

2.1 ELEMENTARY SORTS

- ▶ *rules of the game*
- ▶ *selection sort*
- ▶ *insertion sort*
- ▶ *shuffling*
- ▶ *comparators*

<http://algs4.cs.princeton.edu>

Sorting problem

Ex. Student records in a university.

	Chen	3	A	(991) 878-4944	308 Blair
	Rohde	2	A	(232) 343-5555	343 Forbes
	Gazsi	4	B	(800) 867-5309	101 Brown
item →	Furia	1	A	(766) 093-9873	101 Brown
	Kanaga	3	B	(898) 122-9643	22 Brown
	Andrews	3	A	(664) 480-0023	097 Little
key →	Battle	4	C	(874) 088-1212	121 Whitman

Sort. Rearrange array of N items into ascending order.

Andrews	3	A	(664) 480-0023	097 Little
Battle	4	C	(874) 088-1212	121 Whitman
Chen	3	A	(991) 878-4944	308 Blair
Furia	1	A	(766) 093-9873	101 Brown
Gazsi	4	B	(800) 867-5309	101 Brown
Kanaga	3	B	(898) 122-9643	22 Brown
Rohde	2	A	(232) 343-5555	343 Forbes

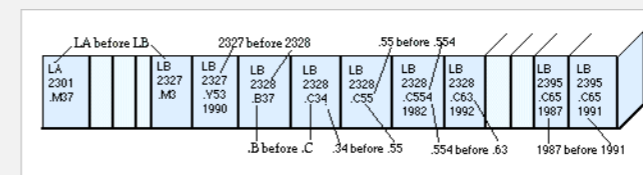
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<http://algs4.cs.princeton.edu>

Sorting applications



Library of Congress numbers



FedEx packages



playing cards



contacts



Hogwarts houses

Sample sort client 1

Goal. Sort **any** type of data.

Ex 1. Sort random real numbers in ascending order.

seems artificial (stay tuned for an application)

```
public class Experiment
{
    public static void main(String[] args)
    {
        int N = Integer.parseInt(args[0]);
        Double[] a = new Double[N];
        for (int i = 0; i < N; i++)
            a[i] = StdRandom.uniform();
        Insertion.sort(a);
        for (int i = 0; i < N; i++)
            StdOut.println(a[i]);
    }
}
```

```
% java Experiment 10
0.08614716385210452
0.09054270895414829
0.10708746304898642
0.21166190071646818
0.363292849257276
0.460954145685913
0.5340026311350087
0.7216129793703496
0.9003500354411443
0.9293994908845686
```

5

Sample sort client 2

Goal. Sort **any** type of data.

Ex 2. Sort strings in alphabetical order.

```
public class StringSorter
{
    public static void main(String[] args)
    {
        String[] a = StdIn.readAllStrings();
        Insertion.sort(a);
        for (int i = 0; i < a.length; i++)
            StdOut.println(a[i]);
    }
}
```

```
% more words3.txt
bed bug dad yet zoo ... all bad yes
```

```
% java StringSorter < words3.txt
all bad bed bug dad ... yes yet zoo
[suppressing newlines]
```

6

Sample sort client 3

Goal. Sort **any** type of data.

Ex 3. Sort the files in a given directory by filename.

```
import java.io.File;

public class FileSorter
{
    public static void main(String[] args)
    {
        File directory = new File(args[0]);
        File[] files = directory.listFiles();
        Insertion.sort(files);
        for (int i = 0; i < files.length; i++)
            StdOut.println(files[i].getName());
    }
}
```

```
% java FileSorter .
Insertion.class
Insertion.java
InsertionX.class
InsertionX.java
Selection.class
Selection.java
Shell.class
Shell.java
ShellX.class
ShellX.java
```

7

Total order

Goal. Sort **any** type of data (for which sorting is well defined).

A **total order** is a binary relation \leq that satisfies:

- Antisymmetry: if both $v \leq w$ and $w \leq v$, then $v = w$.
- Transitivity: if both $v \leq w$ and $w \leq x$, then $v \leq x$.
- Totality: either $v \leq w$ or $w \leq v$ or both.

Ex.

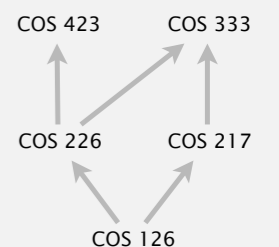
- Standard order for natural and real numbers.
- Chronological order for dates or times.
- Lexicographic order for strings.

Not transitive. Rock-paper-scissors.

Not total. PU course prerequisites.



violates transitivity



violates totality

8

Callbacks

Goal. Sort **any** type of data (for which sorting is well defined).

Q. How can `sort()` know how to compare data of type `Double`, `String`, and `java.io.File` without any information about the type of an item's key?

Callback = reference to executable code.

- Client passes array of objects to `sort()` function.
- The `sort()` function calls object's `compareTo()` method as needed.

Implementing callbacks.

- Java: interfaces.
- C: function pointers.
- C++: class-type functors.
- C#: delegates.
- Python, Perl, ML, Javascript: first-class functions.

9

Callbacks: roadmap

client

```
public class StringSorter
{
    public static void main(String[] args)
    {
        String[] a = StdIn.readAllStrings();
        Insertion.sort(a);
        for (int i = 0; i < a.length; i++)
            StdOut.println(a[i]);
    }
}
```

data-type implementation

```
public class String
implements Comparable<String>
{
    ...
    public int compareTo(String b)
    {
        ...
        return -1;
        ...
        return +1;
        ...
        return 0;
    }
}
```

Comparable interface (built in to Java)

```
public interface Comparable<Item>
{
    public int compareTo(Item that);
}
```

sort implementation

```
public static void sort(Comparable[] a)
{
    int N = a.length;
    for (int i = 0; i < N; i++)
        for (int j = i; j > 0; j--)
            if (a[j].compareTo(a[j-1]) < 0)
                exch(a, j, j-1);
            else break;
}
```

key point: no dependence
on String data type

10

Comparable API

Implement `compareTo()` so that `v.compareTo(w)`

- Defines a total order.
- Returns a negative integer, zero, or positive integer if `v` is less than, equal to, or greater than `w`, respectively.
- Throws an exception if incompatible types (or either is null).



less than (return -1)



equal to (return 0)



greater than (return +1)

Built-in comparable types. Integer, Double, String, Date, File, ...

User-defined comparable types. Implement the Comparable interface.

11

Implementing the Comparable interface

Date data type. Simplified version of `java.util.Date`.

```
public class Date implements Comparable<Date>
{
    private final int month, day, year;

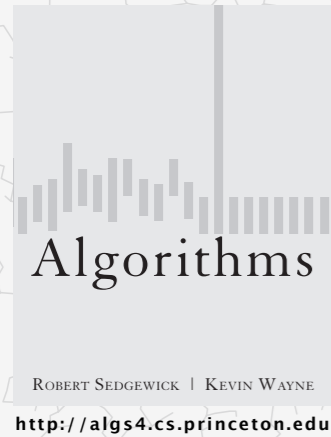
    public Date(int m, int d, int y)
    {
        month = m;
        day = d;
        year = y;
    }
}
```

only compare dates
to other dates

```
public int compareTo(Date that)
{
    if (this.year < that.year ) return -1;
    if (this.year > that.year ) return +1;
    if (this.month < that.month) return -1;
    if (this.month > that.month) return +1;
    if (this.day < that.day ) return -1;
    if (this.day > that.day ) return +1;
    return 0;
}
```

<http://algs4.cs.princeton.edu/12oop/Date.java.html>

12

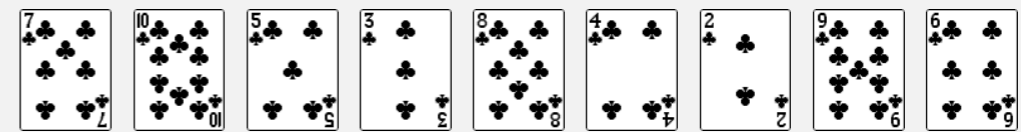


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- ▶ comparators

Selection sort demo

- In iteration i , find index min of smallest remaining entry.
- Swap $a[i]$ and $a[\text{min}]$.



initial

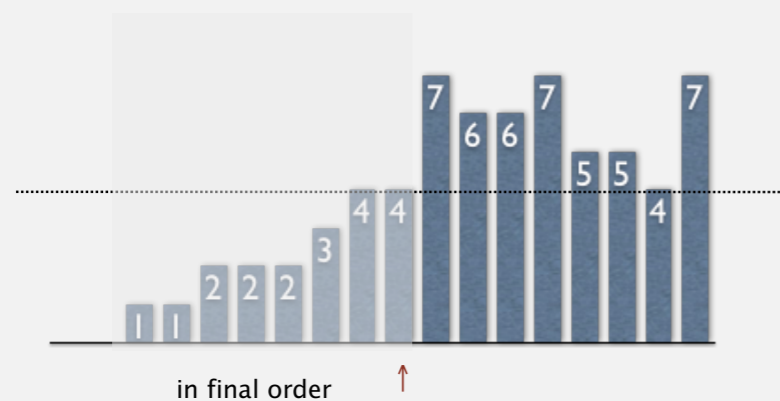


Selection sort

Algorithm. ↑ scans from left to right.

Invariants.

- Entries the left of ↑ (including ↑) fixed and in ascending order.
- No entry to right of ↑ is smaller than any entry to the left of ↑.



Selection sort inner loop

To maintain algorithm invariants:

- Move the pointer to the right.

```
i++;
```



- Identify index of minimum entry on right.

```
int min = i;
for (int j = i+1; j < N; j++)
    if (less(a[j], a[min]))
        min = j;
```



- Exchange into position.

```
exch(a, i, min);
```



Two useful sorting abstractions

Helper functions. Refer to data only through compares and exchanges.

Less. Is item v less than w ?

```
private static boolean less(Comparable v, Comparable w)
{ return v.compareTo(w) < 0; }
```

Exchange. Swap item in array $a[]$ at index i with the one at index j .

```
private static void exch(Object[] a, int i, int j)
{
    Object swap = a[i];
    a[i] = a[j];
    a[j] = swap;
}
```

17

Selection sort: Java implementation

```
public class Selection
{
    public static void sort(Comparable[] a)
    {
        int N = a.length;
        for (int i = 0; i < N; i++)
        {
            int min = i;
            for (int j = i+1; j < N; j++)
                if (less(a[j], a[min]))
                    min = j;
            exch(a, i, min);
        }
    }

    private static boolean less(Comparable v, Comparable w)
    { /* see previous slide */ }

    private static void exch(Object[] a, int i, int j)
    { /* see previous slide */ }
}
```

<http://algs4.cs.princeton.edu/21elementary/Selection.java.html>

18

Generic methods

Oops. The compiler complains.

```
% javac Selection.java
Note: Selection.java uses unchecked or unsafe operations.
Note: Recompile with -Xlint:unchecked for details.
```

```
% javac -Xlint:unchecked Selection.java
Selection.java:83: warning: [unchecked] unchecked call to
compareTo(T) as a member of the raw type java.lang.Comparable
    return (v.compareTo(w) < 0);
                   ^
```

```
1 warning
```

Q. How to fix?

19

Generic methods

Pedantic (type-safe) version. Compiles cleanly.

generic type variable
(value inferred from argument; must be Comparable)

```
public class SelectionPedantic
{
    public static <Key extends Comparable<Key>> void sort(Key[] a)
    { /* as before */ }

    private static <Key extends Comparable<Key>> boolean less(Key v, Key w)
    { /* as before */ }

    private static Object void exch(Object[] a, int i, int j)
    { /* as before */ }
}
```

<http://algs4.cs.princeton.edu/21elementary/SelectionPedantic.java.html>

Remark. Use type-safe version in system code (but not in lecture).

20

Selection sort: animations

20 random items



▲ algorithm position
 ■ in final order
 ■ not in final order

<http://www.sorting-algorithms.com/selection-sort>

21

Selection sort: animations

20 partially-sorted items



▲ algorithm position
 ■ in final order
 ■ not in final order

<http://www.sorting-algorithms.com/selection-sort>

22

Selection sort: quiz

How many compares does selection sort make to sort an array of N keys?

- A. $\sim N$
- B. $\sim 1/4 N^2$
- C. $\sim 1/2 N^2$
- D. $\sim N^2$
- E. *I don't know.*

23

Selection sort: mathematical analysis


Proposition. Selection sort uses $(N-1) + (N-2) + \dots + 1 + 0 \sim N^2/2$ compares and N exchanges.

		a[]										
i	min	0	1	2	3	4	5	6	7	8	9	10
		S	O	R	T	E	X	A	M	P	L	E
0	6	S	O	R	T	E	X	A	M	P	L	E
1	4	A	O	R	T	E	X	S	M	P	L	E
2	10	A	E	R	T	O	X	S	M	P	L	E
3	9	A	E	E	T	O	X	S	M	P	L	R
4	7	A	E	E	L	O	X	S	M	P	T	R
5	7	A	E	E	L	M	X	S	O	P	T	R
6	8	A	E	E	L	M	O	S	X	P	T	R
7	10	A	E	E	L	M	O	P	X	S	T	R
8	8	A	E	E	L	M	O	P	R	S	T	X
9	9	A	E	E	L	M	O	P	R	S	T	X
10	10	A	E	E	L	M	O	P	R	S	T	X

Trace of selection sort (array contents just after each exchange)

Running time insensitive to input. Quadratic time, even if input is sorted.
Data movement is minimal. Linear number of exchanges.

24



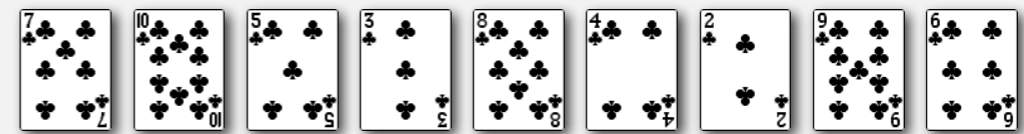
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ROBERT SEDGWICK | KEVIN WAYNE
<http://algs4.cs.princeton.edu>

Insertion sort demo

- In iteration i , swap $a[i]$ with each larger entry to its left.



Insertion sort

Algorithm. ↑ scans from left to right.

Invariants.

- Entries to the left of ↑ (including ↑) are in ascending order.
- Entries to the right of ↑ have not yet been seen.



Insertion sort: inner loop

To maintain algorithm invariants:

- Move the pointer to the right.

```
i++;
```



- Moving from right to left, exchange $a[i]$ with each larger entry to its left.

```
for (int j = i; j > 0; j--)
    if (less(a[j], a[j-1]))
        exch(a, j, j-1);
    else break;
```



Insertion sort: Java implementation

```
public class Insertion
{
    public static void sort(Comparable[] a)
    {
        int N = a.length;
        for (int i = 0; i < N; i++)
            for (int j = i; j > 0; j--)
                if (less(a[j], a[j-1]))
                    exch(a, j, j-1);
                else break;
    }

    private static boolean less(Comparable v, Comparable w)
    { /* as before */ }

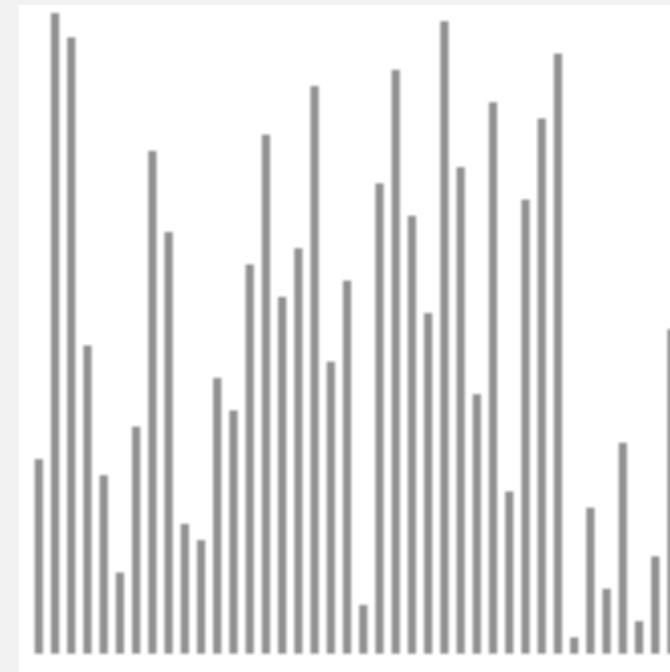
    private static void exch(Object[] a, int i, int j)
    { /* as before */ }
}
```

<http://algs4.cs.princeton.edu/21elementary/Insertion.java.html>

29

Insertion sort: animation

40 random items



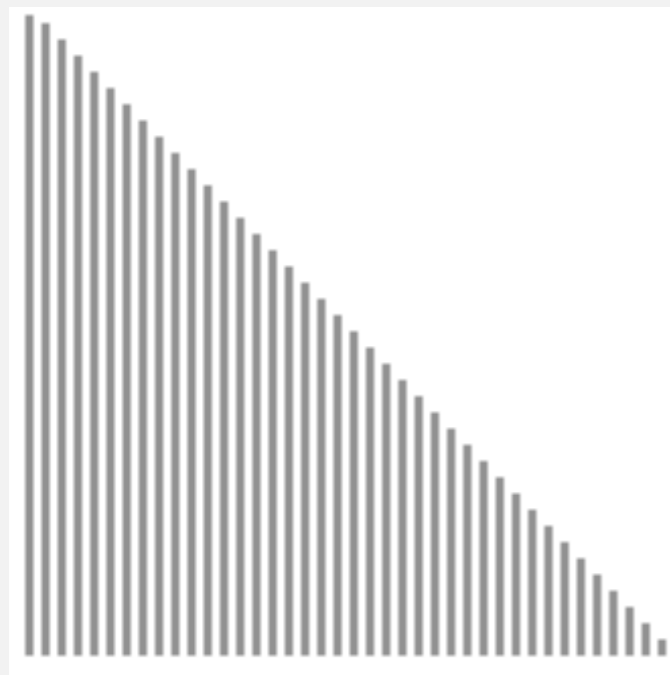
<http://www.sorting-algorithms.com/insertion-sort>

▲ algorithm position
 █████ in order
 █████ not yet seen

30

Insertion sort: animation

40 reverse-sorted items



<http://www.sorting-algorithms.com/insertion-sort>

▲ algorithm position
 █████ in order
 █████ not yet seen

31

Insertion sort: mathematical analysis

Proposition. To sort a randomly-ordered array with distinct keys, insertion sort uses $\sim \frac{1}{4} N^2$ compares and $\sim \frac{1}{4} N^2$ exchanges on average.

Pf. Expect each entry to move halfway back.

		a[]										
i	j	0	1	2	3	4	5	6	7	8	9	10
		S	O	R	T	E	X	A	M	P	L	E
1	0	O	S	R	T	E	X	A	M	P	L	E
2	1	O	R	S	T	E	X	A	M	P	L	E
3	3	O	R	S	T	E	X	A	M	P	L	E
4	0	E	O	R	S	T	X	A	M	P	L	E
5	5	E	O	R	S	T	X	A	M	P	L	E
6	0	A	E	O	R	S	T	X	M	P	L	E
7	2	A	E	M	O	R	S	T	X	P	L	E
8	4	A	E	M	O	P	R	S	T	X	L	E
9	2	A	E	L	M	O	P	R	S	T	X	E
10	2	A	E	E	L	M	O	P	R	S	T	X
		A	E	E	L	M	O	P	R	S	T	X

Trace of insertion sort (array contents just after each insertion)

entries in gray do not move

entry in red is a[j]

entries in black moved one position right for insertion

32

Insertion sort: quiz

How many compares does insertion sort make to sort an array of N distinct keys items in reverse order?

- A. $\sim N$
- B. $\sim 1/4 N^2$
- C. $\sim 1/2 N^2$
- D. $\sim N^2$
- E. *I don't know.*

33

Insertion sort: analysis

Worst case. If the array is in descending order (and no duplicates), insertion sort makes $\sim 1/2 N^2$ compares and $\sim 1/2 N^2$ exchanges.

X T S R P O M L F E A

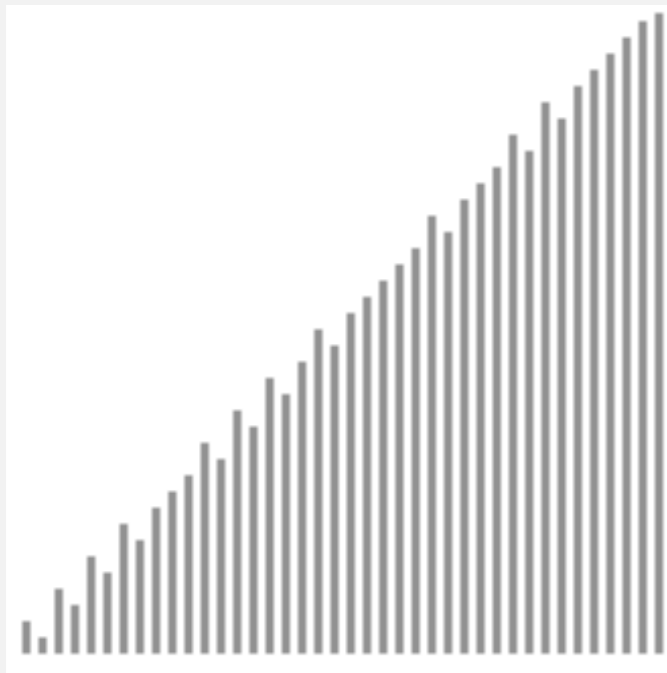
Best case. If the array is in ascending order, insertion sort makes $N-1$ compares and 0 exchanges.

A E E L M O P R S T X

34

Insertion sort: animation

40 partially-sorted items



<http://www.sorting-algorithms.com/insertion-sort>

▲ algorithm position
in order
■ not yet seen

35

Insertion sort: partially-sorted arrays

Def. An **inversion** is a pair of keys that are out of order.

A E E L M O T R X P S

T-R T-P T-S R-P X-P X-S
(6 inversions)

Def. An array is **partially sorted** if the number of inversions is $\leq cN$.

- Ex 1. A sorted array has 0 inversions.
- Ex 2. A subarray of size 10 appended to a sorted subarray of size N .

Proposition. For partially-sorted arrays, insertion sort runs in linear time.

Pf. Number of exchanges equals the number of inversions.

↑
number of compares \leq exchanges + $(N - 1)$

36

Insertion sort: practical improvements

Half exchanges. Shift items over (instead of exchanging).

- Eliminates unnecessary data movement.
- No longer uses only `less()` and `exch()` to access data.

A C H H I M N N P Q X Y K B I N A R Y

Binary insertion sort. Use binary search to find insertion point.

- Number of compares $\sim N \lg N$.
- But still a quadratic number of array accesses.

A C H H I (M) N N P Q X Y K B I N A R Y

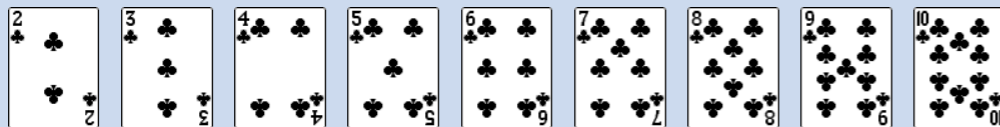
binary search for first key > K

37

Interview question: shuffle an array

Goal. Rearrange array so that result is a uniformly random permutation.

all $N!$ permutations
equally likely



39

2.1 ELEMENTARY SORTS

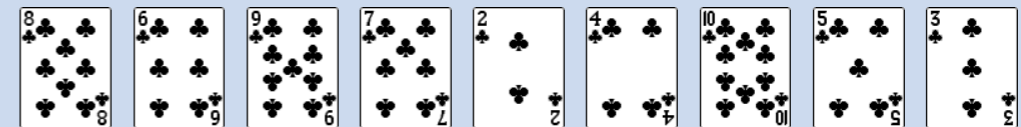


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Interview question: shuffle an array

Goal. Rearrange array so that result is a uniformly random permutation.

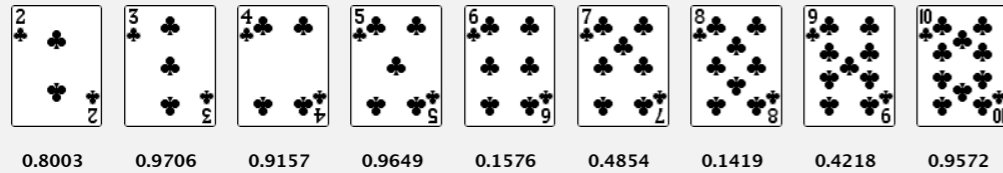
all $N!$ permutations
equally likely



40

Shuffle sort

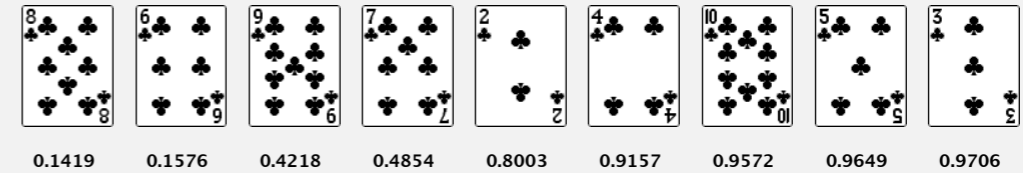
- Generate a random real number for each array entry.
- Sort the array.



41

Shuffle sort

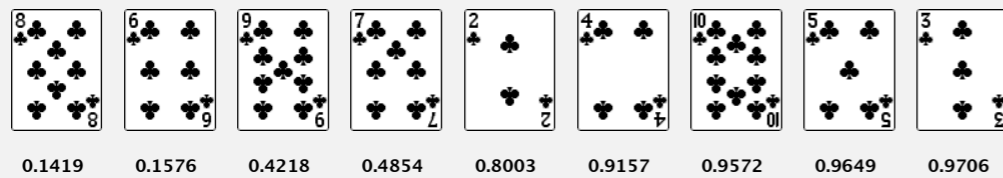
- Generate a random real number for each array entry.
- Sort the array.



42

Shuffle sort

- Generate a random real number for each array entry.
- Sort the array.



Proposition. Shuffle sort produces a uniformly random permutation.

Application. Shuffle columns in a spreadsheet.

← assuming real numbers are uniformly random (and no ties)

43

War story (Microsoft)

[Microsoft antitrust probe by EU](#). Microsoft agreed to provide a randomized ballot screen for users to select browser in Windows 7.

<http://www.browserchoice.eu>

Select your web browser(s)



A fast new browser from Google. Try it now!



Safari for Windows from Apple, the world's most innovative browser.



Your online security is Firefox's top priority. Firefox is free, and made to help you get the most out of the



The fastest browser on Earth. Secure, powerful and easy to use, with excellent privacy protection.



Designed to help you take control of your privacy and browse with confidence. Free from Microsoft.

↑ appeared last 50% of the time

44

War story (Microsoft)

Microsoft antitrust probe by EU. Microsoft agreed to provide a randomized ballot screen for users to select browser in Windows 7.

Solution? Implement shuffle sort by making comparator always return a random answer.

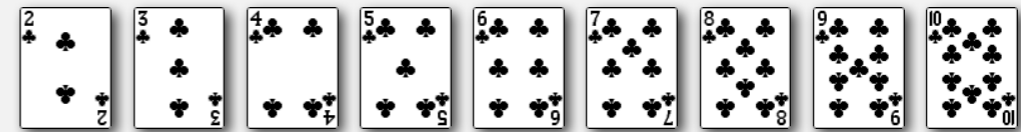
```
public int compareTo(Browser that)
{
    double r = Math.random();
    if (r < 0.5) return -1;
    if (r > 0.5) return +1;
    return 0;
}
```

← browser comparator
(should implement a total order)

45

Knuth shuffle demo

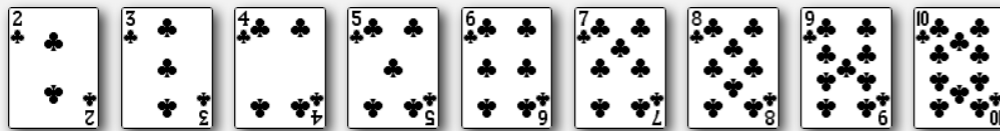
- In iteration i , pick integer r between 0 and i uniformly at random.
- Swap $a[i]$ and $a[r]$.



46

Knuth shuffle

- In iteration i , pick integer r between 0 and i uniformly at random.
- Swap $a[i]$ and $a[r]$.



Proposition. [Fisher-Yates 1938] Knuth shuffling algorithm produces a uniformly random permutation of the input array in linear time.

← assuming integers
uniformly at random

47

Knuth shuffle

- In iteration i , pick integer r between 0 and i uniformly at random.
- Swap $a[i]$ and $a[r]$.

← common bug: between 0 and $N - 1$
correct variant: between i and $N - 1$

```
public class Knuth
{
    public static void shuffle(Object[] a)
    {
        int N = a.length;
        for (int i = 0; i < N; i++)
        {
            int r = StdRandom.uniform(i + 1);
            exch(a, i, r);
        }
    }
}
```

← between 0 and i

<http://algs4.cs.princeton.edu/11model/Knuth.java.html>

48

Broken Knuth shuffle

Q. What happens if integer is chosen between 0 and N-1?

A. Not uniformly random!

instead of
between 0 and i

permutation	Knuth shuffle	broken shuffle
A B C	1/6	4/27
A C B	1/6	5/27
B A C	1/6	5/27
B C A	1/6	5/27
C A B	1/6	4/27
C B A	1/6	4/27

probability of each permutation when shuffling { A, B, C }

49

War story (online poker)

Texas hold'em poker. Software must shuffle electronic cards.



How We Learned to Cheat at Online Poker: A Study in Software Security

http://www.cigital.com/papers/download/developer_gambling.php

50

War story (online poker)

Shuffling algorithm in FAQ at www.planetpoker.com

```
for i := 1 to 52 do begin
  r := random(51) + 1; ← between 1 and 51
  swap := card[r];
  card[r] := card[i];
  card[i] := swap;
end;
```

Bug 1. Random number r never 52 \Rightarrow 52nd card can't end up in 52nd place.

Bug 2. Shuffle not uniform (should be between 1 and i).

Bug 3. random() uses 32-bit seed \Rightarrow 2³² possible shuffles.

Bug 4. Seed = milliseconds since midnight \Rightarrow 86.4 million shuffles.

"The generation of random numbers is too important to be left to chance."

— Robert R. Coveyou

51

War story (online poker)

Best practices for shuffling (if your business depends on it).

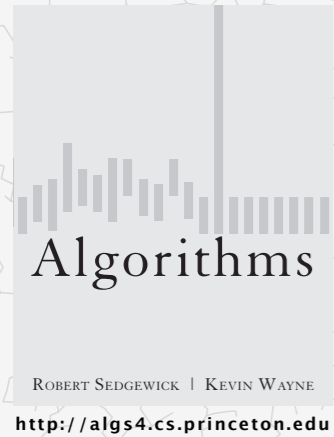
- Use a hardware random-number generator that has passed both the FIPS 140-2 and the NIST statistical test suites.
- Continuously monitor statistic properties: hardware random-number generators are fragile and fail silently.
- Use an unbiased shuffling algorithm.



RANDOM.ORG

Bottom line. Shuffling a deck of cards is hard!

52



2.1 ELEMENTARY SORTS

- ▶ rules of the game
- ▶ selection sort
- ▶ insertion sort
- ▶ shuffling
- ▶ comparators

Sort music library by artist

	Name	Artist	Time	Album
12	Let It Be	The Beatles	4:03	Let It Be
13	Take My Breath Away	BERLIN	4:13	Top Gun - Soundtrack
14	Circle Of Friends	Better Than Ezra	3:27	Empire Records
15	Dancing With Myself	Billy Idol	4:43	Don't Stop
16	Rebel Yell	Billy Idol	4:49	Rebel Yell
17	Piano Man	Billy Joel	5:36	Greatest Hits Vol. 1
18	Pressure	Billy Joel	3:16	Greatest Hits, Vol. II (1978 - 1985) (Disc 2)
19	The Longest Time	Billy Joel	3:36	Greatest Hits, Vol. II (1978 - 1985) (Disc 2)
20	Atomic	Blondie	3:50	Atomic: The Very Best Of Blondie
21	Sunday Girl	Blondie	3:15	Atomic: The Very Best Of Blondie
22	Call Me	Blondie	3:33	Atomic: The Very Best Of Blondie
23	Dreaming	Blondie	3:06	Atomic: The Very Best Of Blondie
24	Hurricane	Bob Dylan	8:32	Desire
25	The Times They Are A-Changin'	Bob Dylan	3:17	Greatest Hits
26	Livin' On A Prayer	Bon Jovi	4:11	Cross Road
27	Beds Of Roses	Bon Jovi	6:35	Cross Road
28	Runaway	Bon Jovi	3:53	Cross Road
29	Rasputin (Extended Mix)	Boney M	5:50	Greatest Hits
30	Have You Ever Seen The Rain	Bonnie Tyler	4:10	Faster Than The Speed Of Night
31	Total Eclipse Of The Heart	Bonnie Tyler	7:02	Faster Than The Speed Of Night
32	Straight From The Heart	Bonnie Tyler	3:41	Faster Than The Speed Of Night
33	Holding Out For A Hero	Bonny Tyler	5:49	Meat Loaf And Friends
34	Dancing In The Dark	Bruce Springsteen	4:05	Born In The U.S.A.
35	Thunder Road	Bruce Springsteen	4:51	Born To Run
36	Born To Run	Bruce Springsteen	4:30	Born To Run
37	Jungleland	Bruce Springsteen	9:34	Born To Run
38	Tunnel Tunnel Tunnel (To Eurobeat)	The Buds	3:57	Forest Gump: The Soundtrack (Disc 2)

Sort music library by song name

	Name	Artist	Time	Album
1	Alive	Pearl Jam	5:41	Ten
2	All Over The World	Pixies	5:27	Bossanova
3	All Through The Night	Cyndi Lauper	4:30	She's So Unusual
4	Allison Road	Gin Blossoms	3:19	New Miserable Experience
5	Amá, Amá, Amá Y Ensancha El...	Extremoduro	2:34	Deltoya (1992)
6	And We Danced	Hooters	3:50	Nervous Night
7	As I Lay Me Down	Sophie B. Hawkins	4:09	Whaler
8	Atomic	Blondie	3:50	Atomic: The Very Best Of Blondie
9	Automatic Lover	Jay-Jay Johanson	4:19	Antenna
10	Baba O'Riley	The Who	5:01	Who's Better, Who's Best
11	Beautiful Life	Ace Of Base	3:40	The Bridge
12	Beds Of Roses	Bon Jovi	6:35	Cross Road
13	Black	Pearl Jam	5:44	Ten
14	Bleed American	Jimmy Eat World	3:04	Bleed American
15	Borderline	Madonna	4:00	The Immaculate Collection
16	Born To Run	Bruce Springsteen	4:30	Born To Run
17	Both Sides Of The Story	Phil Collins	6:43	Both Sides
18	Bouncing Around The Room	Phish	4:09	A Live One (Disc 1)
19	Boys Don't Cry	The Cure	2:35	Staring At The Sea: The Singles 1979-1985
20	Brat	Green Day	1:43	Insomniac
21	Breakdown	Deerheart	3:40	Deerheart
22	Bring Me To Life (Kevin Roen Mix)	Evanescence Vs. Pa...	9:48	
23	Californication	Red Hot Chili Pepp...	1:40	
24	Call Me	Blondie	3:33	Atomic: The Very Best Of Blondie
25	Can't Get You Out Of My Head	Kylie Minogue	3:50	Fever
26	Celebration	Kool & The Gang	3:45	Time Life Music Sounds Of The Seventies - C...
27	Chasing Chasing	Sukhwinder Singh	5:11	Bambay Dreams

Comparable interface: review

Comparable interface: sort using a type's natural order.

```
public class Date implements Comparable<Date>
{
    private final int month, day, year;

    public Date(int m, int d, int y)
    {
        month = m;
        day = d;
        year = y;
    }
    ...
    public int compareTo(Date that)
    {
        if (this.year < that.year ) return -1;
        if (this.year > that.year ) return +1;
        if (this.month < that.month) return -1;
        if (this.month > that.month) return +1;
        if (this.day < that.day ) return -1;
        if (this.day > that.day ) return +1;
        return 0;
    }
}
```

natural order

Comparator interface

Comparator interface: sort using an **alternate order**.

```
public interface Comparator<Key>
{
    int compare(Key v, Key w)    compare keys v and w
}
```

Required property. Must be a **total order**.

string order	example
natural order	Now is the time
case insensitive	is Now the time <small>pre-1994 order for digraphs ch and ll and rr</small>
Spanish language	café cafetero cuarto churro nube ñoño
British phone book	McKinley Mackintosh

57

Comparator interface: system sort

To use with Java system sort:

- Create Comparator object.
- Pass as second argument to Arrays.sort().

```
String[] a;
...
Arrays.sort(a);
...
Arrays.sort(a, String.CASE_INSENSITIVE_ORDER);
...
Arrays.sort(a, Collator.getInstance(new Locale("es")));
...
Arrays.sort(a, new BritishPhoneBookOrder());
...
```

Diagram annotations:
- Arrow from `Arrays.sort(a);` to `String[] a;`: uses natural order
- Arrow from `Arrays.sort(a, String.CASE_INSENSITIVE_ORDER);` to `String.CASE_INSENSITIVE_ORDER`: uses alternate order defined by Comparator<String> object
- Arrow from `Arrays.sort(a, Collator.getInstance(new Locale("es")));` to `Collator.getInstance(new Locale("es"))`: uses alternate order defined by Comparator<String> object
- Arrow from `Arrays.sort(a, new BritishPhoneBookOrder());` to `new BritishPhoneBookOrder()`: uses alternate order defined by Comparator<String> object

Bottom line. Decouples the definition of the data type from the definition of what it means to compare two objects of that type.

58

Comparator interface: using with our sorting libraries

To support comparators in our sort implementations:

- Pass Comparator to both sort() and less(), and use it in less().
- Use Object instead of Comparable.

```
import java.util.Comparator;

public class Insertion
{
    ...

    public static void sort(Object[] a, Comparator comparator)
    {
        int N = a.length;
        for (int i = 0; i < N; i++)
            for (int j = i; j > 0 && less(comparator, a[j], a[j-1]); j--)
                exch(a, j, j-1);
    }

    private static boolean less(Comparator comparator, Object v, Object w)
    { return comparator.compare(v, w) < 0; }
}
```

<http://algs4.cs.princeton.edu/21elementary/Insertion.java.html>

59

Comparator interface: implementing

To implement a comparator:

- Define a (nested) class that implements the Comparator interface.
- Implement the compare() method.
- Provide client access to Comparator.

```
import java.util.Comparator;

public class Student
{
    private final String name;
    private final int section;
    ...

    public static Comparator<Student> byNameOrder()
    { return new ByNameOrder(); }

    private static class ByNameOrder implements Comparator<Student>
    {
        public int compare(Student v, Student w)
        { return v.name.compareTo(w.name); }
    }
    ...
}
```

Diagram annotation:
- Arrow from `byNameOrder()` to `ByNameOrder`: one Comparator for the class

60

Comparator interface: implementing

To implement a comparator:

- Define a (nested) class that implements the Comparator interface.
- Implement the compare() method.
- Provide client access to Comparator.

```
import java.util.Comparator;

public class Student
{
    private final String name;
    private final int section;
    ...

    public static Comparator<Student> bySectionOrder()
    { return new BySectionOrder(); }

    private static class BySectionOrder implements Comparator<Student>
    {
        public int compare(Student v, Student w)
        { return v.section - w.section; }
    }
    ...
}
```

this trick works here since no danger of overflow

61

Comparator interface: implementing

To implement a comparator:

- Define a (nested) class that implements the Comparator interface.
- Implement the compare() method.
- Provide client access to Comparator.

```
Arrays.sort(a, Student.byNameOrder());
```

Andrews	3	A	(664) 480-0023	097 Little
Battle	4	C	(874) 088-1212	121 Whitman
Chen	3	A	(991) 878-4944	308 Blair
Fox	3	A	(884) 232-5341	11 Dickinson
Furia	1	A	(766) 093-9873	101 Brown
Gazsi	4	B	(800) 867-5309	101 Brown
Kanaga	3	B	(898) 122-9643	22 Brown
Rohde	2	A	(232) 343-5555	343 Forbes

```
Arrays.sort(a, Student.bySectionOrder());
```

Furia	1	A	(766) 093-9873	101 Brown
Rohde	2	A	(232) 343-5555	343 Forbes
Andrews	3	A	(664) 480-0023	097 Little
Chen	3	A	(991) 878-4944	308 Blair
Fox	3	A	(884) 232-5341	11 Dickinson
Kanaga	3	B	(898) 122-9643	22 Brown
Battle	4	C	(874) 088-1212	121 Whitman
Gazsi	4	B	(800) 867-5309	101 Brown

62

Stability

A typical application. First, sort by name; then sort by section.

```
Selection.sort(a, Student.byNameOrder());
```

Andrews	3	A	(664) 480-0023	097 Little
Battle	4	C	(874) 088-1212	121 Whitman
Chen	3	A	(991) 878-4944	308 Blair
Fox	3	A	(884) 232-5341	11 Dickinson
Furia	1	A	(766) 093-9873	101 Brown
Gazsi	4	B	(800) 867-5309	101 Brown
Kanaga	3	B	(898) 122-9643	22 Brown
Rohde	2	A	(232) 343-5555	343 Forbes

```
Selection.sort(a, Student.bySectionOrder());
```

Furia	1	A	(766) 093-9873	101 Brown
Rohde	2	A	(232) 343-5555	343 Forbes
Chen	3	A	(991) 878-4944	308 Blair
Fox	3	A	(884) 232-5341	11 Dickinson
Andrews	3	A	(664) 480-0023	097 Little
Kanaga	3	B	(898) 122-9643	22 Brown
Gazsi	4	B	(800) 867-5309	101 Brown
Battle	4	C	(874) 088-1212	121 Whitman

@#%&@! Students in section 3 no longer sorted by name.

A **stable** sort preserves the relative order of items with equal keys.

63

Stability

Which sorting algorithms are stable?

- A. Selection sort.
- B. Insertion sort.
- C. Both A and B.
- D. Neither A nor B.
- E. *I don't know.*

64

Stability: insertion sort

Proposition. Insertion sort is **stable**.

```
public class Insertion
{
    public static void sort(Comparable[] a)
    {
        int N = a.length;
        for (int i = 0; i < N; i++)
            for (int j = i; j > 0 && less(a[j], a[j-1]); j--)
                exch(a, j, j-1);
    }
}
```

i	j	0	1	2	3	4
0	0	B ₁	A ₁	A ₂	A ₃	B ₂
1	0	A ₁	B ₁	A ₂	A ₃	B ₂
2	1	A ₁	A ₂	B ₁	A ₃	B ₂
3	2	A ₁	A ₂	A ₃	B ₁	B ₂
4	4	A ₁	A ₂	A ₃	B ₁	B ₂
		A ₁	A ₂	A ₃	B ₁	B ₂

Pf. Equal items never move past each other.

65

Stability: selection sort

Proposition. Selection sort is **not stable**.

```
public class Selection
{
    public static void sort(Comparable[] a)
    {
        int N = a.length;
        for (int i = 0; i < N; i++)
        {
            int min = i;
            for (int j = i+1; j < N; j++)
                if (less(a[j], a[min]))
                    min = j;
            exch(a, i, min);
        }
    }
}
```

i	min	0	1	2
0	2	B ₁	B ₂	A
1	1	A	B ₂	B ₁
2	2	A	B ₂	B ₁
		A	B ₂	B ₁

Pf by counterexample. Long-distance exchange can move one equal item past another one.

66