# Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE

1.3 BAGS, QUEUES, AND STACKS

- ‣ *stacks*
- ‣ *resizing arrays*
- ‣ *queues*
- ‣ *generics*
- ‣ *iterators*
- ‣ *applications*

Algorithms
FOURTH EDITION

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Stacks and queues

Fundamental data types.
- Value: collection of objects.
- Operations: insert, remove, iterate, test if empty.
- Intent is clear when we insert.
- Which item do we remove?
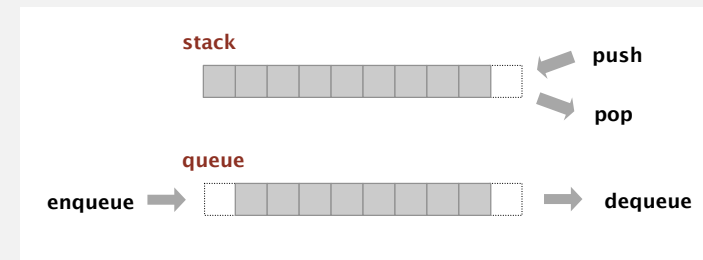
Stack. Examine the item most recently added. ⟵ LIFO = "last in first out"

Queue. Examine the item least recently added. ⟵ FIFO = "first in first out"

Client, implementation, interface

Separate interface and implementation.
Ex: stack, queue, bag, priority queue, symbol table, union-find, ...

Benefits.
- Client can't know details of implementation ⇒
  client has many implementation from which to choose.
- Implementation can't know details of client needs ⇒
  many clients can re-use the same implementation.
- Design: creates modular, reusable libraries.
- Performance: use optimized implementation where it matters.

Client: program using operations defined in interface.

Implementation: actual code implementing operations.

Interface: description of data type, basic operations.

1.3 BAGS, QUEUES, AND STACKS

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Stack API

Warmup API. Stack of strings data type.

push   pop

```
public class StackOfStrings

        StackOfStrings()        create an empty stack

   void push(String item)       insert a new string onto stack

 String pop()                   remove and return the string
                                most recently added

boolean isEmpty()              is the stack empty?

    int size()                  number of strings on the stack
```
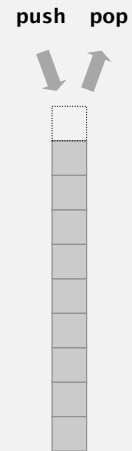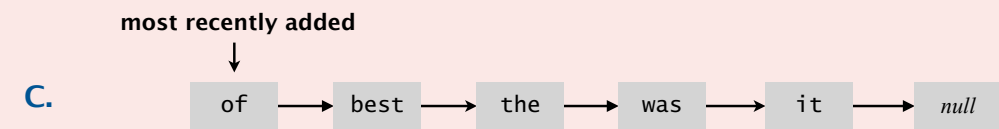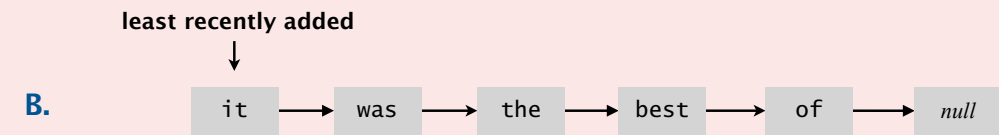
Warmup client. Reverse sequence of strings from standard input.

Stack quiz

How to implement a stack with a singly-linked list?

A.   Can't be done efficiently with a singly-linked list.

least recently added
↓

B.   it → was → the → best → of → null

most recently added
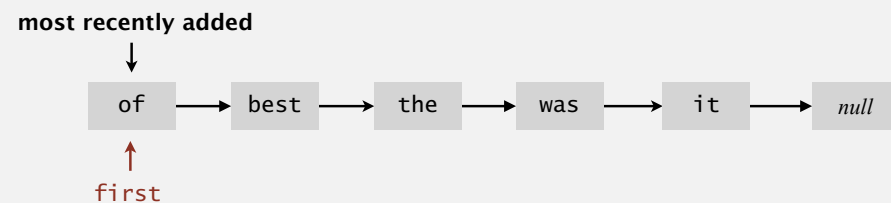↓

C.   of → best → the → was → it → null

D.   I don't know.

Stack: linked-list implementation

- Maintain pointer `first` to first node in a singly-linked list.
- Push new item before `first`.
- Pop item from `first`.

most recently added
↓

of → best → the → was → it → null
↑
first

Stack pop: linked-list implementation

save item to return

```
String item = first.item;
```

delete first node

```
first = first.next;
```

inner class

```
private class Node
{
    String item;
    Node next;
}
```

first → or → be → to null

first → be → to null

return saved item

```
return item;
```
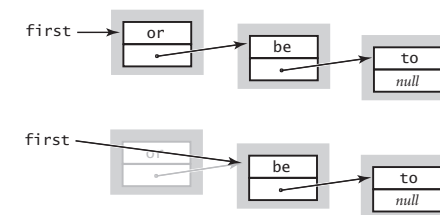
Stack push: linked-list implementation
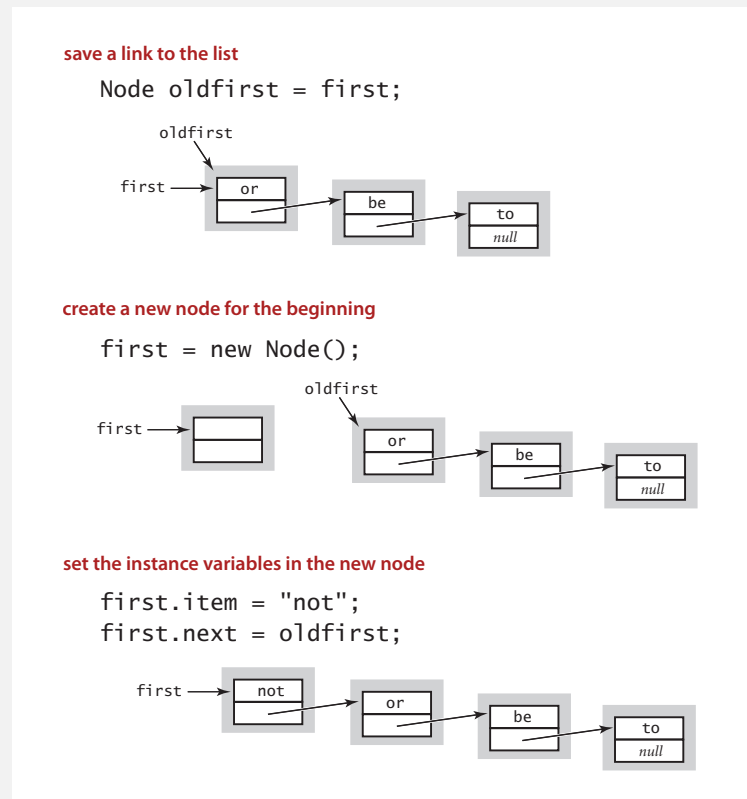
save a link to the list

```
Node oldfirst = first;
```


inner class

```
private class Node
{
    String item;
    Node next;
}
```

create a new node for the beginning

```
first = new Node();
```


set the instance variables in the new node

```
first.item = "not";
first.next = oldfirst;
```

Stack: linked-list implementation in Java

```java
public class LinkedStackOfStrings
{
    private Node first = null;

    private class Node
    {
        String item;
        Node next;
    }

    public boolean isEmpty()
    {  return first == null;  }

    public void push(String item)
    {
        Node oldfirst = first;
        first = new Node();
        first.item = item;
        first.next = oldfirst;
    }

    public String pop()
    {
        String item = first.item;
        first = first.next;
        return item;
    }
}
```

private inner class
(access modifiers for instance
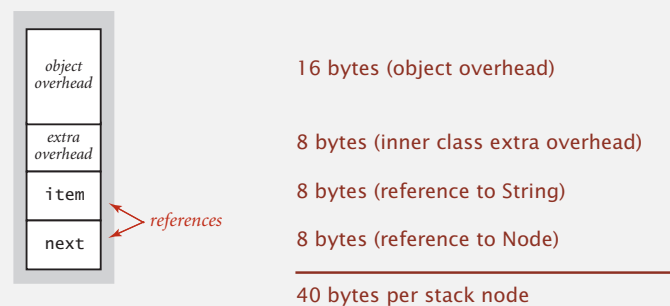variables don't matter)

Stack: linked-list implementation performance

Proposition.   Every operation takes constant time in the worst case.

Proposition. A stack with N items uses $\sim 40\,N$ bytes.

inner class

```
private class Node
{
    String item;
    Node next;
}
```


16 bytes (object overhead)

8 bytes (inner class extra overhead)

8 bytes (reference to String)

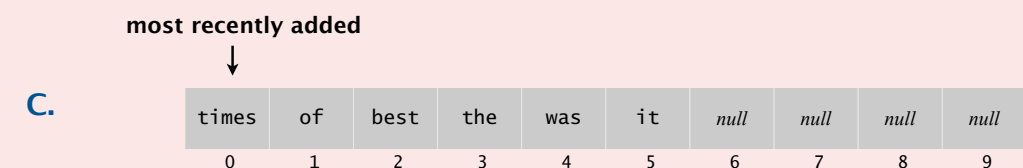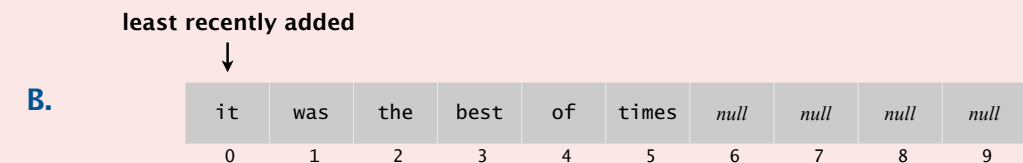8 bytes (reference to Node)

_____

40 bytes per stack node

Remark. This accounts for the memory for the stack
(but not the memory for strings themselves, which the client owns).

Stack quiz

How to implement a fixed-capacity stack with an array?

A.   Can't be done efficiently with an array.

least recently added
↓

B.

it	was	the	best	of	times	null	null	null	null
0	1	2	3	4	5	6	7	8	9

most recently added
↓

C.

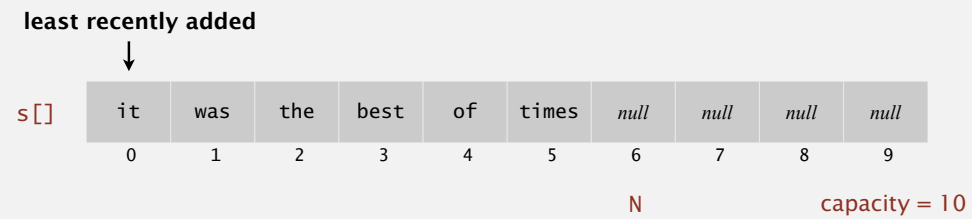| times	of	best	the	was	it	null	null	null	null
0	1	2	3	4	5	6	7	8	9

D.   I don't know.

Fixed-capacity stack: array implementation

- Use array `s[]` to store `N` items on stack.
- `push()`: add new item at `s[N]`.
- `pop()`: remove item from `s[N-1]`.

least recently added

s[]	it	was	the	best	of	times	*null*	*null*	*null*	*null*
	0	1	2	3	4	5	6	7	8	9

N   capacity = 10

Defect. Stack overflows when `N` exceeds capacity. [stay tuned]

Fixed-capacity stack: array implementation

```
public class FixedCapacityStackOfStrings
{
   private String[] s;                          a cheat
   private int N = 0;                          (stay tuned)

   public FixedCapacityStackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

use to index into array;
then increment N

decrement N;
then use to index into array

Stack considerations

Overflow and underflow.
- Underflow: throw exception if pop from an empty stack.
- Overflow: use resizing array for array implementation. [stay tuned]

Null items. We allow null items to be inserted.

Loitering. Holding a reference to an object when it is no longer needed.

```
public String pop()
{  return s[--N];  }
```

loitering

```
public String pop()
{
    String item = s[--N];
    s[N] = null;
    return item;
}
```

this version avoids "loitering":
garbage collector can reclaim memory for
an object only if no outstanding references

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1.3 BAGS, QUEUES, AND STACKS

‣ stacks
‣ **resizing arrays**
‣ queues
‣ generics
‣ iterators
‣ applications

Stack: resizing-array implementation

Problem. Requiring client to provide capacity does not implement API!

Q. How to grow and shrink array?

First try.
- push(): increase size of array s[] by 1.
- pop(): decrease size of array s[] by 1.

Too expensive.   *infeasible for large N*
- Need to copy all items to a new array, for each operation.
- Array accesses to insert first N items $= N + (2 + 4 + \ldots + 2(N-1)) \sim N^2$.

  1 array access per push   *2(k–1) array accesses to expand to size k (ignoring cost to create new array)*

Challenge. Ensure that array resizing happens infrequently.

Stack: resizing-array implementation

Q. How to grow array?   *"repeated doubling"*

A. If array is full, create a new array of twice the size, and copy items.

```
public ResizingArrayStackOfStrings()
{  s = new String[1];  }

public void push(String item)
{
   if (N == s.length) resize(2 * s.length);
   s[N++] = item;
}

private void resize(int capacity)
{
   String[] copy = new String[capacity];
   for (int i = 0; i < N; i++)
      copy[i] = s[i];
   s = copy;
}
```

Array accesses to insert first N = 2^i items. $N + (2 + 4 + 8 + \ldots + N) \sim 3N.$

1 array access per push   *k array accesses to double to size k (ignoring cost to create new array)*

Stack: resizing-array implementation

Q. How to shrink array?

First try.
- push(): double size of array s[] when array is full.
- pop(): halve size of array s[] when array is one-half full.

Too expensive in worst case.
- Consider push-pop-push-pop-... sequence when array is full.
- Each operation takes time proportional to N.

N = 5	to	be	or	not	to	*null*	*null*	*null*

N = 4	to	be	or	not

N = 5	to	be	or	not	to	*null*	*null*	*null*

N = 4	to	be	or	not

Stack: resizing-array implementation

Q. How to shrink array?

Efficient solution.
- push(): double size of array s[] when array is full.
- pop(): halve size of array s[] when array is one-quarter full.

```
public String pop()
{
   String item = s[--N];
   s[N] = null;
   if (N > 0 && N == s.length/4) resize(s.length/2);
   return item;
}
```

Invariant. Array is between 25% and 100% full.

Stack resizing-array implementation: performance

Amortized analysis. Starting from an empty data structure, average running time per operation over a worst-case sequence of operations.

Proposition. Starting from an empty stack, any sequence of M push and pop operations takes time proportional to M.

	best	worst	amortized
construct	1	1	1
push	1	N	1
pop	1	N	1
size	1	1	1

doubling and halving operations

**order of growth of running time
for resizing array stack with N items**

Stack resizing-array implementation: memory usage

Proposition. Uses between $\sim 8\,N$ and $\sim 32\,N$ bytes to represent a stack with N items.
 • $\sim 8\,N$ when full.
 • $\sim 32\,N$ when one-quarter full.

```
public class ResizingArrayStackOfStrings
{
    private String[] s;        ← 8 bytes × array size
    private int N = 0;
    …
}
```

Remark. This accounts for the memory for the stack
(but not the memory for strings themselves, which the client owns).

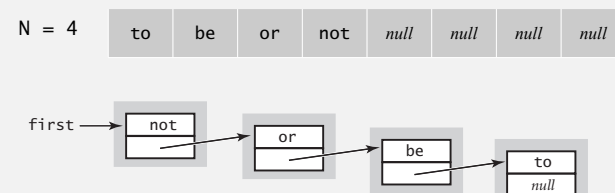## Stack implementations: resizing array vs. linked list

Tradeoffs. Can implement a stack with either resizing array or linked list; client can use interchangeably. Which one is better?

Linked-list implementation.
 • Every operation takes constant time in the worst case.
 • Uses extra time and space to deal with the links.

Resizing-array implementation.
 • Every operation takes constant amortized time.
 • Less wasted space.

N = 4 | to | be | or | not | null | null | null | null |

first → not → or → be → to → null

1.3 BAGS, QUEUES, AND STACKS

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enqueue

```
public class QueueOfStrings

            QueueOfStrings()        create an empty queue

     void   enqueue(String item)    insert a new string onto queue

   String   dequeue()               remove and return the string
                                     least recently added

  boolean   isEmpty()               is the queue empty?

      int   size()                  number of strings on the queue
```
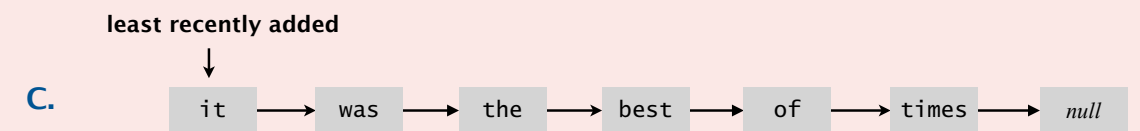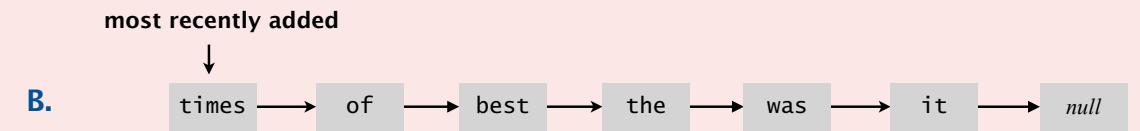
dequeue
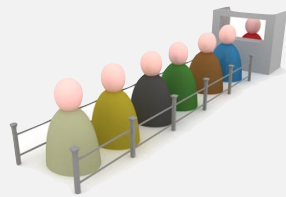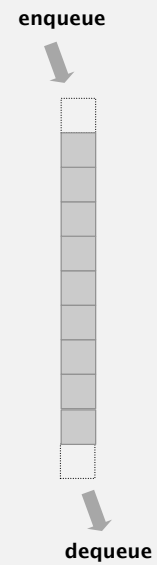
How to implement a queue with a linked list?

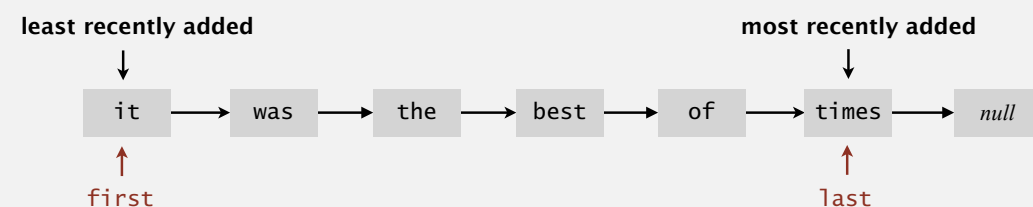**A.** Can't be done efficiently with a singly-linked list.

most recently added
↓

B. times ⟶ of ⟶ best ⟶ the ⟶ was ⟶ it ⟶ *null*

least recently added
↓

C. it ⟶ was ⟶ the ⟶ best ⟶ of ⟶ times ⟶ *null*

D. I don't know.

- Maintain one pointer `first` to first node in a singly-linked list.
- Maintain another pointer `last` to last node.
- Dequeue from `first`.
- Enqueue after `last`.

least recently added
↓

most recently added
↓

it ⟶ was ⟶ the ⟶ best ⟶ of ⟶ times ⟶ *null*

↑   ↑
first   last

---

save item to return

```
String item = first.item;
```

delete first node

```
first = first.next;
```

inner class

```
private class Node
{
    String item;
    Node next;
}
```

first ⟶ to ⟶ be ⟶ or / null   last

first ⟶ be ⟶ or / null   last
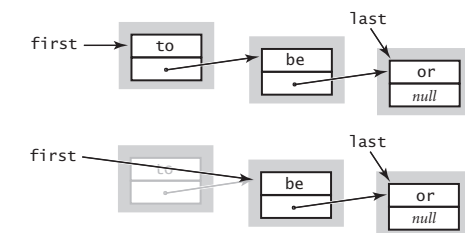
return saved item

```
return item;
```

Remark. Identical code to linked-list stack `pop()`.

Queue enqueue: linked-list implementation

save a link to the last node

```
Node oldlast = last;
```


inner class

```
private class Node
{
    String item;
    Node next;
}
```
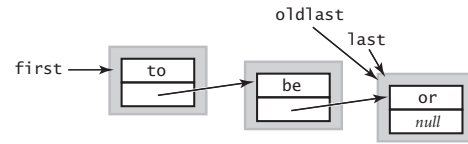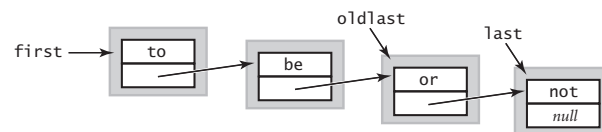
create a new node for the end

```
last = new Node();
last.item = "not";
```
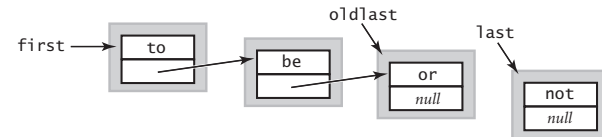

link the new node to the end of the list

```
oldlast.next = last;
```

Queue: linked-list implementation in Java

```java
public class LinkedQueueOfStrings
{
    private Node first, last;

    private class Node
    { /* same as in LinkedStackOfStrings */ }

    public boolean isEmpty()
    { return first == null;  }

    public void enqueue(String item)
    {
        Node oldlast = last;
        last = new Node();
        last.item = item;
        last.next = null;
        if (isEmpty()) first = last;        ← special cases for
        else           oldlast.next = last;    empty queue
    }

    public String dequeue()
    {
        String item = first.item;
        first      = first.next;
        if (isEmpty()) last = null;
        return item;
    }
}
```

Queue quiz 2

How to implement a fixed-capacity queue with an array?

A.   Can't be done efficiently with an array.

least recently added
↓

B.

it	was	the	best	of	times	*null*	*null*	*null*	*null*
0	1	2	3	4	5	6	7	8	9

most recently added
↓

C.

times	of	best	the	was	it	*null*	*null*	*null*	*null*
0	1	2	3	4	5	6	7	8	9

D.   I don't know.

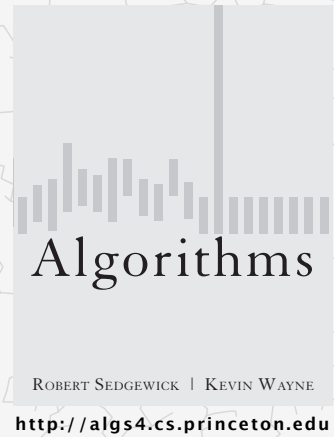## Queue: resizing-array implementation

- Use array `q[]` to store items in queue.
- `enqueue()`: add new item at `q[tail]`.
- `dequeue()`: remove item from `q[head]`.
- Update `head` and `tail` modulo the `capacity`.
- Add resizing array.

	least recently added		**most recently added**							
			↓		↓					

q[]	*null*	*null*	the	best	of	times	*null*	*null*	*null*	*null*
	0	1	2	3	4	5	6	7	8	9

head   tail   capacity = 10

Q. How to resize?

1.3 BAGS, QUEUES, AND STACKS

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▸ stacks
▸ resizing arrays
▸ queues
▸ **generics**
▸ iterators
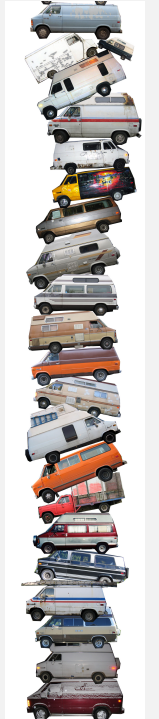▸ applications

---

Parameterized stack

We implemented: `StackOfStrings`.

We also want: `StackOfURLs`, `StackOfInts`, `StackOfVans`, ...

Attempt 1. Implement a separate stack class for each type.
- Rewriting code is tedious and error-prone.
- Maintaining cut-and-pasted code is tedious and error-prone.

@#$*! most reasonable approach until Java 1.5.

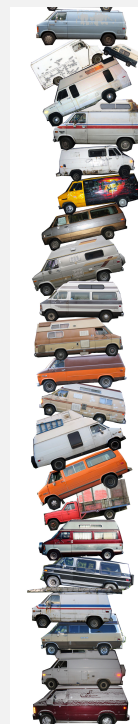---

Parameterized stack

We implemented: `StackOfStrings`.

We also want: `StackOfURLs`, `StackOfInts`, `StackOfVans`, ...

Attempt 2. Implement a stack with items of type `Object`.
- Casting is required in client.
- Casting is error-prone: run-time error if types mismatch.

```
StackOfObjects s = new StackOfObjects();
Apple  a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
a = (Apple) (s.pop());
```

run-time error

Parameterized stack

We implemented: `StackOfStrings`.

We also want: `StackOfURLs`, `StackOfInts`, `StackOfVans`, ...

Attempt 3. Java generics.
- Avoid casting in client.
- Discover type mismatch errors at compile-time instead of run-time.

type parameter

```
Stack<Apple> s = new Stack<Apple>();
Apple  a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
a = s.pop();
```

compile-time error

Guiding principles. Welcome compile-time errors; avoid run-time errors.

Slide 37

Generic stack: linked-list implementation

```
public class LinkedStackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

```
public class Stack<Item>
{
   private Node first = null;

   private class Node
   {
      Item item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(Item item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public Item pop()
   {
      Item item = first.item;
      first = first.next;
      return item;
   }
}
```

generic type name

Slide 38

Generic stack: array implementation

the way it should be

```
public class FixedCapacityStackOfStrings
{
   private String[] s;
   private int N = 0;

   public ..StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

```
public class FixedCapacityStack<Item>
{
   private Item[] s;
   private int N = 0;

   public FixedCapacityStack(int capacity)
   {  s = new Item[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(Item item)
   {  s[N++] = item;  }

   public Item pop()
   {  return s[--N];  }
}
```

@#$*! generic array creation not allowed in Java

Slide 39

Generic stack: array implementation

the way it is

```
public class FixedCapacityStackOfStrings
{
   private String[] s;
   private int N = 0;

   public ..StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

```
public class FixedCapacityStack<Item>
{
   private Item[] s;
   private int N = 0;

   public FixedCapacityStack(int capacity)
   {  s = (Item[]) new Object[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(Item item)
   {  s[N++] = item;  }

   public Item pop()
   {  return s[--N];  }
}
```

the ugly cast

Slide 40

Unchecked cast

```
% javac FixedCapacityStack.java
Note: FixedCapacityStack.java uses unchecked or unsafe operations.
Note: Recompile with -Xlint:unchecked for details.

% javac -Xlint:unchecked FixedCapacityStack.java
FixedCapacityStack.java:26: warning: [unchecked] unchecked cast
found    : java.lang.Object[]
required: Item[]
        a = (Item[]) new Object[capacity];
                     ^
1 warning
```

Q. Why does Java make me cast (or use reflection)?

Short answer. Backward compatibility.

Long answer. Need to learn about type erasure and covariant arrays.

BAD DESIGN!

Generic data types: autoboxing

Q. What to do about primitive types?

Wrapper type.
- Each primitive type has a wrapper object type.
- Ex: `Integer` is wrapper type for `int`.

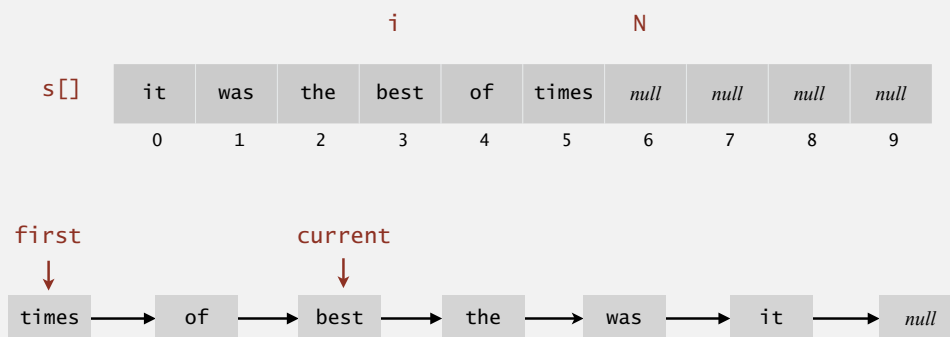Autoboxing. Automatic cast between a primitive type and its wrapper.

```
Stack<Integer> s = new Stack<Integer>();
s.push(17);        // s.push(Integer.valueOf(17));
int a = s.pop();   // int a = s.pop().intValue();
```

Bottom line. Client code can use generic stack for any type of data.

1.3 BAGS, QUEUES, AND STACKS

Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE

http://algs4.cs.princeton.edu

- ▸ *stacks*
- ▸ *resizing arrays*
- ▸ *queues*
- ▸ *generics*
- ▸ *iterators*
- ▸ *applications*

Iteration

Design challenge. Support iteration over stack items by client, without revealing the internal representation of the stack.

	i			N					
s[]	it	was	the	best	of	times	*null*	*null*	*null*	*null*
	0	1	2	3	4	5	6	7	8	9

first → times → of → best → the → was → it → *null*

current → best

Java solution. Make stack implement the `java.lang.Iterable` interface.

Iterators

Q. What is an `Iterable` ?
A. Has a method that returns an `Iterator`.

java.lang.Iterable interface

```
public interface Iterable<Item>
{
    Iterator<Item> iterator();
}
```

Q. What is an `Iterator` ?
A. Has methods `hasNext()` and `next()`.

java.util.Iterator interface

```
public interface Iterator<Item>
{
    boolean hasNext();
    Item next();
    void remove();   ← optional; use
                       at your own risk
}
```

Q. Why make data structures `Iterable` ?
A. Java supports elegant client code.

"foreach" statement (shorthand)

```
for (String s : stack)
    StdOut.println(s);
```

equivalent code (longhand)

```
Iterator<String> i = stack.iterator();
while (i.hasNext())
{
    String s = i.next();
    StdOut.println(s);
}
```

Stack iterator: linked-list implementation

```java
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item>
{
    ...

    public Iterator<Item> iterator() { return new ListIterator(); }

    private class ListIterator implements Iterator<Item>
    {
        private Node current = first;

        public boolean hasNext() {  return current != null;  }
        public void remove()    {  /* not supported */      }
        public Item next()
        {
            Item item = current.item;
            current    = current.next;
            return item;
        }
    }
}
```

throw UnsupportedOperationException

throw NoSuchElementException
if no more items in iteration

first

current

times → of → best → the → was → it → *null*

Stack iterator: array implementation
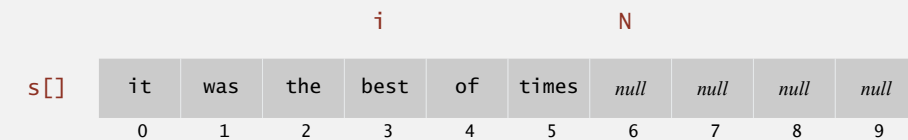
```java
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item>
{
    …

    public Iterator<Item> iterator()
    { return new ReverseArrayIterator(); }

    private class ReverseArrayIterator implements Iterator<Item>
    {
        private int i = N;

        public boolean hasNext() {  return i > 0;        }
        public void remove()     {  /* not supported */  }
        public Item next()       {  return s[--i];       }
    }
}
```

	i				N				

s[]	it	was	the	best	of	times	*null*	*null*	*null*	*null*
	0	1	2	3	4	5	6	7	8	9

Iteration: concurrent modification

Q. What if client modifies the data structure while iterating?

A. A fail-fast iterator throws a java.util.ConcurrentModificationException.

concurrent modification

```java
for (String s : stack)
    stack.push(s);
```

Q. How to detect concurrent modification?

A.

- Count total number of push() and pop() operations in Stack.
- Save counts in *Iterator subclass upon creation.
- If, when calling either next() or hasNext(), the current counts do not equal the saved counts, throw exception.

Algorithms

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1.3 BAGS, QUEUES, AND STACKS

‣ stacks
‣ resizing arrays
‣ queues
‣ generics
‣ iterators
‣ *applications*

Java collections library

List interface. `java.util.List` is API for an sequence of items.

```
public interface List<Item> extends Iterable<Item>

                List()                create an empty list
      boolean   isEmpty()             is the list empty?
          int   size()                number of items
         void   add(Item item)        append item to the end
         Item   get(int index)        return item at given index
         Item   remove(int index)     return and delete item at given index
      boolean   contains(Item item)   does the list contain the given item?
Iterator<Item>  iterator()            iterator over all items in the list

                ...
```

Implementations. `java.util.ArrayList` uses resizing array;
`java.util.LinkedList` uses linked list. ← caveat: only some operations are efficient

---

Java collections library

`java.util.Stack`.
- Supports push(), pop(), and iteration.
- Extends `java.util.Vector`, which implements `java.util.List` interface from previous slide, including `get()` and `remove()`.
- Bloated and poorly-designed API (why?)

Java 1.3 bug report (June 27, 2001)

```
The iterator method on java.util.Stack iterates through a Stack from
the bottom up. One would think that it should iterate as if it were
popping off the top of the Stack.
```

status (closed, will not fix)

```
It was an incorrect design decision to have Stack extend Vector ("is-a"
rather than "has-a"). We sympathize with the submitter but cannot fix
this because of compatibility.
```

Java collections library

`java.util.Stack`.
- Supports push(), pop(), and iteration.
- Extends `java.util.Vector`, which implements `java.util.List` interface from previous slide, including `get()` and `remove()`.
- Bloated and poorly-designed API (why?)

`java.util.Queue.` An interface, not an implementation of a queue.

Best practices. Use our implementations of Stack, Queue, and Bag.

War story (from Assignment 1)

Generate random open sites in an N-by-N percolation system.
- Jenny: pick (i, j) at random; if already open, repeat. Takes $\sim c_1 N^2$ seconds.
- Kenny: create a `java.util.ArrayList` of N^2 closed sites. Pick an index at random and delete. Takes $\sim c_2 N^4$ seconds.
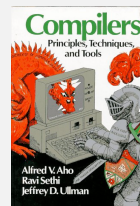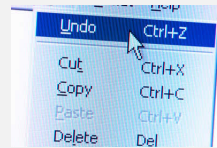

Why is my program so slow?
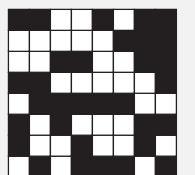
Kenny

Lesson. Don't use a library until you understand its API!

This course. Can't use a library until we've implemented it in class.

Stack applications

- Parsing in a compiler.
- Java virtual machine.
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
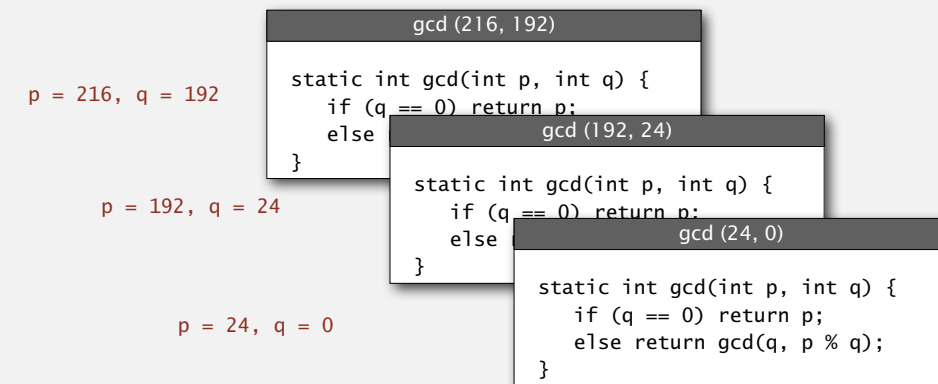- Implementing function calls in a compiler.
- ...

Function calls

How a compiler implements a function.
- Function call: push local environment and return address.
- Return: pop return address and local environment.

Recursive function. Function that calls itself.

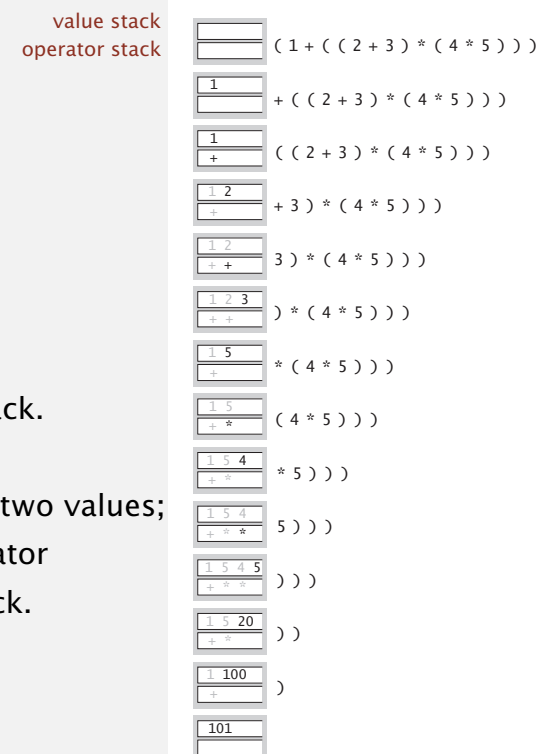Note. Can always use an explicit stack to remove recursion.

p = 216, q = 192

p = 192, q = 24

p = 24, q = 0

```
gcd (216, 192)
static int gcd(int p, int q) {
    if (q == 0) return p;
    else
}
```
```
gcd (192, 24)
static int gcd(int p, int q) {
    if (q == 0) return p;
    else
}
```
```
gcd (24, 0)
static int gcd(int p, int q) {
    if (q == 0) return p;
    else return gcd(q, p % q);
}
```

Arithmetic expression evaluation

Goal. Evaluate infix expressions.

```
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
```

operand   operator

Two-stack algorithm. [E. W. Dijkstra]
- Value: push onto the value stack.
- Operator: push onto the operator stack.
- Left parenthesis: ignore.
- Right parenthesis: pop operator and two values;
  push the result of applying that operator
  to those values onto the operand stack.

Context. An interpreter!

value stack
operator stack

(1 + ((2 + 3) * (4 * 5)))
+ ((2 + 3) * (4 * 5)))
((2 + 3) * (4 * 5)))
+ 3) * (4 * 5)))
3) * (4 * 5)))
) * (4 * 5)))
* (4 * 5)))
(4 * 5)))
* 5)))
5)))
)))
))
)

Dijkstra's two-stack algorithm demo

value stack   operator stack

infix expression
(fully parenthesized)

| (| 1 | + | (| (| 2 | + | 3 |) | * | (| 4 | * | 5 |) |) |) |

operand   operator
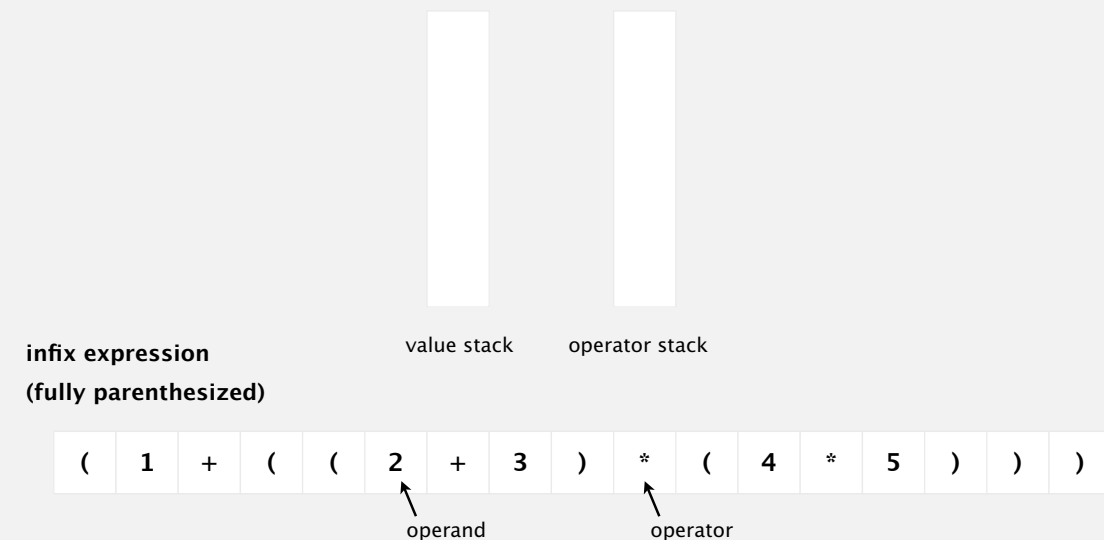
Arithmetic expression evaluation

```
public class Evaluate
{
    public static void main(String[] args)
    {
        Stack<String> ops  = new Stack<String>();
        Stack<Double> vals = new Stack<Double>();
        while (!StdIn.isEmpty()) {
            String s = StdIn.readString();
            if      (s.equals("("))                ;
            else if (s.equals("+"))    ops.push(s);
            else if (s.equals("*"))    ops.push(s);
            else if (s.equals(")"))
            {
                String op = ops.pop();
                if      (op.equals("+")) vals.push(vals.pop() + vals.pop());
                else if (op.equals("*")) vals.push(vals.pop() * vals.pop());
            }
            else vals.push(Double.parseDouble(s));
        }
        StdOut.println(vals.pop());
    }
}
```

```
% java Evaluate
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
101.0
```

Correctness

Q. Why correct?

A. When algorithm encounters an operator surrounded by two values within parentheses, it leaves the result on the value stack.

$$(\ 1 \ + \ (\ (\ 2 \ + \ 3 \ ) \ * \ (\ 4 \ * \ 5 \ ) \ ) \ )$$

as if the original input were:

$$(\ 1 \ + \ (\ 5 \ * \ (\ 4 \ * \ 5 \ ) \ ) \ )$$

Repeating the argument:

```
( 1 + ( 5 * 20 ) )
( 1 + 100 )
101
```

Extensions. More ops, precedence order, associativity.

Stack-based programming languages

Observation 1. Dijkstra's two-stack algorithm computes the same value if the operator occurs after the two values.

$$(\ 1 \ (\ (\ 2 \ 3 \ + \ ) \ (\ 4 \ 5 \ * \ ) \ * \ ) \ + \ )$$

Observation 2. All of the parentheses are redundant!

$$1 \ 2 \ 3 \ + \ 4 \ 5 \ * \ * \ +$$

Jan Lukasiewicz

Bottom line. Postfix or "reverse Polish" notation.

Applications. Postscript, Forth, calculators, Java virtual machine, ...