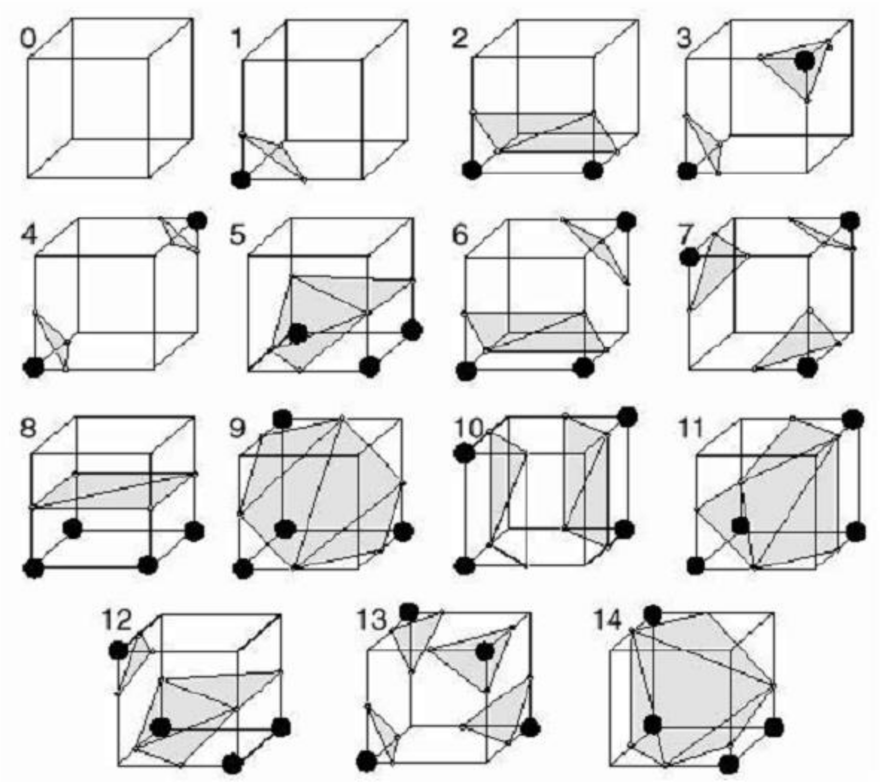
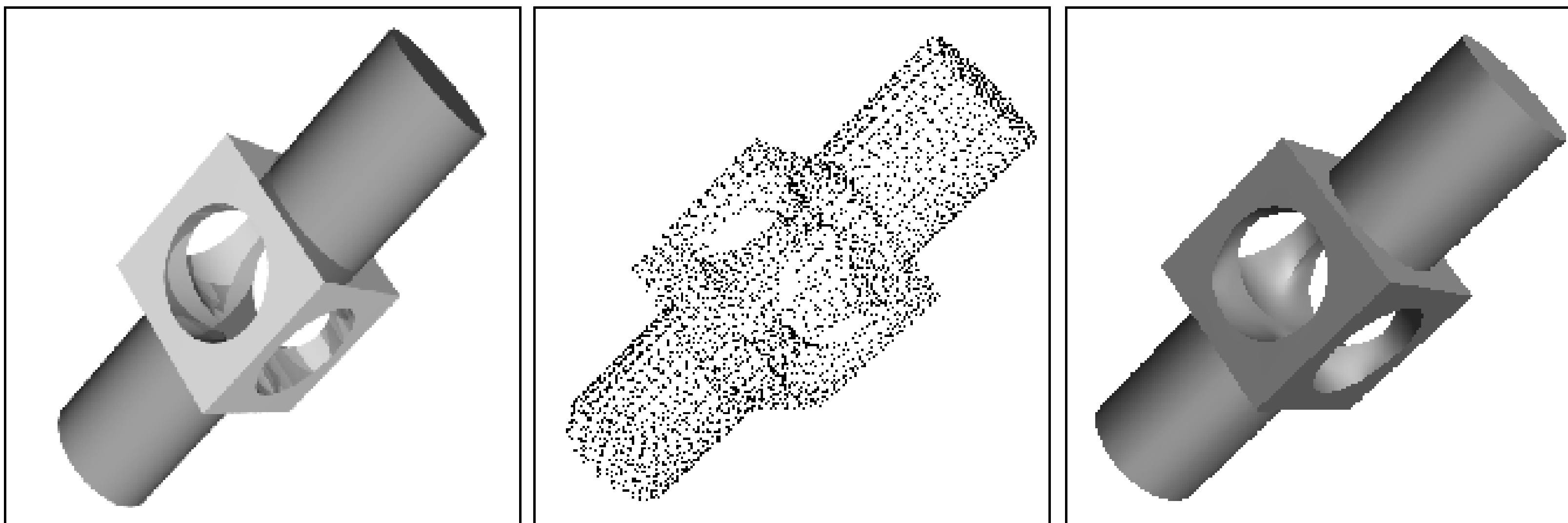
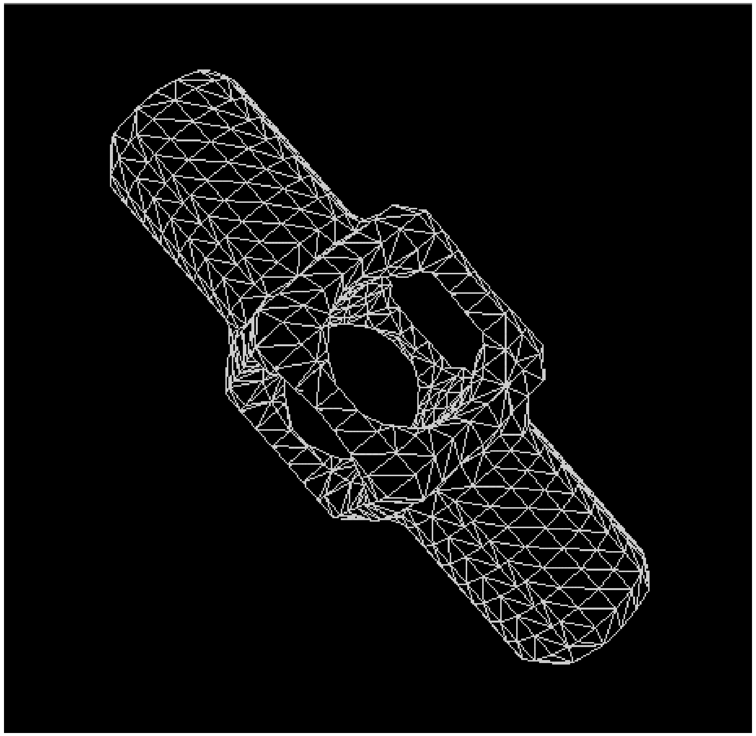


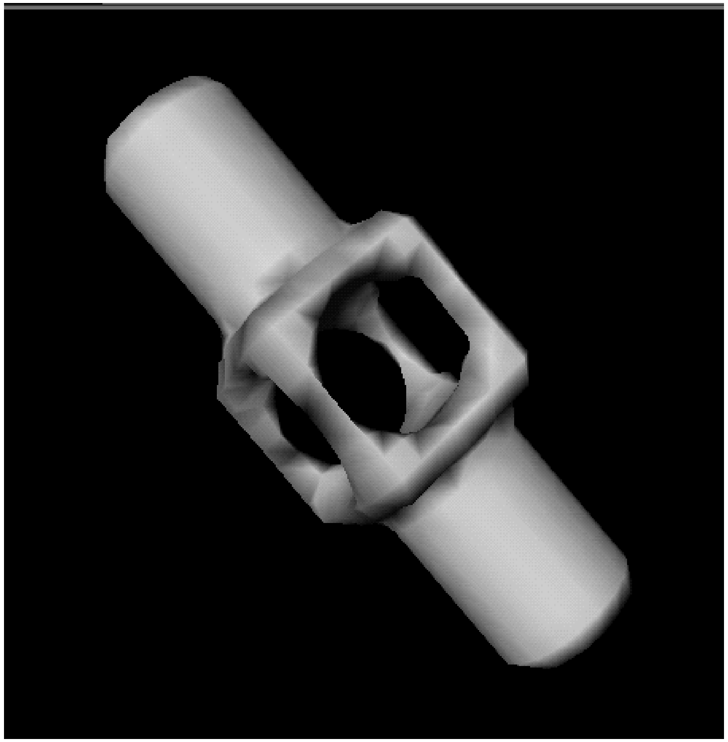
# Sample Results



(a) Unknown surface $U$ (b) Sampled points $X$ (c) Reconstructed surface $S$

# Sample Results





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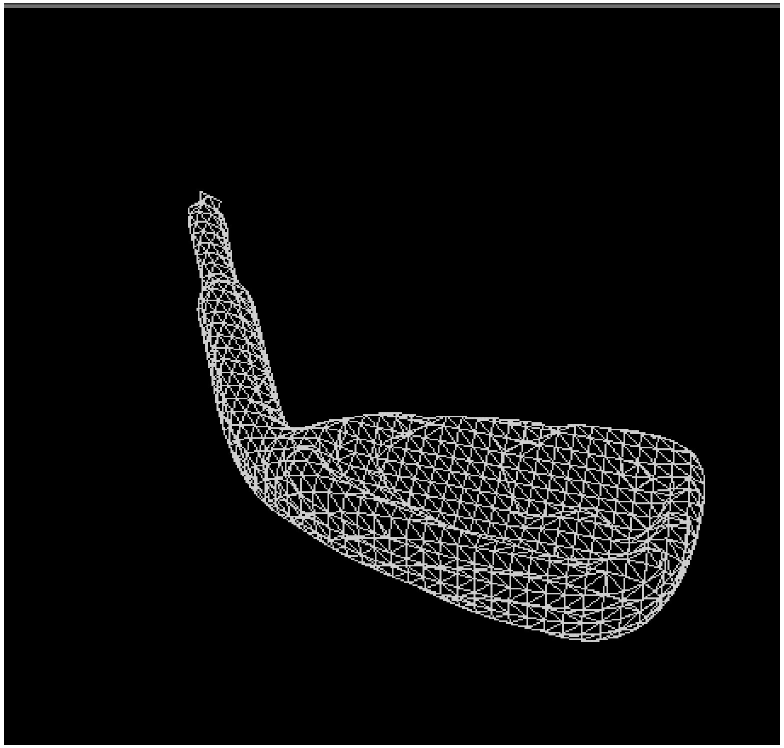

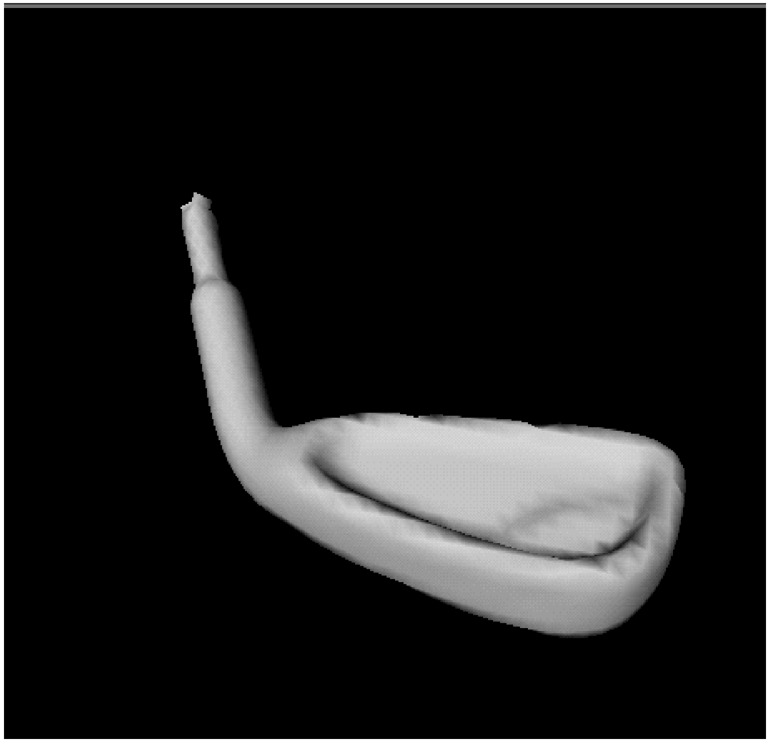

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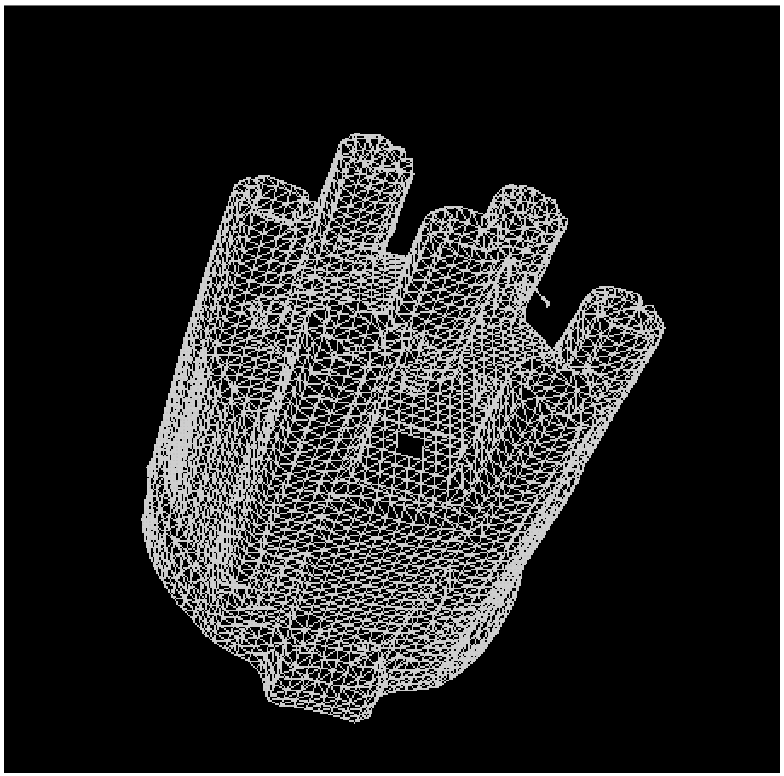
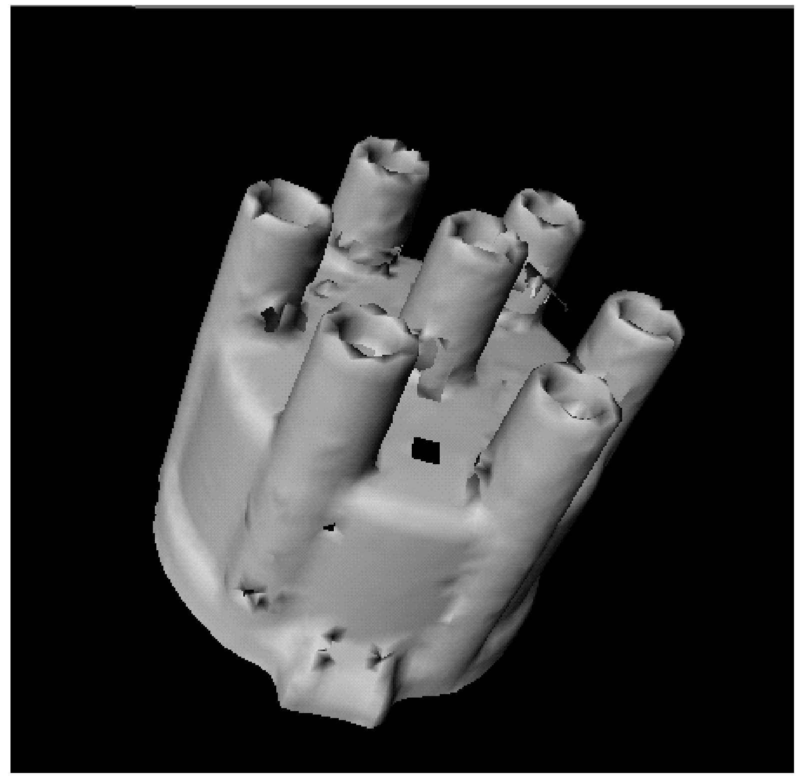
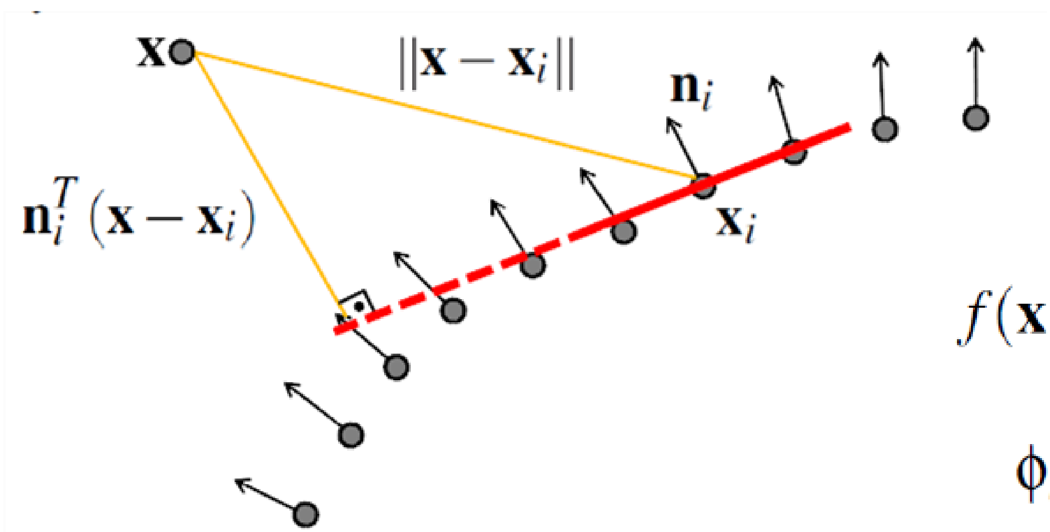




# Moving Least Squares

- Similar, but different implicit function
  - Weighted contribution of nearby points

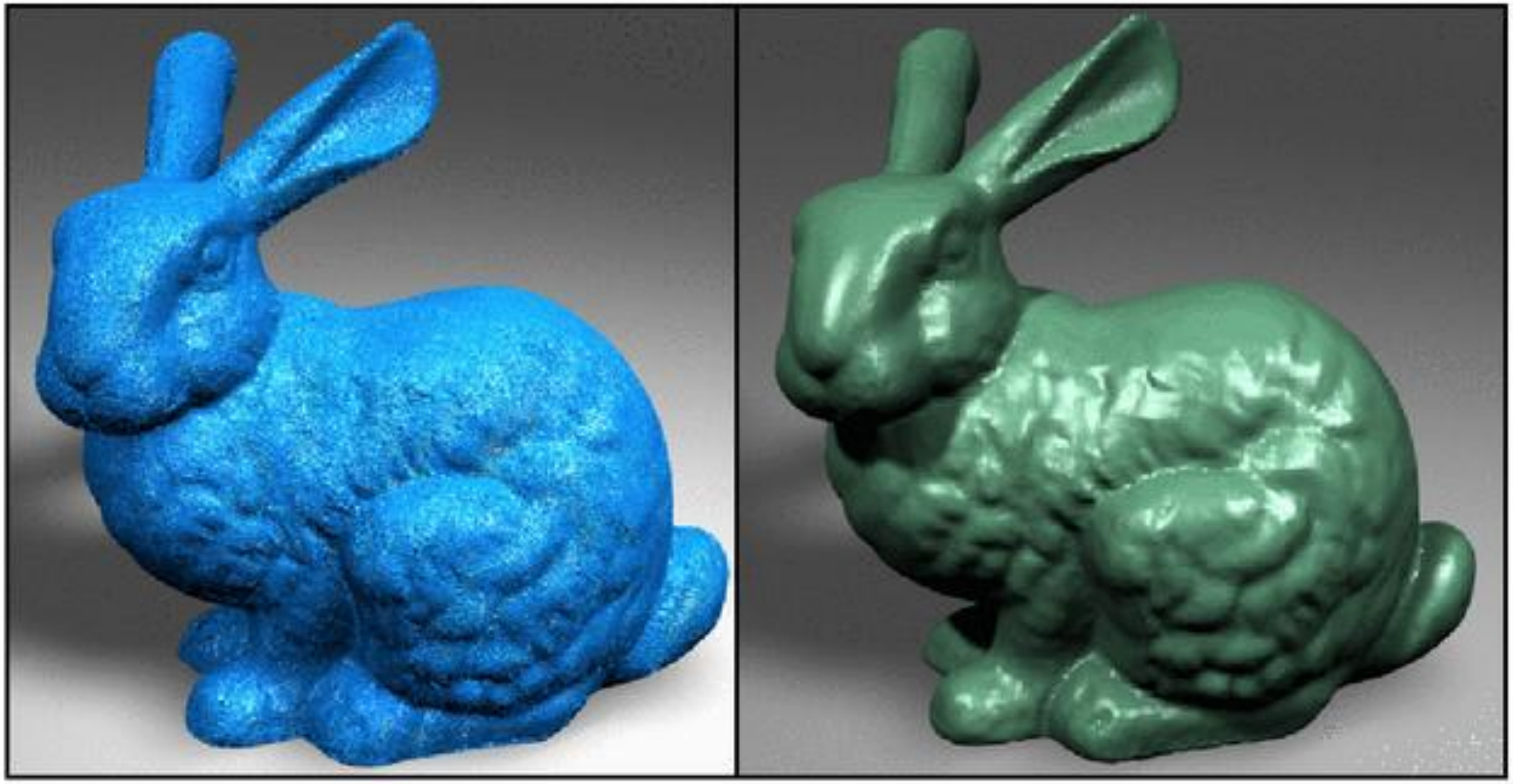



$$f(\mathbf{x}) = \frac{\sum \mathbf{n}_i^T(\mathbf{x}-\mathbf{x}_i)\phi_i(\mathbf{x})}{\sum \phi_i(\mathbf{x})}$$

$$\phi_i(\mathbf{x}) = \phi(\|\mathbf{x}-\mathbf{x}_i\|)$$

$$\phi(x) = e^{-\left(\frac{x}{\sigma_r}\right)^2}$$

# Moving Least Squares



MLS

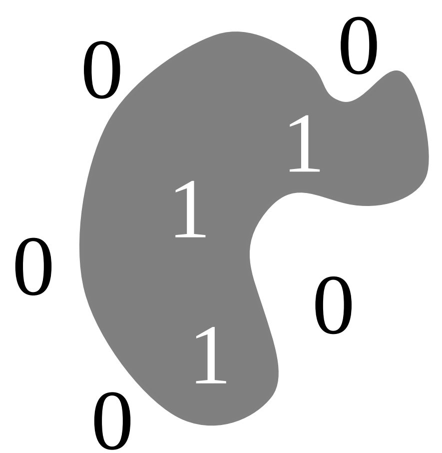
# Possible Approaches

- Explicit Meshing
  - Ball pivoting algorithm
  - Crust
  - etc.
- Implicit Reconstruction
  - Hoppe's algorithm
  - Moving Least Squares (MLS)
  - Poisson surface reconstruction
  - etc.
- Surface fitting
  - Deformable templates
  - etc.

# The Indicator Function

- We reconstruct the surface of the model by solving for the indicator function of the shape.

$$\chi_M(p) = \begin{cases} 1 & \text{if } p \in M \\ 0 & \text{if } p \notin M \end{cases}$$

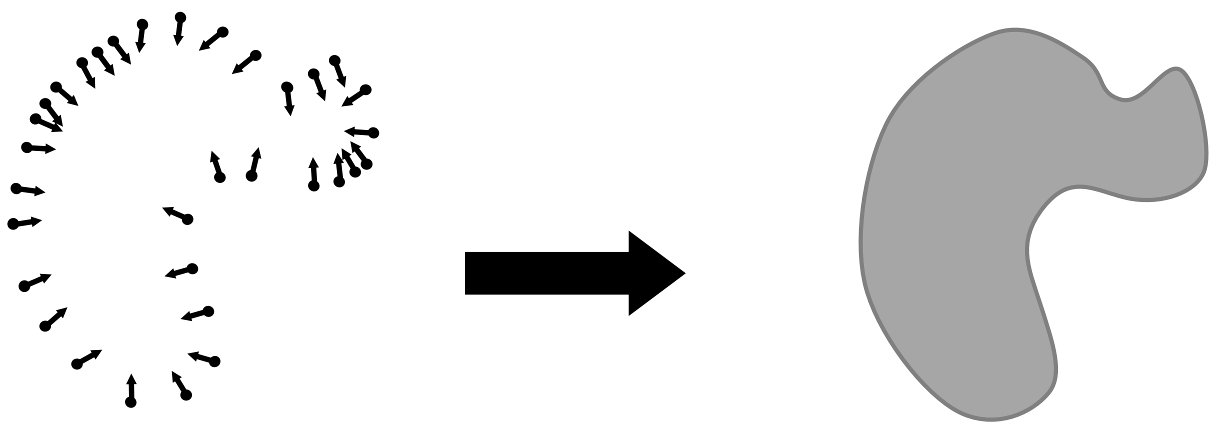



Indicator function

$\chi_M$

# Challenge

- How to construct the indicator function?



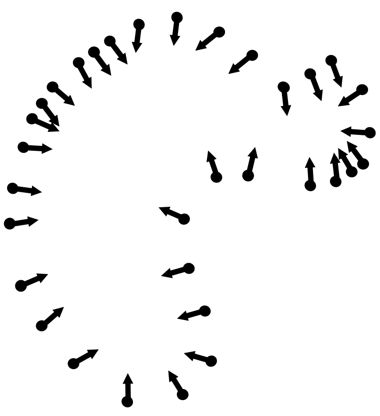
Oriented points

Indicator function

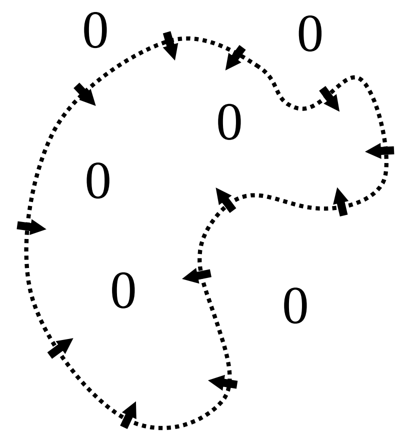
$\chi_M$

# Gradient Relationship

- There is a relationship between the normal field and gradient of indicator function



Oriented points




Indicator gradient

$\nabla \chi_M$

# Integration

- Represent the points by a vector field $\vec{V}$
- Find the function $\chi$ whose gradient best approximates $\vec{V}$ :

$$\min_{\chi} \left\| \nabla \chi - \vec{V} \right\|$$

# Integration as a Poisson Problem

- Represent the points by a vector field $\vec{V}$
- Find the function $\chi$ whose gradient best approximates $\vec{V}$:

$$\min_{\chi} \left\| \nabla \chi - \vec{V} \right\|$$

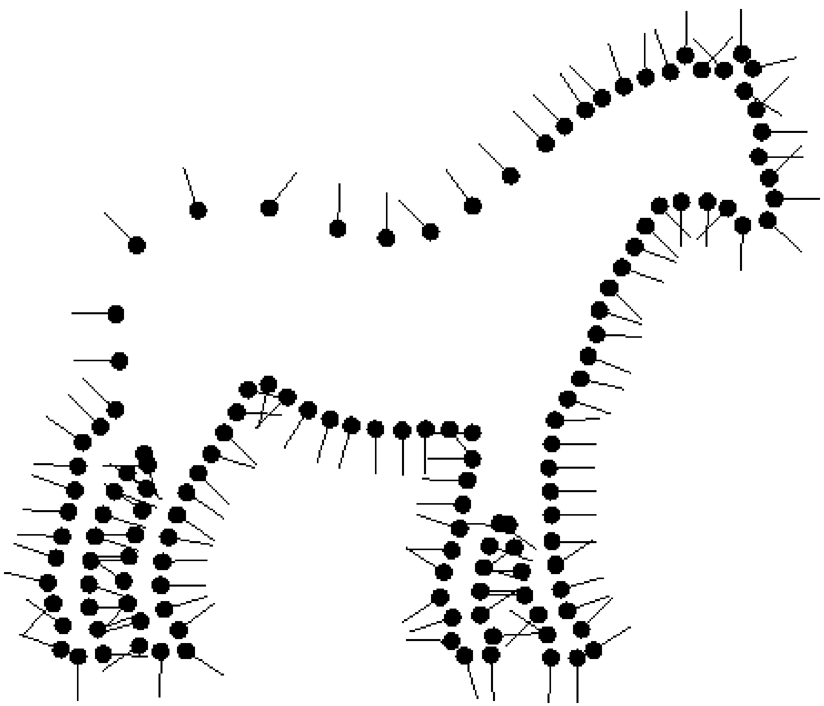
- Applying the divergence operator, we can transform this into a Poisson problem:

$$\nabla \cdot (\nabla \chi) = \nabla \cdot \vec{V} \quad \Leftrightarrow \quad \Delta \chi = \nabla \cdot \vec{V}$$

# Implementation

Given the Points:

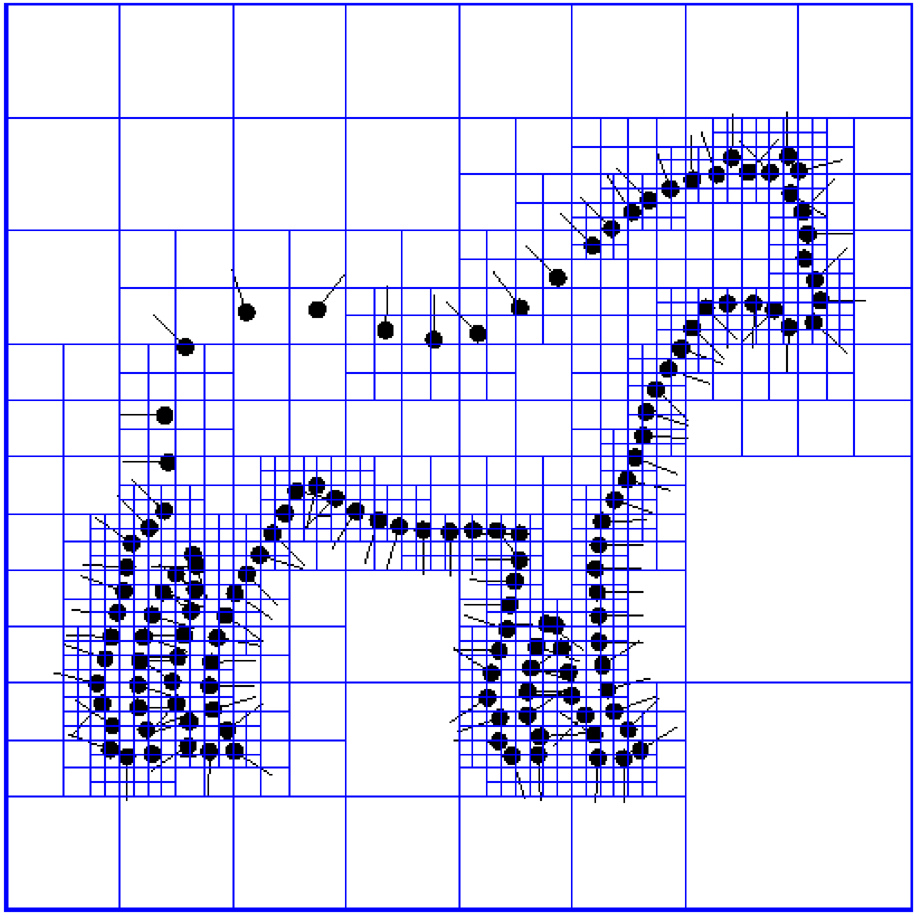
- Set octree
- Compute vector field
- Compute indicator function
- Extract iso-surface



# Implementation: Adapted Octree

Given the Points:

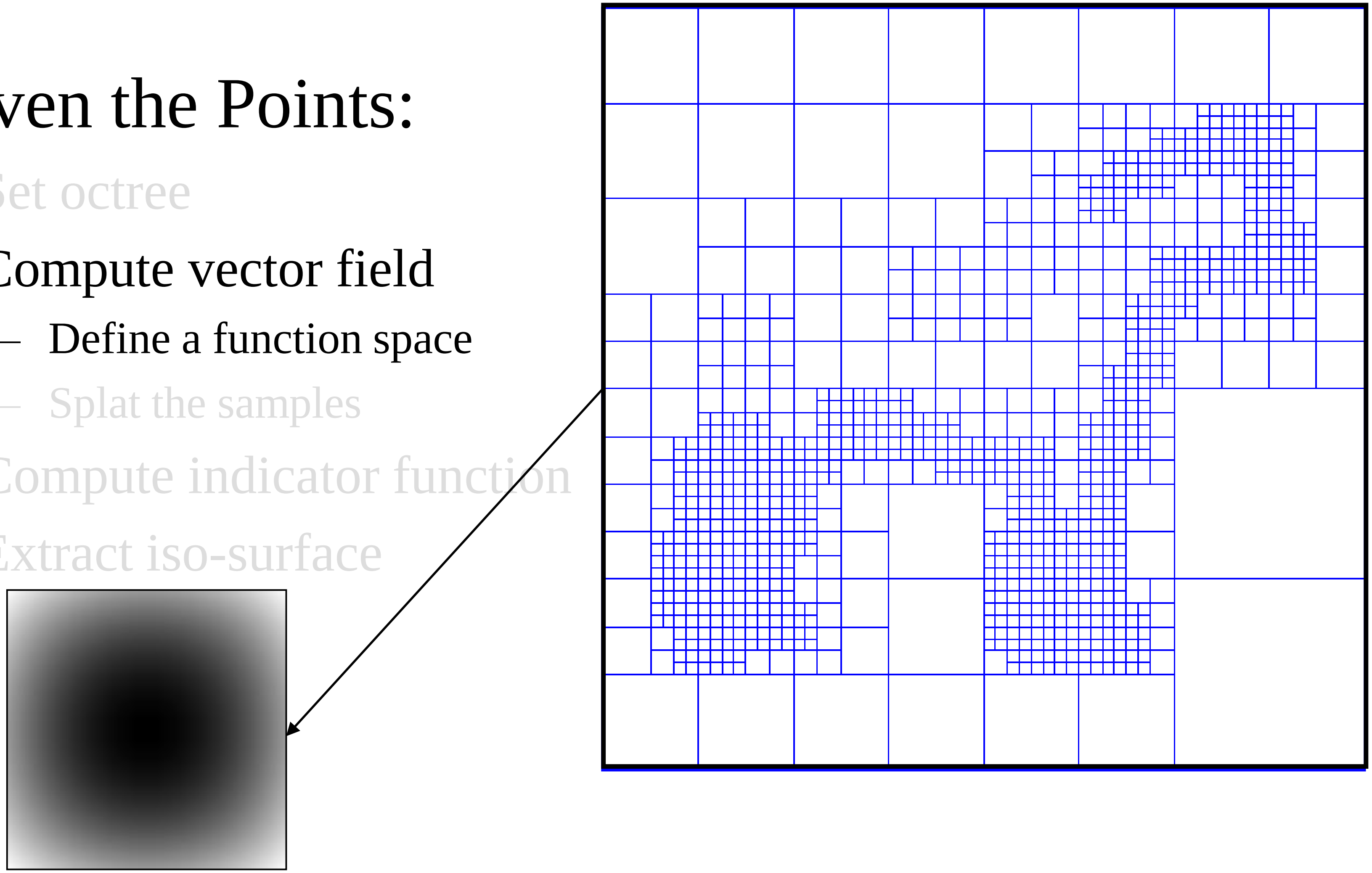
- Set octree
- Compute vector field
- Compute indicator function
- Extract iso-surface



# Implementation: Vector Field

## Given the Points:

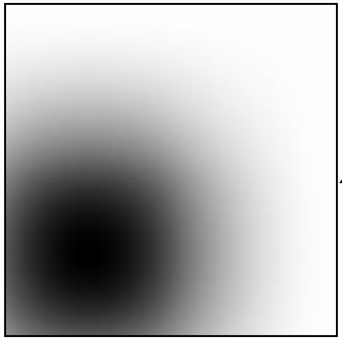
- Set octree
- Compute vector field
  - Define a function space
  - Splat the samples
- Compute indicator function
- Extract iso-surface

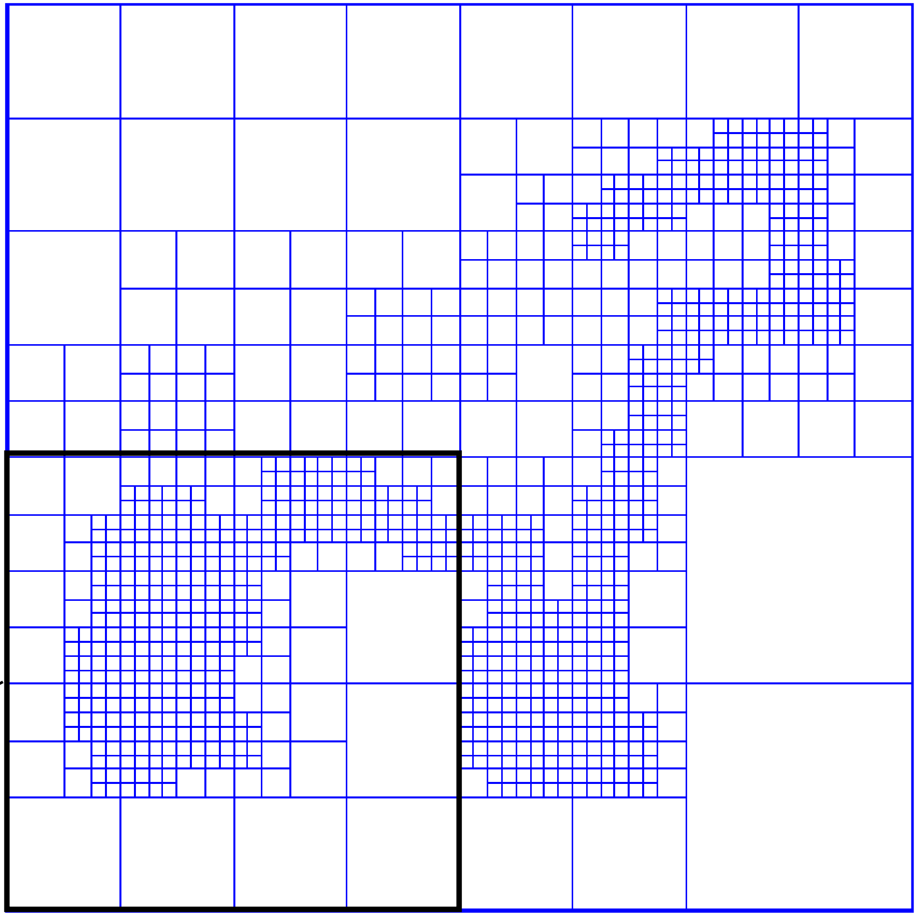


# Implementation: Vector Field

Given the Points:

- Set octree
- Compute vector field
  - Define a function space
  - Splat the samples
- Compute indicator function
- Extract iso-surface

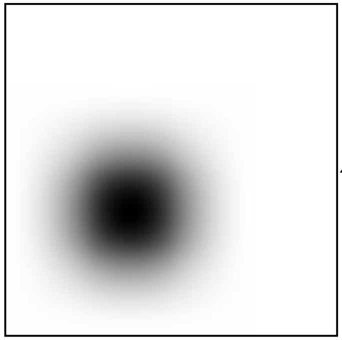


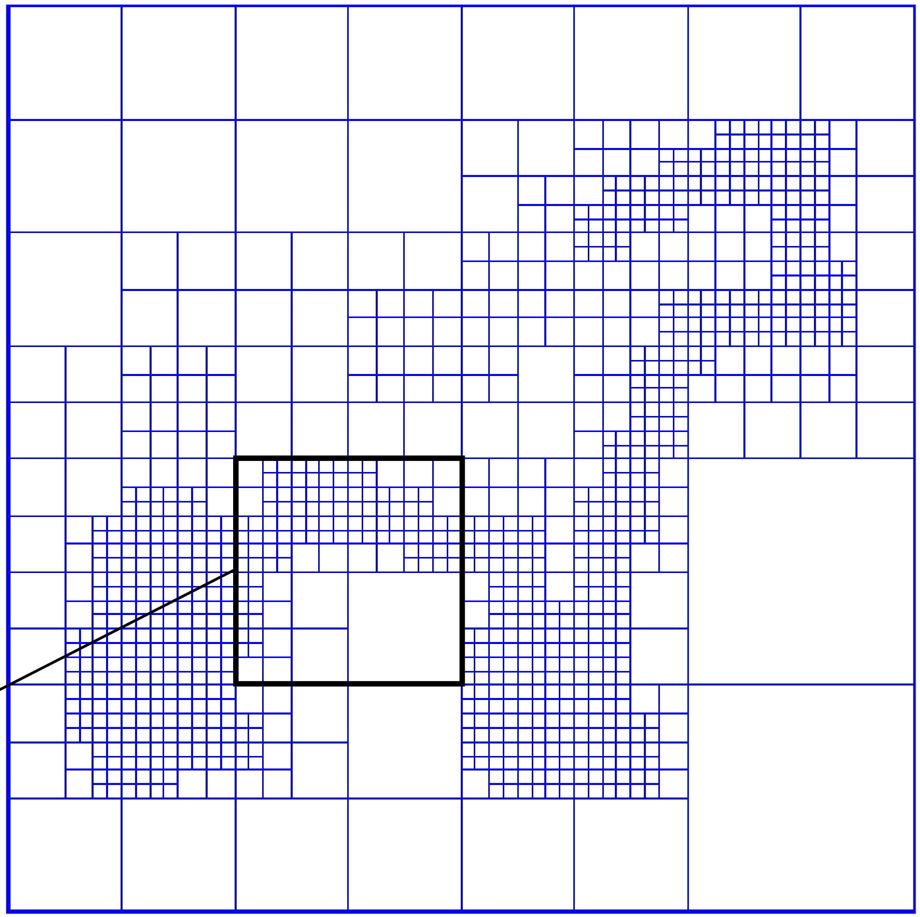


# Implementation: Vector Field

## Given the Points:

- Set octree
- Compute vector field
  - Define a function space
  - Splat the samples
- Compute indicator function
- Extract iso-surface

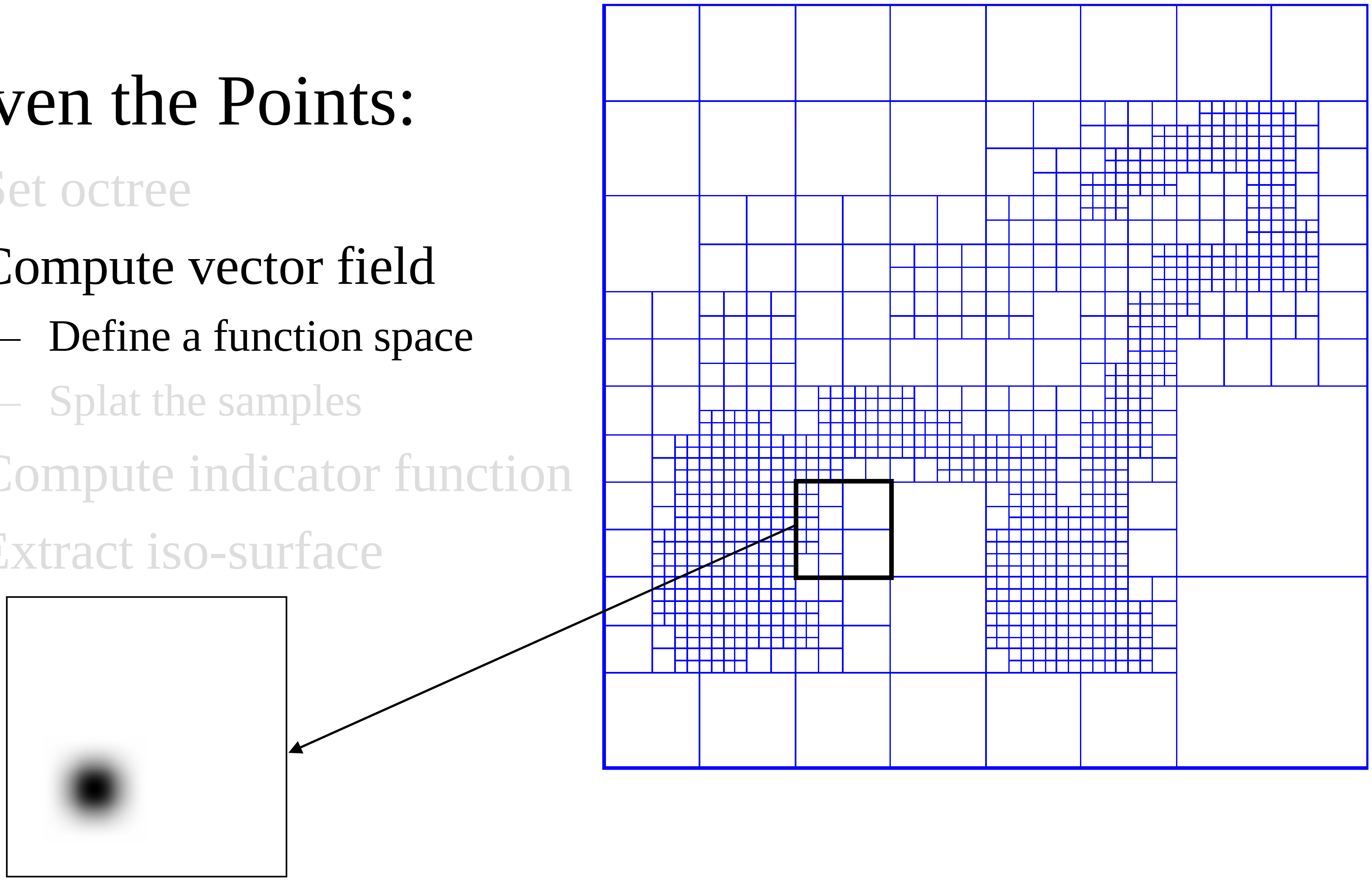




# Implementation: Vector Field

## Given the Points:

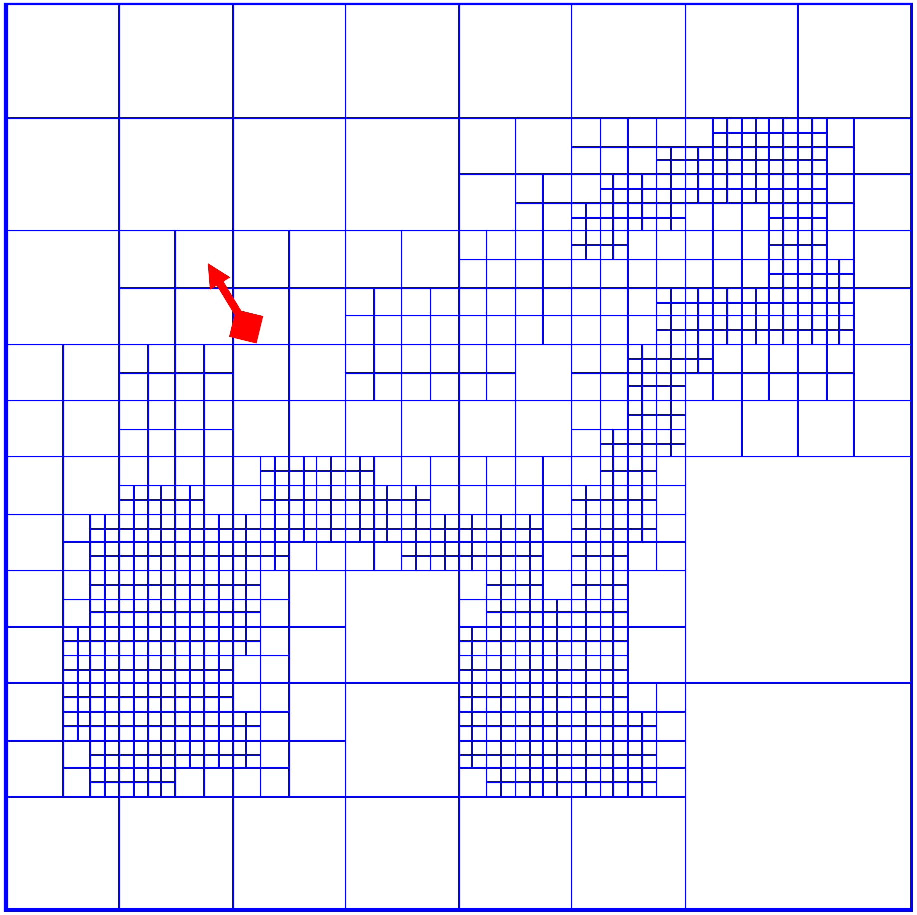
- Set octree
- Compute vector field
  - Define a function space
  - Splat the samples
- Compute indicator function
- Extract iso-surface



# Implementation: Vector Field

## Given the Points:

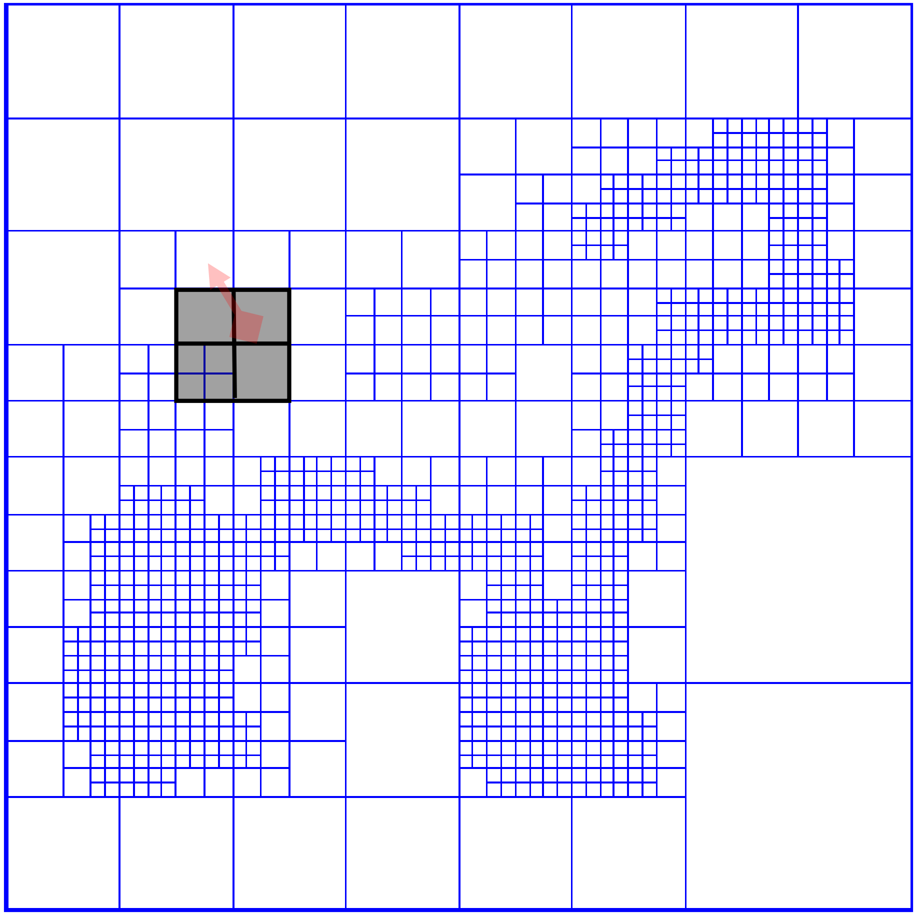
- Set octree
- Compute vector field
  - Define a function basis
  - Splat the samples
- Compute indicator function
- Extract iso-surface



# Implementation: Vector Field

## Given the Points:

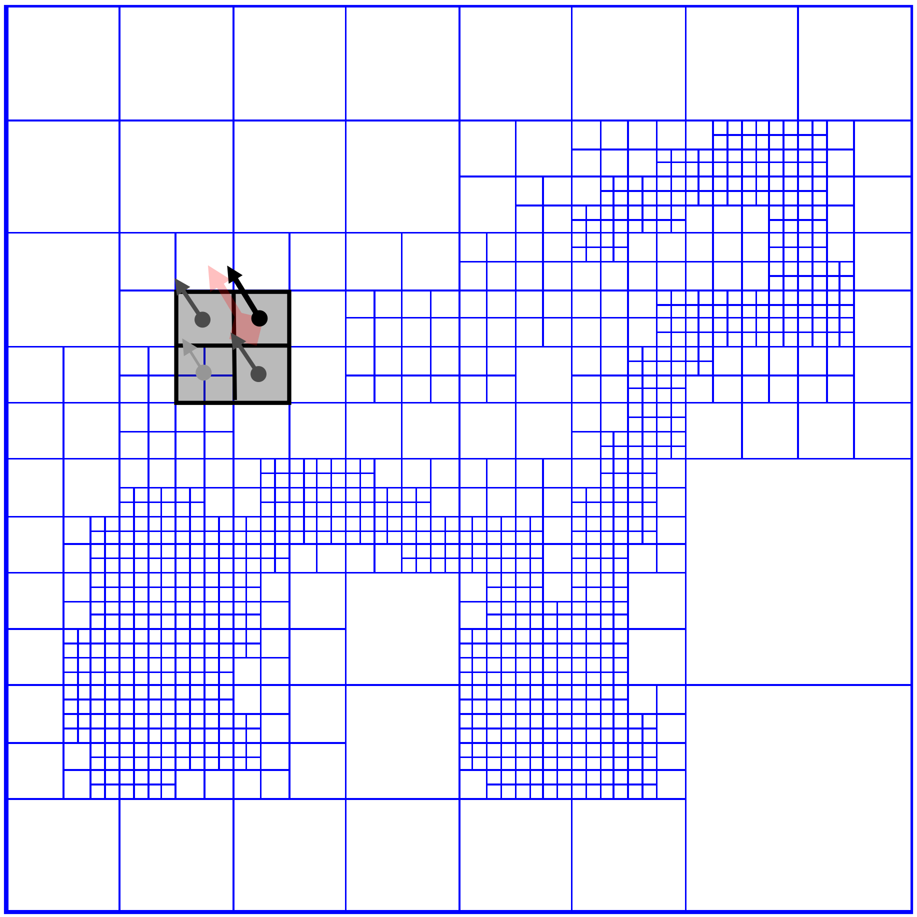
- Set octree
- Compute vector field
  - Define a function basis
  - Splat the samples
- Compute indicator function
- Extract iso-surface



# Implementation: Vector Field

Given the Points:

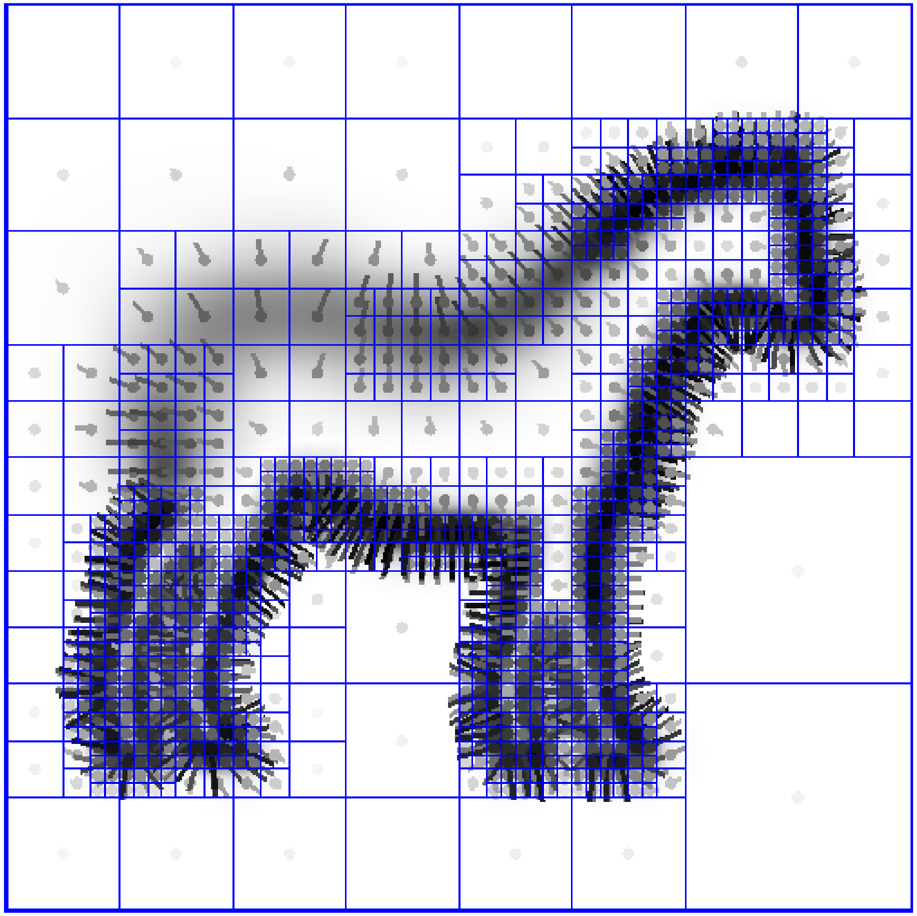
- Set octree
- Compute vector field
  - Define a function basis
  - Splat the samples
- Compute indicator function
- Extract iso-surface



# Implementation: Vector Field

## Given the Points:

- Set octree
- Compute vector field
  - Define a function space
  - Splat the samples
- Compute indicator function
- Extract iso-surface



# Implementation: Indicator Function

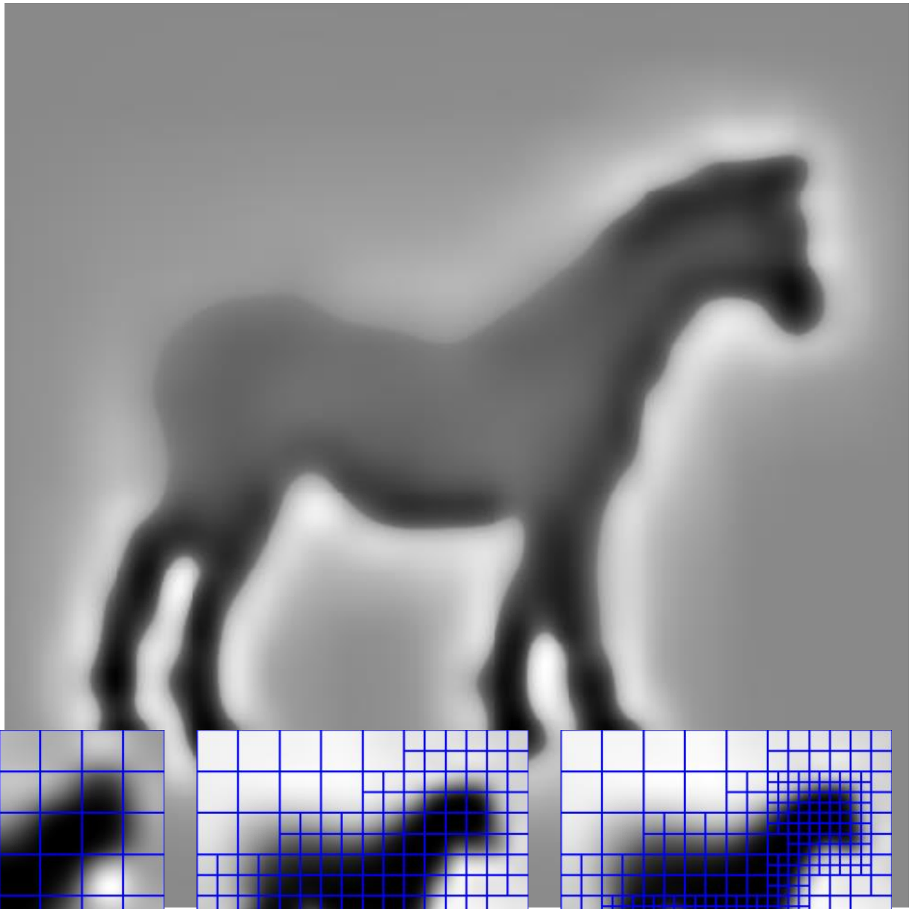
Given the Points:

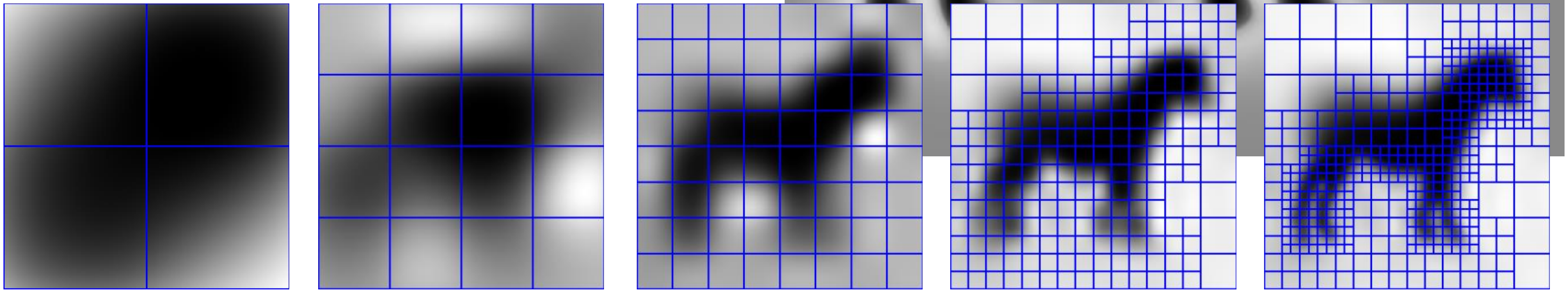
- Set octree
- Compute vector field
- Compute indicator function
  - Compute divergence
  - Solve Poisson equation
- Extract iso-surface

# Implementation: Indicator Function

## Given the Points:

- Set octree
- Compute vector field
- Compute indicator function
  - Compute divergence
  - Solve Poisson equation
- Extract iso-surface

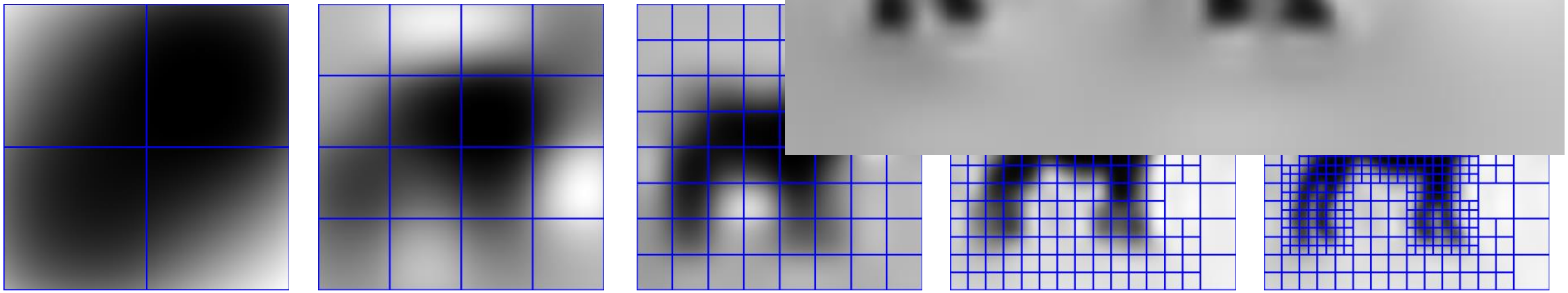




# Implementation: Indicator Function

## Given the Points:

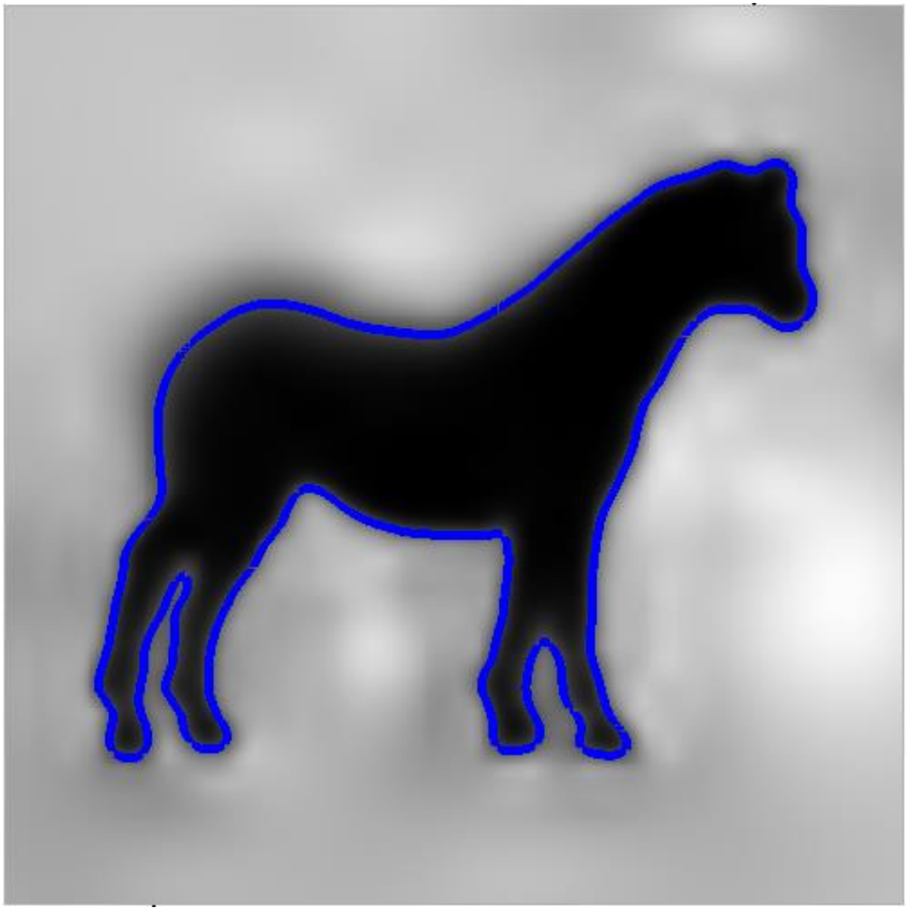
- Set octree
- Compute vector field
- Compute indicator function
  - Compute divergence
  - Solve Poisson equation
- Extract iso-surface



# Implementation: Surface Extraction

## Given the Points:

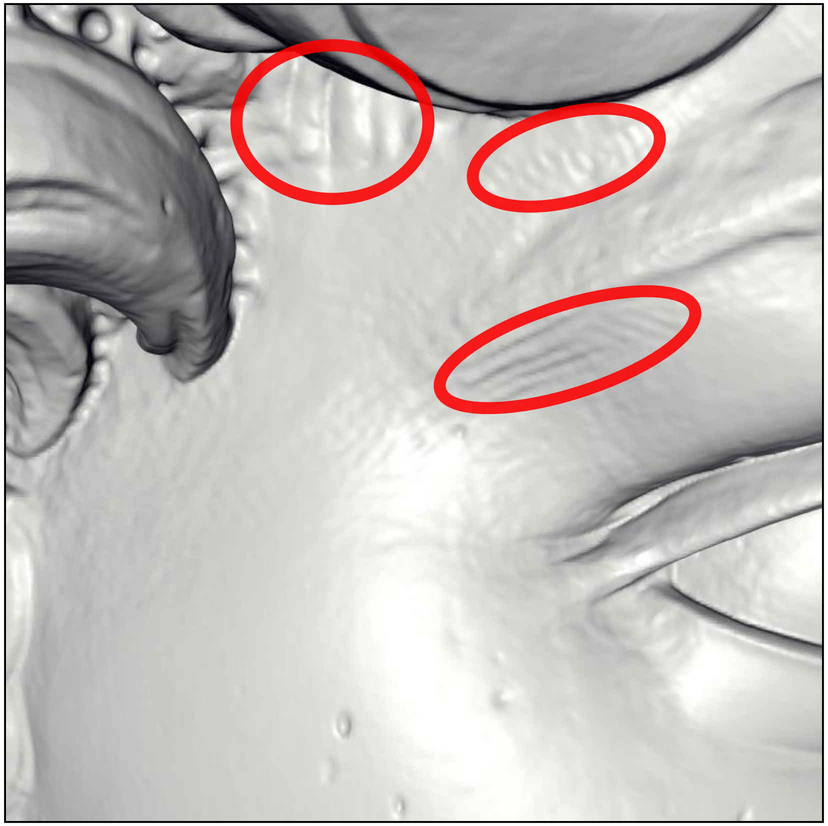
- Set octree
- Compute vector field
- Compute indicator function
- Extract iso-surface



# Michelangelo's David

- 215 million data points from 1000 scans
- 22 million triangle reconstruction
- Maximum tree depth of 11
- Compute Time: 2.1 hours
- Peak Memory: 6600MB

# David – Chisel marks



# David – Drill Marks

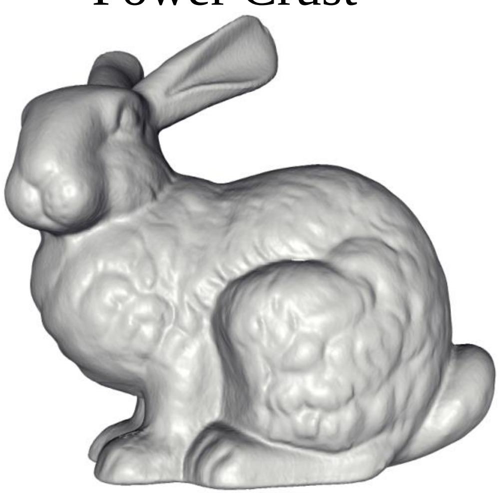
# David – Eye

# Stanford Bunny

Power Crust

FastRBF

MPU

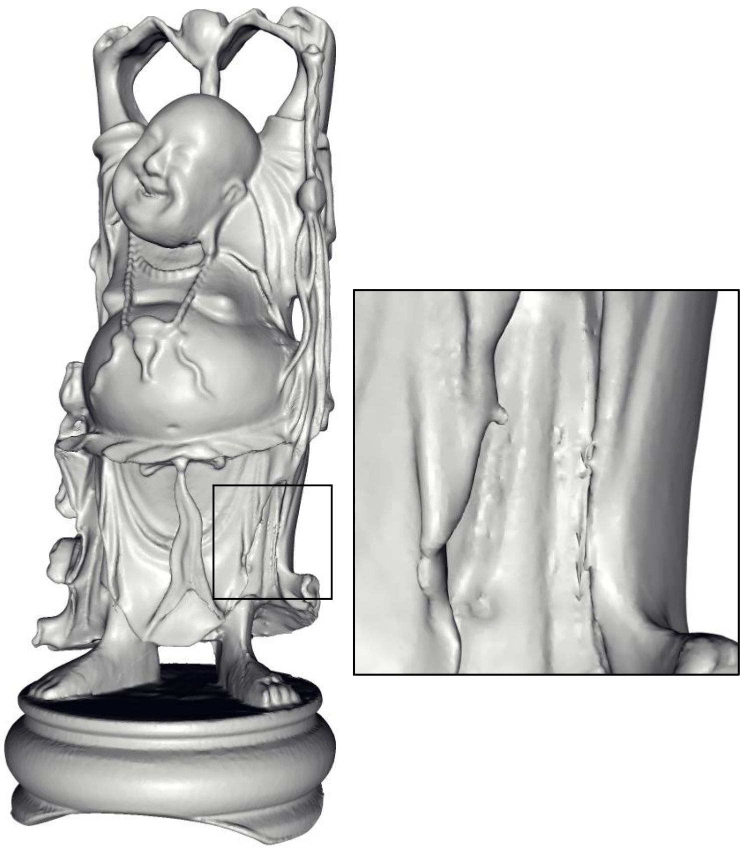


VRIP

FFT Reconstruction

Our Method

# VRIP Comparison



VRIP

Our Method

Questions?