COS126 Number Systems Activity — Booksite 5.1

Instead of "ones, tens, hundreds, ..." places, binary has "ones, twos, fours, eights, ..." places.

Base	Digits	#digits	"1000" in this base converted to decimal	"205" in this base converted to decimal
decimal	$0, 1, 2 \ldots, 8, 9$	10	$10^3 = 1000$	$2 \times 10^2 + 0 \times 10^1 + 5 \times 10^0 = 205$
binary	$0, 1$	2	$2^3 = 8$	n/a
hexadecimal	$0, \ldots, 9, A, \ldots, F$	16	$16^3 = 4096$	$2 \times 16^2 + 0 \times 16^1 + 5 \times 16^0$ $= 2 \times 256 + 0 + 5 = 517$ dec.
octal	$0, 1, 2, 3, 4, 5, 6, 7$		$8^3 = 512$	$2 \times 8^2 + 0 \times 8^1 + 5 \times 8^0$ $= 2 \times 64 + 0 \times 8 + 5 \times 1 = 133$ dec.

1. What is the binary integer 101, represented in decimal? $4 + 1 =$ **5**

2. What is the binary integer 1010, represented in decimal? $8 + 2 =$ **10**.
(How is this related to the previous answer?) Twice as much as 101

3. What is the binary integer 10100, represented in decimal? **20**.
(What is the pattern?) Again twice as much since all ones became twice as valuable

4. What is the binary integer 101001, represented in decimal? **41**. Twice as much plus one.
(Could you write a program to use this approach?) Yes, and it is useful in LFSR!

5. What is the decimal integer 116, represented in binary? Use either of two common approaches:

   - Work right to left; start by determining the rightmost bit.

   - Work left to right; start by determining how many bits this binary number will have.

   Right to left: see "Converting from decimal to base b" on booksite §5.1. 116 is even, so ends in a 0, preceded by representation of $116/2 = 58$. 58 even so it ends in a 0, etc. $\Rightarrow$ **1110100**
   Left to right: biggest power of 2 that fits (≤ 116) is 64, leaving 116-64 = 52. Biggest power of 2 in this remainder is 32. Keep going with remainders, 116=64+32+16+4 = binary **1110100**.

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6. What are the hexadecimal numbers C, D, and E, expressed in binary? These are twelve, thirteen, fourteen, which are **1100, 1101, 1110**.

7. Express the hexadecimal number C0DE as a sum of 4 terms corresponding to the 4 digits. What is the value of this expression when converted to binary? Note that $16 = 2^4, 16^3 = 2^{12}$ and $\times 2$ shifts us left by one position. C0DE is $12 \times 16^3 + 0 \times 16^2 + 13 \times 16^1 + 14 \times 16^0 = 12 \times 2^{12} + 13 \times 2^4 + 14$ $= 1100\ 0000\ 0000\ 0000 + 1101\ 0000 + 1110 =$ **1100 0000 1101 1110** $(C\ 0\ D\ E)$

8. What is the binary number 100100110, represented in hexadecimal? (Avoid using decimal.) Reverse the previous process. 1 0010 0110 and converting each 4 bits to a hex digit, **126**

9. Optional: what is the value of DEE+24 in hexadecimal? (Avoid using decimal.) **E12**, use long addition working right to left

Bitwise Operators (In Q10 thru Q14, all numbers are in binary)

10. What is the binary value of 1010 | 110? **1110**

11. What is the binary value of 1010 & 110? **10**

12. What is the binary value of 1010 << 10? **101000**

13. What is the binary value of 1010 >> 10? **10**

14. What is the binary value of 1010 ∧ 110? **1100**

15. What is the value, expressed in hexadecimal, of C05126 ∧ CBE245 ∧ C05126? (What is the trick?) Since the order of inputs to xor doesn't matter, this equals CBE245 ∧ C05126 ∧ C05126. Since anything xor'ed with itself is 0, this is CBE245 ∧ 0 = **CBE245**

16-bit Two's-Complement Representations

16. What is the complement of 0101 0000 1100 1111? **1010 1111 0011 0000**

17. Give the **16-bit two's-complement** binary representation of the decimal integer 116 (Use question 5) **0000 0000 0111 0100**

18. Give the 16-bit two's-complement binary representation of the decimal integer −116 First complement the bits of +116, then add one, giving **1111 1111 1000 1100**

19. What is the 16-bit two's-complement **hexadecimal** representation of the decimal integer −116? Like Q8 (converting each 4 bits to a hex digit) **FF8C**

20. What is the decimal representation of the 16-bit two's-complement hexadecimal number FFFE? Since the first bit is 1, this number is negative. Call this negative number X. Then the binary representation of the positive number $-X$ is obtained by flipping bits (0000 0000 0000 0001) and adding one (0000 0000 0000 0010). So $-X$ is 2, i.e. X is $-\mathbf{2}$.

Challenges (Read Booksite §5.1)

21. What should the binary numbers 0.1 and 0.01 represent? In decimal these are 10^{-1} and 10^{-2}. In binary these are likewise $2^{-1} = 1/2$ and $2^{-2} = 1/4$

22. What are the powers of nine in octal? What are the powers of seventeen in hexadecimal?

23. Booksite exercises 5.1.18, 5.1.23, 5.1.25, Booksite creative exercises 5.1.6, 5.1.29