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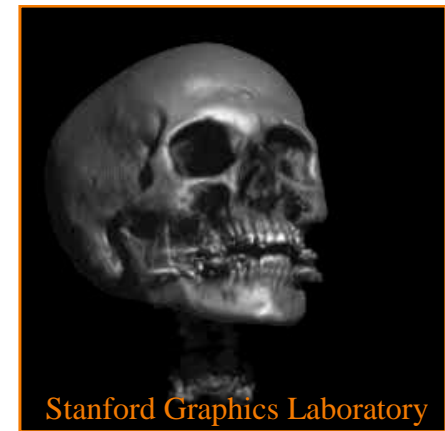
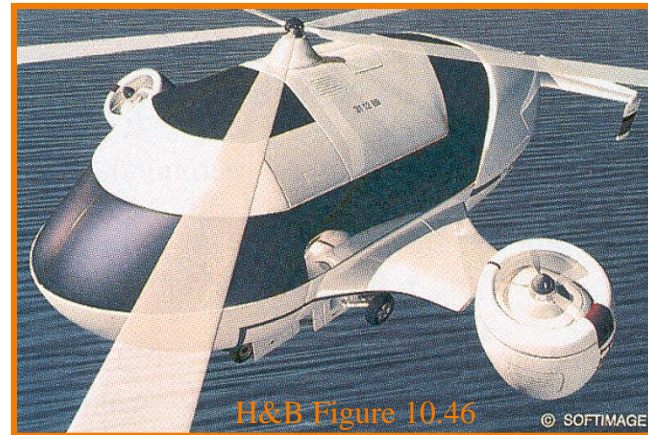
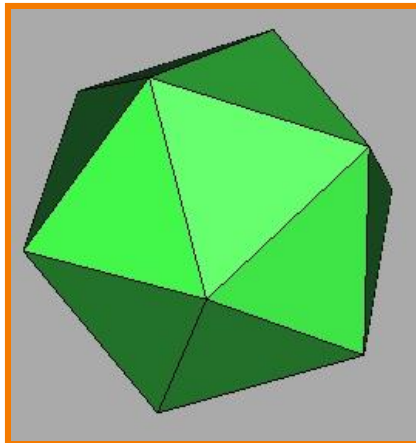
# 3D Object Representations

COS 526, Fall 2014  
Princeton University

# 3D Object Representations



- How do we ...
  - Represent 3D objects in a computer?
  - Acquire computer representations of 3D objects?
  - Manipulate computer representations of 3D objects?

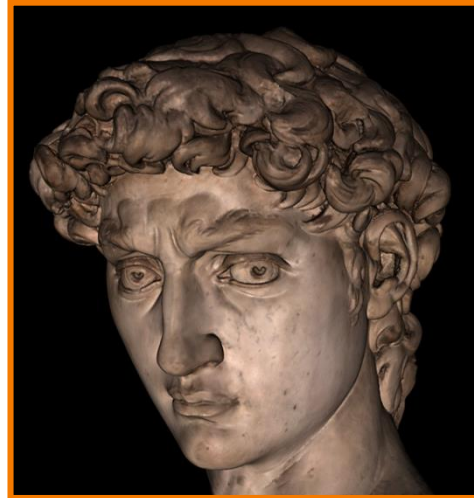


# 3D Object Representations



What can we do with a 3D object representation?

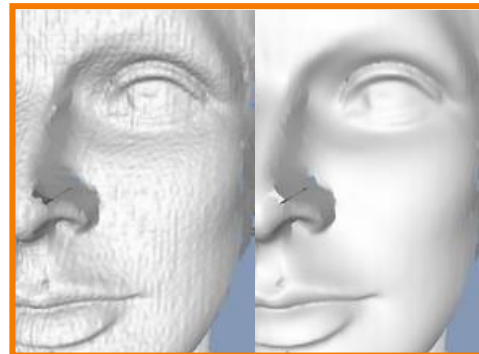
- Edit
- Transform
- Smooth
- Render
- Animate
- Morph
- Compress
- Transmit
- Analyze
- etc.



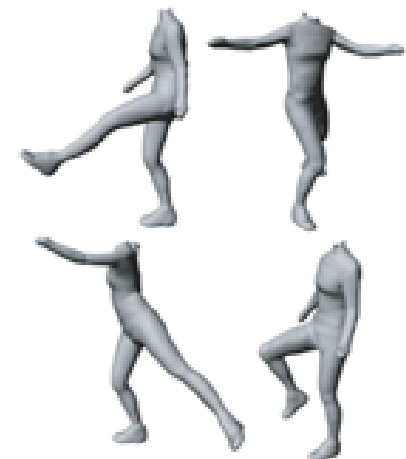
Digital Michelangelo



Pirates of the Caribbean



Thouis "Ray" Jones



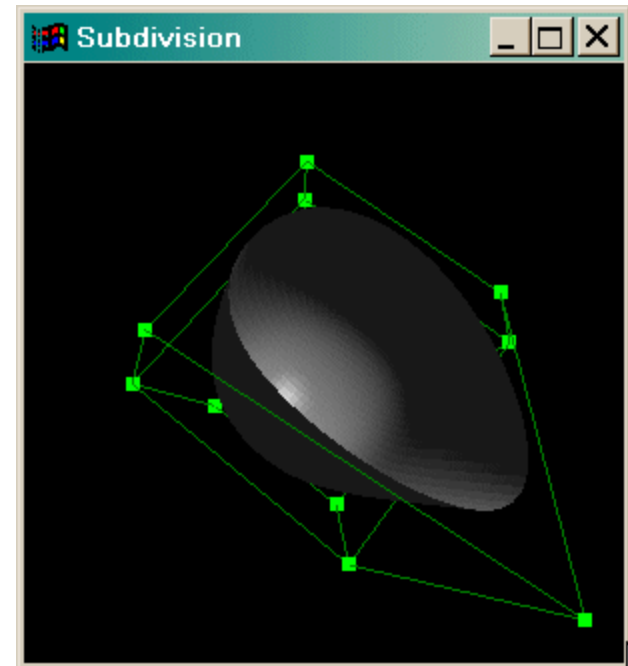
Sand et al.

# 3D Object Representations

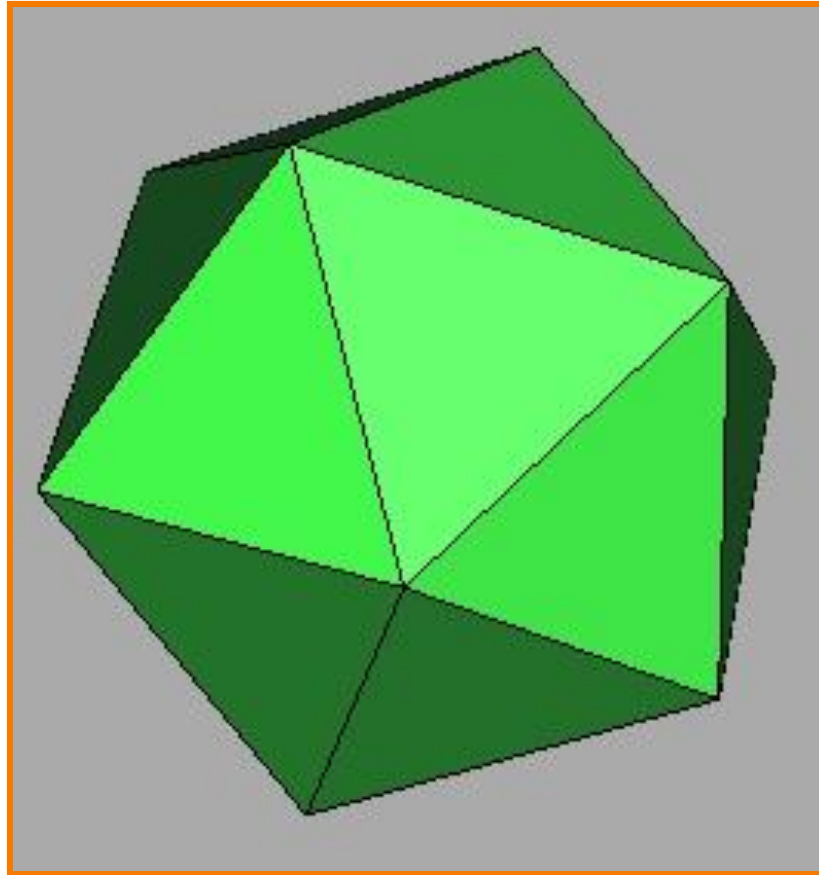


Desirable properties depend on intended use

- Easy to acquire
- Accurate
- Concise
- Intuitive editing
- Efficient editing
- Efficient display
- Efficient intersections
- Guaranteed validity
- Guaranteed smoothness
- etc.

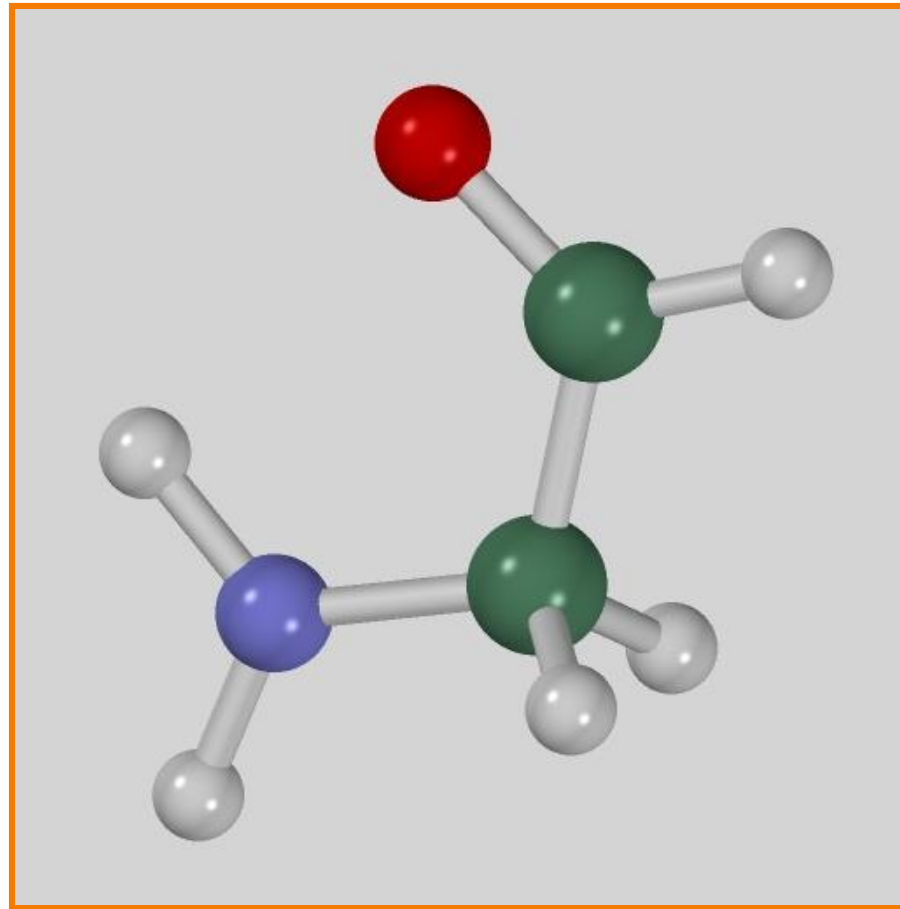


# 3D Object Representations



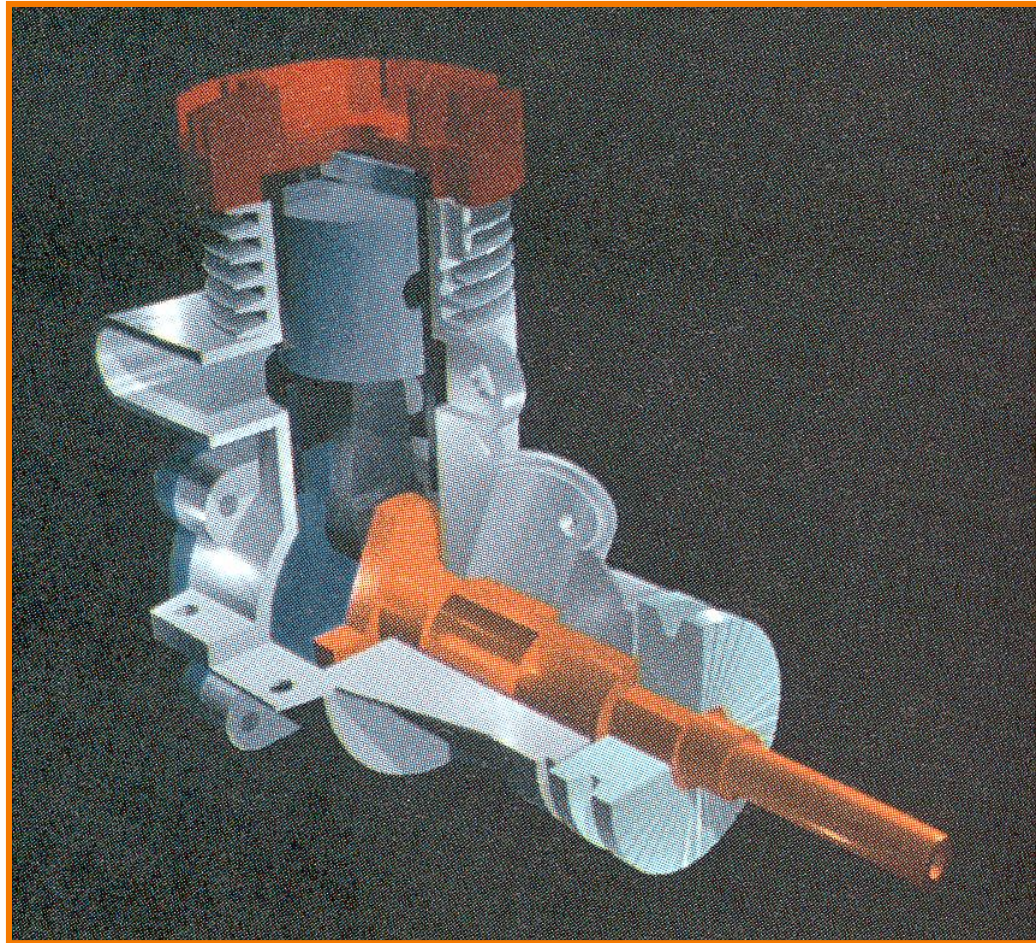
How can this object be represented in a computer?

# 3D Object Representations



How about this one?

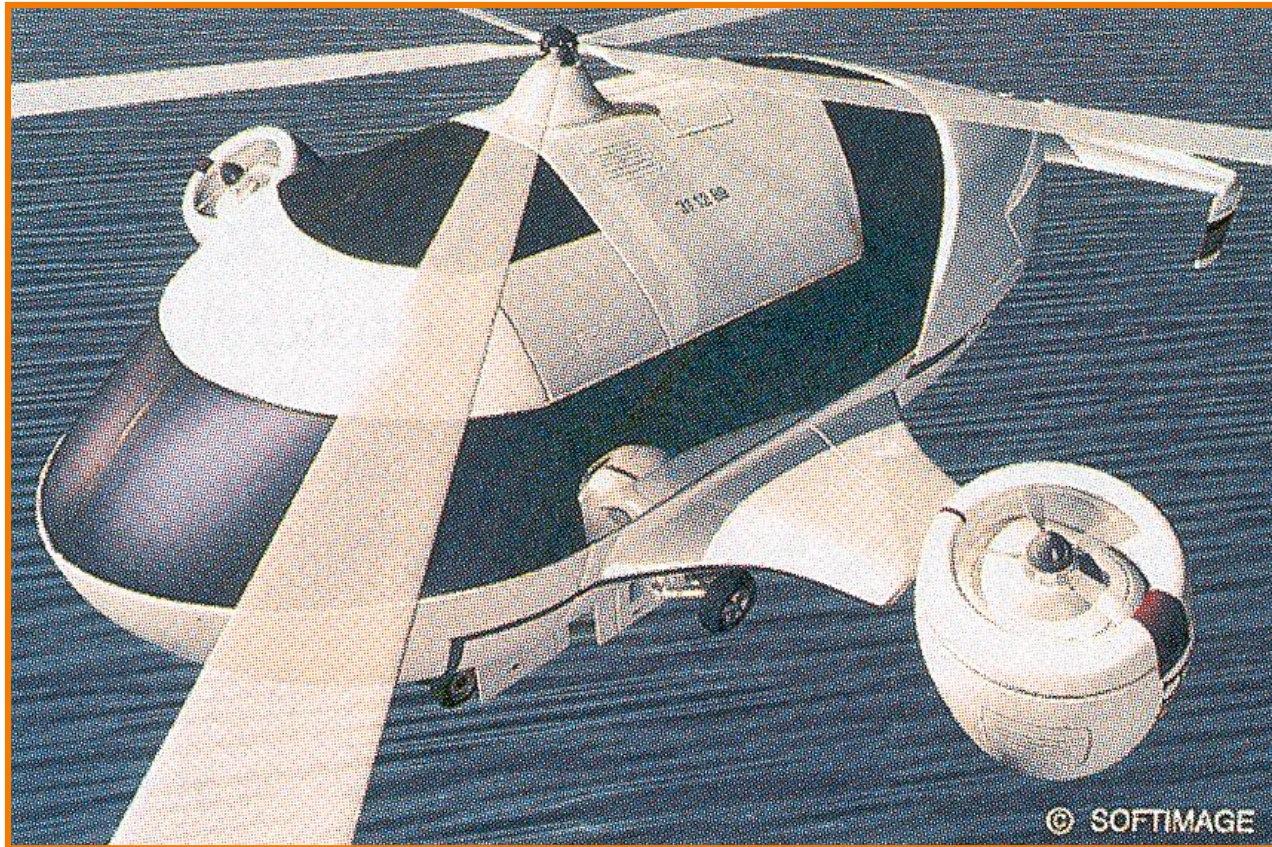
# 3D Object Representations



This one?

H&B Figure 9.9

# 3D Object Representations



H&B Figure 10.46

This one?



# 3D Object Representations



This one?

Stanford Graphics Laboratory

# 3D Object Representations



This one?

# 3D Object Representations



- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

# Equivalence of Representations



- Thesis:
  - Each representation has enough expressive power to model the shape of any geometric object
  - It is possible to perform all geometric operations with any fundamental representation
- Analogous to Turing-equivalence
  - Computers and programming languages are Turing-equivalent, but each has its benefits...

# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- Manipulation
- Animation
- Analysis

Data structures determine algorithms

# Why Different Representations?



- Efficiency
  - Representational complexity (e.g. volume vs. surface)
  - Computational complexity (e.g.  $O(n^2)$  vs  $O(n^3)$  )
  - Space/time trade-offs (e.g. z-buffer)
  - Numerical accuracy/stability (e.g. degree of polynomial)
- Simplicity
  - Ease of acquisition
  - Hardware acceleration
  - Software creation and maintenance
- Usability
  - Designer interface vs. computational engine

# 3D Object Representations

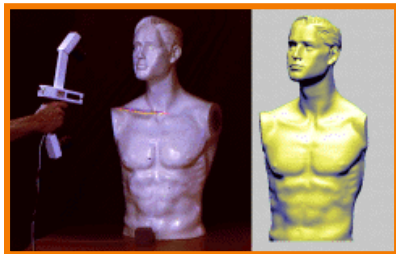


- Points
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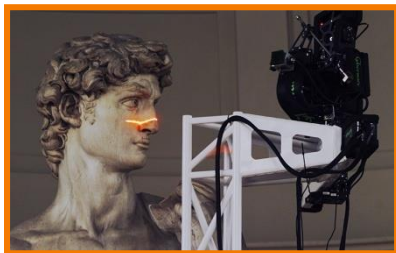
# Range Image

Set of 3D points mapping to pixels of depth image

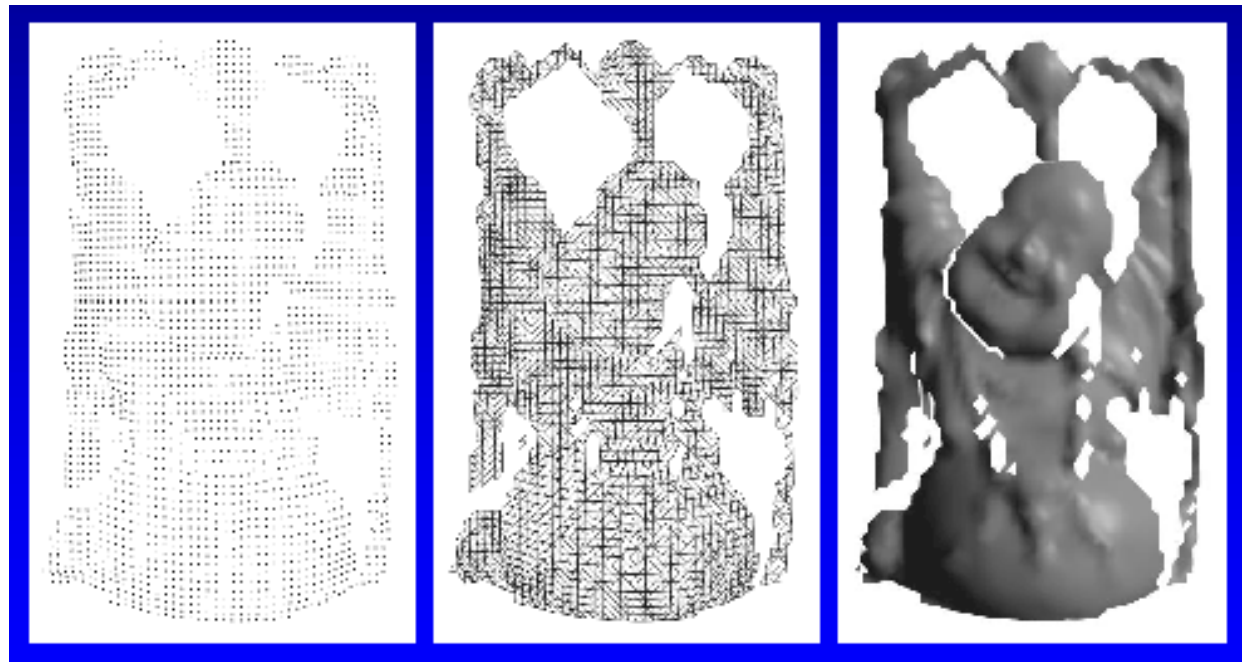
- Can be acquired from range scanner



Cyberware



Stanford



Range Image

Tessellation

Range Surface



# Point Cloud



Unstructured set of 3D point samples

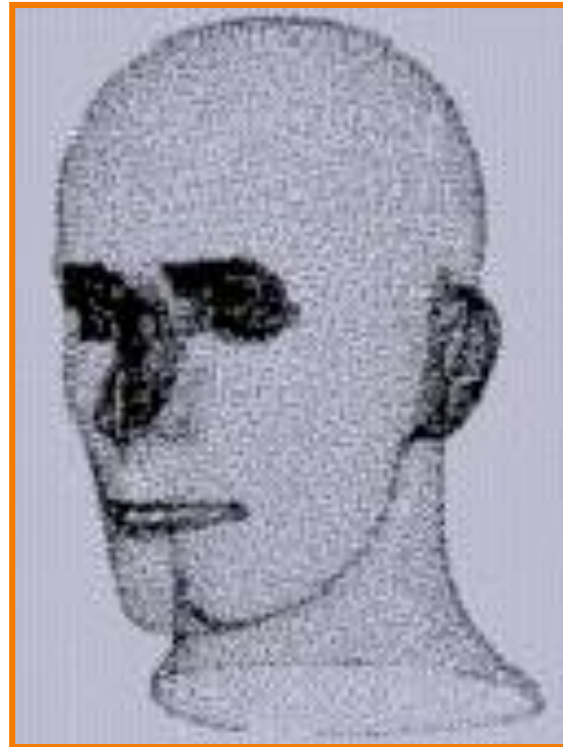
- Acquired from range finder, computer vision, etc



Polhemus



Microscribe-3D



Hoppe



Hoppe

# 3D Object Representations

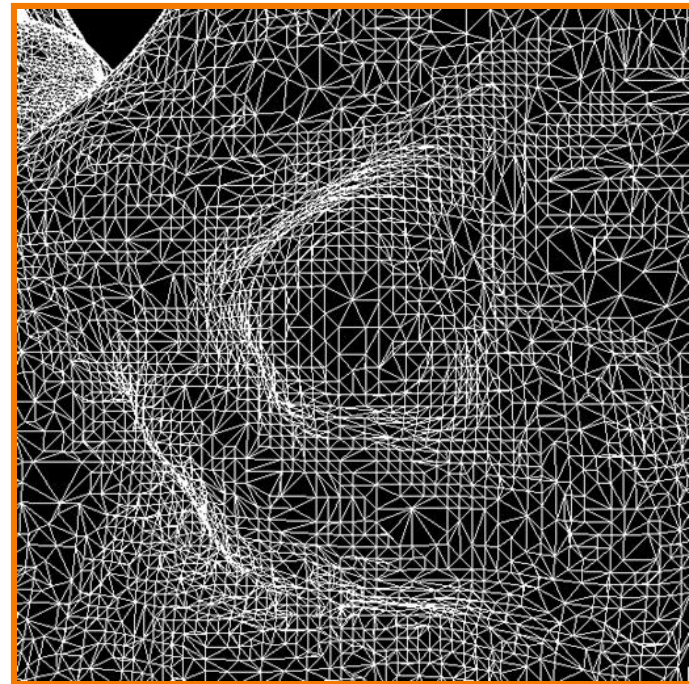


- Points
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# Polygonal Mesh



Connected set of polygons (usually triangles)

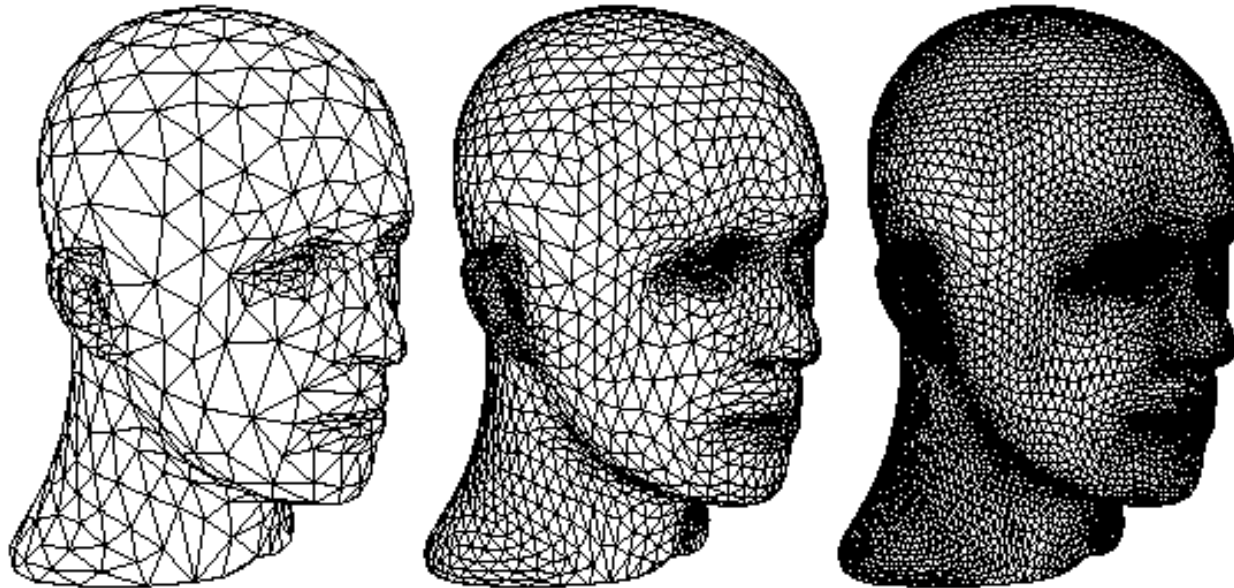


# Subdivision Surface



## Coarse mesh & subdivision rule

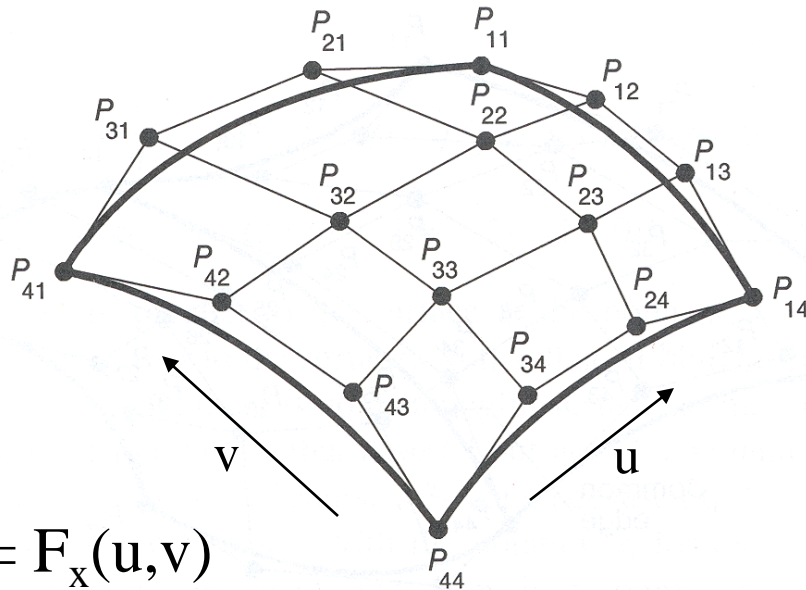
- Smooth surface is **limit** of sequence of refinements



# Parametric Surface

## Tensor-product spline patches

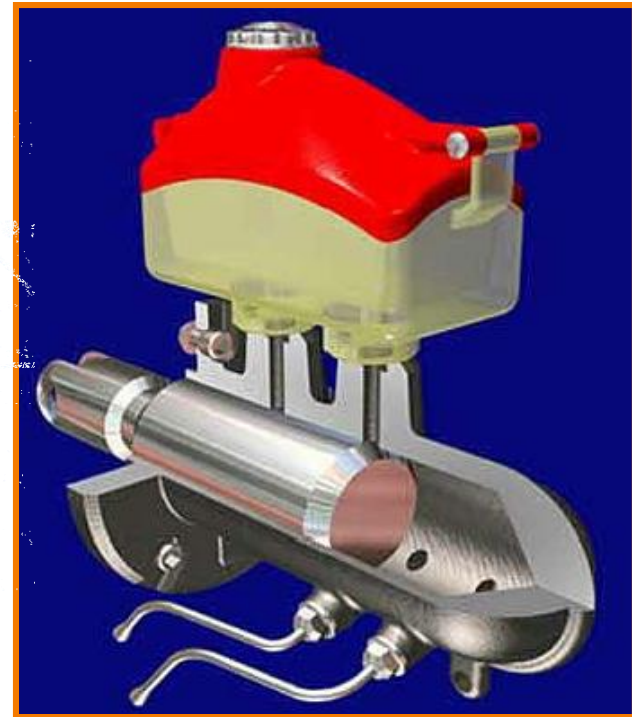
- Each patch is parametric function
- Careful constraints to maintain continuity



$$x = F_x(u,v)$$

$$y = F_y(u,v)$$

$$z = F_z(u,v)$$

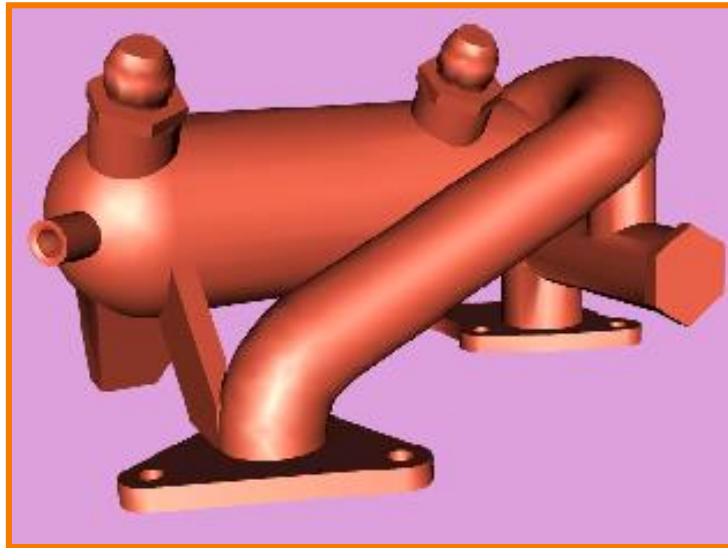


FvDFH Figure 11.44

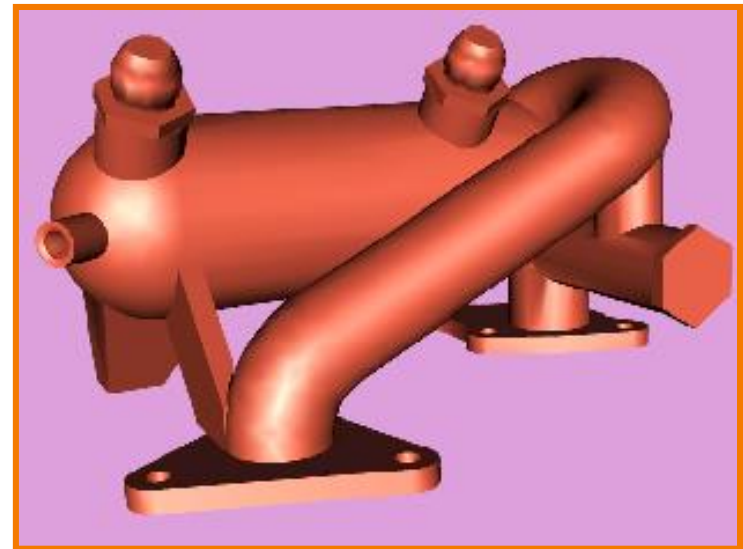
# Implicit Surface



Set of all points satisfying:  $F(x,y,z) = 0$



Polygonal Model



Implicit Model

# 3D Object Representations



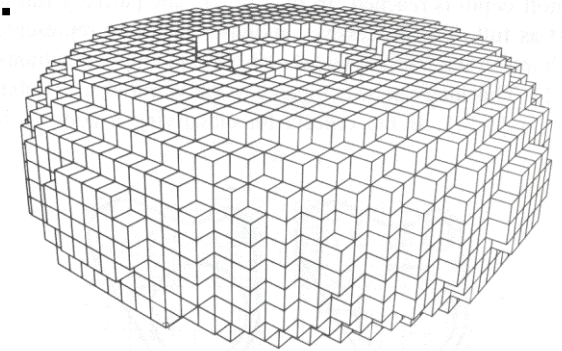
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# Voxel grid

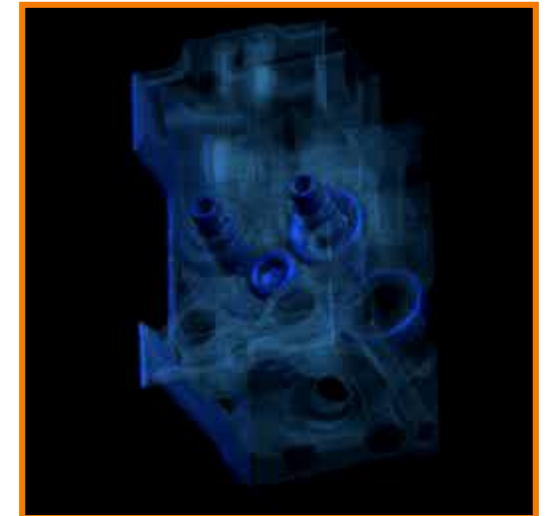


Uniform volumetric grid of samples:

- Occupancy  
(object vs. empty space)
- Density
- Color
- Other function  
(speed, temperature, etc.)
  
- Often acquired via  
simulation or from  
CAT, MRI, etc.



FvDFH Figure 12.20

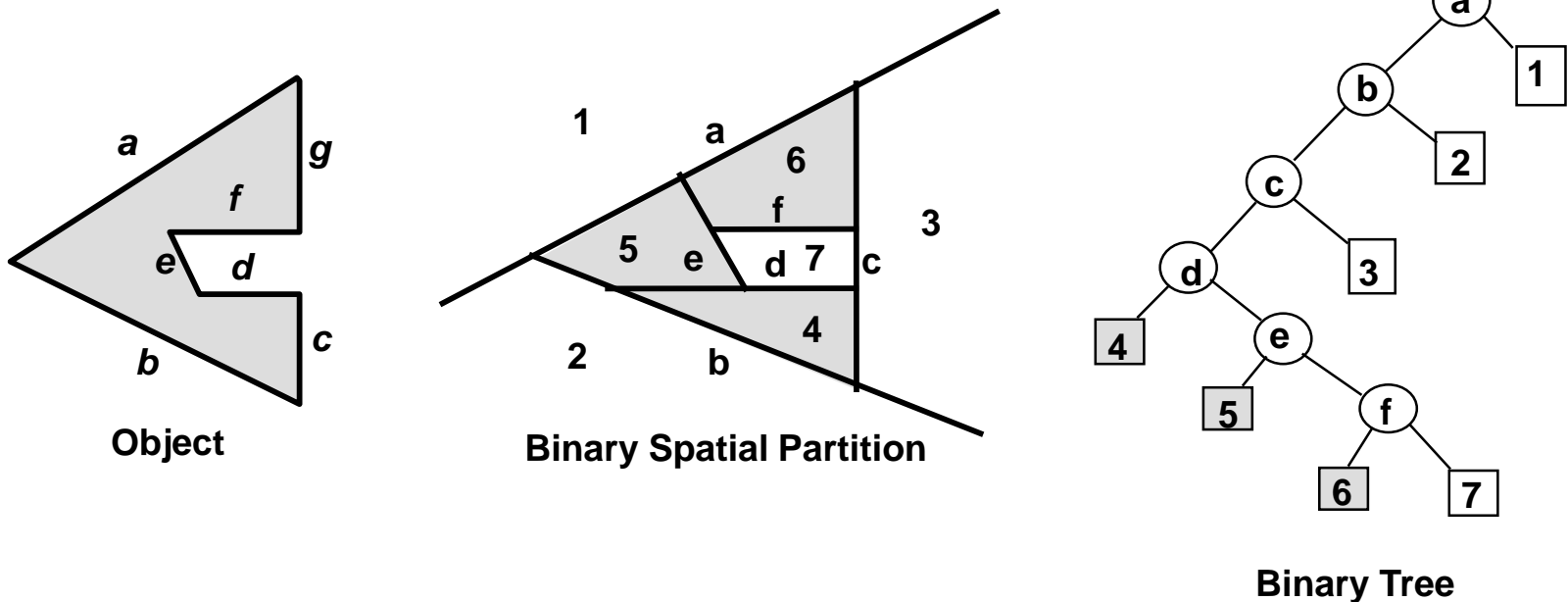




# BSP Tree

Hierarchical **B**inary **S**pace **P**artition with solid/empty cells labeled

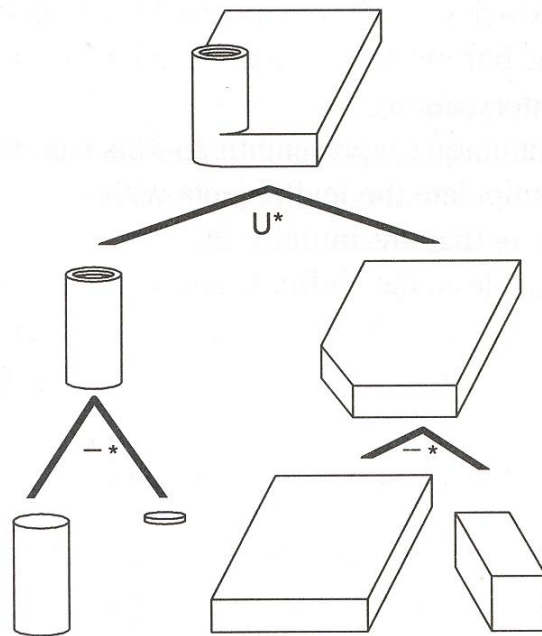
- Constructed from polygonal representations



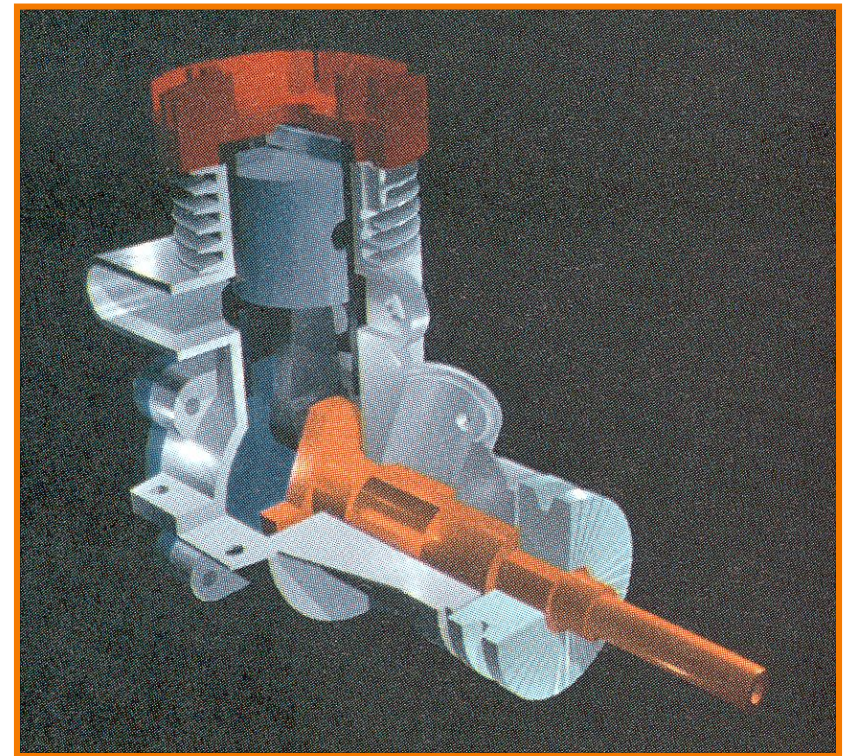
# CSG



**C**onstructive **S**olid **G**eometry: set operations (union, difference, intersection) applied to simple shapes



FvDFH Figure 12.27

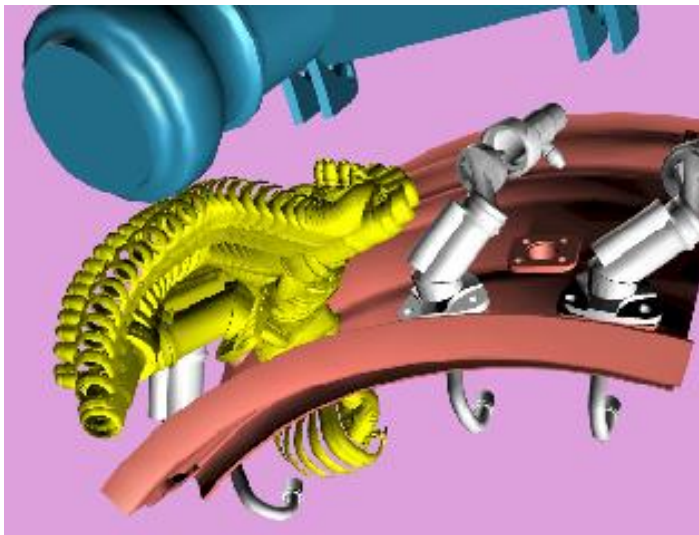


H&B Figure 9.9

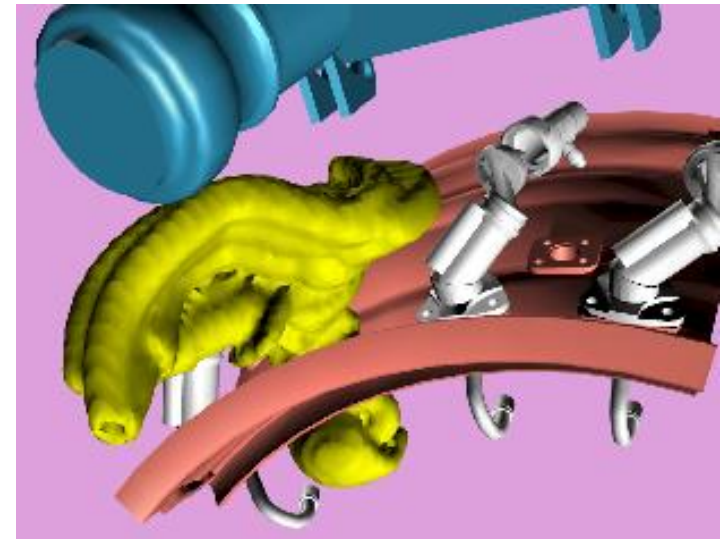
# Sweep



Solid swept by curve along trajectory



Removal Path



Sweep Model

Bill Lorensen  
SIGGRAPH 99  
Course #4 Notes

# 3D Object Representations



- Points
  - Range image
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# Scene Graph



Union of objects at leaf nodes

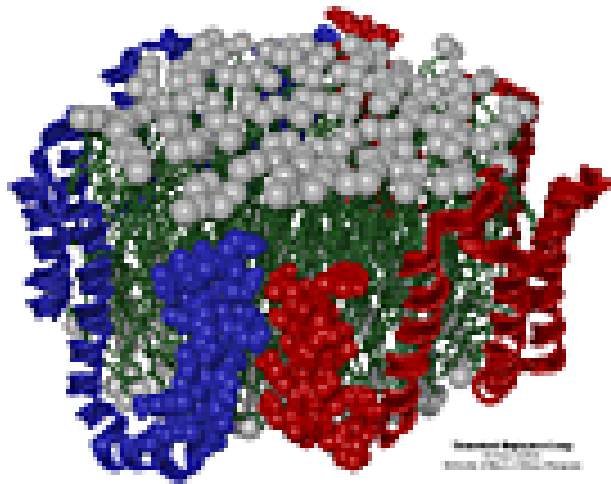


Bell Laboratories

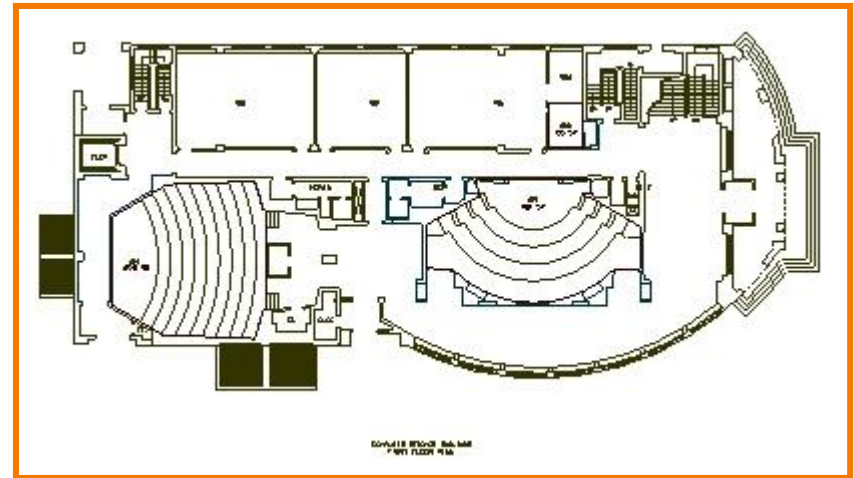


[avalon.viewpoint.com](http://avalon.viewpoint.com)

# Application Specific



**Apo A-1**  
*(Theoretical Biophysics Group,  
University of Illinois at Urbana-Champaign)*



**Architectural Floorplan**  
*(CS Building, Princeton University)*

# 3D Object Representations



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# 3D Object Representations



## ➤ Points

- Range image
- Point cloud

## • Surfaces

- Polygonal mesh
- Subdivision
- Parametric
- Implicit

## • Solids

- Voxels
- BSP tree
- CSG
- Sweep

## • High-level structures

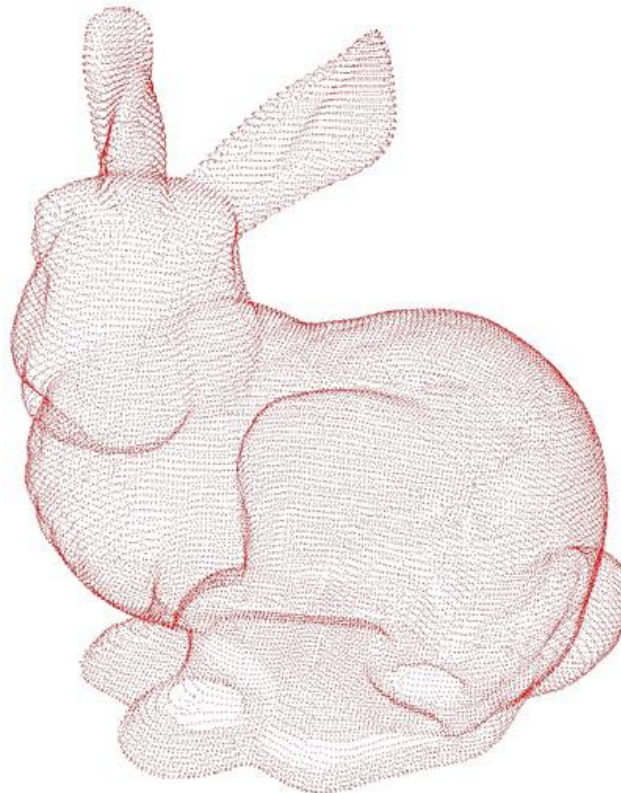
- Scene graph
- Application specific

# Point Clouds



Represent surface by a set of points

- Each point is represented by  $(x, y, z)$
- No connectivity between points

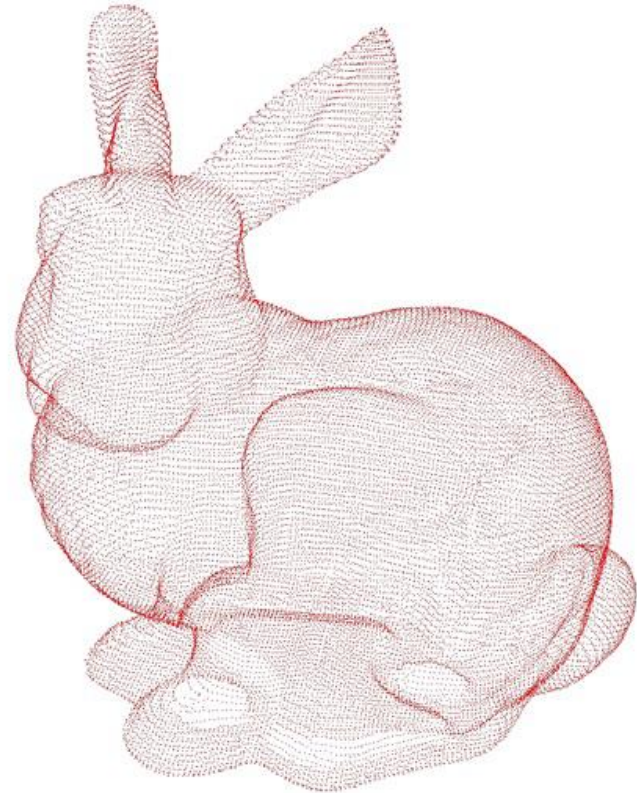


# Point Clouds



## Properties?

- Easy to acquire
- Accurate
- Concise
- Intuitive editing
- Efficient editing
- Efficient display
- Efficient intersections
- Guaranteed validity
- Guaranteed smoothness
- etc.

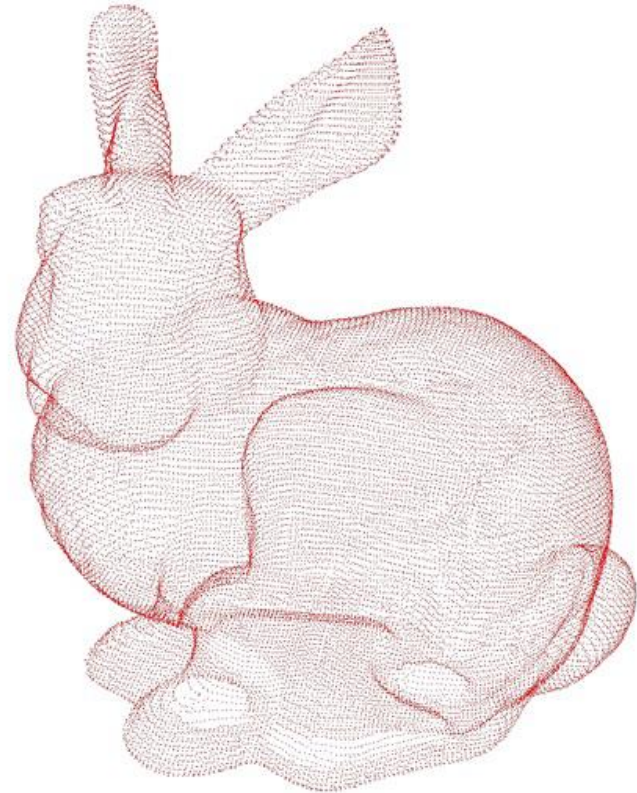


# Point Clouds



## Properties?

- **Easy to acquire**
- Accurate
- Concise
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- Efficient editing
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- Guaranteed validity
- Guaranteed smoothness
- etc.



# Point Cloud Acquisition



## Passive

- Structure from motion

## Active

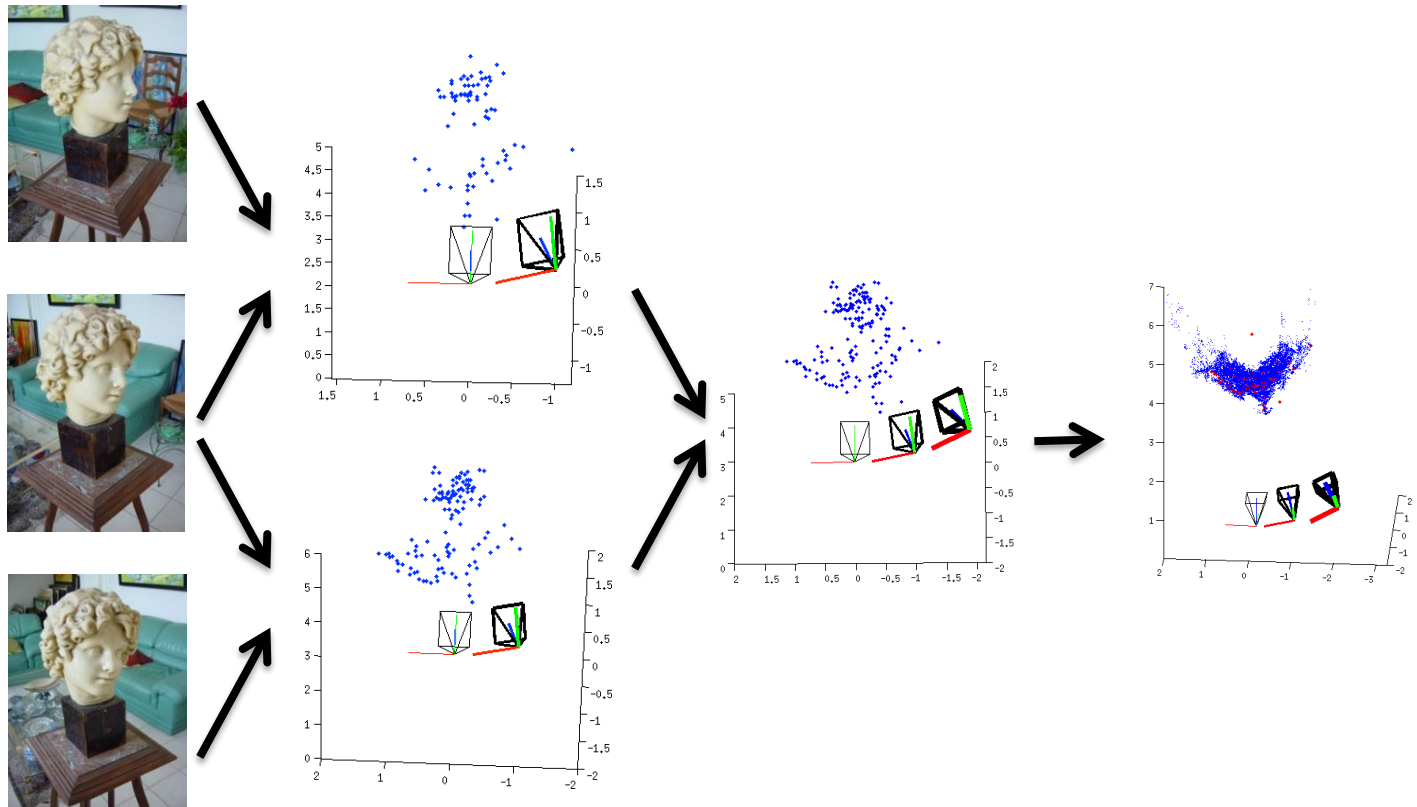
- Touch probes
- Reflectance scanning
  - Time of flight
  - Triangulation
    - Laser
    - Structured light



# Structure from Motion



Solve for 3D structure of pixel correspondences in multiple images



Structure from Motion (SfM)

Multi-view Stereo (MVS)

# Structure from Motion



## Advantages:

- Has been demonstrated for large photo collections
- Passive

## Disadvantages:

- Only works for points where pixel correspondences can be found



# Touch Probes



Capture points on object with tracked tip of probe

- Physical contact with the object
- Manual or computer-guided





# Touch Probes



## Advantages:

- Can be very precise
- Can scan **any** solid surface

## Disadvantages:

- Slow, small scale
- Can't use on fragile objects

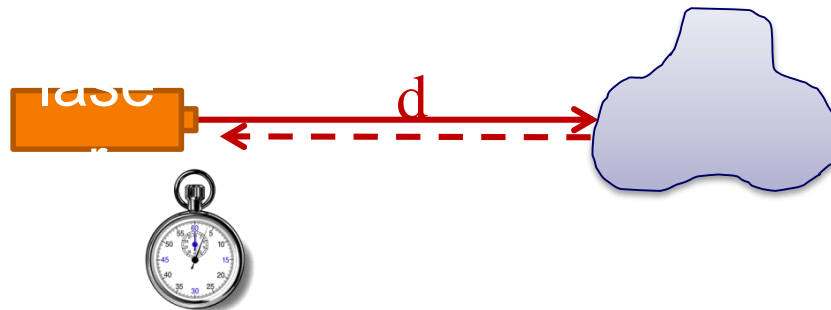
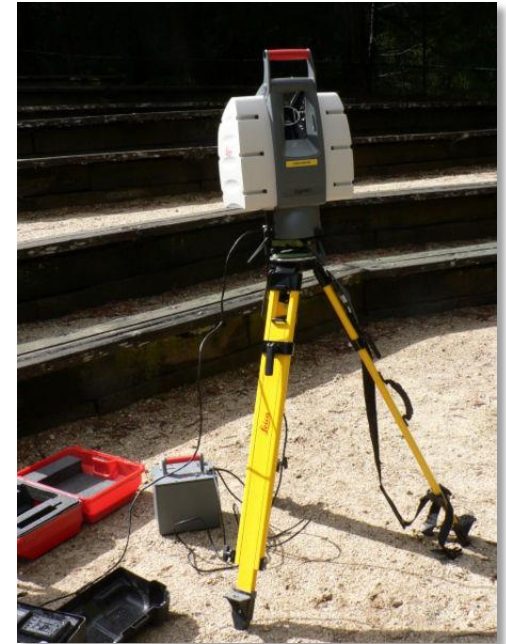


# Time of Flight Laser Scanning



Measures the time it takes  
the laser beam to hit the object  
and come back

e.g., LIDAR



$$d = 0.5 t \cdot c$$

# Time of Flight Laser Scanning

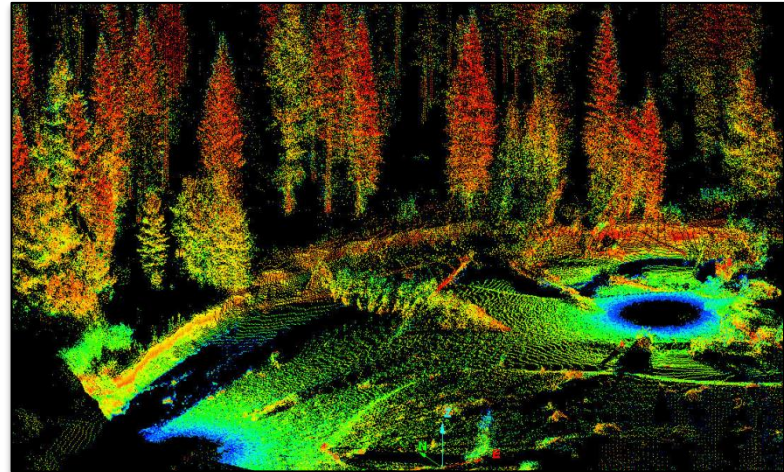
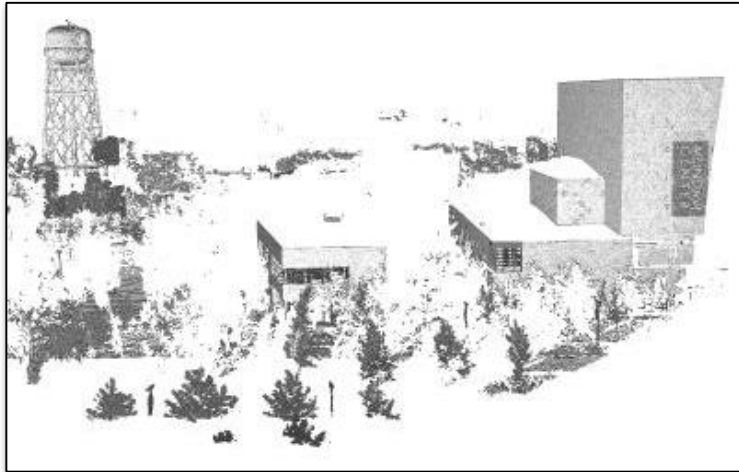


## Advantages

- Accommodates large range – up to several miles (suitable for buildings, rocks)

## Disadvantages

- Lower accuracy



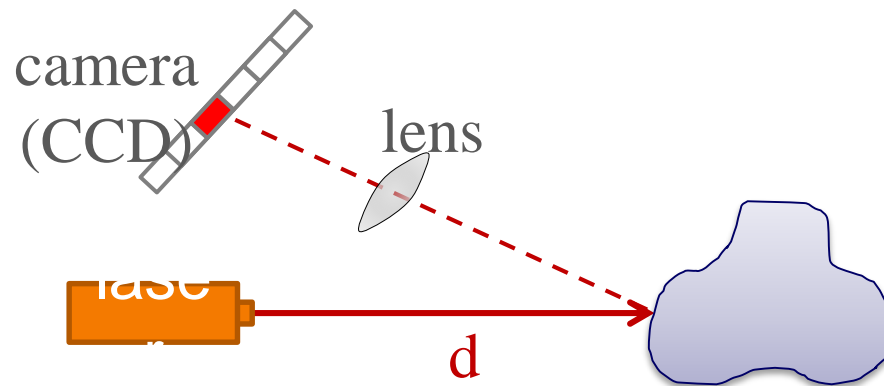
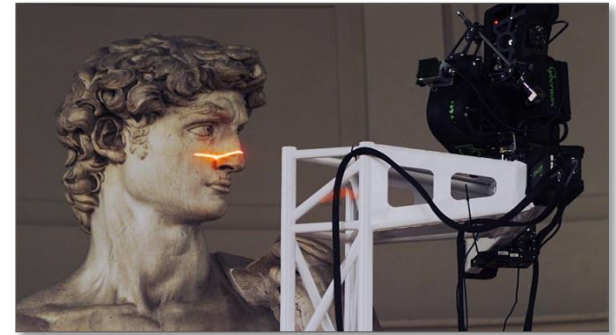
# Triangulation Laser Scanning



System includes calibrated laser beam and camera

Laser dot is photographed

The location of the dot in the image allows triangulation:  
tells distance to the object



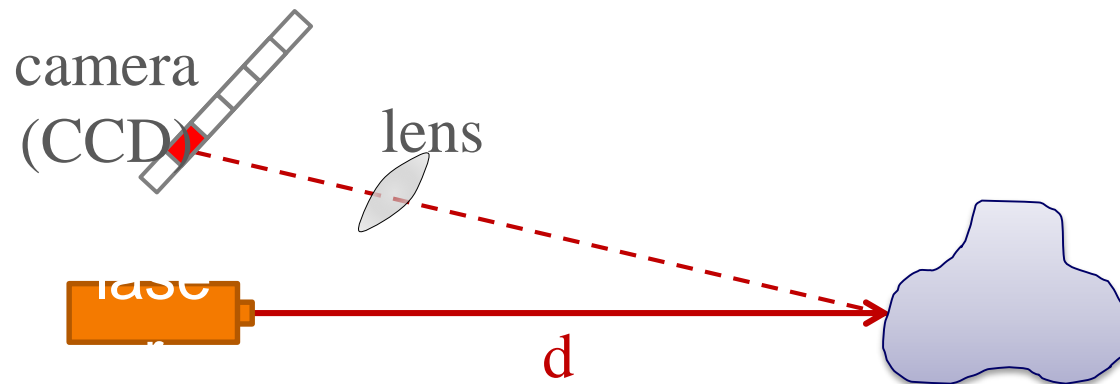
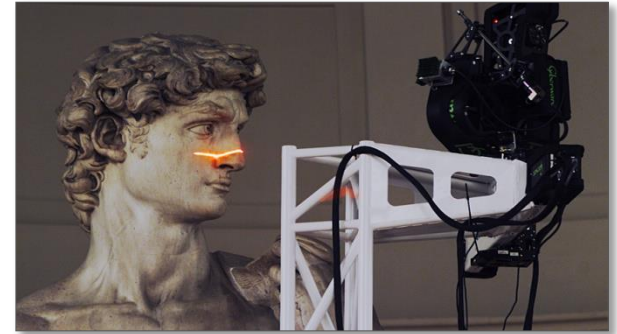
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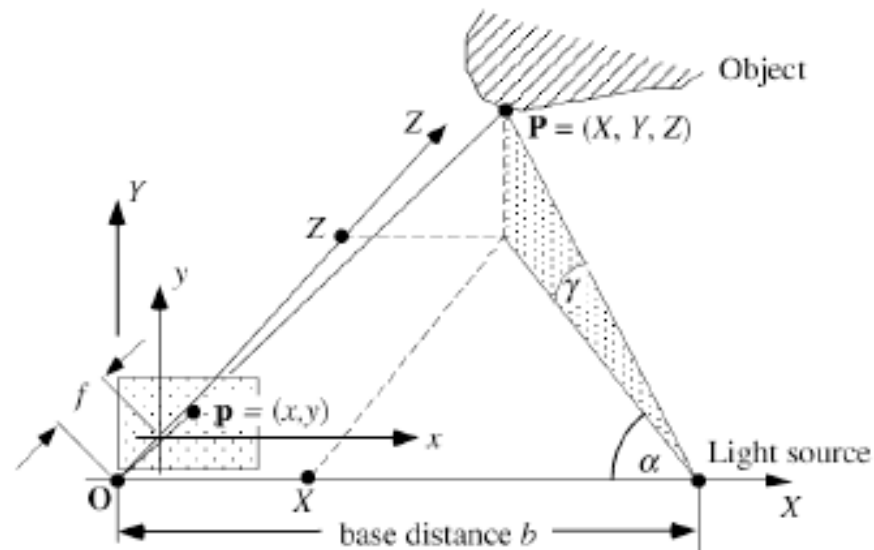
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# Triangulation Laser Scanning



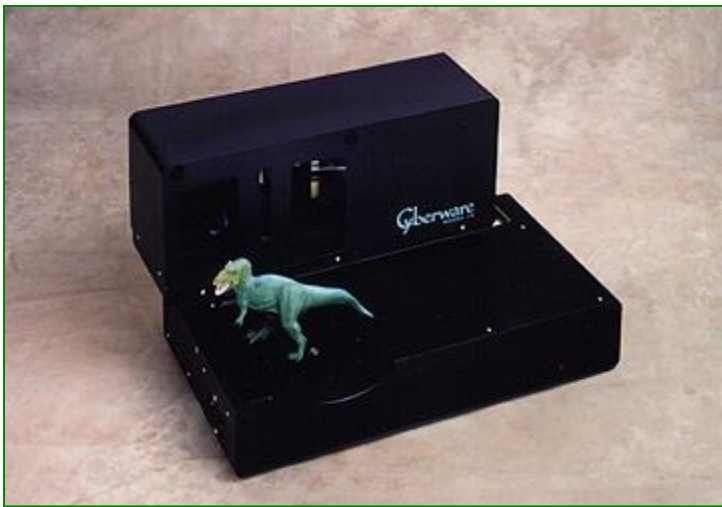
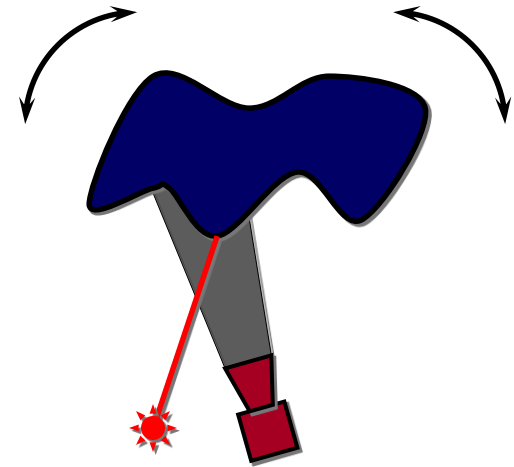
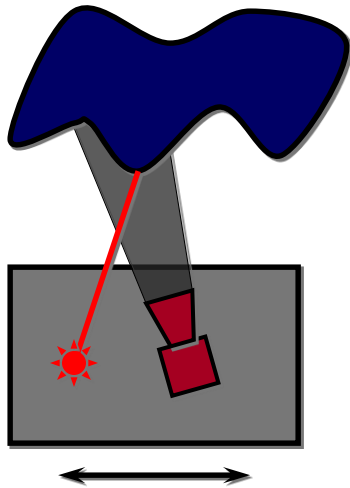
The ray theorem (of central projection) tells us that  $\frac{X}{x} = \frac{Z}{f} = \frac{Y}{y}$ , and from the trigonometry of right triangles we know that  $\tan \alpha = \frac{Z}{b-X}$ . It follows that

$$Z = \frac{X}{x} \cdot f = \tan \alpha \cdot (b - X) \quad \text{and} \quad X \cdot \left( \frac{f}{x} + \tan \alpha \right) = \tan \alpha \cdot b$$

The solution is

$$X = \frac{\tan \alpha \cdot b \cdot x}{f + x \cdot \tan \alpha}, \quad Y = \frac{\tan \alpha \cdot b \cdot y}{f + x \cdot \tan \alpha}, \quad Z = \frac{\tan \alpha \cdot b \cdot f}{f + x \cdot \tan \alpha}$$

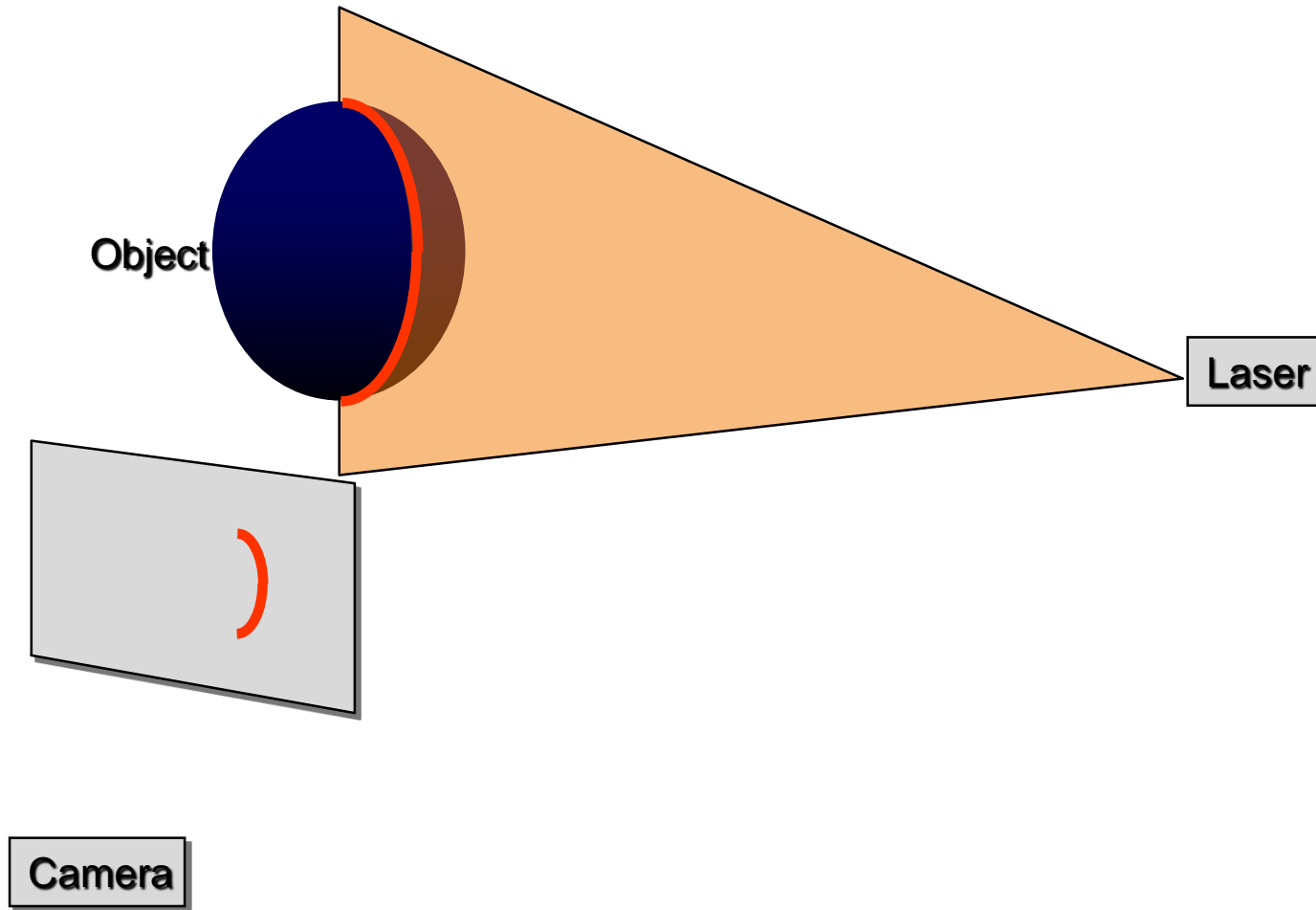
# Triangulation Laser Scanning



# Triangulation Laser Scanning



## Stripe triangulation

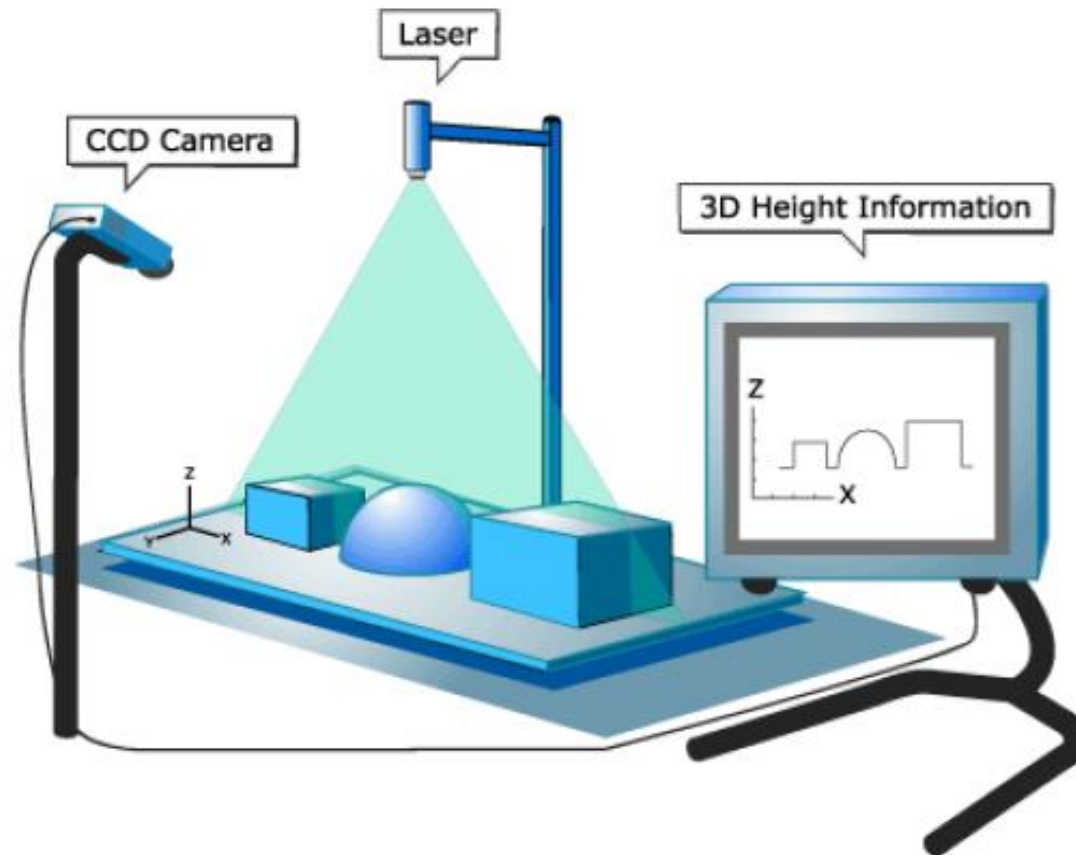




# Triangulation Laser Scanning



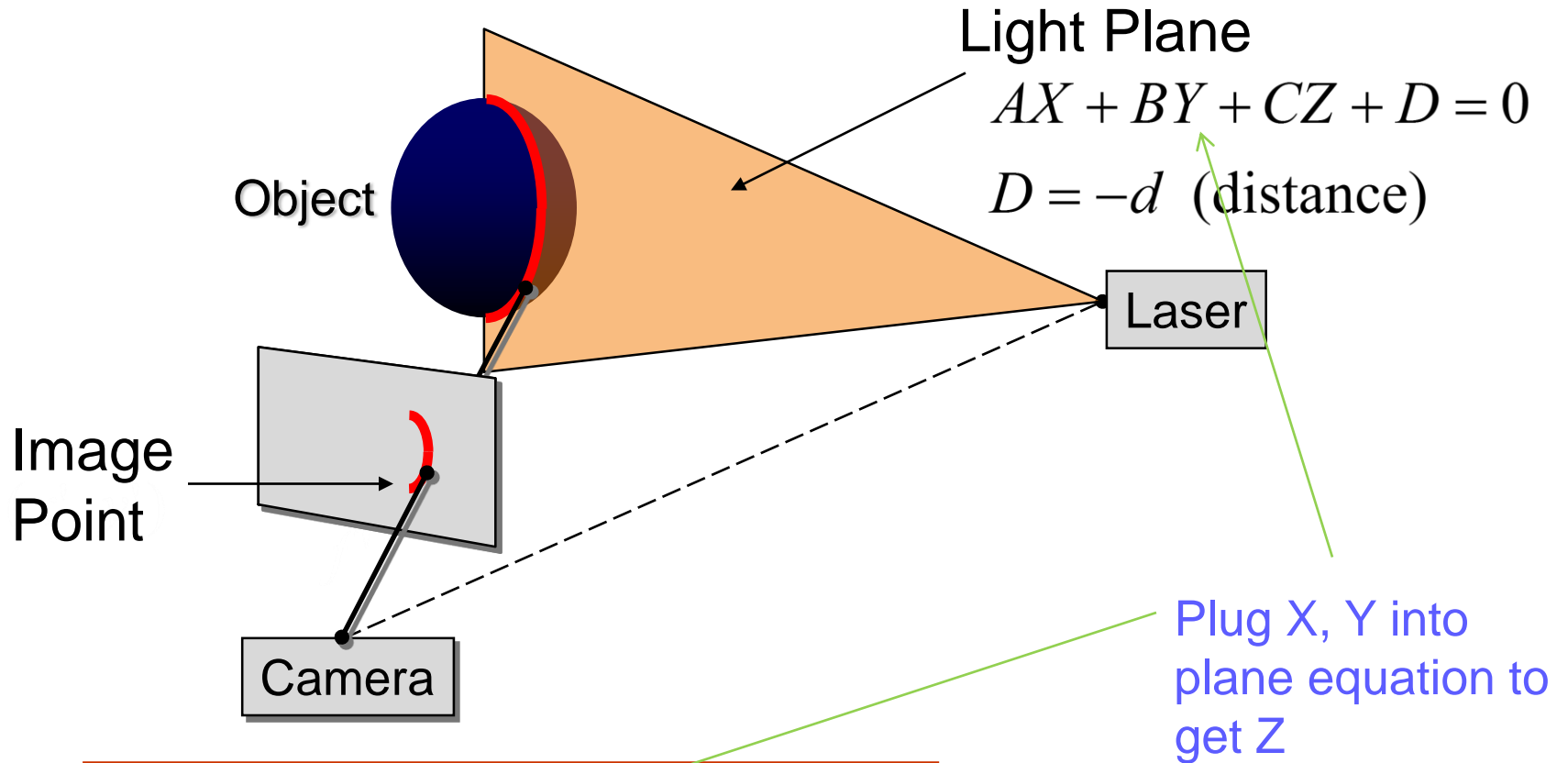
## Stripe triangulation



# Triangulation Laser Scanning



## Stripe triangulation



$$\begin{aligned} X &= x'Z / f' \\ Y &= y'Z / f' \end{aligned} \quad Z = \frac{-Df'}{Ax' + By' + Cf'}$$

# Triangulation Laser Scanning



## Advantages

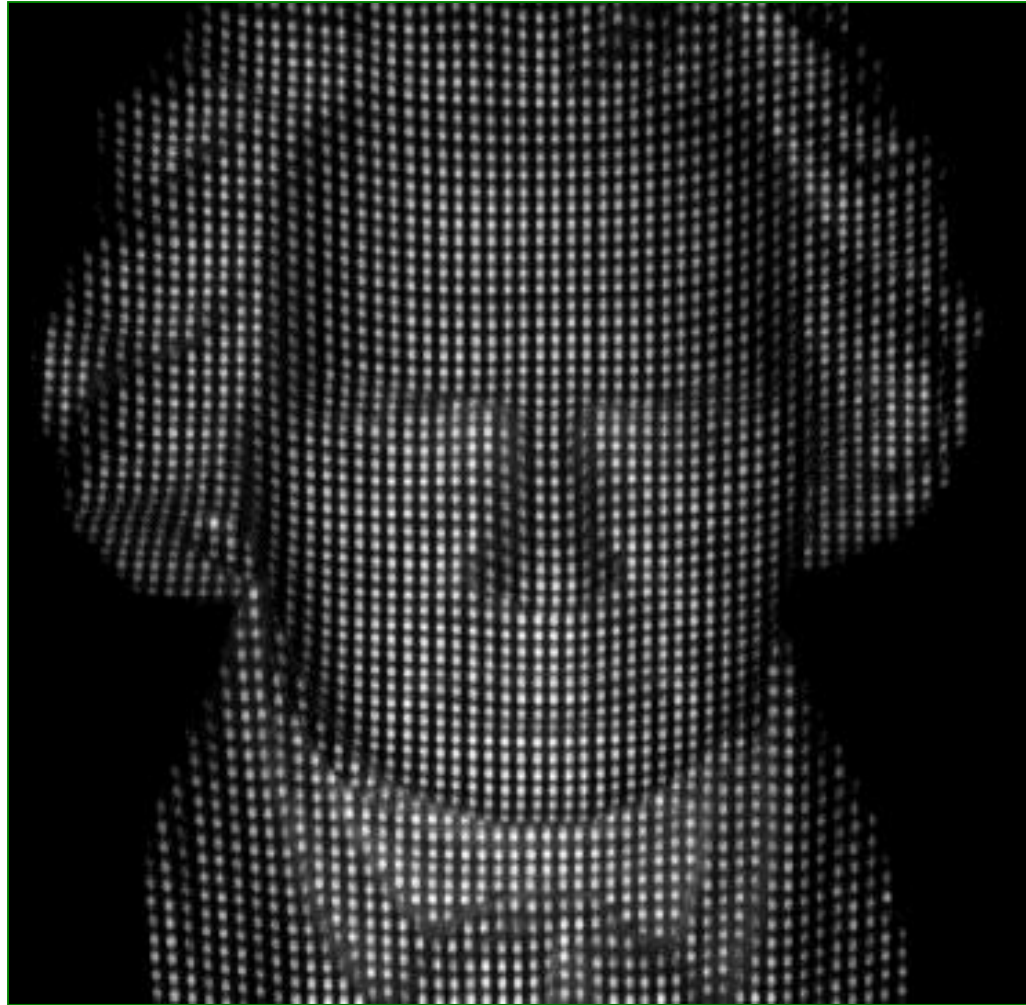
- Very precise (tens of microns)

## Disadvantages

- Small distances (meters)
- Inaccessible regions



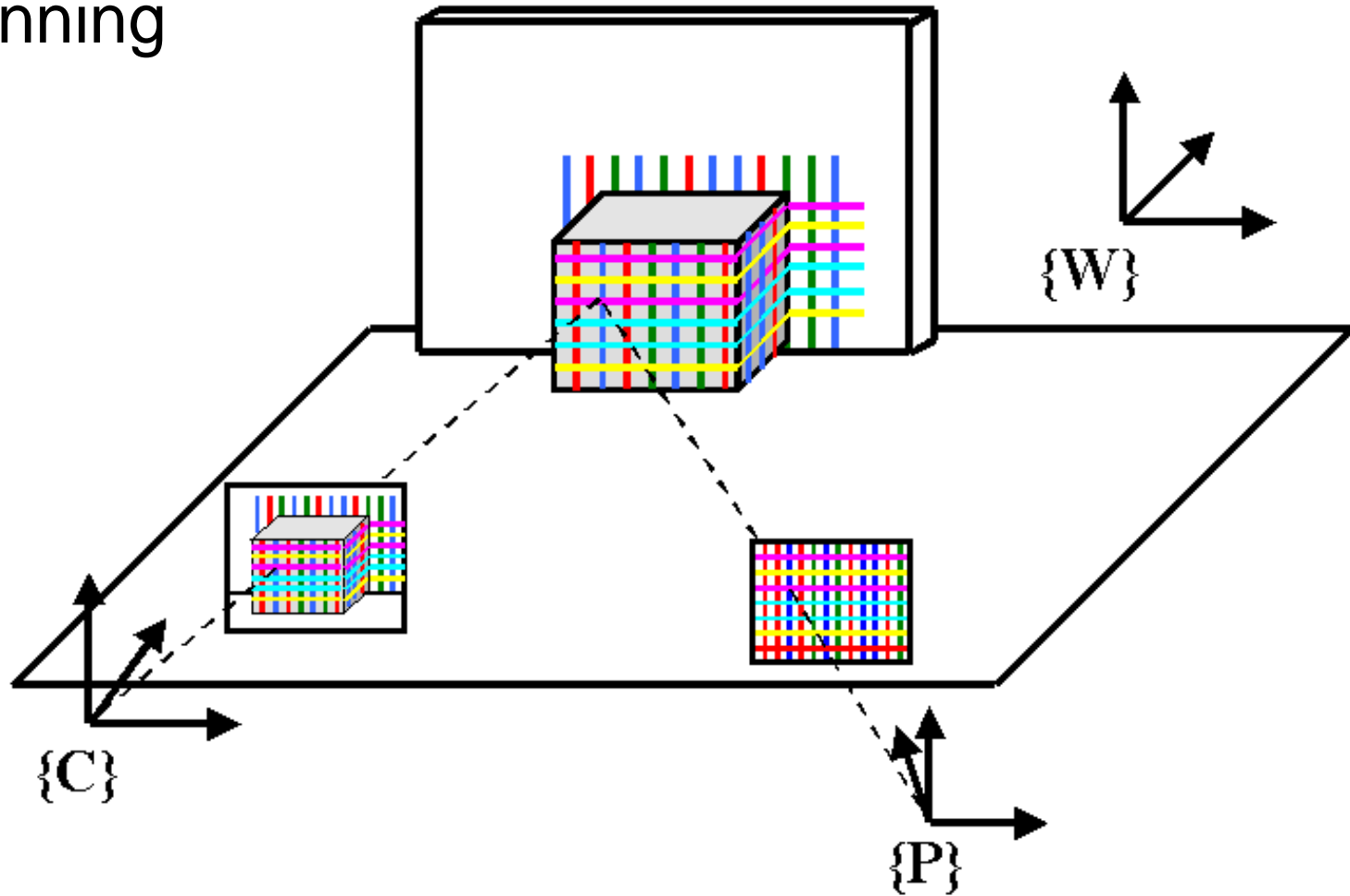
# Multi-Stripe Triangulation



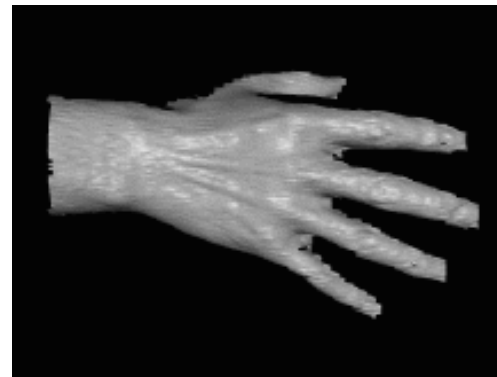
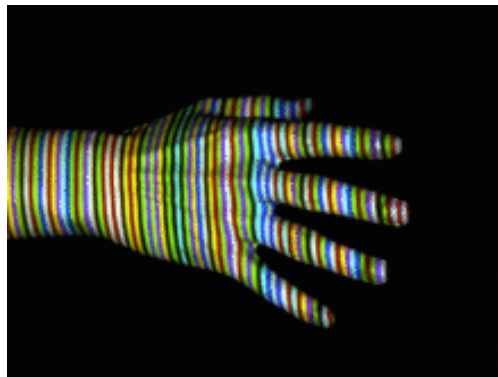
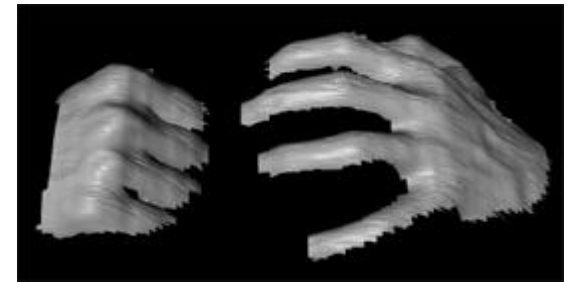
# Color-Coded Stripe Triangulation



Active  
Scanning



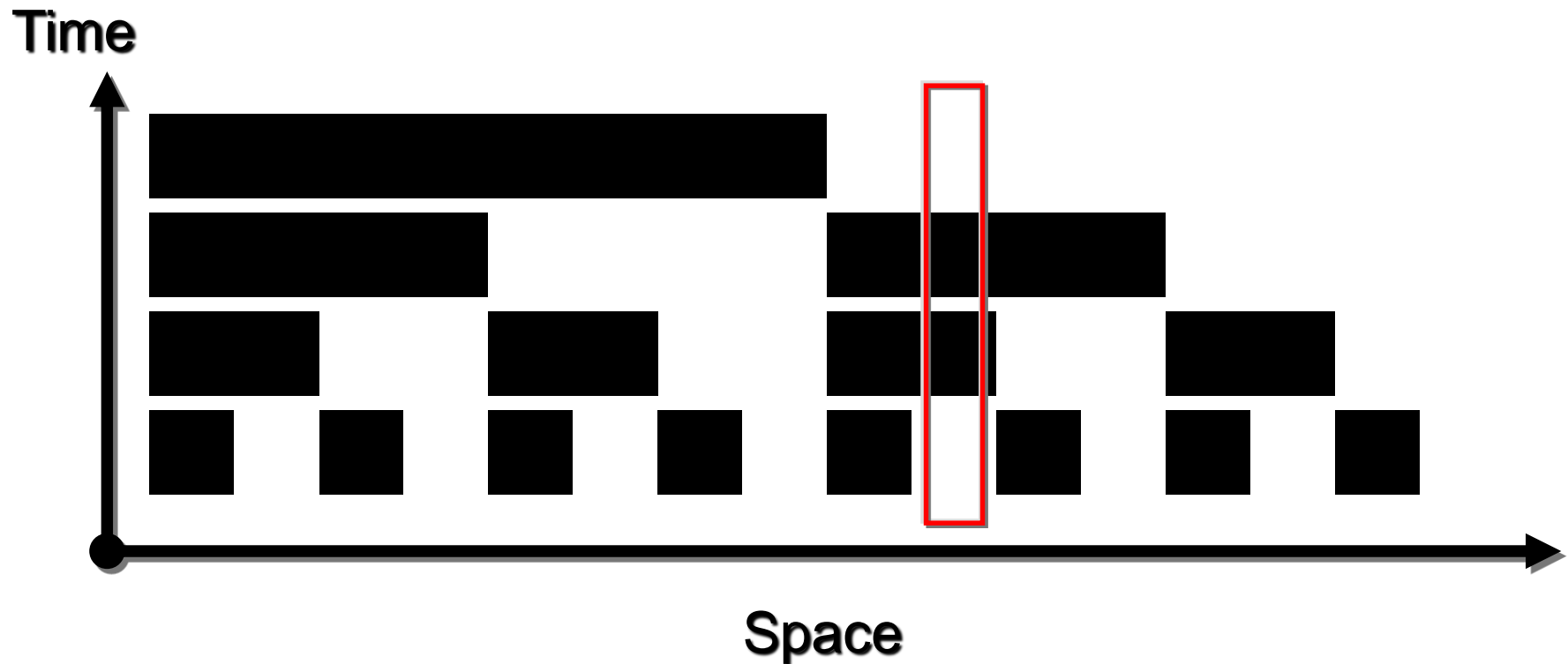
# Color-Coded Stripe Triangulation



# Time-Coded Stripe Triangulation



Assign each stripe a unique illumination code over time

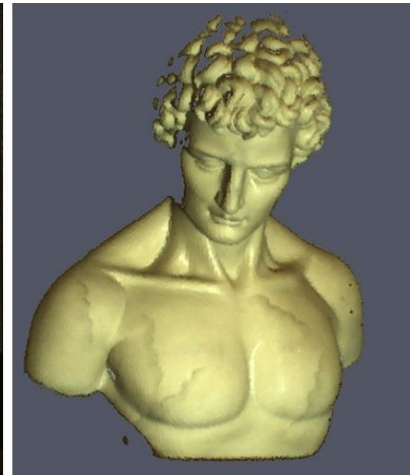
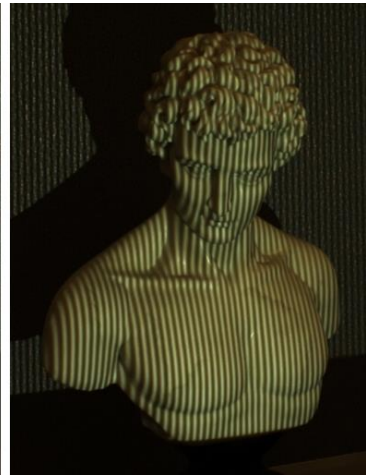
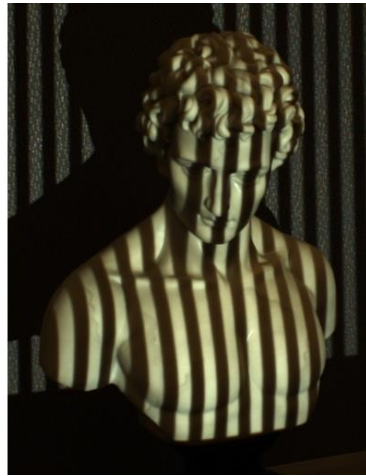


# Time-Coded Stripe Triangulation

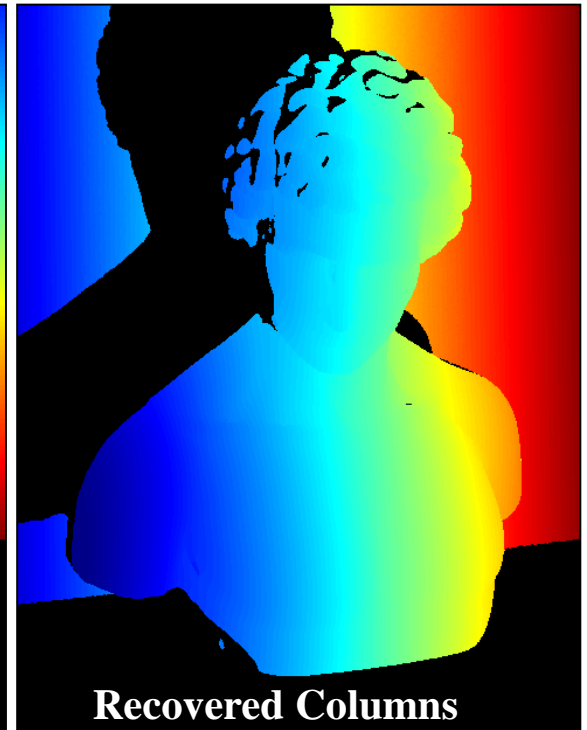
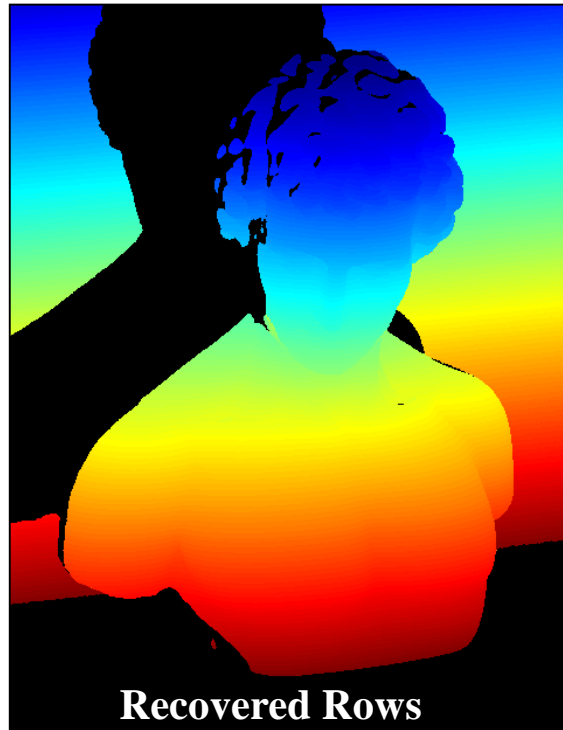




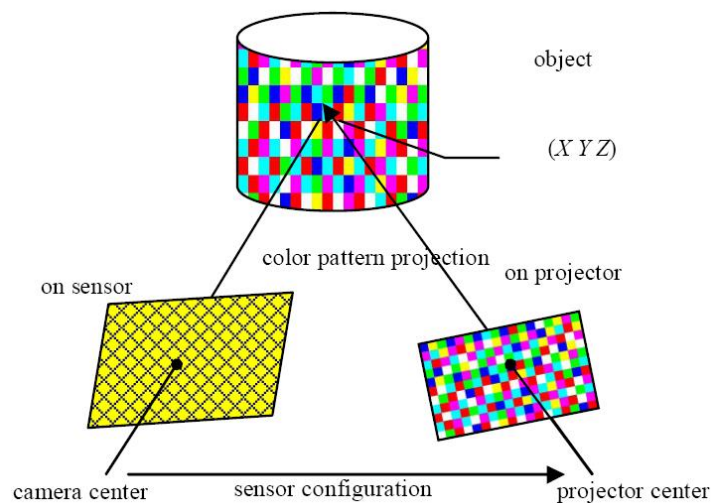
# Time-Coded Stripe Triangulation



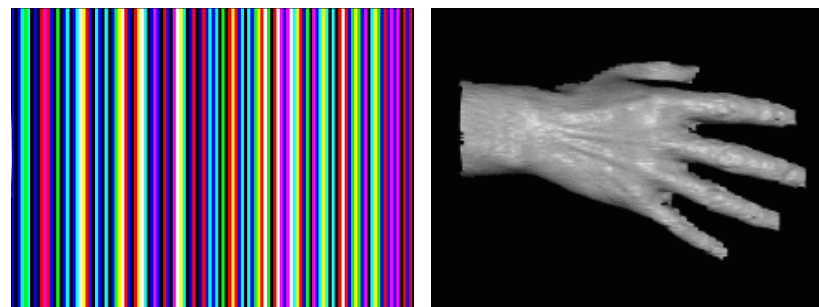
# Time-Coded Stripe Triangulation



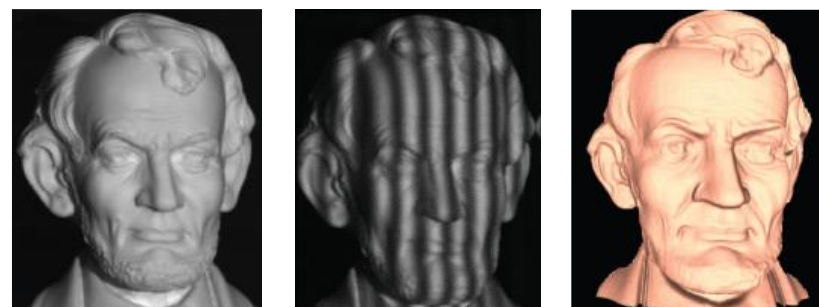
# Structured Light Patterns



“Single-shot” patterns (N-arrays, grids, random, etc.)

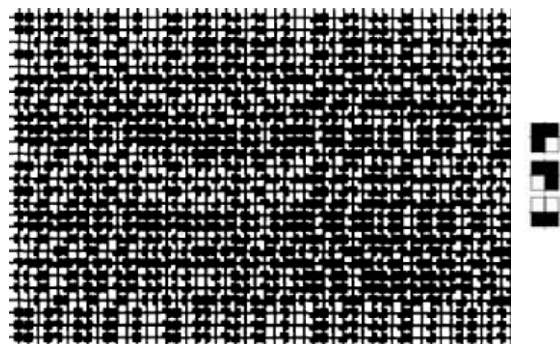


De Bruijn sequences [Zhang et al. 2002]



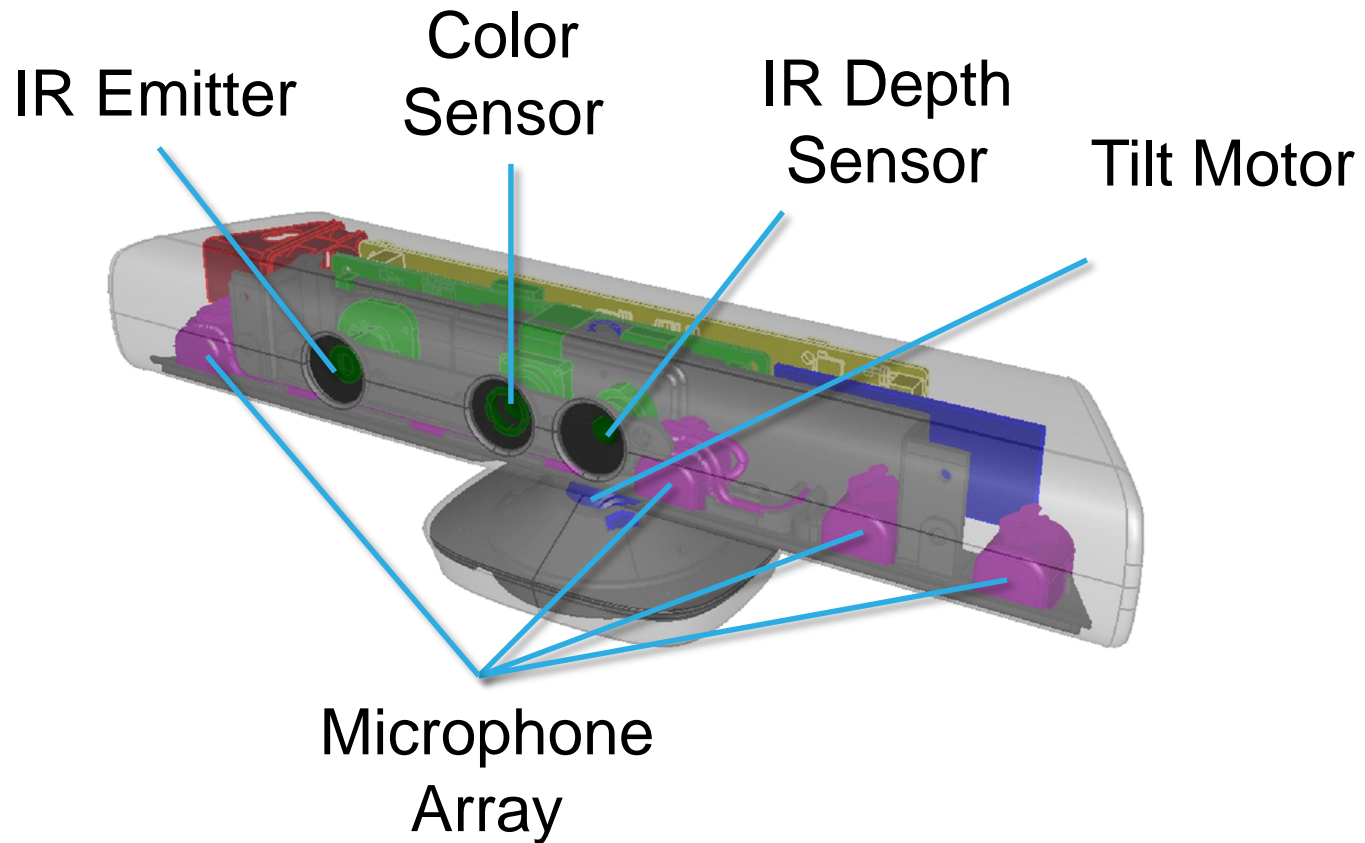
Phase-shifting [Zhang et al. 2004]

Spatial encoding strategies [Chen et al. 2007]



Pseudorandom and M-arrays [Griffin 1992]

# Structured Light Scanning: Kinect



# Structured Light Scanning: Kinect



Projected IR Pattern



# Structured Light Scanning: Kinect



Depth Map



RGB Image

# Structured Light Scanning: Kinect



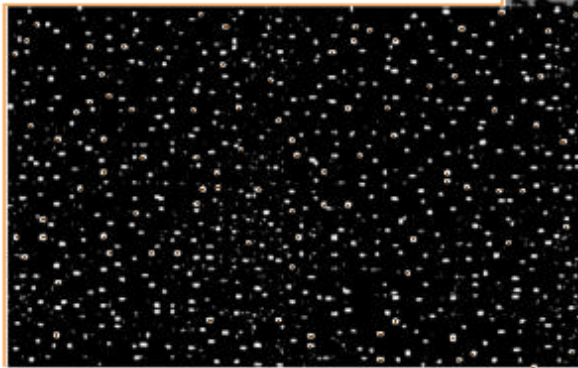
How the Kinect Depth Sensor Works in 2 Minutes

<http://www.youtube.com/watch?v=uq9SEJxZiUg>

# Structured Light Scanning: Kinect



<http://users.dickinson.edu/~jmac/selected-talks/kinect.pdf>





# Structured Light Scanning

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## Advantages:

- Very fast – 2D pattern at once

## Disadvantages:

- Prone to noise

# Next Time



## Point cloud processing and surface reconstruction

