Surface Reconstruction
From Unorganized Point Sets

COS 526, Fall 2014

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Point Sets

Absolute Geometries

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Problem Statement

• Given a set of sample points in three dimensions produce a simplicial surface that captures the “most reasonable shape” the points were sampled from.
Applications

- Computer graphics,
- Medical imaging
- Cartography
- Compression
- Reverse engineering
- Urban modeling
- etc.
Desirable Properties

- Interpolate input points
- Handle arbitrary genus
- Generally smooth
- Retain sharp features
- Watertight surface
Challenges

Even, noiseless sampling

Noisy sampling: interpolation

Noisy sampling: estimation

orientation?

Thin surfaces

Uneven sampling

Small features and topology

gap?

which one?
Possible Approaches?

Surface Reconstruction Algorithm
Possible Approaches

• Explicit Meshing
  – Ball pivoting algorithm
  – Crust
  – etc.

• Implicit Reconstruction
  – Hoppe’s algorithm
  – Moving Least Squares (MLS)
  – Poisson surface reconstruction
  – etc.

• Surface fitting
  – Deformable templates
  – etc.
Possible Approaches

• Explicit Meshing
  – Ball pivoting algorithm
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  – etc.
The Ball Pivoting Algorithm

- Pick a ball radius, roll ball around surface, connect what it hits
The Ball Pivoting Algorithm

Initial seed triangle

Active edge

Point on front
The Ball Pivoting Algorithm

Ball pivoting around active edge

Active edge

Point on front
The Ball Pivoting Algorithm

Ball pivoting around active edge

Active edge

Point on front
The Ball Pivoting Algorithm

Ball pivoting around active edge

Active edge

• Point on front
The Ball Pivoting Algorithm

Ball pivoting around active edge

Active edge

• Point on front
The Ball Pivoting Algorithm

Ball pivoting around active edge

Active edge
- Point on front
- Internal point
The Ball Pivoting Algorithm

Boundary edge

Ball pivoting around active edge

No pivot found

Active edge
- Point on front
- Internal point
The Ball Pivoting Algorithm

Boundary edge

Ball pivoting around active edge

Active edge
- Point on front
- Internal point
The Ball Pivoting Algorithm

Boundary edge

Ball pivoting around active edge

No pivot found

Active edge
- Point on front
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The Ball Pivoting Algorithm

Boundary edge

Ball pivoting around active edge

Active edge
- Point on front
- Internal point
The Ball Pivoting Algorithm

Possible problems?
The Ball Pivoting Algorithm

Possible problems?

Undersampling

Small Concavities
The Ball Pivoting Algorithm

Possible problems?

Self-intersection?  Watertight?
Possible Approaches

• **Explicit Meshing**
  – Ball pivoting algorithm
  – **Crust**
  – etc.

• **Implicit Reconstruction**
  – Hoppe’s algorithm
  – Moving Least Squares (MLS)
  – Poisson surface reconstruction
  – etc.

• **Surface fitting**
  – Deformable templates
  – etc.
Crust

Aims to find adjacent surface without a parameter specifying feature sizes
Definitions

Delaunay Triangulation, Voronoi Diagram
Definitions

Medial Axis: of surface F is the closure of points that have more than one closest point in F.
The Intuition behind Crust

The Voronoi Cells of a dense sampling are thin and long.

The Medial Axis is the extension of Voronoi Diagram for continuous surfaces in the sense that the Voronoi Diagram of S Can be defined as the set of points with more than one closest point in S. (S = Sample Point Set)
Crust in 2D

Input: \( P = \) Set of sample points in the plane
Output: \( E = \) Set of edges connecting points in \( P \)

The Algorithm

Compute the Voronoi vertices of \( P = V \)
Calculate the Delaunay of \( (P \cup V) \)
Pick the edges \((p,q)\) where both \(p, q\) are in \(P\)
Sample Output
Crust in 3D

- Some Voronoi vertices lie neither near the surface nor near the medial axis
- Keep the “poles”
Crust in 3D

• Compute the 3D Voronoi diagram of the sample points.
• For each sample point \( s \), pick the farthest vertex \( v \) of its Voronoi cell, and the farthest vertex \( v' \) such that angle \( vsv' \) exceeds 90 degrees.
• Compute the Voronoi diagram of the sample points and the "poles", the Voronoi vertices chosen in the second step.
• Add a triangle on each triple of sample points with neighboring cells in the second Voronoi diagram.
Sample Output
Possible Approaches

• Explicit Meshing
  – Ball pivoting algorithm
  – Crust
  – etc.

• Implicit Reconstruction
  – Hoppe’s algorithm
  – Moving Least Squares (MLS)
  – Poisson surface reconstruction
  – etc.

• Surface fitting
  – Deformable templates
  – etc.
Implicit Reconstruction

• Main idea:
  – Compute an implicit function $f(p)$ (negative outside, positive inside)
  – Extract surface where $f(p)=0$
Hoppe et al’s Algorithm

1. Tangent Plane Estimation
2. Consistent tangent plane orientation
3. Signed distance function computation
4. Surface extraction
Tangent Plane Estimation

- Principal Component Analysis (PCA)
  - Extract points \{q_i\} in neighborhood
  - Compute covariance matrix \( M \)
  - Analyze eigenvalues and eigenvectors of \( M \) (via SVD)

\[
M = \frac{1}{n} \sum_{i=1}^{n} \begin{bmatrix}
q_i^x q_i^x & q_i^x q_i^y & q_i^x q_i^z \\
q_i^y q_i^x & q_i^y q_i^y & q_i^y q_i^z \\
q_i^z q_i^x & q_i^z q_i^y & q_i^z q_i^z 
\end{bmatrix}
\]

Covariance Matrix

\[
M = \mathbf{U} \mathbf{S} \mathbf{U}'
\]

\[
S = \begin{bmatrix}
\lambda_a & 0 & 0 \\
0 & \lambda_b & 0 \\
0 & 0 & \lambda_c
\end{bmatrix}
\]

\[
U = \begin{bmatrix}
A_x & A_y & A_z \\
B_x & B_y & B_z \\
C_x & C_y & C_z
\end{bmatrix}
\]

Eigenvalues & Eigenvectors
Tangent Plane Estimation
Tangent Plane Estimation
Tangent Plane Estimation

• Eigenvectors are “Principal Axes of Inertia”
• Eigenvalues are variances of the point distribution in those directions
Tangent Plane Estimation

- Surface normal is estimated by eigenvector (principal axis) associated with smallest eigenvalue.
Consistent Tangent Plane Orientation

- Traverse nearest neighbor graph flipping normals for consistency
  - Greedy propagation algorithm (minimum spanning tree of normal similarity)

\[ 1 - |\hat{n}_i \cdot \hat{n}_j| \]
Signed Distance Function

- \( f(p) \) is signed distance to tangent plane of closest point sample

\[
\begin{align*}
\{ \text{Compute } z \text{ as the projection of } p \text{ onto } T_p(x_i) \} \\
& z \leftarrow o_i - ((p - o_i) \cdot \hat{n}_i) \hat{n}_i \\
\text{if } d(z, X) < \rho + \delta \text{ then} \\
& f(p) \leftarrow (p - o_i) \cdot \hat{n}_i \quad \{= \pm \|p - z\|\} \\
\text{else} \\
& f(p) \leftarrow \text{undefined} \\
\text{endif}
\end{align*}
\]
Signed Distance Function

- \( f(p) \) is signed distance to tangent plane of closest point sample

Ravikrishna Bvs Kolluri
Surface Extraction

- Extract triangulated surface where $f(p)=0$
  - e.g., Marching Cubes
Surface Extraction

- Extract triangulated surface where $f(p) = 0$
  - e.g., Marching Cubes
Sample Results

(a) Unknown surface $U$
(b) Sampled points $X$
(c) Reconstructed surface $S$
Sample Results
Sample Results
Sample Results
Moving Least Squares

• Similar, but different implicit function
  – Weighted contribution of nearby points

\[
f(x) = \frac{\sum n_i^T (x - x_i) \phi_i(x)}{\sum \phi_i(x)}
\]

\[
\phi_i(x) = \phi(||x - x_i||)
\]

\[
\phi(x) = e^{-\left(\frac{x}{\sigma_r}\right)^2}
\]
Moving Least Squares

MLS
Possible Approaches

• Explicit Meshing
  – Ball pivoting algorithm
  – Crust
  – etc.

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  – etc.

• Surface fitting
  – Deformable templates
  – etc.
The Indicator Function

- We reconstruct the surface of the model by solving for the indicator function of the shape.

\[ \chi_M(p) = \begin{cases} 
1 & \text{if } p \in M \\
0 & \text{if } p \notin M 
\end{cases} \]
Challenge

- How to construct the indicator function?
Gradient Relationship

- There is a relationship between the normal field and gradient of indicator function.

Oriented points

Indicator gradient $\nabla \chi_M$
Integration

• Represent the points by a vector field $\vec{V}$
• Find the function $\chi$ whose gradient best approximates $\vec{V}$:

$$\min_\chi \| \nabla \chi - \vec{V} \|$$
Integration as a Poisson Problem

• Represent the points by a vector field $\vec{V}$

• Find the function $\chi$ whose gradient best approximates $\vec{V}$:

$$\min_{\chi} \| \nabla \chi - \vec{V} \|$$

• Applying the divergence operator, we can transform this into a Poisson problem:

$$\nabla \cdot (\nabla \chi) = \nabla \cdot \vec{V} \iff \Delta \chi = \nabla \cdot \vec{V}$$
Implementation

Given the Points:

- Set octree
- Compute vector field
- Compute indicator function
- Extract iso-surface
Implementation: Adapted Octree

Given the Points:

- Set octree
- Compute vector field
- Compute indicator function
- Extract iso-surface
Implementation: Vector Field

Given the Points:

- Set octree
- Compute vector field
  - Define a function space
  - Splat the samples
- Compute indicator function
- Extract iso-surface
Implementation: Vector Field

Given the Points:

- Set octree
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Implementation: Vector Field

Given the Points:

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- Compute indicator function
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Implementation: Vector Field

Given the Points:

- Set octree
- Compute vector field
  - Define a function basis
  - Splat the samples
- Compute indicator function
- Extract iso-surface
Implementation: Vector Field

Given the Points:

- Set octree
- Compute vector field
  - Define a function basis
  - Splat the samples
- Compute indicator function
- Extract iso-surface
Implementation: Vector Field

Given the Points:

- Set octree
- Compute vector field
  - Define a function basis
  - Splat the samples
- Compute indicator function
- Extract iso-surface
Implementation: Vector Field

Given the Points:
- Set octree
- Compute vector field
  - Define a function space
  - Splat the samples
- Compute indicator function
- Extract iso-surface
Implementation: Indicator Function

Given the Points:

- Set octree
- Compute vector field
- **Compute indicator function**
  - Compute divergence
  - Solve Poisson equation
- Extract iso-surface
Implementation: Indicator Function

Given the Points:

- Set octree
- Compute vector field
- Compute indicator function
  - Compute divergence
  - Solve Poisson equation
- Extract iso-surface
Implementation: Indicator Function

Given the Points:
- Set octree
- Compute vector field
- Compute indicator function
  - Compute divergence
  - Solve Poisson equation
- Extract iso-surface
Implementation: Surface Extraction

Given the Points:

- Set octree
- Compute vector field
- Compute indicator function
- Extract iso-surface
Michelangelo’s David

- 215 million data points from 1000 scans
- 22 million triangle reconstruction
- Maximum tree depth of 11
- Compute Time: 2.1 hours
- Peak Memory: 6600MB
David – Chisel marks
David – Drill Marks
David – Eye
Stanford Bunny

Power Crust  FastRBF  MPU

VRIP  FFT Reconstruction  Our Method
VRIP Comparison

VRIP

Our Method
Questions?