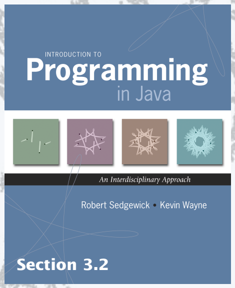

Section 3.2

http://introcs.cs.princeton.edu

10. Creating Data Types

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10. Creating Data Types

- Overview
- Point charges
- Turtle graphics
- Complex numbers

CS.10.A.CreatingDTs.Overview

Object-oriented programming (OOP)

Object-oriented programming (OOP).
- Create your own data types.
- Use them in your programs (manipulate *objects*).

An object holds a data type value.
Variable names refer to objects.

Examples (stay tuned for details)

data type	*set of values*	*examples of operations*	
`Color`	three 8-bit integers	get red component, brighten	🟥⬛⬜🟦
`Picture`	2D array of colors	get/set color of pixel (i, j)	
`String`	sequence of characters	length, substring, compare	C A T A G C G C

An abstract data type is a data type whose representation is *hidden from the client.*

Impact: We can use ADTs without knowing implementation details.
- Previous lecture: how to write client programs for several useful ADTs
- This lecture: how to implement your own ADTs

Implementing a data type

To create a data type, you need provide code that
- Defines the set of values (instance variables).
- Implements operations on those values (methods).
- Creates and initialize new objects (constructors).

In Java, a data-type implementation is known as a *class.*

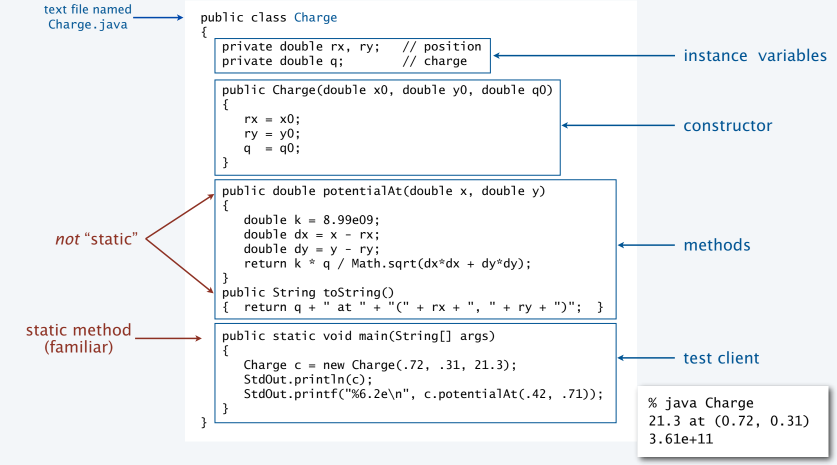
Instance variables
- Declarations associate variable names with types.
- Set of type values is "set of values".

Methods
- Like static methods.
- Can refer to instance variables.

Constructors
- Methods with the same name as the type.
- No return type declaration.
- Invoked by new, returns object of the type.

A Java class

Anatomy of a Class



text file named `Charge.java`

```
public class Charge
{
   private double rx, ry;   // position
   private double q;        // charge

   public Charge(double x0, double y0, double q0)
   {
      rx = x0;
      ry = y0;
      q  = q0;
   }

   public double potentialAt(double x, double y)
   {
      double k = 8.99e09;
      double dx = x - rx;
      double dy = y - ry;
      return k * q / Math.sqrt(dx*dx + dy*dy);
   }
   public String toString()
   {  return q + " at " + "(" + rx + ", " + ry + ")";  }

   public static void main(String[] args)
   {
      Charge c = new Charge(.72, .31, 21.3);
      StdOut.println(c);
      StdOut.printf("%6.2e\n", c.potentialAt(.42, .71));
   }
}
```

instance variables

constructor

methods

test client

not "static"

static method (familiar)

```
% java Charge
21.3 at (0.72, 0.31)
3.61e+11
```

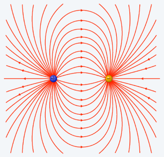
10. Creating Data Types

- Overview
- **Point charges**
- Turtle graphics
- Complex numbers

ADT for point charges

A point charge is an idealized model of a particle that has an electric charge.

An ADT allows us to write Java programs that manipulate point charges.



Values

	examples	
position (x, y)	(.53, .63)	(.13, .94)
electrical charge	21.3	81.9

API (operations)

public class Charge	
Charge(double x0, double y0, double q0)	
double potentialAt(double x, double y)	*electric potential at (x, y) due to charge*
String toString()	*string representation of this charge*

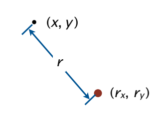
Crash course on electric potential

Electric potential is a measure of the effect of a point charge on its surroundings.

- It increases in proportion to the charge value.
- It decreases in proportion to the *inverse of the distance* from the charge.

Mathematically,

- Suppose a point charge c is located at (r_x, r_y) and has charge q.
- Let r be the distance between (x, y) and (r_x, r_y)
- Let $V_c(x,y)$ be the potential at (x, y) due to c.
- Then $\boxed{V_c(x,y) = k\frac{q}{r}}$ where $k = 8.99\times10^9$ is a normalizing factor.



Q. What happens when multiple charges are present?

A. The potential at a point is the *sum* of the potentials due to the individual charges.

Note: Similar laws hold in many other situations. ← Example. *N*-body (an inverse *square* law).

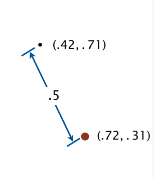
Point charge implementation: Test client

Best practice. Begin by implementing a simple test client.

```
public static void main(String[] args)
{
   Charge c = new Charge(.72, .31, 20.1);
   StdOut.println(c);
   StdOut.printf("%6.2e\n", c.potentialAt(.42, .71));
}
```

$$V_c(x,y) = k\frac{q}{r}$$

$$r = \sqrt{(r_x - x)^2 + (r_y - y)^2}$$
$$= \sqrt{.3^2 + .4^2} = .5$$
$$V_c(.42,.71) = 8.99 \times 10^9 \frac{20.1}{.5}$$
$$= 3.6 \times 10^{11}$$



```
% java Charge
21.3 at (0.72, 0.31)
3.61e+11
```

What we *expect*, once the implementation is done.

Point charge implementation: Instance variables

Instance variables define data-type values.

Values		*examples*	
	position (*x*, *y*)	(.53, .63)	(.13, .94)
	electrical charge	21.3	81.9

```
public class Charge
{
   private final double rx, ry;
   private final double q;
...
}
```

Modifiers control access.

- `private` denies clients access and therefore makes data type abstract.
- `final` disallows any change in value and therefore makes variable *immutable.*

stay tuned

Key to OOP. Each *object* has instance-variable values.

Point charge implementation: Constructor

Constructors create and initialize new objects.

```
public class Charge
{
...
  public Charge(double x0, double y0, double q0)
  {
     rx = x0;
     ry = y0;
     q  = q0;
  }
...
}
```

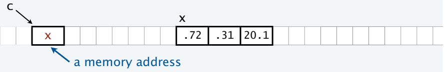
references to instance variables, which are *not* declared within the constructor

Clients use new to invoke constructors.

- Pass arguments as in a method call.
- Return value is reference to new object.

Possible memory representation of

```
Charge c = new Charge(.72, .31, 20.1);
```



Point charge implementation: Methods

Methods define data-type operations (implement APIs).

API

`public class Charge`	
`Charge(double x0, double y0, double q0)`	
`double potentialAt(double x, double y)`	*electric potential at (x, y) due to charge*
`String toString()`	*string representation of this charge*

```
public class Charge
{
  ...
   public double potentialAt(double x, double y)
   {
      double k = 8.99e09;
      double dx = x - rx;
      double dy = y - ry;
      return k * q / Math.sqrt(dx*dx + dy*dy);
   }

   public String toString()
   {  return q + " at " + "(" + rx + ", " + ry + ")";  }
  ...
}
```

$$V_C(x,y) = k\frac{q}{r}$$

Key to OOP. An instance variable reference in a class method *refers to the value for the object that was used to invoke the method.*

Point charge implementation

text file named
Charge.java

```
public class Charge
{
   private double rx, ry;   // position
   private double q;        // charge
```

instance variables

```
   public Charge(double x0, double y0, double q0)
   {
      rx = x0;
      ry = y0;
      q  = q0;
   }
```

constructor

```
   public double potentialAt(double x, double y)
   {
      double k = 8.99e09;
      double dx = x - rx;
      double dy = y - ry;
      return k * q / Math.sqrt(dx*dx + dy*dy);
   }
   public String toString()
   {  return q + " at " + "(" + rx + ", " + ry + ")";  }
```

methods

```
   public static void main(String[] args)
   {
      Charge c = new Charge(.72, .31, 20.1);
      System.out.println(c);
      StdOut.printf("%6.2e\n", c.potentialAt(.42, .71));
   }
}
```

test client

```
% java Charge
21.3 at (0.72, 0.31)
3.61e+11
```

Point charge client: Potential visualization (helper methods)

Read point charges from StdIn.
- Uses Charge like any other type.
- Returns an array of Charges

```
public static Charge[] readCharges()
{
   int N = StdIn.readInt();
   Charge[] a = new Charge[N];
   for (int i = 0; i < N; k++)
   {
      double x0 = StdIn.readDouble();
      double y0 = StdIn.readDouble();
      double q0 = StdIn.readDouble();
      a[i] = new Charge(x0, y0, q0);
   }
   return a;
}
```

Convert potential values to a color.
- Convert V to an 8-bit integer.
- Use grayscale.

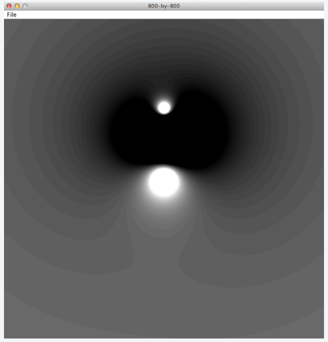
```
public static Color toColor(double V)
{
   V = 128 + V / 2.0e10;
   int t = 0;
   if (V > 255) t = 255;
   else if (V >= 0) t = (int) V;
   return new Color(t, t, t);
}
```

V										
t	0	1	...	37	38	39	...	128	...	254	255
	█	█		▓	▓	▓		▒		□	□

Point charge client: Potential visualization

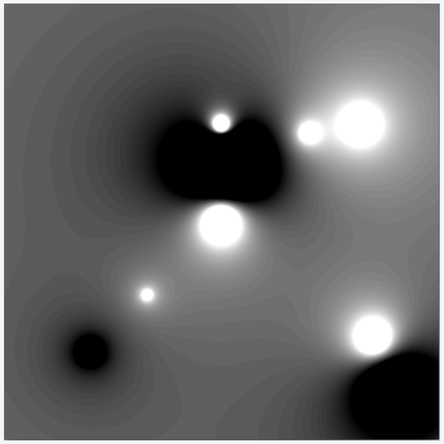
```
import java.awt.Color;
public class Potential
{
   public static Charge[] readCharges()
   { // See previous slide. }
   public static Color toColor()
   { // See previous slide. }
   public static void main(String[] args)
   {
      Charge[] a = readCharges();
      int SIZE = 800;
      Picture pic = new Picture(SIZE, SIZE);
      for (int col = 0; col < SIZE; col++)
         for (int row = 0; row < SIZE; row++)
         {
            double V = 0.0;
            for (int k = 0; k < a.length; k++)
            {
               double x = 1.0 * col / SIZE;
               double y = 1.0 * row / SIZE;
               V += a[k].potentialAt(x, y);
            }
            pic.set(col, SIZE-1-row, toColor(V));
        }
      pic.show();
   }
}
```

```
% more charges3.txt
3
.51 .63 -100
.50 .50   40
.50 .72   20
% java Potential < charges3.txt
```



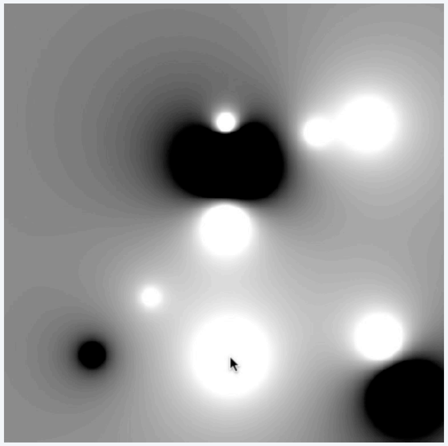
Potential visualization I

```
% more charges9.txt
9
.51 .63 -100
.50 .50   40
.50 .72   20
.33 .33    5
.20 .20  -10
.70 .70   10
.82 .72   20
.85 .23   30
.90 .12  -50

% java Potential < charges9.txt
```



Potential visualization II: A moving charge

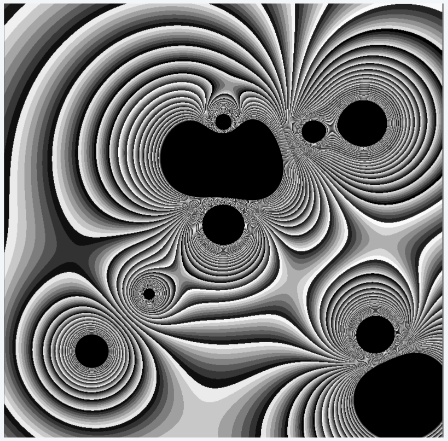
```
% more charges9.txt
9
.51 .63 -100
.50 .50   40
.50 .72   20
.33 .33    5
.20 .20  -10
.70 .70   10
.82 .72   20
.85 .23   30
.90 .12  -50

% java PotentialWithMovingCharge < charges9.txt
```



Potential visualization III: Discontinuous color map

```
public static Color toColor(double V)
{
   V = 128 + V / 2.0e10;
   int t = 0;
   if (V > 255) t = 255;
   else if (V >= 0) t = (int) V;
   t = t*37 % 255
   return new Color(t, t, t);
}
```

V											
t	0	1	2	3	4	5	6	7	8	9	...
	█	▓	▓	▒	▒	░	░	█	▓	▓	

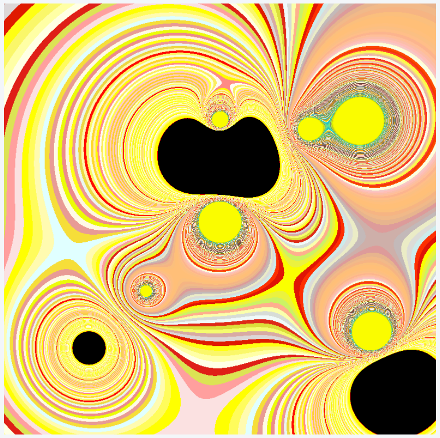


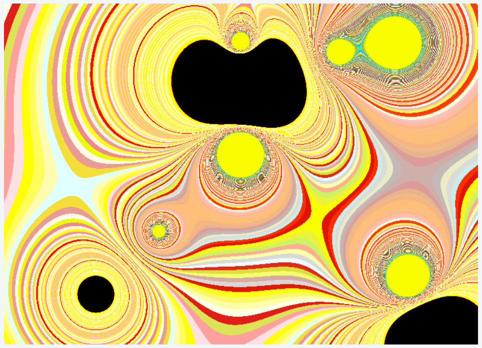
Potential visualization IV: Arbitrary discontinuous color map (a bug!)

If you are an *artist*
- Choose 255 beautiful colors.
- Put them in an array.
- Index with t to pick a color.

If you are an *computer scientist*
- Play with colors.
- Maybe you'll hit on something...

```
public static Color toColor(double V)
{
   V = 128 + V / 2.0e10;
   int t = 0;
   if (V > 255) t = 255;
   else if (V >= 0) t = (int) V;
   return Color.getHSBColor(t, t, t);
   return new Color(t, t, t);
}
```





Pop quiz 1 on OOP

Q. Fix the serious bug in this code:

```
public class Charge
{
   private double rx, ry;
   private double q;
   public Charge (double x0, double y0, double q0)
   {
      double rx = x0;
      double ry = y0;
      double q = q0;
   }
}
```

COMPUTER SCIENCE
SEDGEWICK / WAYNE

10. Creating Data Types

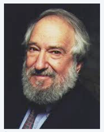
- Overview
- Point charges
- **Turtle graphics**
- Complex numbers

CS.10.C.CreatingDTs.Turtles

ADT for turtle graphics

A turtle is an idealized model of a plotting device.

An ADT allows us to write Java programs that manipulate turtles.



Seymour Papert
1928–

Values

position (*x*, *y*)	(.5, .5)	(.25, .75)	(.22, .12)
orientation	90°	135°	10°
			

API (operations)

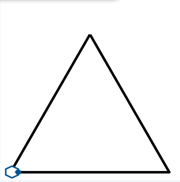
public class Turtle	
Turtle(double x0, double y0, double q0)	
void turnLeft(double delta)	*rotate* `delta` *degrees counterclockwise*
void goForward(double step)	*move distance* `step`*, drawing a line*

Turtle graphics implementation: Test client

Best practice. Begin by implementing a simple test client.

```
public static void main(String[] args)
{
   Turtle turtle = new Turtle(0.0, 0.0, 0.0);
   turtle.goForward(1.0);
   turtle.turnLeft(120.0);
   turtle.goForward(1.0);
   turtle.turnLeft(120.0);
   turtle.goForward(1.0);
   turtle.turnLeft(120.0);
}
```

```
% java Turtle
```



Note: Client drew triangle without computing $\sqrt{3}$

What we *expect*, once the implementation is done.

Turtle implementation: Instance variables and constructor

Instance variables define data-type values.

Constructors create and initialize new objects.

```
public class Turtle
{
   private double x, y;
   private double angle;

   public Turtle(double x0, double y0, double a0)
   {
      x = x0;
      y = y0;
      angle = a0;
   }
...
}
```

instance variables are *not* `final`

Values

position (*x*, *y*)	(.5, .5)	(.75, .75)	(.22, .12)
orientation	90°	135°	10°
			

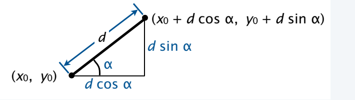
Turtle implementation: Methods

Methods define data-type operations (implement APIs).

```
public class Turtle
{
 ...
   public void turnLeft(double delta)
   { angle += delta; }
   public void goForward(double d)
   {
      double oldx = x;
      double oldy = y;
      x += d * Math.cos(Math.toRadians(angle));
      y += d * Math.sin(Math.toRadians(angle));
      StdDraw.line(oldx, oldy, x, y);
   }
 ...
}
```

API

`public class Turtle`	
`Turtle(double x0, double y0, double q0)`	
`void turnLeft(double delta)`	
`void goForward(double step)`	



Turtle implementation

text file named `Turtle.java`

```
public class Turtle
{
   private double x, y;
   private double angle;

   public Turtle(double x0, double y0, double a0)
   {
      x = x0;
      y = y0;
      angle = a0;
   }

   public void turnLeft(double delta)
   { angle += delta; }
   public void goForward(double d)
   {
      double oldx = x;
      double oldy = y;
      x += d * Math.cos(Math.toRadians(angle));
      y += d * Math.sin(Math.toRadians(angle));
      StdDraw.line(oldx, oldy, x, y);
   }

   public static void main(String[] args)
   {
     Turtle turtle = new Turtle(0.0, 0.0, 0.0);
     turtle.goForward(1.0); turtle.turnLeft(120.0);
     turtle.goForward(1.0); turtle.turnLeft(120.0);
     turtle.goForward(1.0); turtle.turnLeft(120.0);
   }
}
```

Text

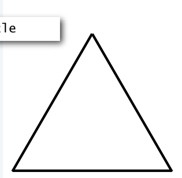
instance variables

constructor

methods

test client

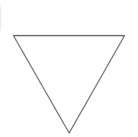
```
% java Turtle
```



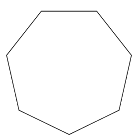
Turtle client: N-gon

```
public class Ngon
{
   public static void main(String[] args)
   {
      int N        = Integer.parseInt(args[0]);
      double angle = 360.0 / N;
      double step  = Math.sin(Math.toRadians(angle/2.0));
      Turtle turtle = new Turtle(0.5, 0, angle/2.0);
      for (int i = 0; i < N; i++)
      {
         turtle.goForward(step);
         turtle.turnLeft(angle);
      }
   }
}
```

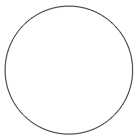
```
% java Ngon 3
```



```
% java Ngon 7
```



```
% java Ngon 1440
```

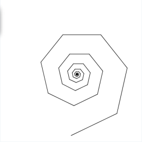


Turtle client: Spira Mirabilis

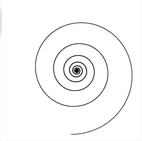
```
public class Spiral
{
   public static void main(String[] args)
   {
      int N        = Integer.parseInt(args[0]);
      double decay = Integer.parseInt(args[1]);
      double angle = 360.0 / N;
      double step  = Math.sin(Math.toRadians(angle/2.0));
      Turtle turtle = new Turtle(0.5, 0, angle/2.0);
      for (int i = 0; i < 10 * N; i++)
      {
         step /= decay;
         turtle.goForward(step);
         turtle.turnLeft(angle);
      }
   }
}
```

`% java Spiral 3 1.2`

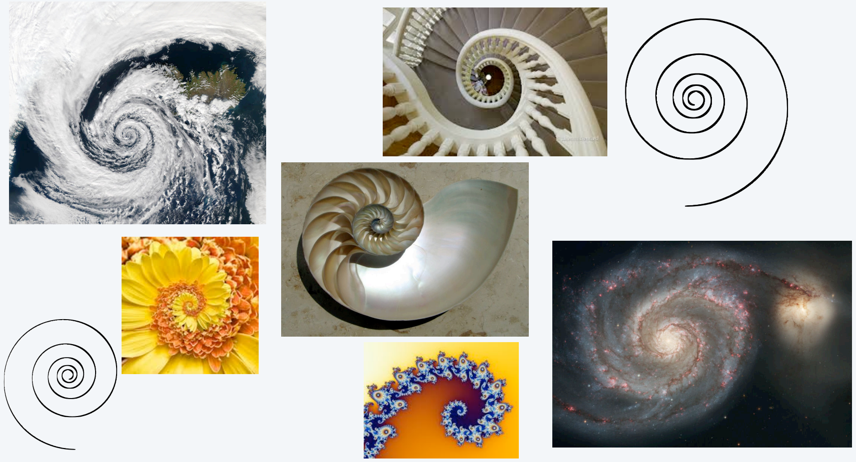
`% java Spiral 7 1.2`



`% java Spiral 1440 1.0004`



Spira Mirabilis in the wild



COMPUTER SCIENCE
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10. Creating Data Types

- Overview
- Point charges
- Turtle graphics
- **Complex numbers**

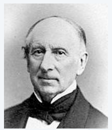
CS.10.D.CreatingDTs.Mandelbrot

Crash course in complex numbers

A complex number is a number of the form $a + bi$ where a and b are real and $i \equiv \sqrt{-1}$.

Complex numbers are a *quintessential mathematical abstraction* that have been used for centuries to give insight into real-world problems not easily addressed otherwise.

Leonhard Euler
1707–1783



A. L. Cauchy
1789–1857

To perform *algebraic operations* on complex numbers, use real algebra, replace i^2 by -1 and collect terms.

- Addition example: $(3 + 4i) + (-2 + 3i) = 1 + 7i$.
- Multiplication example: $(3 + 4i) \times (-2 + 3i) = -18 + i$.

Example: $|\, 3 + 4i \,| = 5$

The *magnitude* or *absolute value* of a complex number $a + bi$ is $|a + bi| = \sqrt{a^2 + b^2}$.

Applications: Signal processing, control theory, quantum mechanics, analysis of algorithms...

ADT for complex numbers

A complex number is a number of the form $a + bi$ where a and b are real and $i \equiv \sqrt{-1}$.

An ADT allows us to write Java programs that manipulate complex numbers.

Values

complex number	$3 + 4i$	$-2 + 2i$
real part	3.0	−2.0
imaginary part	4.0	2.0

API (operations)

`public class Complex`	
`Complex(double real, double imag)`	
`Complex plus(Complex b)`	*sum of this number and* `b`
`Complex times(Complex b)`	*product of this number and* `b`
`double abs()`	*magnitude*
`String toString()`	*string representation*

Complex number data type implementation: Test client

Best practice. Begin by implementing a simple test client.

```
public static void main(String[] args)
{
   Complex a = new Complex( 3.0, 4.0);
   Complex b = new Complex(-2.0, 3.0);
   StdOut.println("a = " + a);
   StdOut.println("b = " + b);
   StdOut.println("a * b = " + a.times(b));
}
```

$$
\begin{aligned}
a &= v + wi \\
b &= x + yi \\
a \times b &= vx + vyi + wxi + wyi^2 \\
&= vx - wy + (vy + wx)i
\end{aligned}
$$

```
% java Complex
a = 3.0 + 4.0i
b = -2.0 + 3.0i
a * b = -18.0 + 1.0i
```

What we *expect*, once the implementation is done.

Complex number data type implementation: Instance variables and constructor

Instance variables define data-type values.

Constructors create and initialize new objects.

```
public class Complex
{
   private final double re;
   private final double im;

   public Complex(double real, double imag)
   {
      re = real;
      im = imag;
   }
...
}
```

instance variables are `final`

Values

complex number	$3 + 4i$	$-2 + 2i$
real part	3.0	−2.0
imaginary part	4.0	2.0

Complex number data type implementation: Methods

Methods define data-type operations (implement APIs).

```
public class Complex
{
 ...
   public Complex plus(Complex b)
   {
      double real = re + b.re;
      double imag = im + b.im;
      return new Complex(real, imag);
   }
   public Complex times(Complex b)
   {
      double real = a.re * b.re - a.im * b.im;
      double imag = a.re * b.im + a.im * b.re;
      return new Complex(real, imag);
   }
   public double abs()
   {  return Math.sqrt(re*re + im*im);  }

   public String toString()
   {  return re + " + " + im + "i";  }
 ...
}
```

$$
\begin{aligned}
a &= v + wi \\
b &= x + yi \\
a \times b &= vx + vyi + wxi + wyi^2 \\
&= vx - wy + (vy + wx)i
\end{aligned}
$$

API

`public class Complex`	
`Complex(double real, double imag)`	
`Complex plus(Complex b)`	*sum of this number and* `b`
`Complex times(Complex b)`	*product of this number and* `b`
`double abs()`	*magnitude*
`String toString()`	*string representation*

Complex number data type implementation

text file named
`Complex.java`

```
public class Complex
{
   private double re;
   private double im;

   public Complex(double real, double imag)
   {  re = real; im = imag;  }
   public Complex plus(Complex b)
   {
      double real = re + b.re;
      double imag = im + b.im;
      return new Complex(real, imag);
   }
   public Complex times(Complex b)
   {
      double real = a.re * b.re - a.im * b.im;
      double imag = a.re * b.im + a.im * b.re;
      return new Complex(real, imag);
   }
   public double abs()
   {  return Math.sqrt(re*re + im*im);  }
   public String toString()
   {  return re + " + " + im + "i";  }
   public static void main(String[] args)
   {
      Complex a = new Complex( 3.0, 4.0);
      Complex b = new Complex(-2.0, 3.0);
      StdOut.println("a = " + a);
      StdOut.println("b = " + b);
      StdOut.println("a * b = " + a.times(b));
   }
}
```

instance variables

constructor

methods

test client

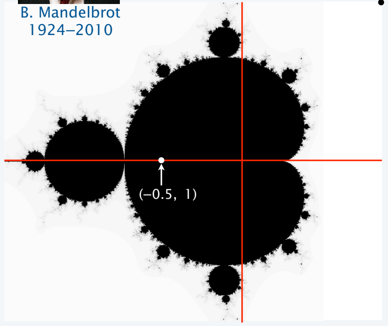
```
% java Complex
a = 3.0 + 4.0i
b = -2.0 + 3.0i
a * b = -18.0 + 1.0i
```

The Mandelbrot set

The *Mandelbrot set* is a set of complex numbers.

- Represent each complex number $x + yi$ by a point (x, y) in the plane.
- If a point is *in* the set, we color it BLACK.
- If a point is *not* in the set, we color it WHITE.

B. Mandelbrot
1924–2010



Examples

- *In* the set: $-0.5 + 0i$.
- *Not* in the set: $1 + i$.

Challenge

- No simple formula exists for testing whether a number is in the set.
- Instead, the set is defined by an *algorithm*.

Determining whether a point is in the Mandelbrot set

Is a complex number z_0 in the set?

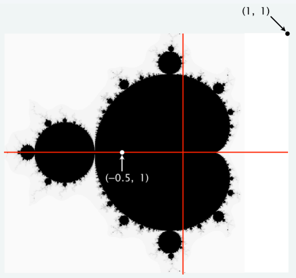
- Iterate $z_{t+1} = (z_t)^2 + z_0$.
- If $|z_t|$ *diverges to infinity*, z_0 is *not in* the set.
- If not, z_0 is *in* the set.

t	z_t
0	$-1/2 + 0i$
1	$-1/4 + 0i$
2	$-7/16 + 0i$
3	$-79/256 + 0i$
4	$-26527/65536 + 0i$

converges to 0
$z = -1/2 + 0i$ is *in* the set

t	z_t
0	$1 + i$
1	$1 + 3i$
2	$-7 + 7i$
3	$1 - 97i$
4	$-9407 - 193i$

diverges to infinity
$z = 1 + i$ is *not* in the set



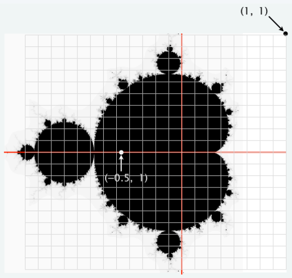
$(1+i)^2 + (1+i) = 1 + 2i + i^2 + 1 + i = 1+3i$

$(1+3i)^2 + (1+i) = 1 + 6i + 9i^2 + 1 + i = -7+7i$

Plotting the Mandelbrot set

Practical issues

- Cannot plot infinitely many points.
- Cannot iterate infinitely many times.



Approximate solution for first issue

- Sample from an N-by-N grid of points in the plane.
- Zoom in to see more detail (stay tuned!).

Approximate solution for second issue

- Fact: if $| z_t | > 2$ for any t, then z *is not* in the set.
- Pseudo-fact: if $| z_{255} | \leq 2$ then z is "likely" in the set.

Important note: Solutions imply significant computation.

Complex number client: Mandelbrot set visualization (helper method)

Mandelbrot function of a complex number.

- Returns WHITE if the number is not in the set.
- Returns BLACK if the number is (probably) in the set.

```
public static Color mand(Complex z0)
{
   Complex z = z0;
   for (int t = 0; t < 255; t++)
   {
      if (z.abs() > 2.0) return Color.WHITE;
      z = z.times(z);
      z = z.plus(z0);
   }
   return Color.BLACK;
}
```

For a more dramatic picture,
`return new Color(255-t, 255-t, 255-t)`
or colors picked from a color table.

Complex number client: Mandelbrot set visualization

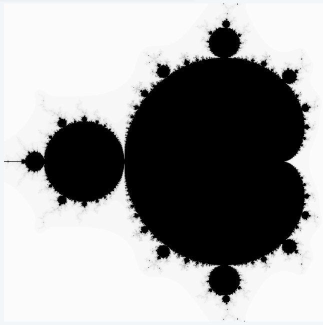
```
import java.awt.Color;
public class Mandelbrot
{
   public static Color mand(Complex z0)
   { // See previous slide. }
   public static void main(String[] args)
   {
      double xc   = Double.parseDouble(args[0]);
      double yc   = Double.parseDouble(args[1]);
      double size = Double.parseDouble(args[2]);
      int N = 512;
      Picture pic = new Picture(N, N);

      for (int col = 0; col < N; col++)
         for (int row = 0; row < N; row++)
         {
            double x0 = xc - size/2 + size*col/N;
            double y0 = yc - size/2 + size*row/N;
            Complex z0 = new Complex(x0, y0);
            Color color = mand(z0);
            pic.set(col, N-1-row, color);
         }
      pic.show();
   }
}
```

scale to screen coordinates

(0, 0) is upper left corner

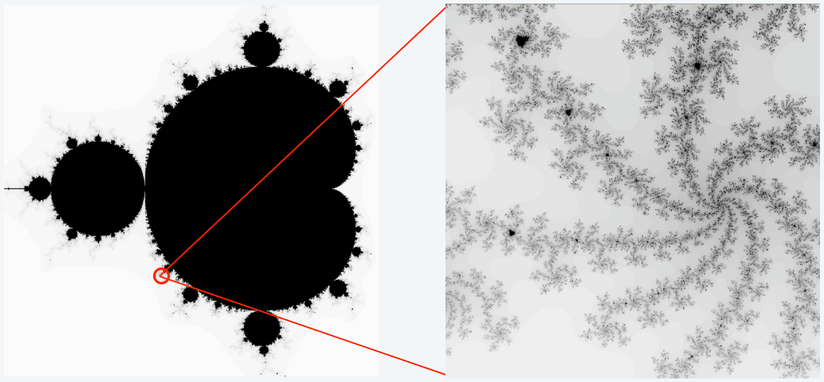
```
% java Mandelbrot -.5 0 2
```



Mandelbrot Set

```
% java Mandelbrot -.5 0 2
```

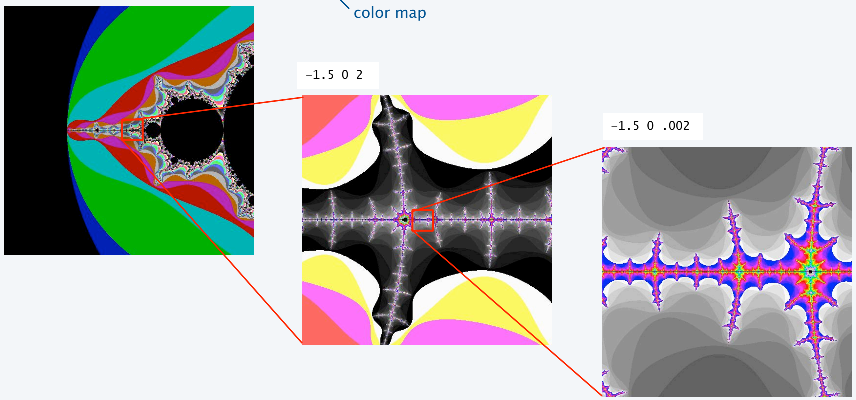
```
% java Mandelbrot .1045 -.637 .01
```

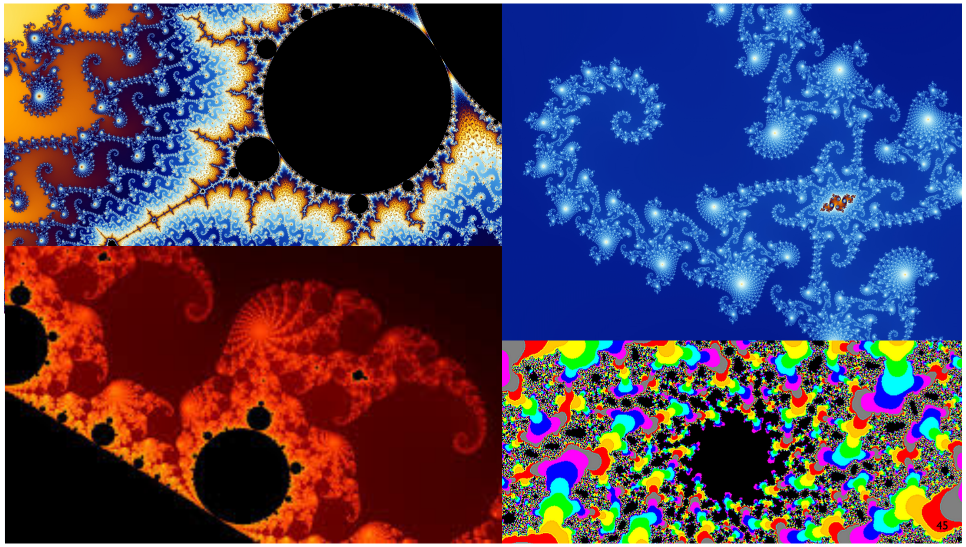


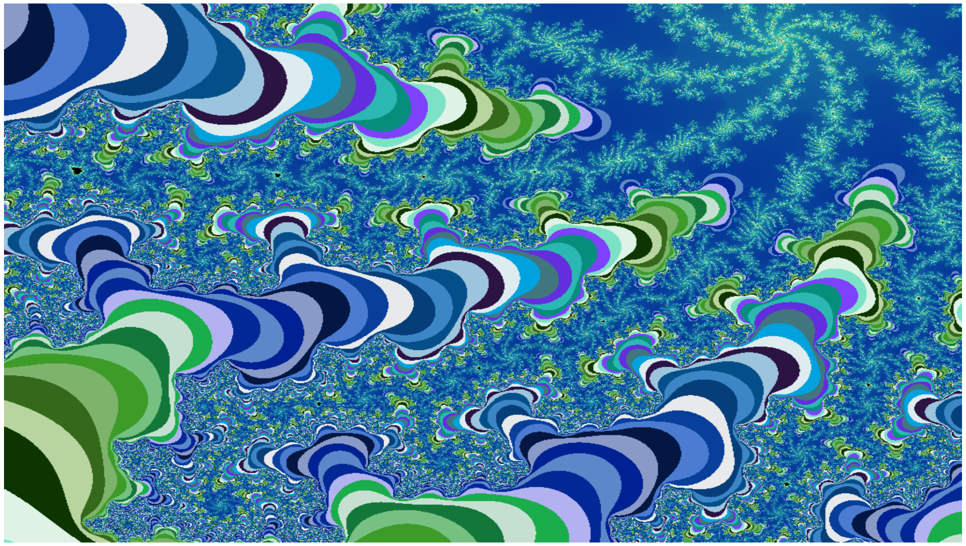
Mandelbrot Set

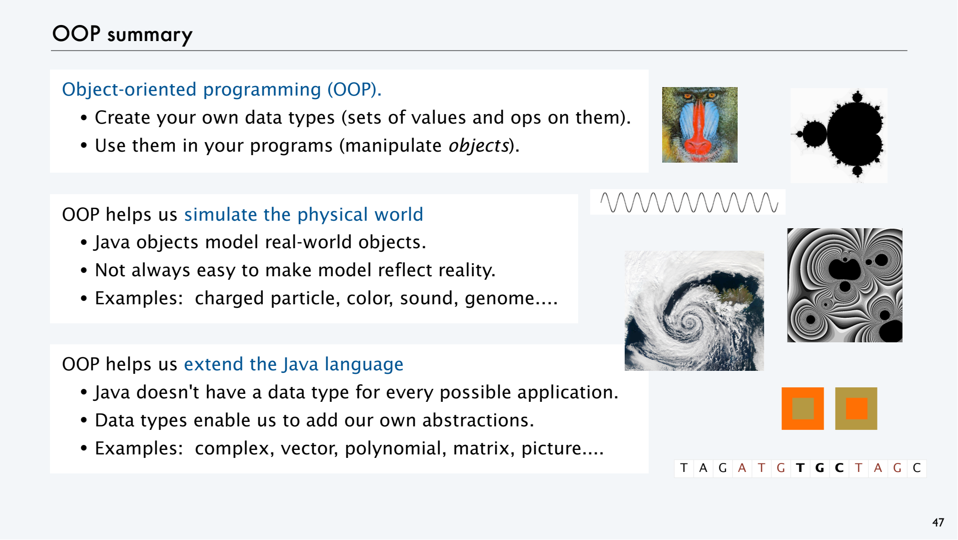
```
% java ColorMandelbrot -.5 0 2 < mandel.txt
```

color map









OOP summary

Object-oriented programming (OOP).
- Create your own data types (sets of values and ops on them).
- Use them in your programs (manipulate *objects*).

OOP helps us simulate the physical world
- Java objects model real-world objects.
- Not always easy to make model reflect reality.
- Examples: charged particle, color, sound, genome....

OOP helps us extend the Java language
- Java doesn't have a data type for every possible application.
- Data types enable us to add our own abstractions.
- Examples: complex, vector, polynomial, matrix, picture....

COMPUTER SCIENCE
SEDGEWICK / WAYNE

INTRODUCTION TO
Programming
in Java
An Interdisciplinary Approach
Robert Sedgewick · Kevin Wayne

Section 3.2

http://introcs.cs.princeton.edu

10. Creating Data Types