COMPUTER SCIENCE
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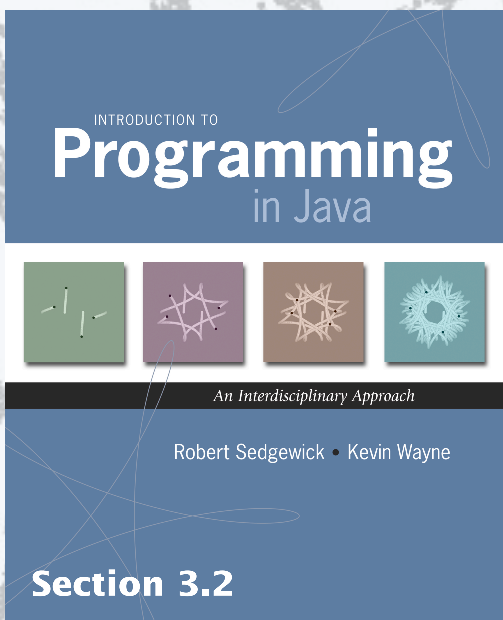


http://introcs.cs.princeton.edu

10. Creating Data Types

10. Creating Data Types

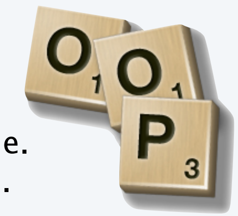
- Overview
- Point charges
- Turtle graphics
- Complex numbers

Object-oriented programming (OOP)

Object-oriented programming (OOP).

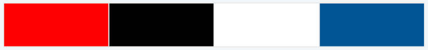
- Create your own data types.
- Use them in your programs (manipulate *objects*).

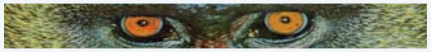
An object holds a data type value.
Variable names refer to objects.



Examples (stay tuned for details)

data type	*set of values*	*examples of operations*
`Color`	three 8-bit integers	get red component, brighten
`Picture`	2D array of colors	get/set color of pixel (i, j)
`String`	sequence of characters	length, substring, compare





C A T A G C G C

An abstract data type is a data type whose representation is *hidden from the client*.

Impact: We can use ADTs without knowing implementation details.

- Previous lecture: how to write client programs for several useful ADTs
- This lecture: how to implement your own ADTs

Implementing a data type

To create a data type, you need provide code that

- Defines the set of values (instance variables).
- Implements operations on those values (methods).
- Creates and initialize new objects (constructors).

In Java, a data-type implementation is known as a *class*.

Instance variables

- Declarations associate variable names with types.
- Set of type values is "set of values".

Methods

- Like static methods.
- Can refer to instance variables.

Constructors

- Methods with the same name as the type.
- No return type declaration.
- Invoked by new, returns object of the type.

A Java class

Anatomy of a Class

text file named Charge.java

```
public class Charge
{
   private double rx, ry;    // position
   private double q;         // charge

   public Charge(double x0, double y0, double q0)
   {
      rx = x0;
      ry = y0;
      q  = q0;
   }

   public double potentialAt(double x, double y)
   {
      double k = 8.99e09;
      double dx = x - rx;
      double dy = y - ry;
      return k * q / Math.sqrt(dx*dx + dy*dy);
   }
   public String toString()
   {  return q + " at " + "(" + rx + ", " + ry + ")";  }

   public static void main(String[] args)
   {
      Charge c = new Charge(.72, .31, 21.3);
      StdOut.println(c);
      StdOut.printf("%6.2e\n", c.potentialAt(.42, .71));
   }
}
```

instance variables

constructor

methods

not "static"

static method (familiar)

test client

```
% java Charge
21.3 at (0.72, 0.31)
3.61e+11
```

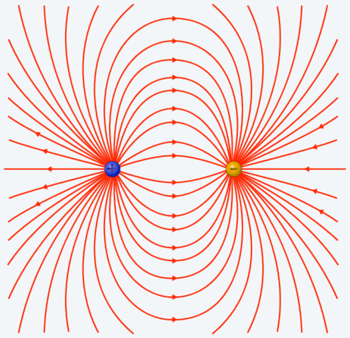
10. Creating Data Types

- Overview
- Point charges
- Turtle graphics
- Complex numbers

ADT for point charges

A point charge is an idealized model of a particle that has an electric charge.

An ADT allows us to write Java programs that manipulate point charges.



Values

	examples	
position (*x*, *y*)	(.53, .63)	(.13, .94)
electrical charge	21.3	81.9

API (operations)

public class Charge	
Charge(double x0, double y0, double q0)	
double potentialAt(double x, double y)	*electric potential at (x, y) due to charge*
String toString()	*string representation of this charge*

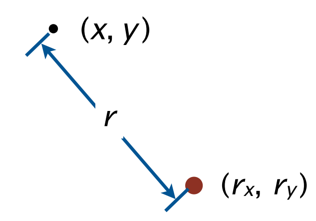
Crash course on electric potential

Electric potential is a measure of the effect of a point charge on its surroundings.

- It increases in proportion to the charge value.
- It decreases in proportion to the *inverse of the distance* from the charge.

Mathematically,

- Suppose a point charge c is located at (r_x, r_y) and has charge q.
- Let r be the distance between (x, y) and (r_x, r_y)
- Let $V_c(x,y)$ be the potential at (x, y) due to c.
- Then $V_c(x, y) = k\frac{q}{r}$ where $k = 8.99 \times 10^9$ is a normalizing factor.



Q. What happens when multiple charges are present?

A. The potential at a point is the *sum* of the potentials due to the individual charges.

Note: Similar laws hold in many other situations. ← Example. *N*-body (an inverse *square* law).

Point charge implementation: Test client

Best practice. Begin by implementing a simple test client.


```
public static void main(String[] args)
{
   Charge c = new Charge(.72, .31, 20.1);
   StdOut.println(c);
   StdOut.printf("%6.2e\n", c.potentialAt(.42, .71));
}
```

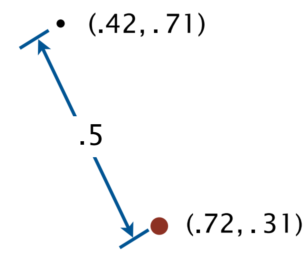
$$V_c(x, y) = k\frac{q}{r}$$

$$r = \sqrt{(r_x - x)^2 + (r_y - y)^2}$$

$$= \sqrt{.3^2 + .4^2} = .5$$

$$V_c(.42, .71) = 8.99 \times 10^9 \frac{20.1}{.5}$$

$$= 3.6 \times 10^{11}$$




```
% java Charge
21.3 at (0.72, 0.31)
3.61e+11
```

← What we *expect*, once the implementation is done.

Point charge implementation: Instance variables

Instance variables define data-type values.

Values		*examples*	
	position (*x*, *y*)	(.53, .63)	(.13, .94)
	electrical charge	21.3	81.9

```
public class Charge
{
   private final double rx, ry;
   private final double q;
...
}
```

Modifiers control access.

- `private` denies clients access and therefore makes data type abstract.
- `final` disallows any change in value and therefore makes variable *immutable*.

stay tuned

Key to OOP. Each *object* has instance-variable values.

Point charge implementation: Constructor

Constructors create and initialize new objects.


```
public class Charge
{
...
  public Charge(double x0, double y0, double q0)
   {
      rx = x0;
      ry = y0;
      q  = q0;
   }
...
}
```

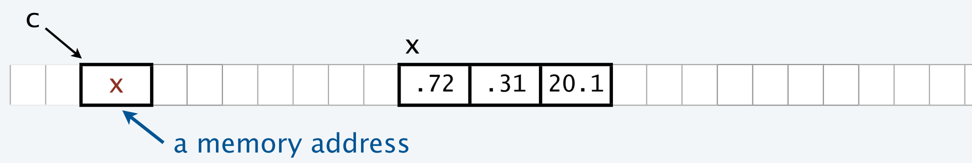
references to instance variables, which are *not* declared within the constructor

Clients use new to invoke constructors.

- Pass arguments as in a method call.
- Return value is reference to new object.

Possible memory representation of

`Charge c = new Charge(.72, .31, 20.1);`



Point charge implementation: Methods

Methods define data-type operations (implement APIs).

API

public class Charge	
Charge(double x0, double y0, double q0)	
double potentialAt(double x, double y)	*electric potential at (x, y) due to charge*
String toString()	*string representation of this charge*


```
public class Charge
{
  ...
   public double potentialAt(double x, double y)
   {
      double k = 8.99e09;
      double dx = x - rx;
      double dy = y - ry;
      return k * q / Math.sqrt(dx*dx + dy*dy);
   }

   public String toString()
   {  return q + " at " + "(" + rx + ", " + ry + ")";  }
  ...
}
```

$$V_C(x, y) = k\frac{q}{r}$$

Key to OOP. An instance variable reference in a class method *refers to the value for the object that was used to invoke the method.*

Point charge implementation

text file named Charge.java →

```
public class Charge
{
   private double rx, ry;    // position
   private double q;         // charge

   public Charge(double x0, double y0, double q0)
   {
      rx = x0;
      ry = y0;
      q  = q0;
   }

   public double potentialAt(double x, double y)
   {
      double k = 8.99e09;
      double dx = x - rx;
      double dy = y - ry;
      return k * q / Math.sqrt(dx*dx + dy*dy);
   }
   public String toString()
   {  return q + " at " + "(" + rx + ", " + ry + ")";  }

   public static void main(String[] args)
   {
      Charge c = new Charge(.72, .31, 20.1);
      System.out.println(c);
      StdOut.printf("%6.2e\n", c.potentialAt(.42, .71));
   }
}
```

instance variables

constructor

methods

test client

```
% java Charge
21.3 at (0.72, 0.31)
3.61e+11
```

Point charge client: Potential visualization (helper methods)

Read point charges from StdIn.

- Uses Charge like any other type.
- Returns an array of Charges

Convert potential values to a color.

- Convert V to an 8-bit integer.
- Use grayscale.

```
public static Charge[] readCharges()
{
   int N = StdIn.readInt();
   Charge[] a = new Charge[N];
   for (int i = 0; i < N; k++)
   {
      double x0 = StdIn.readDouble();
      double y0 = StdIn.readDouble();
      double q0 = StdIn.readDouble();
      a[i] = new Charge(x0, y0, q0);
   }
   return a;
}
```

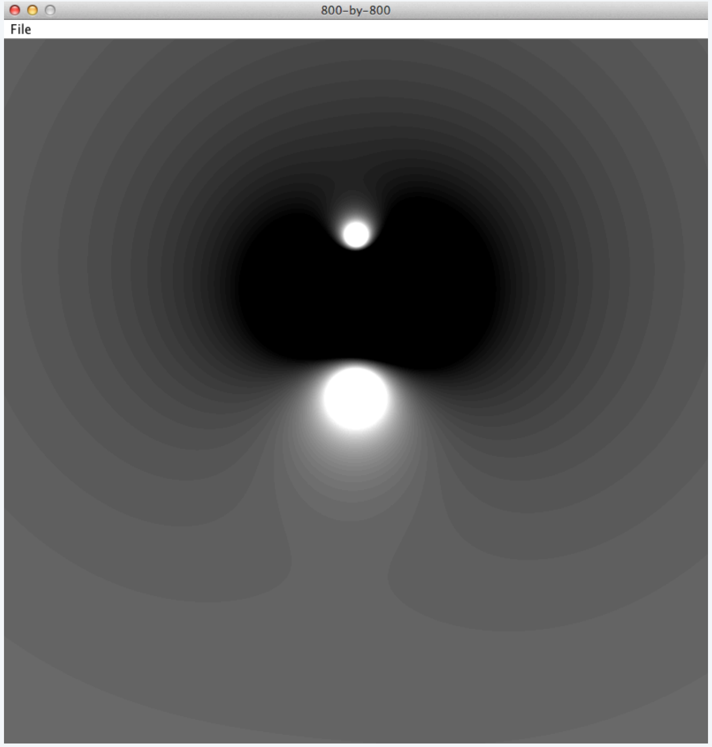
```
public static Color toColor(double V)
{
   V = 128 + V / 2.0e10;
   int t = 0;
   if (V > 255) t = 255;
   else if (V >= 0) t = (int) V;
   return new Color(t, t, t);
}
```

V											
t	0	1	...	37	38	39	...	128	...	254	255

Point charge client: Potential visualization

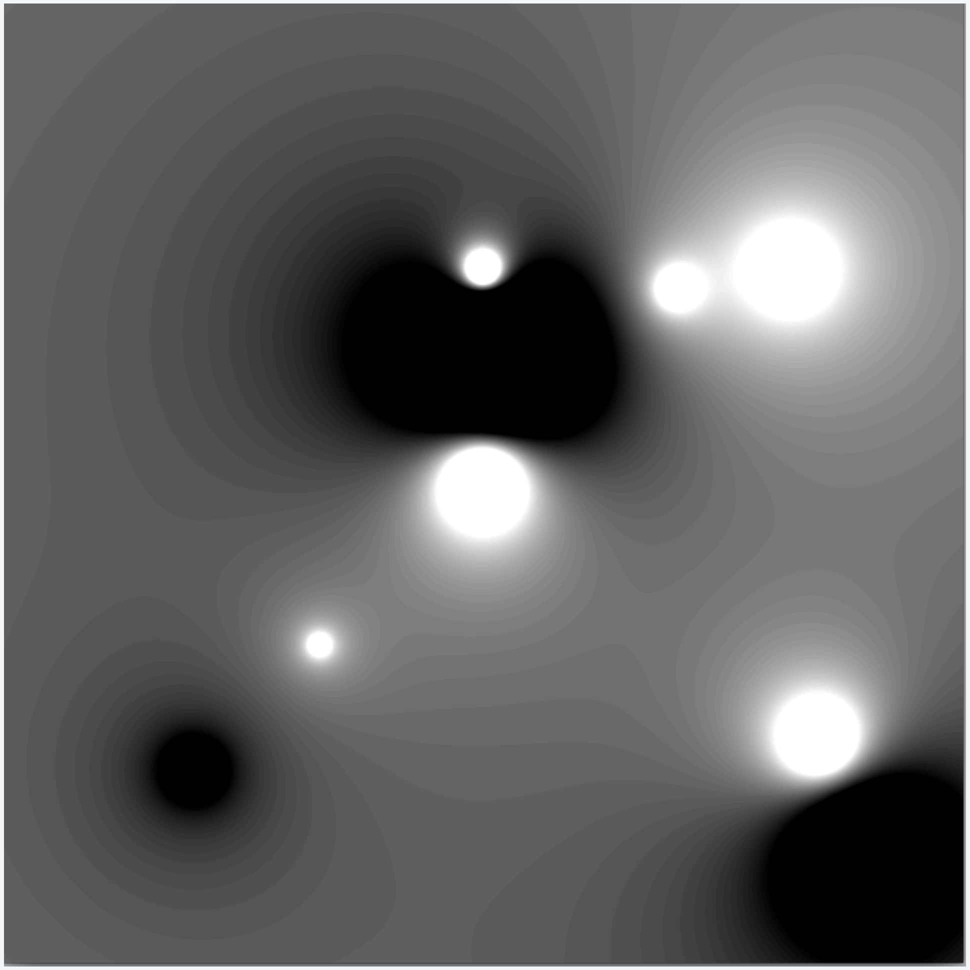
```
import java.awt.Color;
public class Potential
{
   public static Charge[] readCharges()
   { // See previous slide. }
   public static Color toColor()
   { // See previous slide. }
   public static void main(String[] args)
   {
      Charge[] a = readCharges();
      int SIZE = 800;
      Picture pic = new Picture(SIZE, SIZE);
      for (int col = 0; col < SIZE; col++)
         for (int row = 0; row < SIZE; row++)
         {
            double V = 0.0;
            for (int k = 0; k < a.length; k++)
            {
               double x = 1.0 * col / SIZE;
               double y = 1.0 * row / SIZE;
               V += a[k].potentialAt(x, y);
            }
            pic.set(col, SIZE-1-row, toColor(V));
        }
     pic.show();
   }
}
```

```
% more charges3.txt
3
.51 .63 -100
.50 .50   40
.50 .72   20
% java Potential < charges3.txt
```



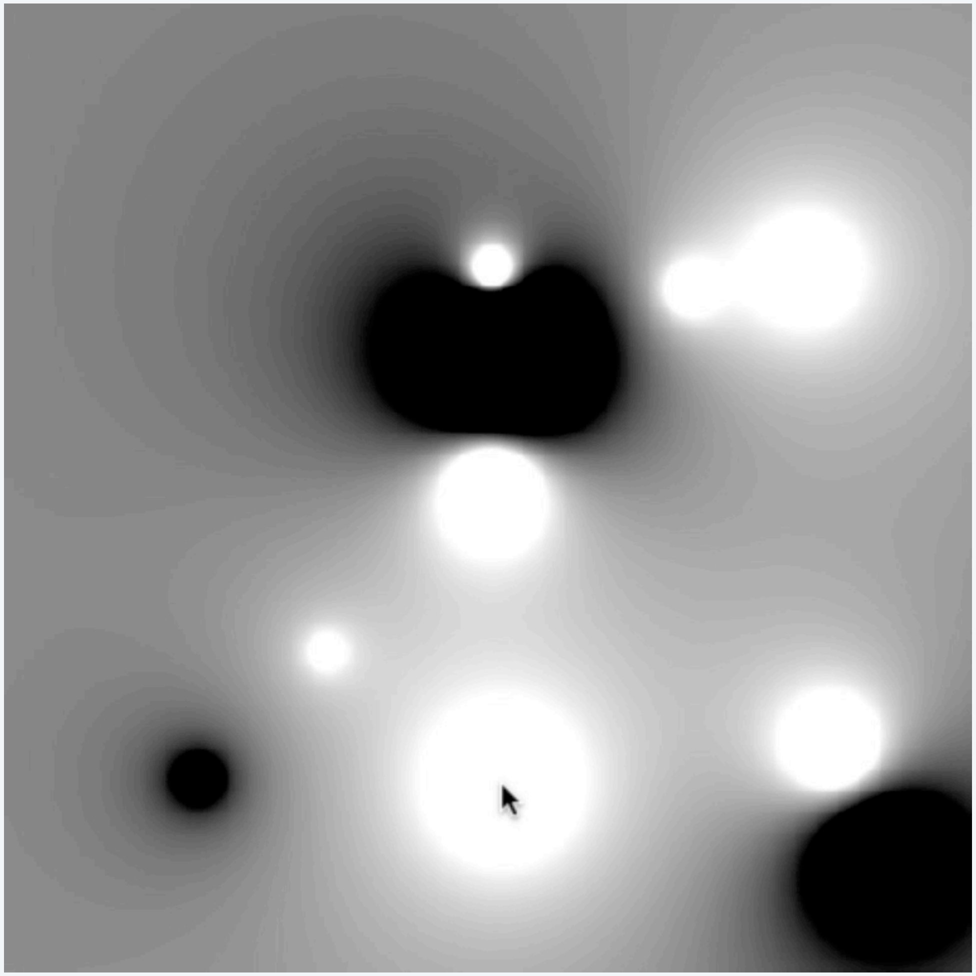
Potential visualization I

```
% more charges9.txt
9
.51 .63 -100
.50 .50   40
.50 .72   20
.33 .33    5
.20 .20  -10
.70 .70   10
.82 .72   20
.85 .23   30
.90 .12  -50

% java Potential < charges9.txt
```



Potential visualization II: A moving charge

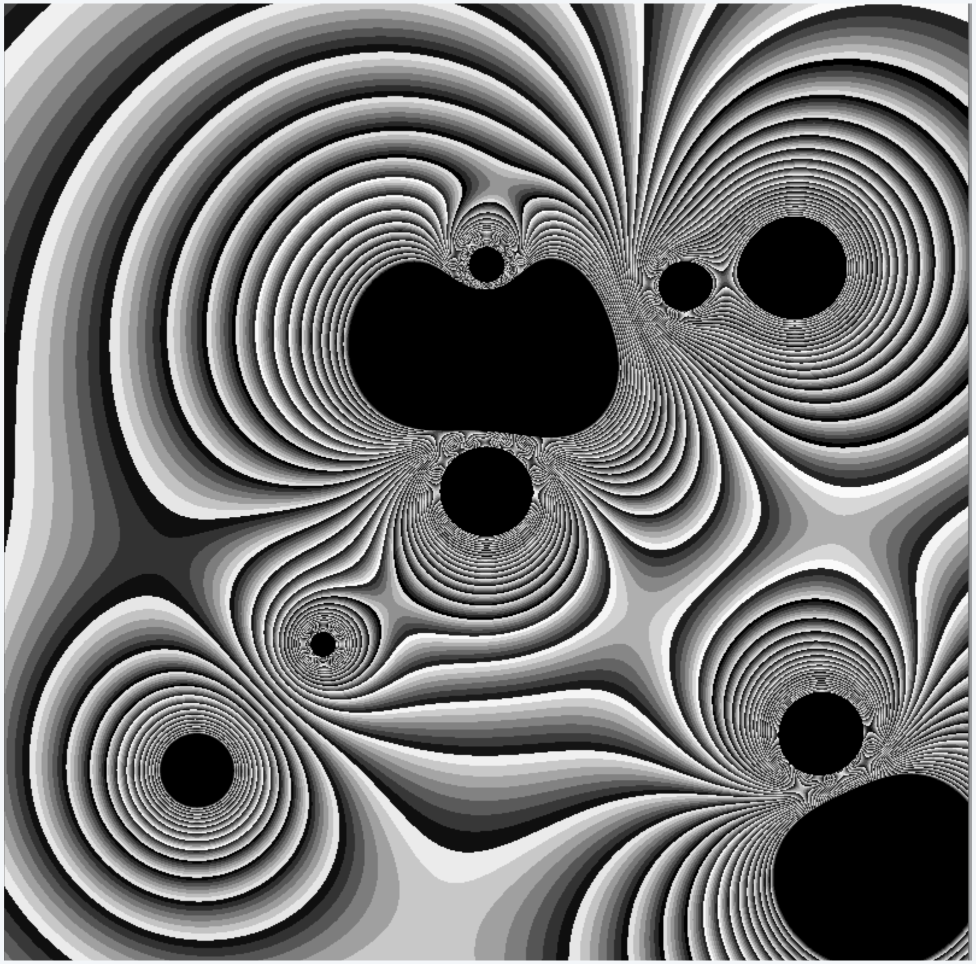
```
% more charges9.txt
9
.51 .63 -100
.50 .50   40
.50 .72   20
.33 .33    5
.20 .20  -10
.70 .70   10
.82 .72   20
.85 .23   30
.90 .12  -50

% java PotentialWithMovingCharge < charges9.txt
```



Potential visualization III: Discontinuous color map

```
public static Color toColor(double V)
{
   V = 128 + V / 2.0e10;
   int t = 0;
   if (V > 255) t = 255;
   else if (V >= 0) t = (int) V;
   t = t*37 % 255
   return new Color(t, t, t);
}
```

V											
t	0	1	2	3	4	5	6	7	8	9	...



Potential visualization IV: Arbitrary discontinuous color map (a bug!)

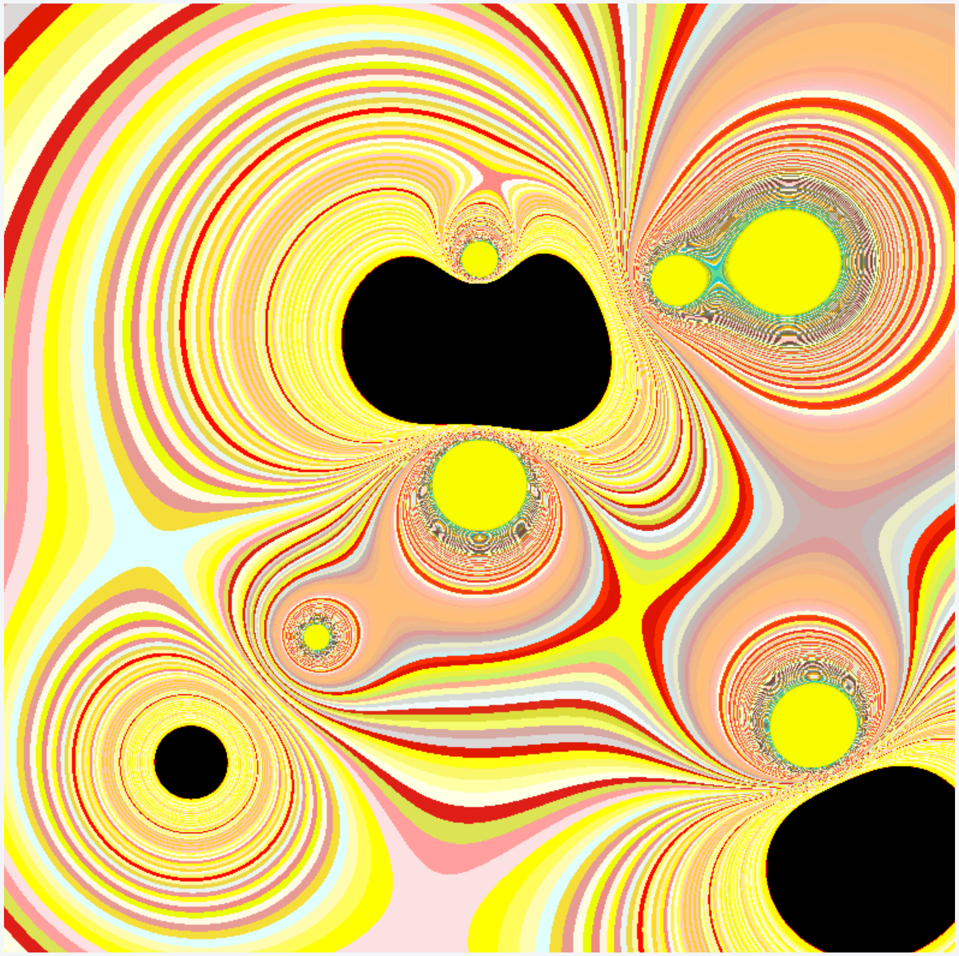
If you are an *artist*

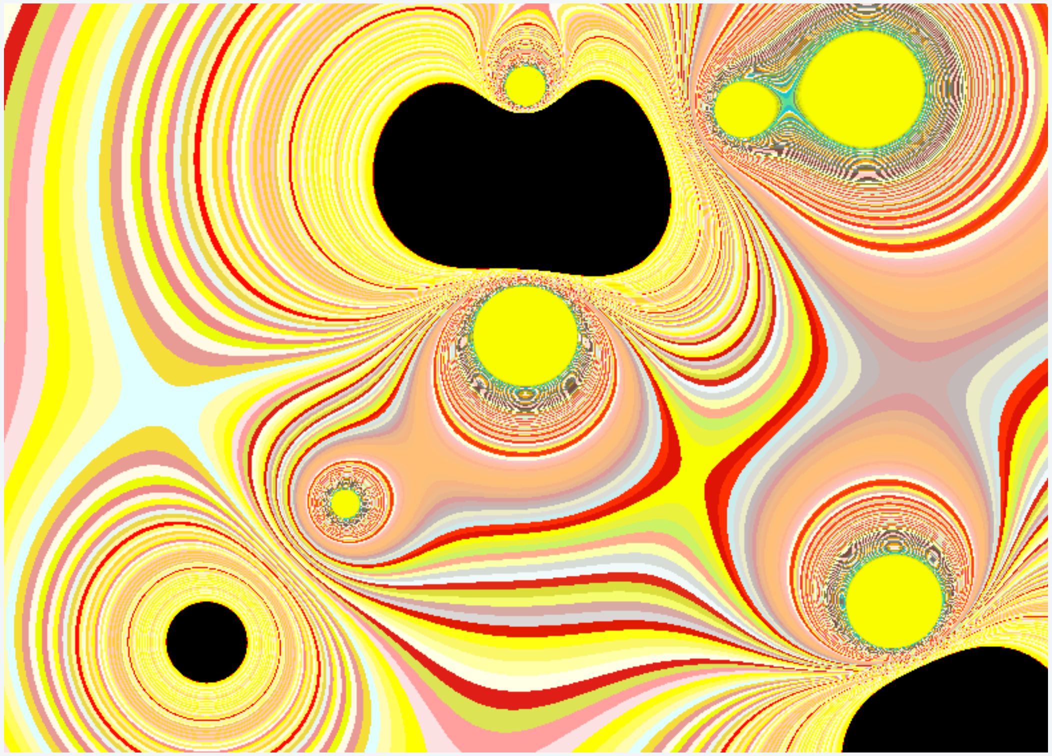
- Choose 255 beautiful colors.
- Put them in an array.
- Index with t to pick a color.

If you are an *computer scientist*

- Play with colors.
- Maybe you'll hit on something...

```
public static Color toColor(double V)
{
   V = 128 + V / 2.0e10;
   int t = 0;
   if (V > 255) t = 255;
   else if (V >= 0) t = (int) V;
   return Color.getHSBColor(t, t, t);
   return new Color(t, t, t);
}
```





Pop quiz 1 on OOP

Q. Fix the serious bug in this code:

```
public class Charge
{
   private double rx, ry;
   private double q;
   public Charge (double x0, double y0, double q0)
   {
      double rx = x0;
      double ry = y0;
      double q = q0;
   }
}
```

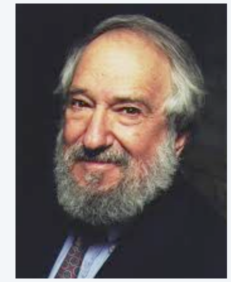
10. Creating Data Types

- Overview
- Point charges
- **Turtle graphics**
- Complex numbers

ADT for turtle graphics

A turtle is an idealized model of a plotting device.

An ADT allows us to write Java programs that manipulate turtles.



Seymour Papert
1928–

Values

position (x, y)	(.5, .5)	(.25, .75)	(.22, .12)
orientation	90°	135°	10°
			

API (operations)

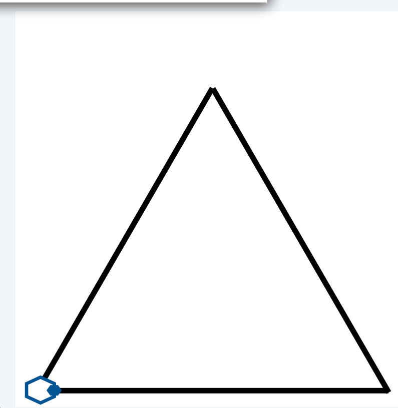
`public class Turtle`	
`Turtle(double x0, double y0, double q0)`	
`void turnLeft(double delta)`	*rotate* `delta` *degrees counterclockwise*
`void goForward(double step)`	*move distance* `step`*, drawing a line*

Turtle graphics implementation: Test client

Best practice. Begin by implementing a simple test client.


```
public static void main(String[] args)
{
   Turtle turtle = new Turtle(0.0, 0.0, 0.0);
   turtle.goForward(1.0);
   turtle.turnLeft(120.0);
   turtle.goForward(1.0);
   turtle.turnLeft(120.0);
   turtle.goForward(1.0);
   turtle.turnLeft(120.0);
}
```

```
% java Turtle
```



Note: Client drew triangle without computing $\sqrt{3}$

What we *expect*, once the implementation is done.

Turtle implementation: Instance variables and constructor

Instance variables define data-type values.

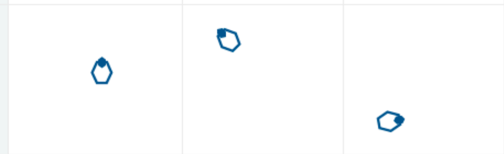
Constructors create and initialize new objects.


```
public class Turtle
{
    private double x, y;
    private double angle;

    public Turtle(double x0, double y0, double a0)
    {
       x = x0;
       y = y0;
       angle = a0;
    }
...
}
```

instance variables are *not* `final`

Values

position (*x*, *y*)	(.5, .5)	(.75, .75)	(.22, .12)
orientation	90°	135°	10°



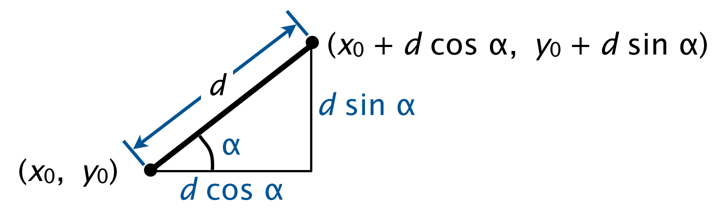
Turtle implementation: Methods

Methods define data-type operations (implement APIs).

```
public class Turtle
{
 ...
   public void turnLeft(double delta)
   { angle += delta; }
   public void goForward(double d)
   {
      double oldx = x;
      double oldy = y;
      x += d * Math.cos(Math.toRadians(angle));
      y += d * Math.sin(Math.toRadians(angle));
      StdDraw.line(oldx, oldy, x, y);
   }
 ...
}
```

API

public class Turtle	
Turtle(double x0, double y0, double q0)	
void turnLeft(double delta)	
void goForward(double step)	



Turtle implementation

text file named `Turtle.java`

```
public class Turtle
{
   private double x, y;
   private double angle;

   public Turtle(double x0, double y0, double a0)
   {
      x = x0;
      y = y0;
      angle = a0;
   }

   public void turnLeft(double delta)
   { angle += delta; }
   public void goForward(double d)
   {
      double oldx = x;
      double oldy = y;
      x += d * Math.cos(Math.toRadians(angle));
      y += d * Math.sin(Math.toRadians(angle));
      StdDraw.line(oldx, oldy, x, y);
   }

   public static void main(String[] args)
   {
     Turtle turtle = new Turtle(0.0, 0.0, 0.0);
     turtle.goForward(1.0); turtle.turnLeft(120.0);
     turtle.goForward(1.0); turtle.turnLeft(120.0);
     turtle.goForward(1.0); turtle.turnLeft(120.0);
   }
}
```

Text

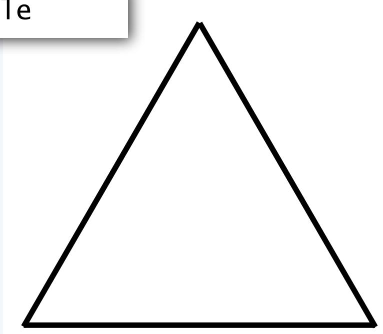
instance variables

constructor

methods

test client

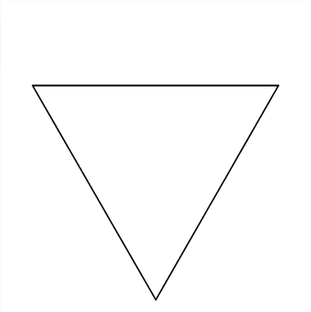
```
% java Turtle
```



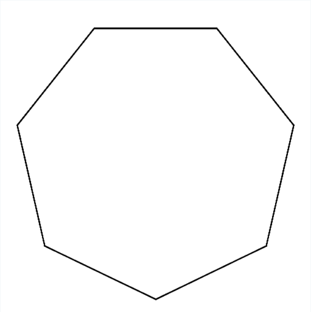
Turtle client: N-gon

```
public class Ngon
{
   public static void main(String[] args)
   {
      int N        = Integer.parseInt(args[0]);
      double angle = 360.0 / N;
      double step  = Math.sin(Math.toRadians(angle/2.0));
      Turtle turtle = new Turtle(0.5, 0, angle/2.0);
      for (int i = 0; i < N; i++)
      {
         turtle.goForward(step);
         turtle.turnLeft(angle);
      }
   }
}
```

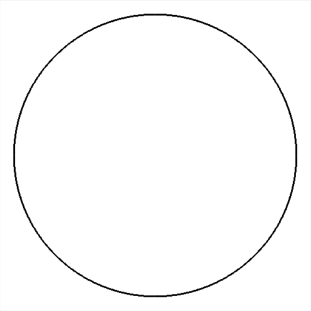
```
% java Ngon 3
```



```
% java Ngon 7
```



```
% java Ngon 1440
```

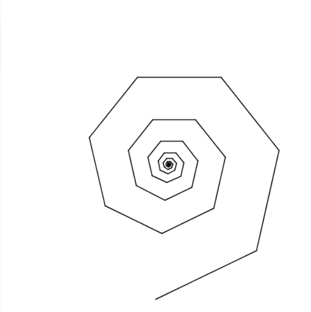


Turtle client: Spira Mirabilis

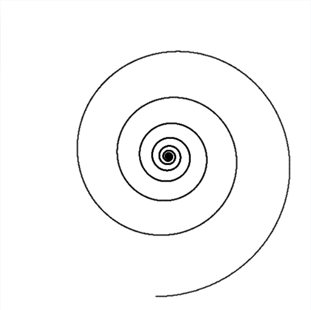
```
public class Spiral
{
   public static void main(String[] args)
   {
      int N        = Integer.parseInt(args[0]);
      double decay = Integer.parseInt(args[1]);
      double angle = 360.0 / N;
      double step  = Math.sin(Math.toRadians(angle/2.0));
      Turtle turtle = new Turtle(0.5, 0, angle/2.0);
      for (int i = 0; i < 10 * N; i++)
      {
         step /= decay;
         turtle.goForward(step);
         turtle.turnLeft(angle);
      }
   }
}
```

```
% java Spiral 3 1.2
```

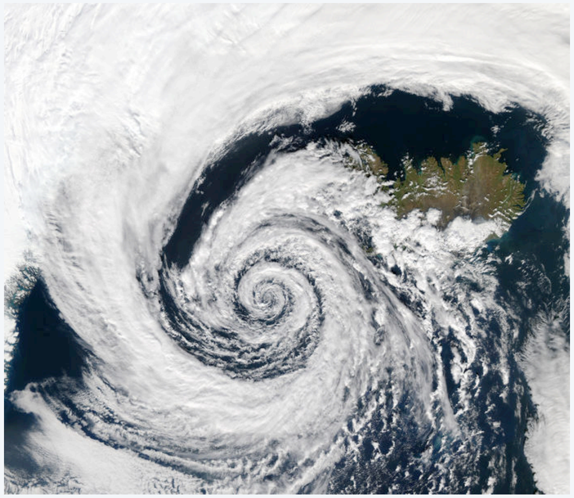
```
% java Spiral 7 1.2
```

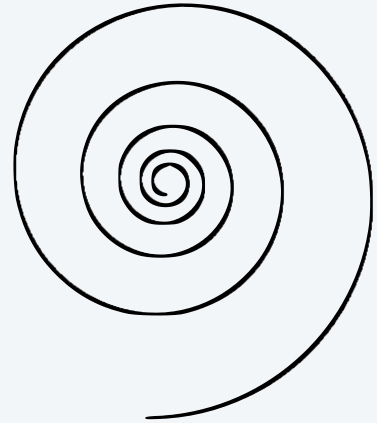


```
% java Spiral 1440 1.0004
```

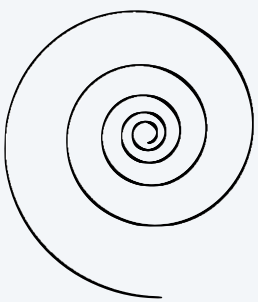
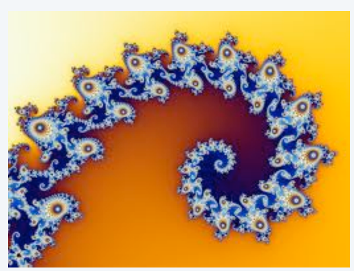


Spira Mirabilis in the wild







10. Creating Data Types

- Overview
- Point charges
- Turtle graphics
- Complex numbers

Crash course in complex numbers

A complex number is a number of the form $a + bi$ where a and b are real and $i \equiv \sqrt{-1}$.

Complex numbers are a *quintessential mathematical abstraction* that have been used for centuries to give insight into real-world problems not easily addressed otherwise.

Leonhard Euler
1707–1783

A. L. Cauchy
1789–1857

To perform *algebraic operations* on complex numbers, use real algebra, replace i^2 by -1 and collect terms.

- Addition example: $(3 + 4i) + (-2 + 3i) = 1 + 7i$.
- Multiplication example: $(3 + 4i) \times (-2 + 3i) = -18 + i$.

Example: $|3 + 4i| = 5$

The *magnitude* or *absolute value* of a complex number $a + bi$ is $|a + bi| = \sqrt{a^2 + b^2}$.

Applications: Signal processing, control theory, quantum mechanics, analysis of algorithms...

ADT for complex numbers

A complex number is a number of the form $a + bi$ where a and b are real and $i \equiv \sqrt{-1}$.

An ADT allows us to write Java programs that manipulate complex numbers.

Values

complex number	$3 + 4i$	$-2 + 2i$
real part	3.0	−2.0
imaginary part	4.0	2.0

API (operations)

`public class Complex`	
`Complex(double real, double imag)`	
`Complex plus(Complex b)`	*sum of this number and* `b`
`Complex times(Complex b)`	*product of this number and* `b`
`double abs()`	*magnitude*
`String toString()`	*string representation*

Complex number data type implementation: Test client

Best practice. Begin by implementing a simple test client.


```
public static void main(String[] args)
{
   Complex a = new Complex( 3.0, 4.0);
   Complex b = new Complex(-2.0, 3.0);
   StdOut.println("a = " + a);
   StdOut.println("b = " + b);
   StdOut.println("a * b = " + a.times(b));
}
```

$$a = v + wi$$
$$b = x + yi$$
$$a \times b = vx + vyi + wxi + wyi^2$$
$$= vx - wy + (vy + wx)i$$

```
% java Complex
a = 3.0 + 4.0i
b = -2.0 + 3.0i
a * b = -18.0 + 1.0i
```

What we *expect*, once the implementation is done.

Complex number data type implementation: Instance variables and constructor

Instance variables define data-type values.

Constructors create and initialize new objects.


```
public class Complex
{
   private final double re;
   private final double im;

   public Complex(double real, double imag)
   {
      re = real;
      im = imag;
   }
...
}
```

instance variables *are* final

Values

complex number	$3 + 4i$	$-2 + 2i$
real part	3.0	–2.0
imaginary part	4.0	2.0

Complex number data type implementation: Methods

Methods define data-type operations (implement APIs).


```
public class Complex
{
 ...
   public Complex plus(Complex b)
   {
      double real = re + b.re;
      double imag = im + b.im;
      return new Complex(real, imag);
   }
   public Complex times(Complex b)
   {
      double real = a.re * b.re - a.im * b.im;
      double imag = a.re * b.im + a.im * b.re;
      return new Complex(real, imag);
   }
   public double abs()
   {  return Math.sqrt(re*re + im*im);  }

   public String toString()
   {  return re + " + " + im + "i";  }
 ...
}
```

$$a = v + wi$$

$$b = x + yi$$

$$a \times b = vx + vyi + wxi + wyi^2$$

$$= vx - wy + (vy + wx)i$$

API

public class Complex	
Complex(double real, double imag)	
Complex plus(Complex b)	*sum of this number and* b
Complex times(Complex b)	*product of this number and* b
double abs()	*magnitude*
String toString()	*string representation*

Complex number data type implementation

text file named `Complex.java`

```
public class Complex
{
   private double re;
   private double im;

   public Complex(double real, double imag)
   {  re = real; im = imag;  }
   public Complex plus(Complex b)
   {
      double real = re + b.re;
      double imag = im + b.im;
      return new Complex(real, imag);
   }
   public Complex times(Complex b)
   {
      double real = a.re * b.re - a.im * b.im;
      double imag = a.re * b.im + a.im * b.re;
      return new Complex(real, imag);
   }
   public double abs()
   {  return Math.sqrt(re*re + im*im);  }
   public String toString()
   {  return re + " + " + im + "i";  }
   public static void main(String[] args)
   {
      Complex a = new Complex( 3.0, 4.0);
      Complex b = new Complex(-2.0, 3.0);
      StdOut.println("a = " + a);
      StdOut.println("b = " + b);
      StdOut.println("a * b = " + a.times(b));
   }
}
```

instance variables

constructor

methods

test client

```
% java Complex
a = 3.0 + 4.0i
b = -2.0 + 3.0i
a * b = -18.0 + 1.0i
```

The Mandelbrot set

The *Mandelbrot set* is a set of complex numbers.

- Represent each complex number $x + yi$ by a point (x, y) in the plane.
- If a point is *in* the set, we color it BLACK.
- If a point is *not* in the set, we color it WHITE.

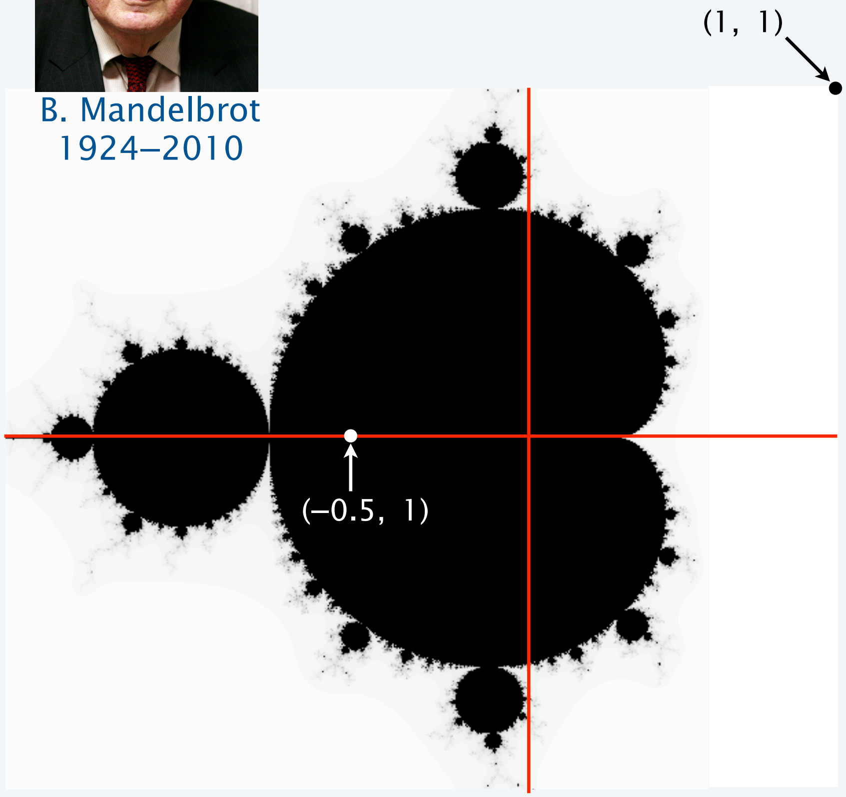
Examples

- *In* the set: $-0.5 + 0i$.
- *Not* in the set: $1 + i$.

Challenge

- No simple formula exists for testing whether a number is in the set.
- Instead, the set is defined by an *algorithm*.

B. Mandelbrot
1924–2010



Determining whether a point is in the Mandelbrot set

Is a complex number z_0 in the set?

- Iterate $z_{t+1} = (z_t)^2 + z_0$.
- If $|z_t|$ *diverges to infinity*, z_0 is *not in* the set.
- If not, z_0 is *in* the set.

t	z_t
0	$-1/2 + 0i$
1	$-1/4 + 0i$
2	$-7/16 + 0i$
3	$-79/256 + 0i$
4	$-26527/65536 + 0i$

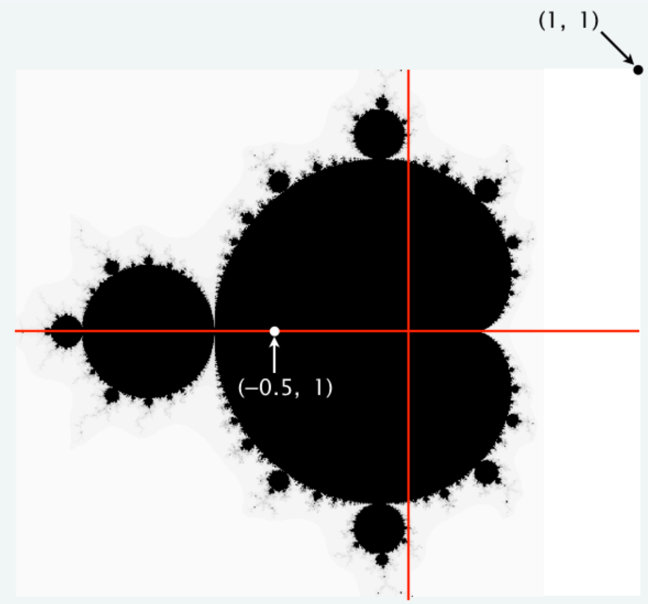
converges to 0

$z = -1/2 + 0i$ is *in* the set

t	z_t
0	$1 + i$
1	$1 + 3i$
2	$-7 + 7i$
3	$1 - 97i$
4	$-9407 - 193i$

diverges to infinity

$z = 1 + i$ is *not* in the set



$(1+i)^2 + (1+i) = 1 + 2i + i^2 + 1 + i = 1+3i$

$(1+3i)^2 + (1+i) = 1 + 6i + 9i^2 + 1 + i = -7+7i$

Plotting the Mandelbrot set

Practical issues

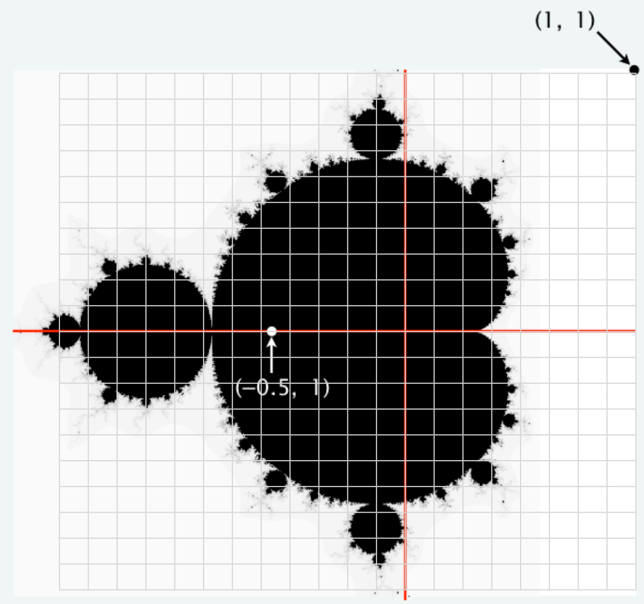
- Cannot plot infinitely many points.
- Cannot iterate infinitely many times.

Approximate solution for first issue

- Sample from an N-by-N grid of points in the plane.
- Zoom in to see more detail (stay tuned!).

Approximate solution for second issue

- Fact: if $| z_t | > 2$ for any t, then z *is not* in the set.
- Pseudo-fact: if $| z_{255} | \leq 2$ then z is "likely" in the set.



Important note: Solutions imply significant computation.

Complex number client: Mandelbrot set visualization (helper method)

Mandelbrot function of a complex number.

- Returns WHITE if the number is not in the set.
- Returns BLACK if the number is (probably) in the set.

```
public static Color mand(Complex z0)
{
   Complex z = z0;
   for (int t = 0; t < 255; t++)
   {
      if (z.abs() > 2.0) return Color.WHITE;
      z = z.times(z);
      z = z.plus(z0);
   }
   return Color.BLACK;
}
```

For a more dramatic picture,

```
return new Color(255-t, 255-t, 255-t)
```

or colors picked from a color table.

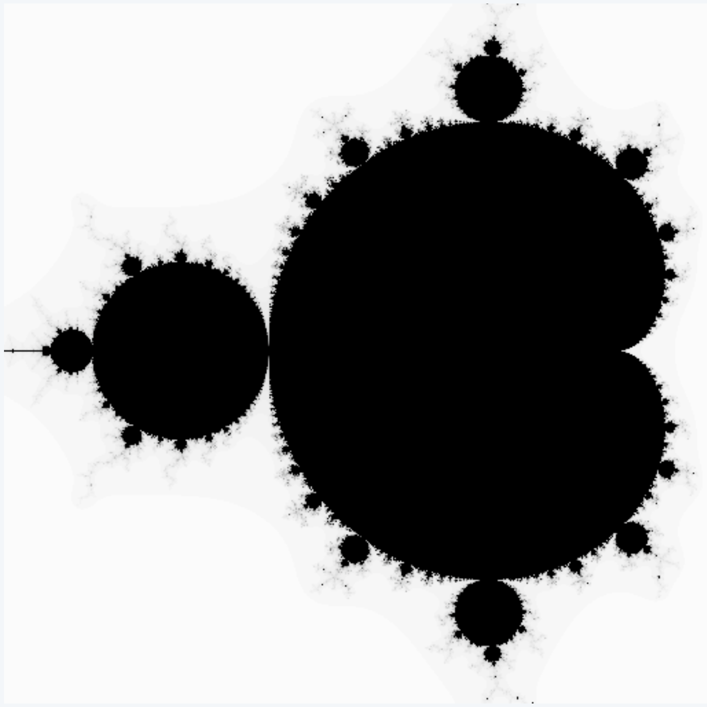
Complex number client: Mandelbrot set visualization

```
import java.awt.Color;
public class Mandelbrot
{
   public static Color mand(Complex z0)
   { // See previous slide. }
   public static void main(String[] args)
   {
      double xc   = Double.parseDouble(args[0]);
      double yc   = Double.parseDouble(args[1]);
      double size = Double.parseDouble(args[2]);
      int N = 512;
      Picture pic = new Picture(N, N);

      for (int col = 0; col < N; col++)
         for (int row = 0; row < N; row++)
         {
            double x0 = xc - size/2 + size*col/N;
            double y0 = yc - size/2 + size*row/N;
            Complex z0 = new Complex(x0, y0);
            Color color = mand(z0);
            pic.set(col, N-1-row, color);
         }
      pic.show();
   }
}
```

scale to screen coordinates

(0, 0) is upper left corner

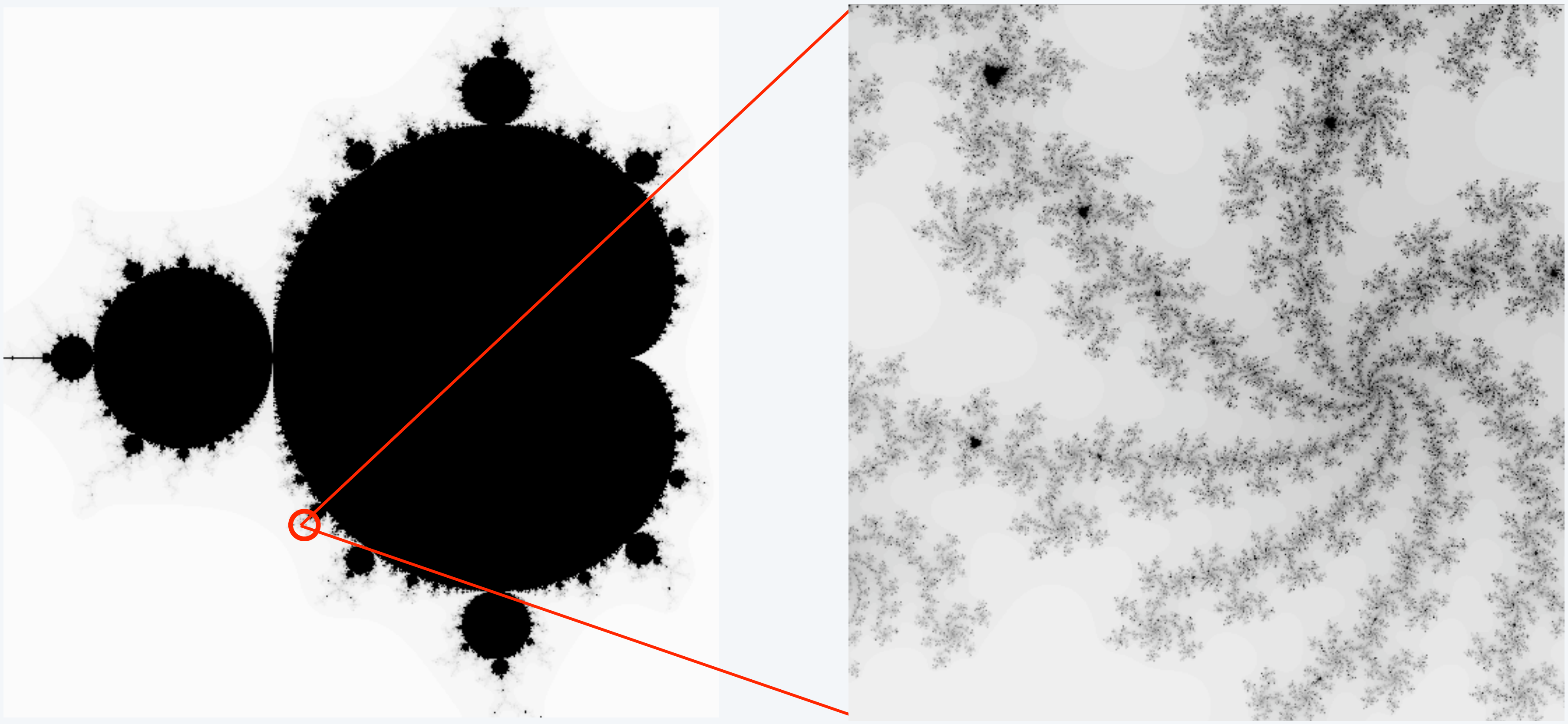
```
% java Mandelbrot -.5 0 2
```



Mandelbrot Set

```
% java Mandelbrot -.5 0 2
```

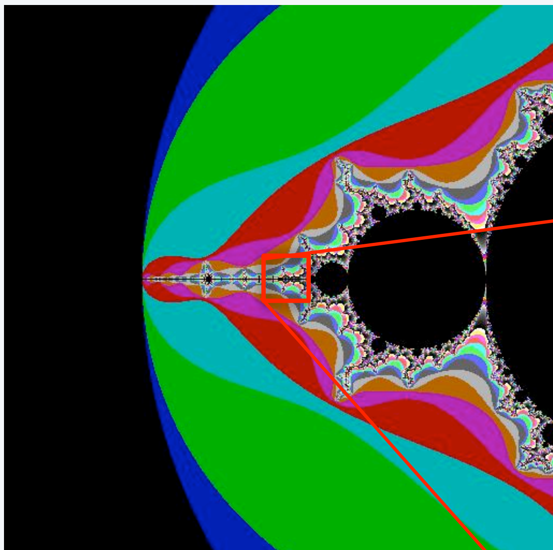
```
% java Mandelbrot .1045 -.637 .01
```



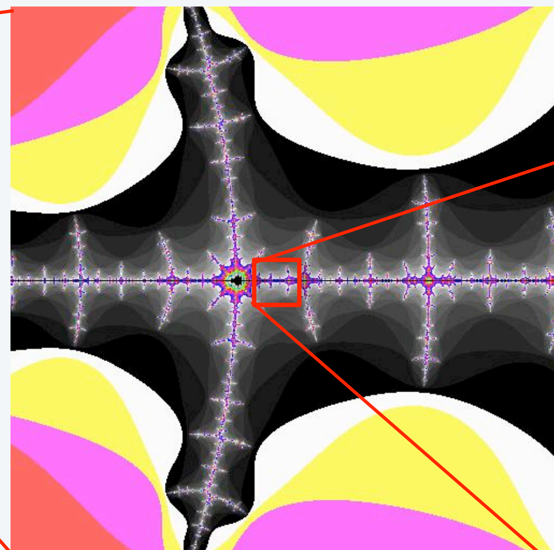
Mandelbrot Set

```
% java ColorMandelbrot -.5 0 2 < mandel.txt
```

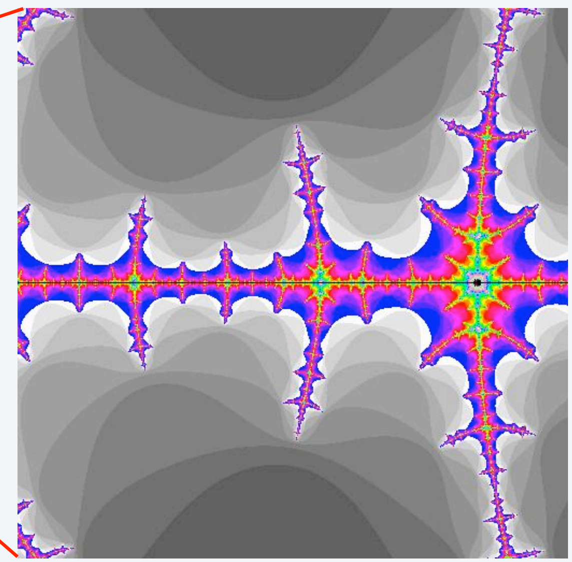
color map

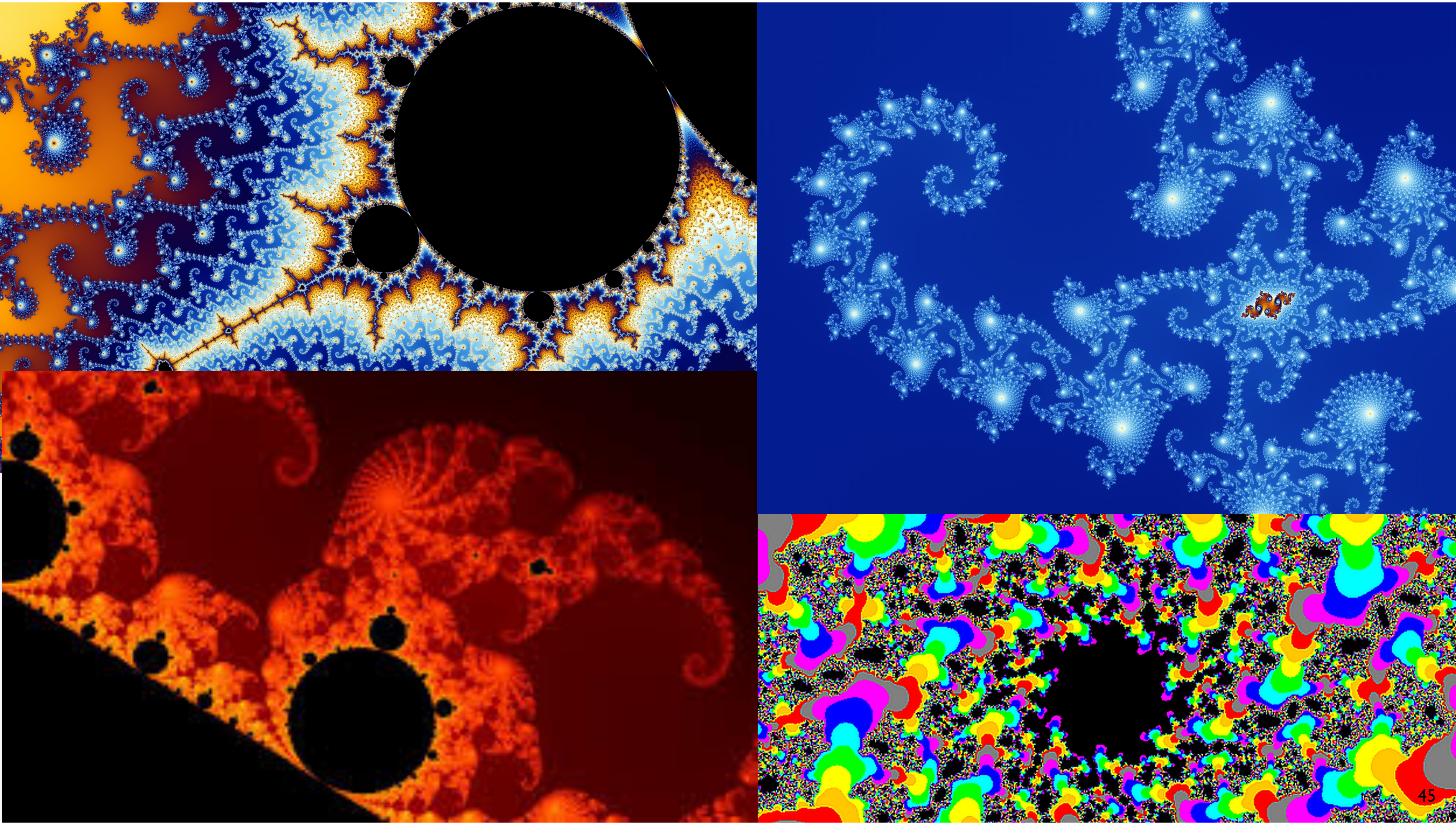


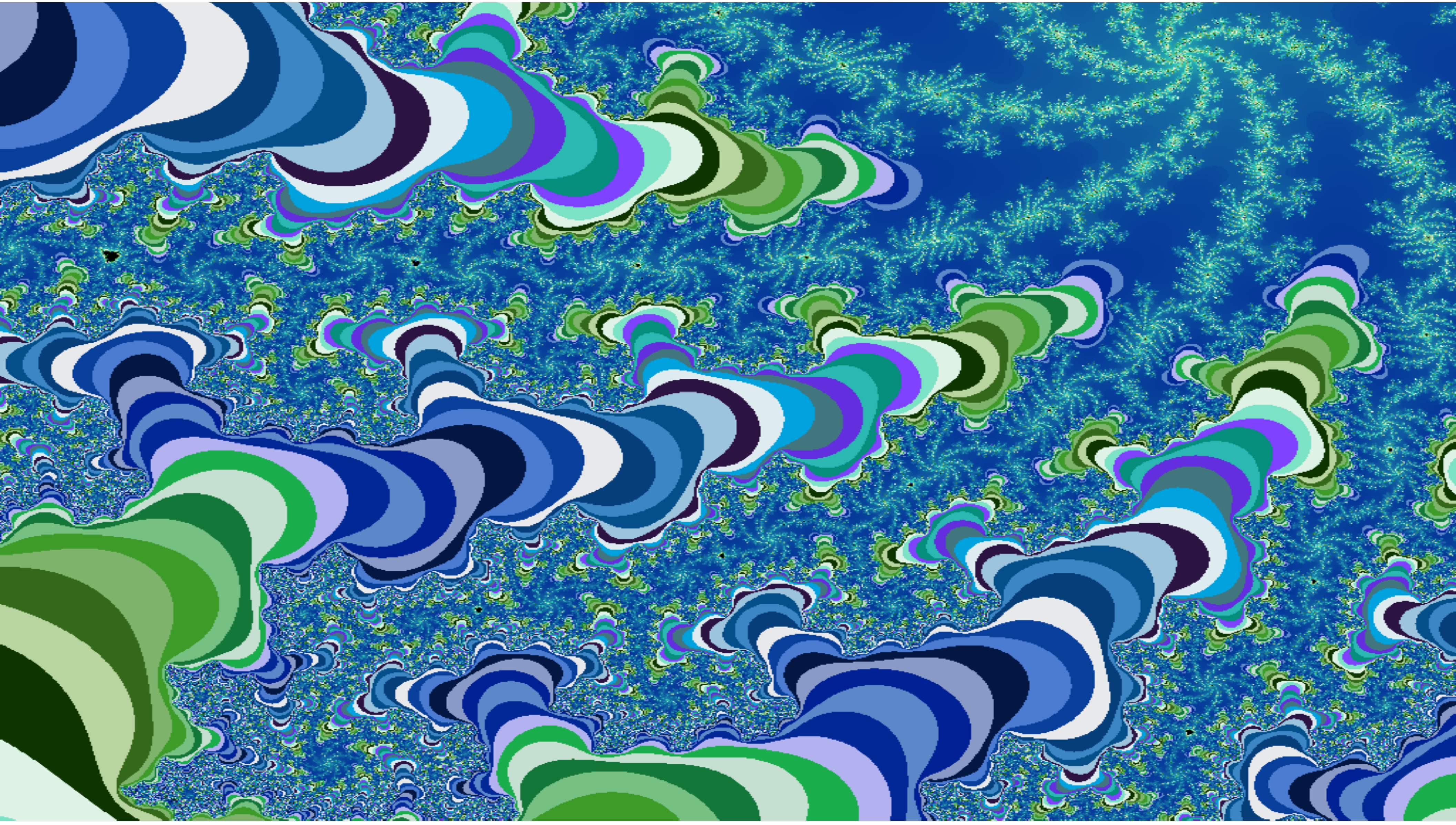
```
-1.5 0 2
```



```
-1.5 0 .002
```



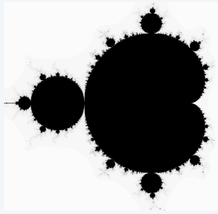




OOP summary

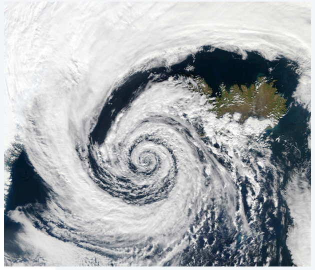
Object-oriented programming (OOP).

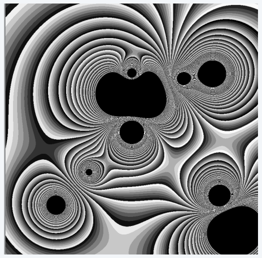
- Create your own data types (sets of values and ops on them).
- Use them in your programs (manipulate *objects*).



OOP helps us simulate the physical world

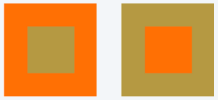
- Java objects model real-world objects.
- Not always easy to make model reflect reality.
- Examples: charged particle, color, sound, genome....





OOP helps us extend the Java language

- Java doesn't have a data type for every possible application.
- Data types enable us to add our own abstractions.
- Examples: complex, vector, polynomial, matrix, picture....



T	A	G	A	T	G	**T**	**G**	**C**	T	A	G	C

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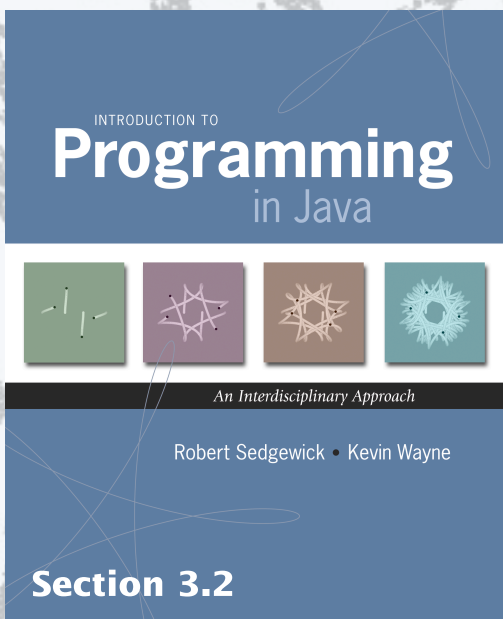

http://introcs.cs.princeton.edu

10. Creating Data Types