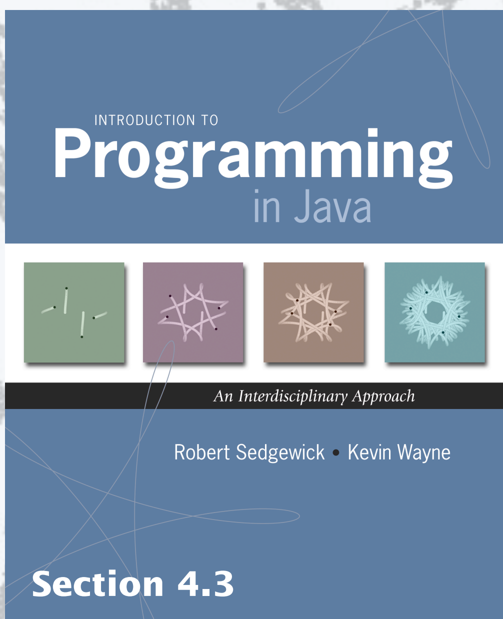


http://introcs.cs.princeton.edu

14. Stacks and Queues

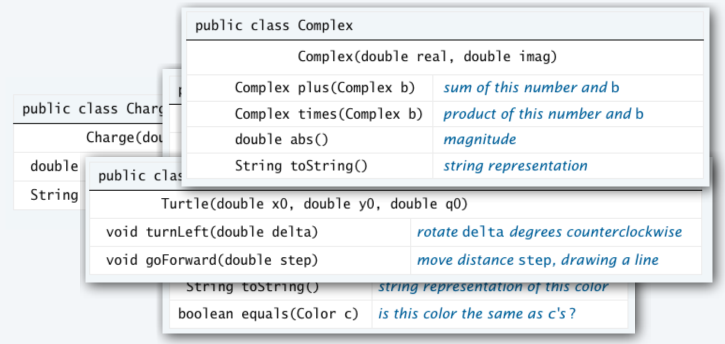
14. Stacks and Queues

- **APIs**
- Clients
- Strawman implementation
- Linked lists
- Implementations

Data types and data structures

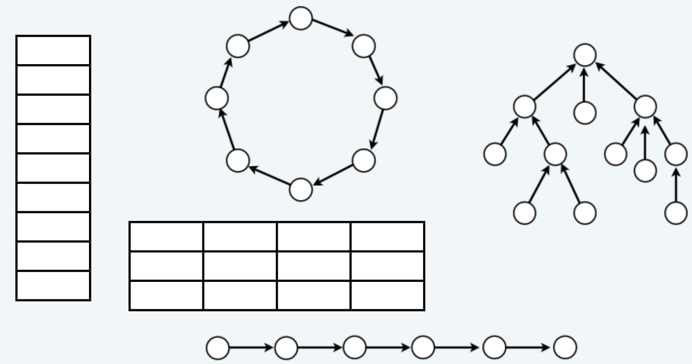
Data types

- Set of values.
- Set of operations on those values.
- Some are built in to Java: `int`, `double`, `String`, . . .
- Most are not: `Complex`, `Picture`, `Charge`, . . .



Data structures

- Represent data.
- Represent relationships among data.
- Some are built in to Java: 1D arrays, 2D arrays, . . .
- Most are not: linked list, circular list, tree, . . .



Design challenge for every data type: Which data structure to use?

- Resource 1: How much memory is needed?
- Resource 2: How much time do data-type methods use?

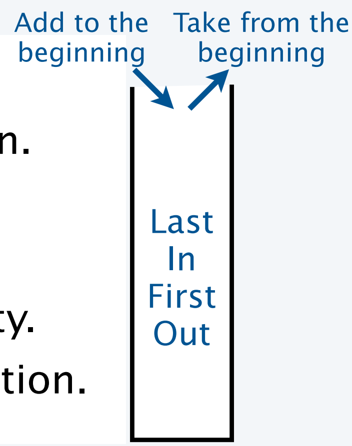
Stack and Queue APIs

A collection is an ADT whose values are a multiset of items, all of the same type.

Two fundamental collection ADTs differ in just a detail of the specification of their operations.

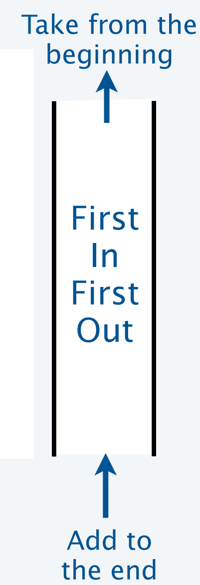
Stack operations

- Add an item to the collection.
- Remove and return the item *most* recently added (LIFO).
- Test if the collection is empty.
- Return the size of the collection.



Queue operations

- Add an item to the collection.
- Remove and return the item *least* recently added (FIFO).
- Test if the collection is empty.
- Return the size of the collection.



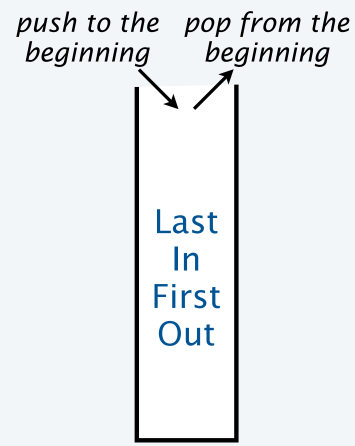
Stacks and queues both arise naturally in countless applications.

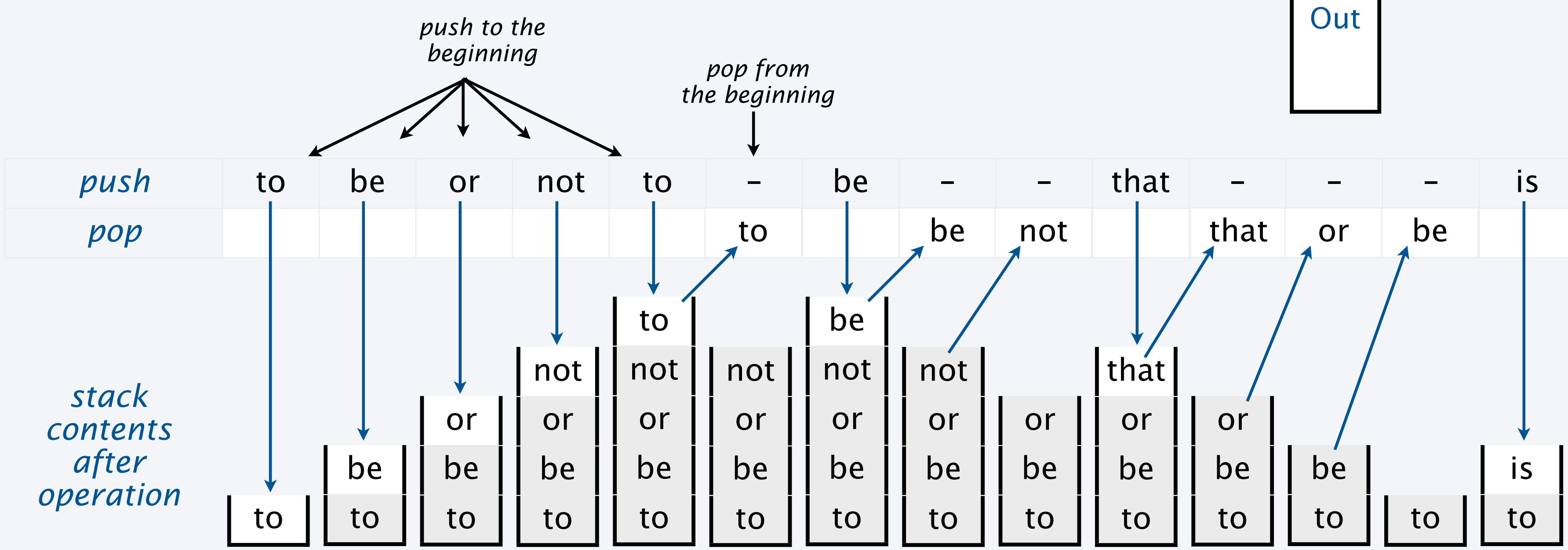
A key characteristic. No limit on the size of the collection.

Example of stack operations

Push. Add an item to the collection.

Pop. Remove and return the item *most* recently added.

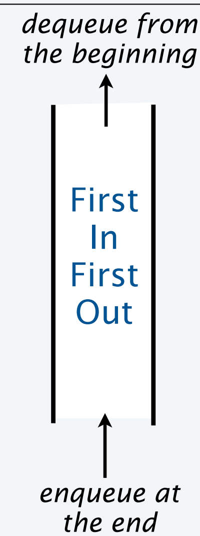




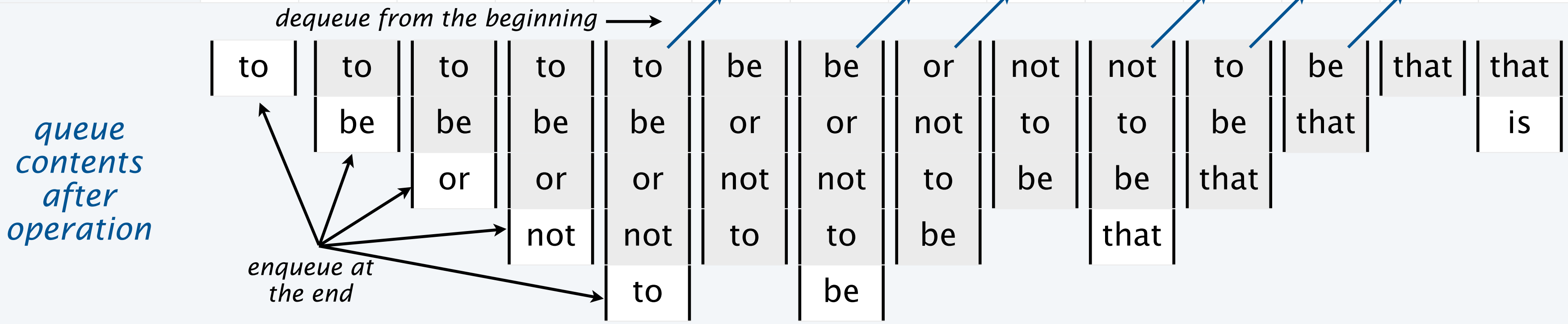
Example of queue operations

Enqueue. Add an item to the collection.

Dequeue. Remove and return the item *least* recently added.



enqueue	to	be	or	not	to	–	be	–	–	that	–	–	–	is
dequeue						to		be	or		not	to	be	



Parameterized data types

Goal. Simple, safe, and clear client code for collections of any type of data.

Java approach: Parameterized data types (generics)
- Use placeholder type name in definition.
- Substitute concrete type for placeholder in clients. ← stay tuned for examples

Stack API

public class Stack<Item>	
Stack<Item>()	*create a stack of objects, all of type* Item
void push(Item item)	*add* item *to stack*
Item pop()	*remove and return the item most recently pushed*
boolean isEmpty()	*is the stack empty?*
int size()	*# of objects on the stack*

Queue API

public class Queue<Item>	
Queue<Item>()	*create a queue of objects, all of type* Item
void enqueue(Item item)	*add* item *to queue*
Item dequeue()	*remove and return the item least recently enqueued*
boolean isEmpty()	*is the queue empty?*
int size()	*# of objects on the queue*

Performance specifications

Challenge. Provide guarantees on performance.

Goal. Simple, safe, clear, and *efficient* client code.

Typically required for client code to be *scalable*

Performance specifications

- All operations are constant-time.
- Memory use is proportional to the size of the collection, when it is nonempty.
- No limits within the code on the collection size.

Java. Any implementation of the API implements the stack/queue abstractions.

RS+KW. Implementations that do not meet performance specs *do not* implement the abstractions.

14. Stacks and Queues

- APIs
- **Clients**
- Strawman implementation
- Linked lists
- Implementations

Stack and queue applications

Queues

- First-come-first-served resource allocation.
- Asynchronous data transfer (StdIn, StdOut).
- Dispensing requests on a shared resource (printer, processor).
- Simulations of the real world (guitar string, traffic analysis, ...)

Stacks

- Last-come-first-served processes (browser, e-mail).
- Function calls in programming languages.
- Basic mechanism in interpreters, compilers.
- . . .

Queue client example: Read all strings from StdIn into an array

Challenge

- Can't store strings in array before creating the array.
- Can't create the array without knowing how many strings are in the input stream.
- Can't know how many strings are in the input stream without reading them all.

Solution: Use a Queue<String>.

```
public class QEx
{
    public static String[] readAllStrings()
    { // See next slide. }

    public static void main(String[] args)
    {
        String[] words = readAllStrings();
        for (int i = 0; i < words.length; i++)
            StdOut.println(words[i]);
    }
}
```

Note: StdIn has this functionality

```
% more moby.txt
moby dick
herman melville
call me ishmael some years ago never
mind how long precisely having
little or no money
...
```

```
% java QEx < moby.txt
moby
dick
herman
melville
call
me
ishmael
some
years
...
```

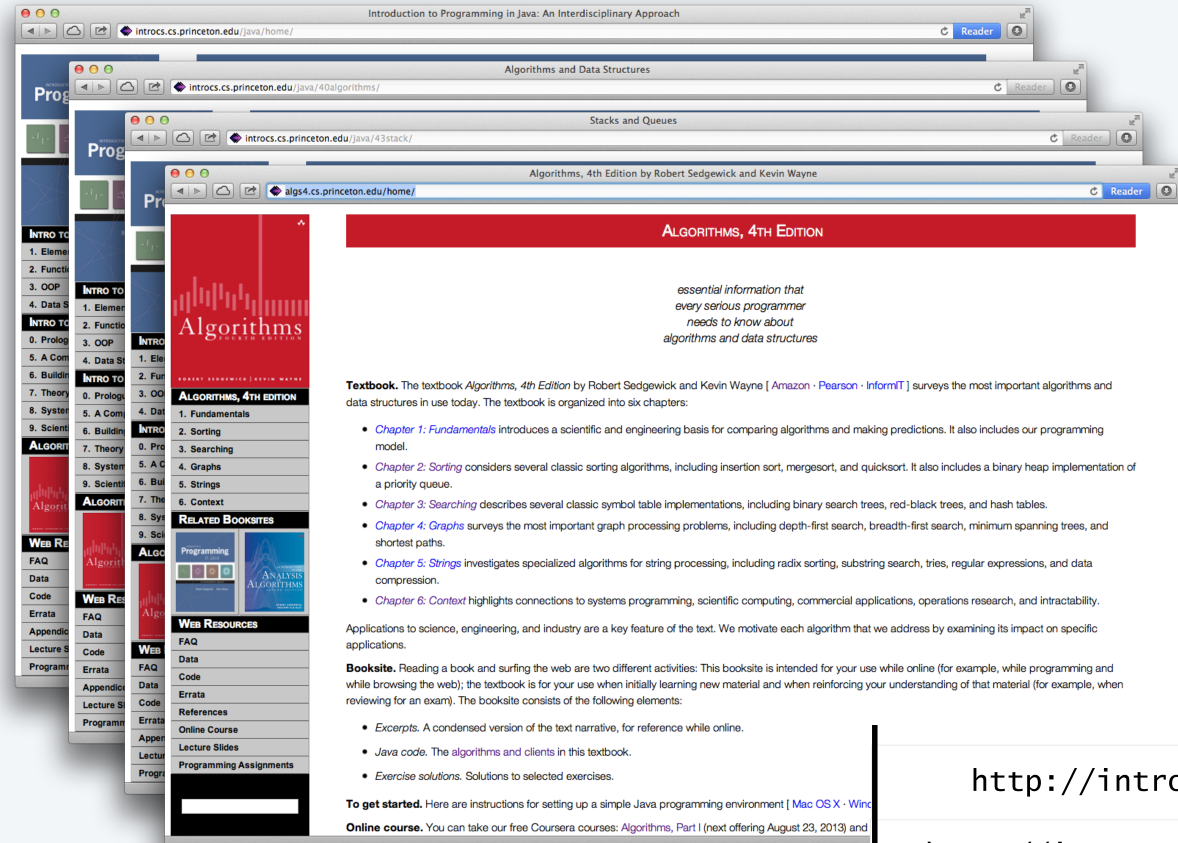
Queue client example: Read all strings from StdIn into an array

Solution: Use a Queue<String>.

- Store strings in the queue.
- Get the size when all have been read from StdIn.
- Create an array of that size.
- Copy the strings into the array.

```
public class QEx
{
    public static String[] readAllStrings()
    {
        Queue<String> q = new Queue<String>();
        while (!StdIn.isEmpty())
            q.enqueue(StdIn.readString());
        int N = q.size();
        String[] words = new String[N];
        for (int i = 0; i < N; i++)
            words[i] = q.dequeue();
        return words;
    }
    public static void main(String[] args)
    {
        String[] words = readAllStrings();
        for (int i = 0; i < words.length; i++)
            StdOut.println(words[i]);
    }
}
```

Stack example: "Back" button in a browser



Typical scenario

- Visit a page.
- Click a link to another page.
- Click a link to another page.
- Click a link to another page.
- Click "back" button.
- Click "back" button.
- Click "back" button.

http://introcs.cs.princeton.edu/java/43stack/
http://introcs.cs.princeton.edu/java/40algorithms/
http://introcs.cs.princeton.edu/java/home/

Autoboxing

Challenge. Use a *primitive* type in a parameterized ADT.

Wrapper types

- Each primitive type has a wrapper reference type.
- Wrapper type has larger set of operations than primitive type.
  Example: `Integer.parseInt()`.
- Values of wrapper types are objects.
- Wrapper type can be used in a parameterized ADT.

primitive type	*wrapper type*
`int`	`Integer`
`long`	`Long`
`double`	`Double`
`boolean`	`Boolean`

Autoboxing. Automatic cast from primitive type to wrapper type.

Auto-unboxing. Automatic cast from wrapper type to primitive type.

Simple client code →
(no casts)

```
Stack<Integer> stack = new Stack<Integer>();
stack.push(17);         // Autobox      (int -> Integer)
int a = stack.pop();    // Auto-unbox (Integer -> int)
```

Stack client example: Postfix expression evaluation

Infix. Standard way of writing arithmetic expressions, using parentheses for precedence.

Example. (1 + ((2 + 3) * (4 * 5))) = (1 + (5 * 20)) = 101

Jan Łukasiewicz
1878–1956

Postfix. Write operator *after* operands (instead of in between them).

Example. 1 2 3 + 4 5 * * + ← also called "reverse Polish" notation (RPN)

Remarkable fact. No parentheses are needed!

There is only one way to parenthesize a postfix expression.

1 2 3 + 4 5 * * +

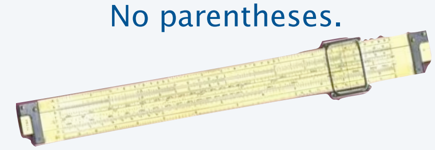
1 (2 + 3) 4 5 * * + ← find first operator, convert to infix, enclose in ()

1 ((2 + 3) * (4 * 5)) +

(1 + ((2 + 3) * (4 * 5)))

iterate, treating subexpressions in parentheses as atomic

HP-35 (1972)
First handheld calculator.
"Enter" means "push".
No parentheses.

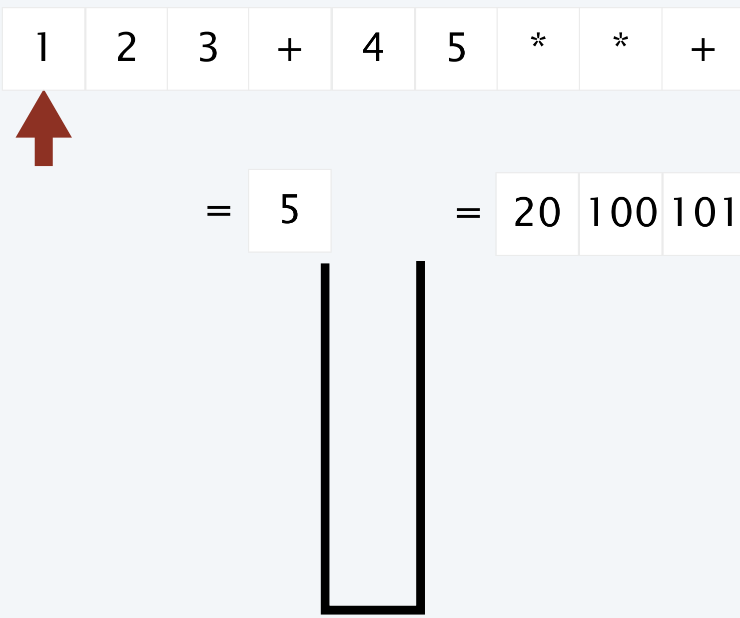


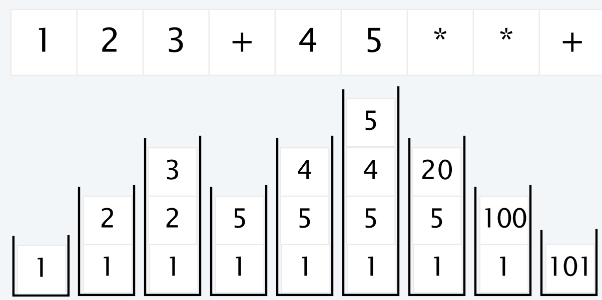
Made slide rules obsolete (!)

Next. With a stack, postfix expressions are easy to evaluate.

Postfix arithmetic expression evaluation

Algorithm
- While input stream is nonempty, read a token.
- Value: Push onto the stack.
- Operator: Pop operand(s), apply operator, push the result.





Stack client example: Postfix expression evaluation

```
public class Postfix
{
   public static void main(String[] args)
   {
      Stack<Double> stack = new Stack<Double>();
      while (!StdIn.isEmpty())
      {
         String token = StdIn.readString();
         if (token.equals("*"))
             stack.push(stack.pop() * stack.pop());
         else if (token.equals("+"))
             stack.push(stack.pop() + stack.pop());
         else if (token.equals("-"))
             stack.push(- stack.pop() + stack.pop());
         else if (token.equals("/"))
             stack.push((1.0/stack.pop()) * stack.pop());
         else if (token.equals("sqrt"))
             stack.push(Math.sqrt(stack.pop()));
         else
             stack.push(Double.parseDouble(token));
      }
      StdOut.println(stack.pop());
   }
}
```

```
% java Postfix
1 2 3 + 4 5 * * +
101
```

```
% java Postfix
1 5 sqrt + 2 /
1.618033988749895
```

$$\frac{1+\sqrt{5}}{2}$$

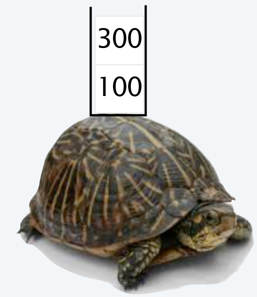
Perspective

- Easy to add operators of all sorts.
- Can do infix with two stacks (see text).
- Could output TOY program.
- Indicative of how Java compiler works.

Real-world stack application: PostScript

PostScript (Warnock-Geschke, 1980s): A turtle with a stack.

- Postfix program code (push literals; functions pop arguments).
- Add commands to drive virtual graphics machine.
- Add loops, conditionals, functions, types, fonts, strings....



PostScript code

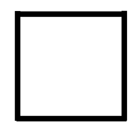
push(100)

call "moveto" (takes args from stack)

```
100 100 moveto
100 300 lineto
300 300 lineto
300 100 lineto
stroke
```

define a path

draw the path



A simple virtual machine, but not a toy

- Easy to specify published page.
- Easy to implement on various specific printers.
- Revolutionized world of publishing.

Another stack machine: The JVM (Java Virtual Machine)!

14. Stacks and Queues

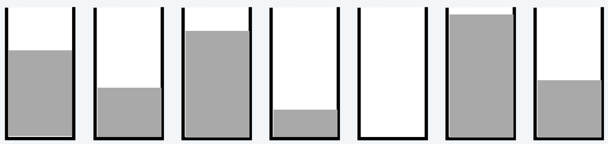
- APIs
- Clients
- **Strawman implementation**
- Linked lists
- Implementations

Strawman ADT for pushdown stacks

Warmup: simplify the ADT

- Implement only for items of type `String`.
- Have client provide a stack *capacity* in the constructor.

values



Strawman API

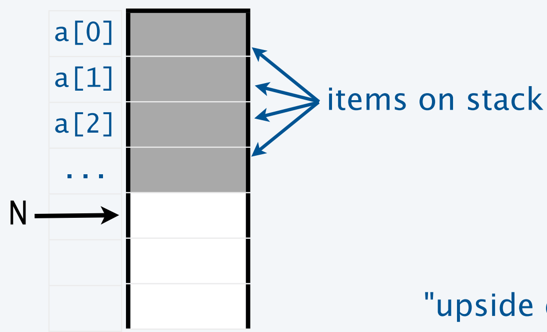
`public class StrawStack`	
`StrawStack(int max)`	*create a stack of capacity* `max`
`void push(String item)`	*add* `item` *to stack*
`String pop()`	*return the string most recently pushed*
`boolean isEmpty()`	*is the stack empty?*
`int size()`	*number of strings on the stack*

Rationale. Allows us to represent the collection with an array of strings.

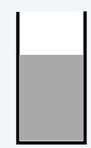
Strawman implementation: Instance variables and constructor

Data structure choice. Use an array to hold the collection.

```
public class StrawStack
{
   private String[] a;
   private int N = 0;

   public StrawStack(int max)
   {  a = new String[max];  }
...
}
```



"upside down"
representation of



Strawman stack implementation: Test client


```
public static void main(String[] args)
{
   int max = Integer.parseInt(args[0]);
   StrawStack stack = new StrawStack(max);
   while (!StdIn.isEmpty())
   {
      String item = StdIn.readString();
      if (item.equals("-"))
         stack.push(item);
      else
         StdOut.print(stack.pop());
   }
   StdOut.println();
}
```

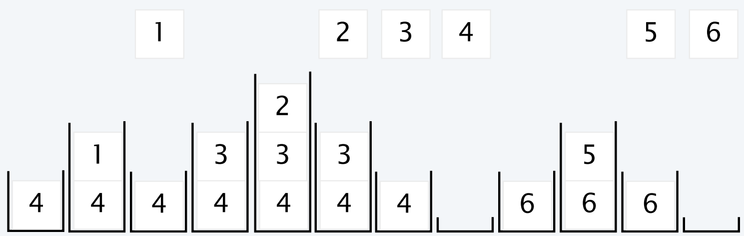
```
% more tobe.txt
to be or not to - be - - that - - - is

% java StrawStack 20 < tobe.txt
to be not that or be
```

What we *expect*, once the implementation is done.

Self-assessment 1 on stacks

Q. Can we always insert pop() commands to make items come out in sorted order?

Example 1.	6	5	4	3	2	1	–	–	–	–	–	–
Example 2.	1	–	2	–	3	–	4	–	5	–	6	–
Example 3.	4	1	–	3	2	–	–	–	6	5	–	–

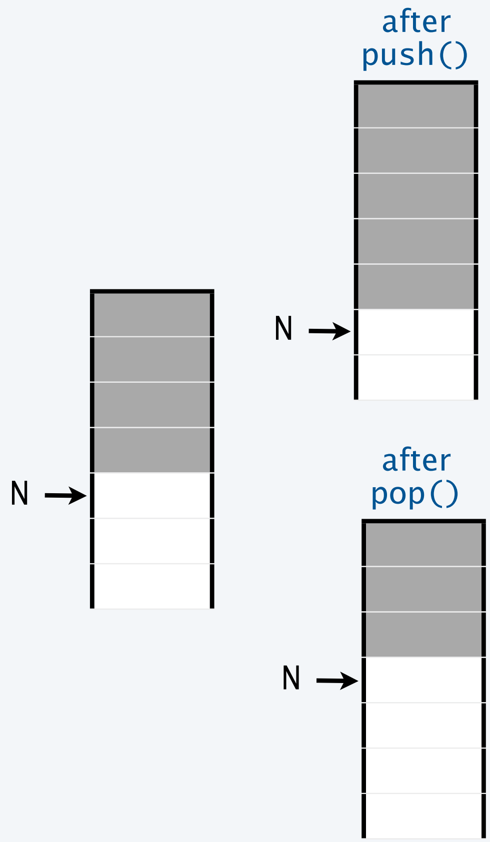


Strawman implementation: Methods

Methods define data-type operations (implement APIs).

```
public class StrawStack
{
...
   public boolean isEmpty()
   {  return (N == 0); }

   public void push(Object item)
   {  a[N++] = item;   }

   public String pop()
   {  return a[--N];   }

   public int size()
   {  return N;   }
...
}
```

all constant-time one-liners!



Strawman pushdown stack implementation

```
public class StrawStack
{
   private String[] a;
   private int N = 0;

   public StrawStack(int max)
   {  a = new String[max];  }

   public boolean isEmpty()
   {  return (N == 0);   }

   public void push(String item)
   {  a[N++] = item;  }

   public String pop()
   {  return a[--N];  }

   public int size()
   {  return N;  }

   public static void main(String[] args)
   {
      int max = Integer.parseInt(args[0]);
      StrawStack stack = new StrawStack(max);
      while (!StdIn.isEmpty())
      {
         String item = StdIn.readString();
         if (item.compareTo("-") != 0)
            stack.push(item);
         else
            StdOut.print(stack.pop());
      }
      StdOut.println();
   }
}
```

instance variables

constructor

methods

test client

```
% more tobe.txt
to be or not to - be - - that - - - is

% java StrawStack 20 < tobe.txt
to be not that or be
```

Trace of strawman stack implementation (array representation)

push	to	be	or	not	to	–	be	–	–	that	–	–	–	is
pop						to		be	not		that	or	be	

stack contents after operation

a[0]	to	to	to	to	to	to	to	to	to	to	to	to	to	to
a[1]		be	be	be	be	be	be	be	be	be	be	be	be	is
a[2]			or	or	or	or	or	or	or	or	or	or	or	or
a[3]				not	not	not	not	not	not	that	that	that	that	that
a[4]					to	to	be	be	be	be	be	be	be	be
a[5]														
a[6]														
a[7]														
a[8]														
a[9]														
a[10]														
a[11]														
a[12]														
a[13]														
a[14]														
a[15]														
a[16]														
a[17]														
a[18]														
a[19]														

N

Significant wasted space when stack size is not near the capacity (typical).

Benchmarking the strawman stack implementation

StrawStack implements a *fixed-capacity collection that behaves like a stack* if the data fits.

It does *not* implement the stack API or meet the performance specifications.

StrawStack requires client to provide capacity

Stack API

StrawStack works only for strings

public class Stack<Item>	
Stack<Item>() ✘	*create a stack of objects, all of type* Item
void push(Item item)	*add* item *to stack*
✘ Item pop()	*remove and return the item most recently pushed*
boolean isEmpty()	*is the stack empty?*
int size()	*# of objects on the stack*

Performance specifications

- All operations are constant-time. ✓
- Memory use is proportional to the size of the collection, when it is nonempty. ✘
- No limits within the code on the collection size. ✘

Nice try, but need a new *data structure.*

14. Stacks and Queues

- APIs
- Clients
- Strawman implementation
- **Linked lists**
- Implementations

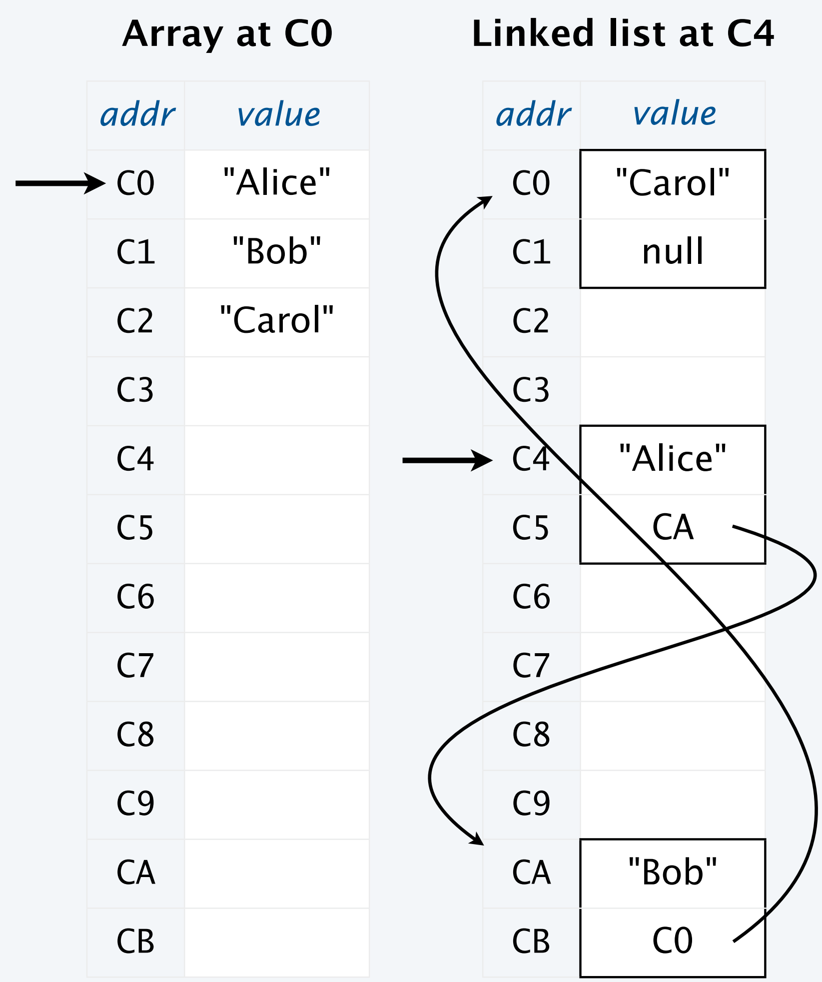
Data structures: sequential vs. linked

Sequential data structure

- Put objects next to one another.
- TOY: consecutive memory cells.
- Java: array of objects.
- Fixed size, arbitrary access. ← *i*th element

Linked data structure

- Associate with each object a link to another one.
- TOY: link is memory address of next object.
- Java: link is reference to next object.
- Variable size, sequential access. ← *next* element
- Overlooked by novice programmers.
- Flexible, widely used method for organizing data.



Simplest singly-linked data structure: linked list

Linked list

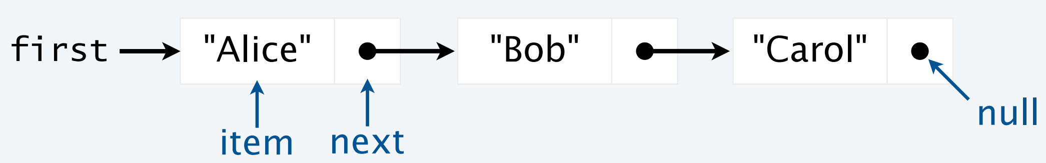
- A recursive data structure.
- Def. A *linked list* is null or a reference to a *node*.
- Def. A *node* is a data type that contains a reference to a node.
- Unwind recursion: A linked list is a sequence of nodes.

```
private class Node
{
  private String item;
  private Node next;
}
```

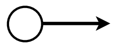
Representation

- Use a private nested class Node to implement the node abstraction.
- For simplicity, start with nodes having two values: a `String` and a Node.

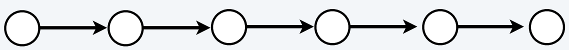
A linked list



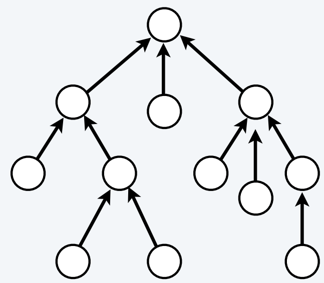
Singly-linked data structures

Even with just one link () a wide variety of data structures are possible.

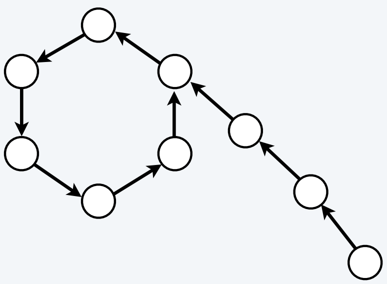
Linked list (this lecture)



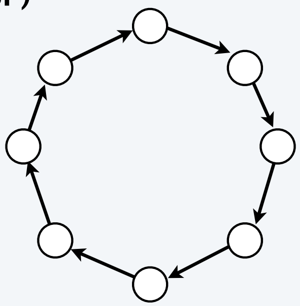
Tree



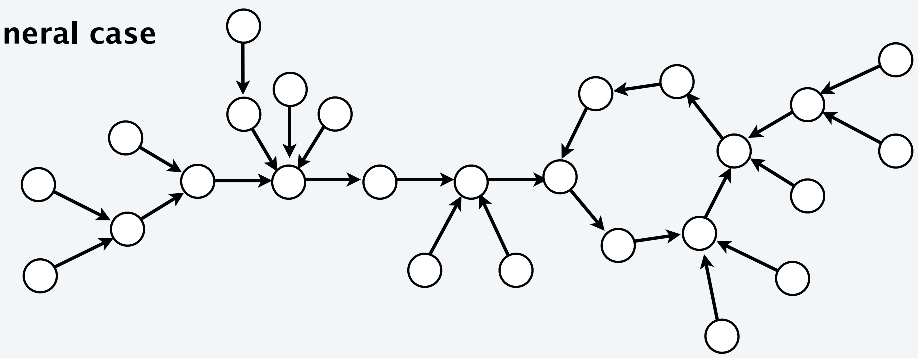
Rho



Circular list (TSP)



General case

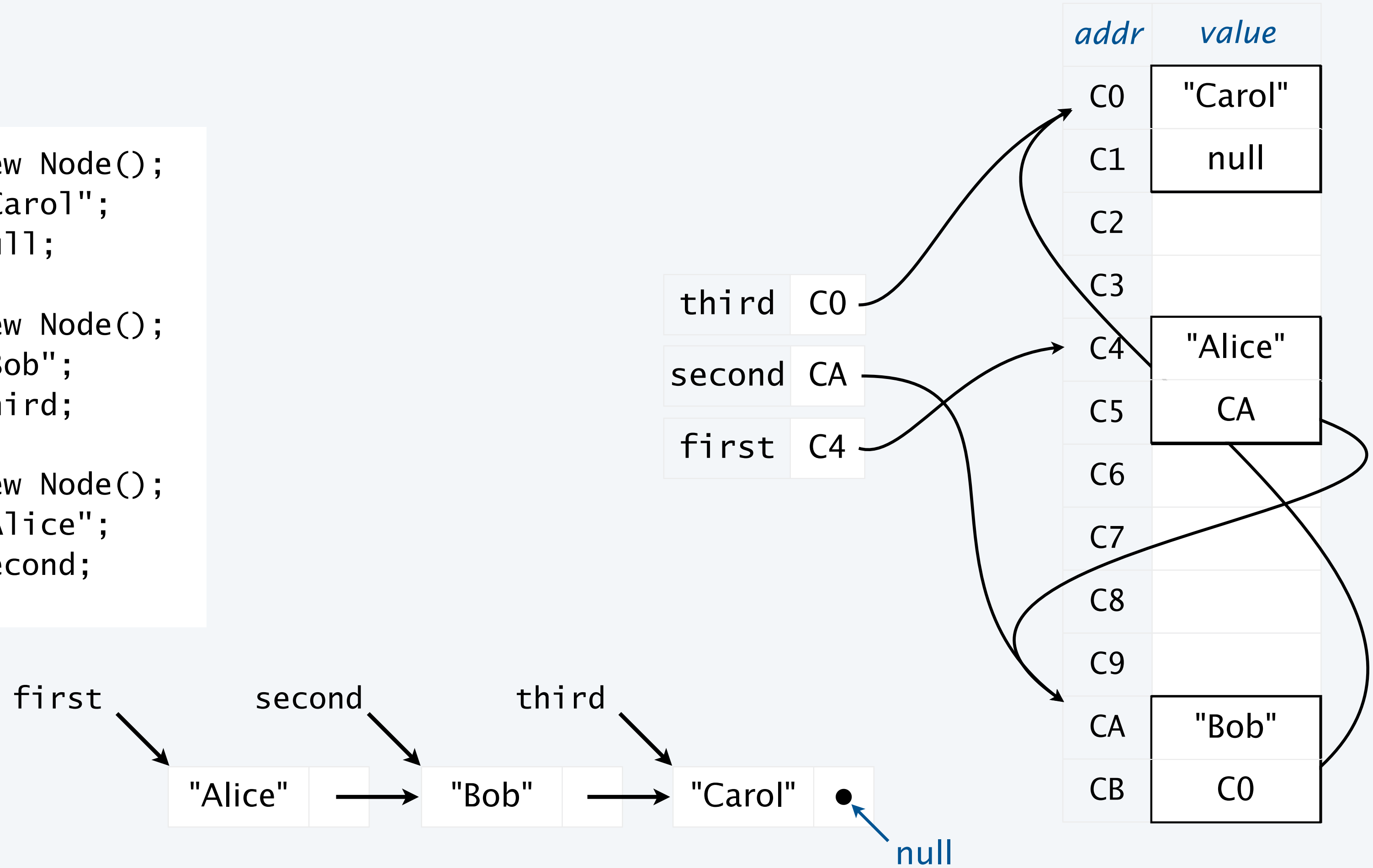


Multiply linked structures: many more possibilities!

From the point of view of a particular object, all of these structures look the same.

Building a linked list

```
Node third  = new Node();
third.item  = "Carol";
third.next  = null;

Node second = new Node();
second.item = "Bob";
second.next = third;

Node first  = new Node();
first.item  = "Alice";
first.next  = second;
```



List processing code

Standard operations for processing data structured as a singly-linked list

- Add a node at the beginning.
- Remove and return the node at the beginning.
- Add a node at the end (requires a reference to the last node).
- Traverse the list (visit every node, in sequence).

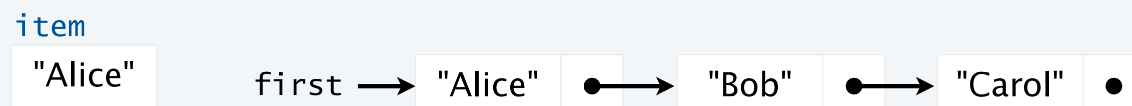
An operation that calls for a *doubly*-linked list (slightly beyond our scope)

- Remove and return the node at the end.

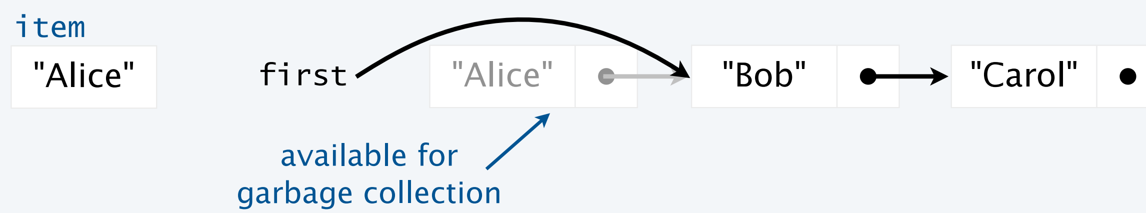
List processing code: Remove and return the first item

Goal. Remove and return the first item in a linked list `first`.


```
item = first.item;
```



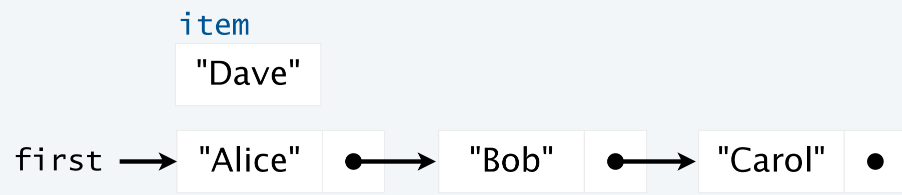

```
first = first.next;
```



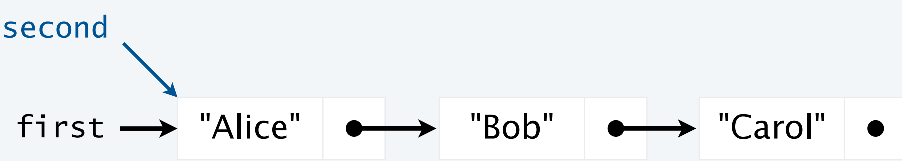

```
return item;
```

List processing code: Add a new node at the beginning

Goal. Add `item` to a linked list `first`.




```
Node second = first;
```



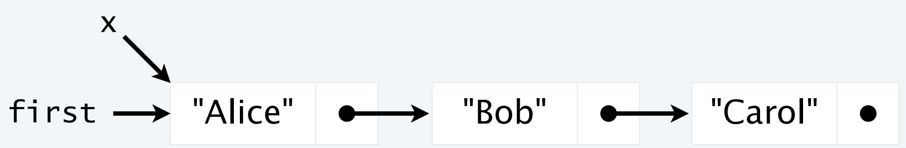

```
first = new Node();
```


```
first.item = item;
first.next = second;
```

List processing code: Traverse a list

Goal. Visit every node on a linked list `first`.

```
Node x = first;
while (x != null)
{
   StdOut.println(x.item);
   x = x.next;
}
```



StdOut

```
Alice
Bob
Carol
```

Self-assessment 1 on linked lists

Q. What is the effect of the following code (not-so-easy question)?

```
...
Node list = null;
while (!StdIn.isEmpty())
{
   Node old = list;
   list = new Node();
   list.item = StdIn.readString();
   list.next = old;
}
for (Node t = list; t != null; t = t.next)
   StdOut.println(t.item);
...
```

Self-assessment 2 on stacks

Q. Give code that uses a stack to print the strings from `StdIn` on `StdOut`, in reverse order.

Self-assessment 2 on linked lists

Q. What is the effect of the following code (not-so-easy question)?

```
...
Node list = new Node();
list.item = StdIn.readString();
Node last = list;
while (!StdIn.isEmpty())
{
   last.next = new Node();
   last = last.next;
   last.item = StdIn.readString();
}
...
```

COMPUTER SCIENCE
SEDGEWICK / WAYNE

14. Stacks and Queues

- APIs
- Clients
- Strawman implementation
- Linked lists
- **Implementations**

ADT for pushdown stacks: review

A pushdown stack is an idealized model of a LIFO storage mechanism.

An ADT allows us to write Java programs that use and manipulate pushdown stacks.

API

public class Stack<Item>	
Stack<Item>()	*create a stack of objects, all of type* Item
void push(Item item)	*add* item *to stack*
Item pop()	*remove and return the item most recently pushed*
boolean isEmpty()	*is the stack empty?*
int size()	*# of objects on the stack*

Performance specifications

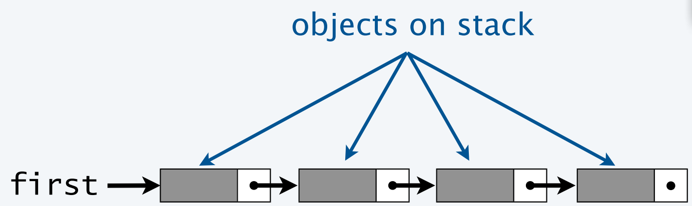
- All operations are constant-time.
- Memory use is proportional to the size of the collection, when it is nonempty.
- No limits within the code on the collection size.

Pushdown stack implementation: Instance variables and constructor

Data structure choice. Use a linked list to hold the collection.

```
public class Stack<Item>
{
   private Node first = null;
   private int N = 0;

   private class Node
   {
      private Item item;
      private Node next;
   }
...
}
```

use in place of concrete type



Annoying exception (not a problem here).
- Can't declare an array of `Item` objects (don't ask why).
- Need cast: `Item[] a = (Item[]) new Object[N]`

Stack implementation: Test client


```
public static void main(String[] args)
{
   Stack<String> stack = new Stack<String>();
   while (!StdIn.isEmpty())
   {
      String item = StdIn.readString();
      if (item.equals("-"))
         System.out.print(stack.pop());
      else
         stack.push(item);
   }
   StdOut.println();
}
```

```
% more tobe.txt
to be or not to - be - - that - - - is

% java Stack < tobe.txt
to be not that or be
```

What we *expect*, once the implementation is done.

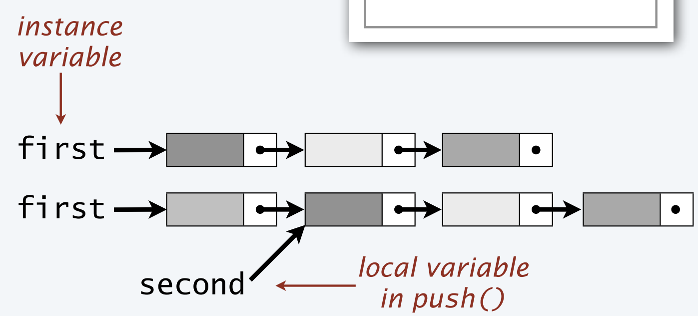
Stack implementation: Methods

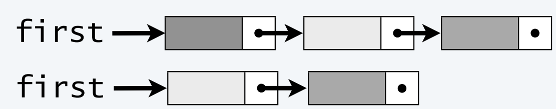
Methods define data-type operations (implement the API).

```
public class Stack<Item>
{
...
   public boolean isEmpty()
   {  return first == null;  }
   public void push(Item item)
   {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
      N++;
   }
   public Item pop()
   {
      Item item = first.item;
      first = first.next;
      N--;
      return item;
   }
   public int size()
   {  return N;  }
...
}
```

add a new node
to the beginning of the list

remove and return
first item on list





Stack implementation

```
public class Stack<Item>
{
   private Node first = null;
   private int N = 0;
   private class Node
   {
      private Item item;
      private Node next;
   }
   public boolean isEmpty()
   {  return first == null;  }
   public void push(Item item)
   {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
      N++;
   }
   public Item pop()
   {
      Item item = first.item;
      first = first.next;
      N--;
      return item;
   }
   public int size()
   {  return N;  }
   public static void main(String[] args)
   { // See earlier slide  }
}
```

instance variables

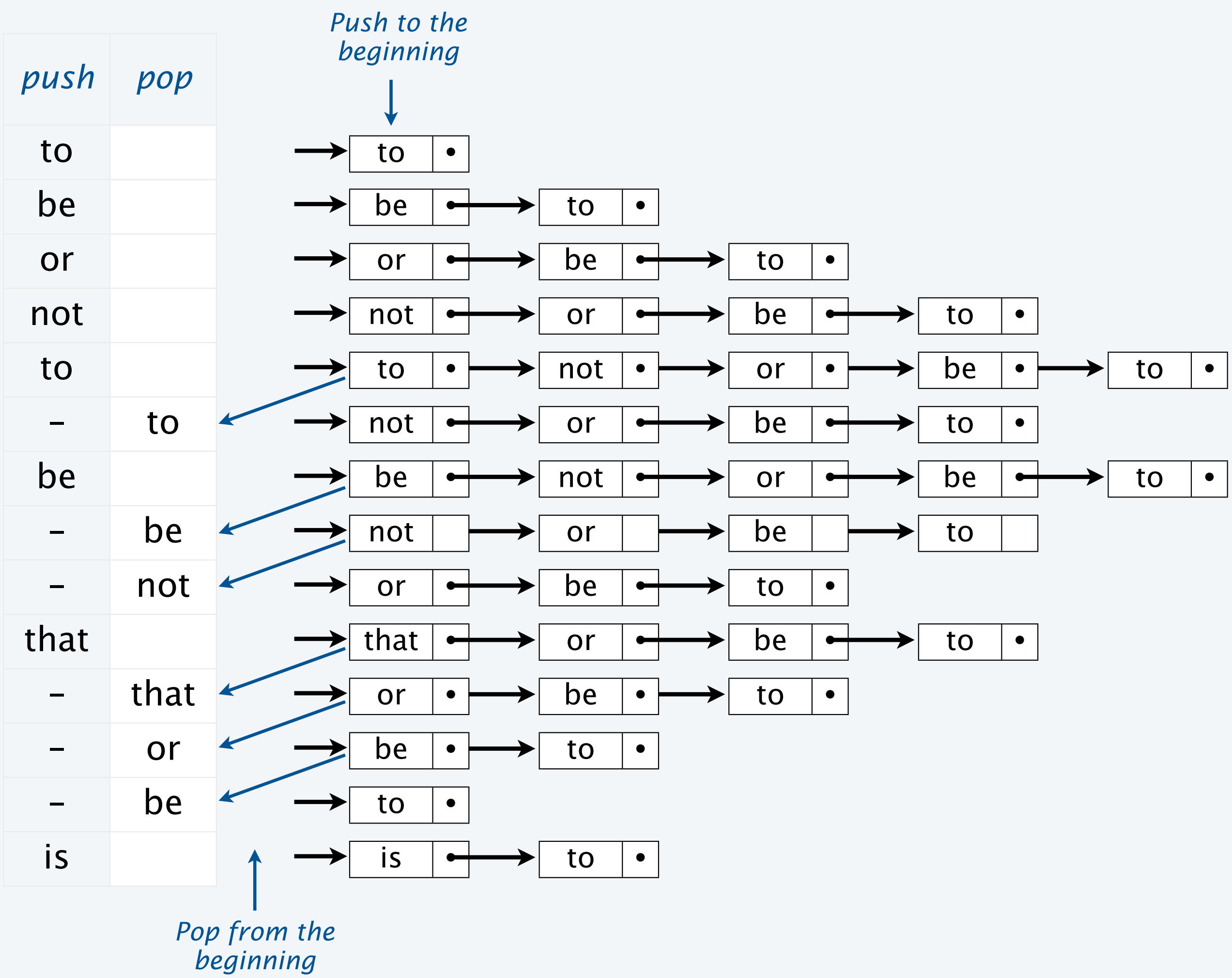
nested class

methods

test client

```
% more tobe.txt
to be or not to - be - - that - - - is

% java Stack < tobe.txt
to be not that or be
```

Trace of stack implementation (linked list representation)



Benchmarking the stack implementation

Stack implements the stack abstraction.

It *does* implement the API and meet the performance specifications.

Stack API

public class Stack<Item>	
Stack<Item>()	*create a stack of objects, all of type* Item
void push(Item item)	*add* item *to stack*
Item pop()	*remove and return the item most recently pushed*
boolean isEmpty()	*is the stack empty?*
int size()	*# of objects on the stack*

✓

Performance specifications

- All operations are constant-time. ✓
- Memory use is proportional to the size of the collection, when it is nonempty. ✓
- No limits within the code on the collection size. ✓

Made possible by *linked data structure.*

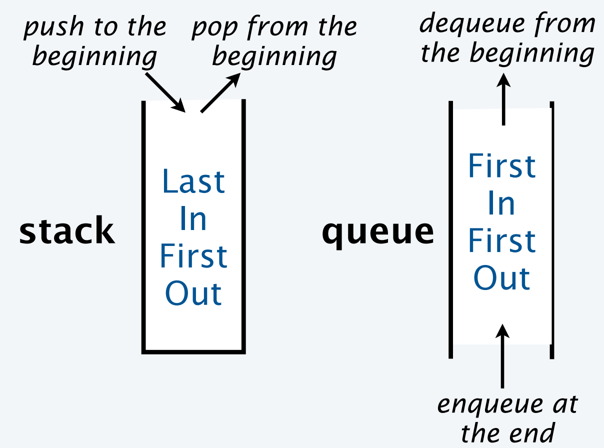
dequeue(): same code as pop()
enqueue(): slightly more complicated

Also possible to implement the *queue* abstraction with a singly-linked list (see text).

Summary

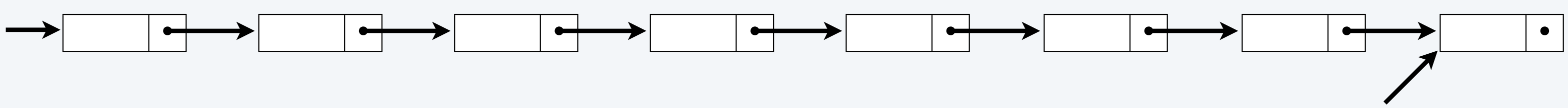
Stacks and queues

- Fundamental collection abstractions.
- Differ only in order in which items are removed.
- Performance specifications: Constant-time for all operations and space proportional to number of objects.



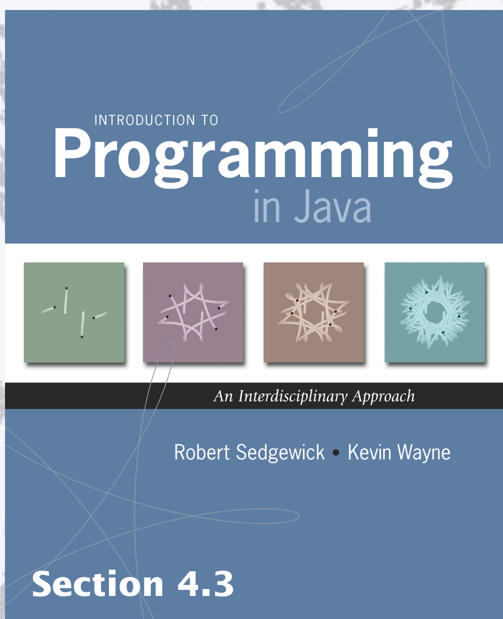
Linked structures

- Fundamental alternative to sequential structures.
- Enable implementations of the stack/queue abstractions *that meet performance specifications*.



Next: *Symbol tables*

COMPUTER SCIENCE
SEDGEWICK / WAYNE



http://introcs.cs.princeton.edu

14. Stacks and Queues