Shape Analysis

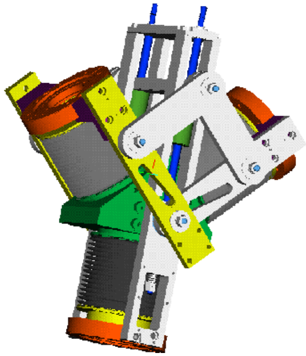
COS 429

Princeton University

Motivation

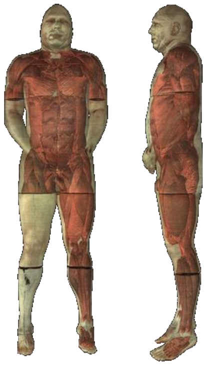
Large repositories of 3D data are available

Computer Graphics



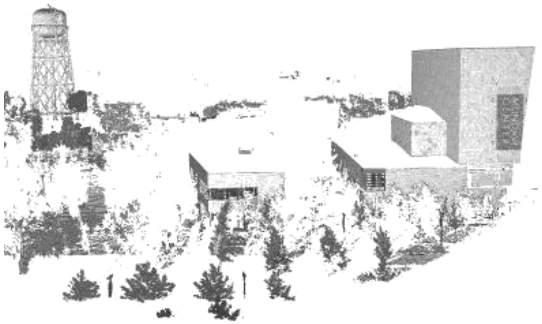
Mechanical CAD

Anthropometry



Medicine

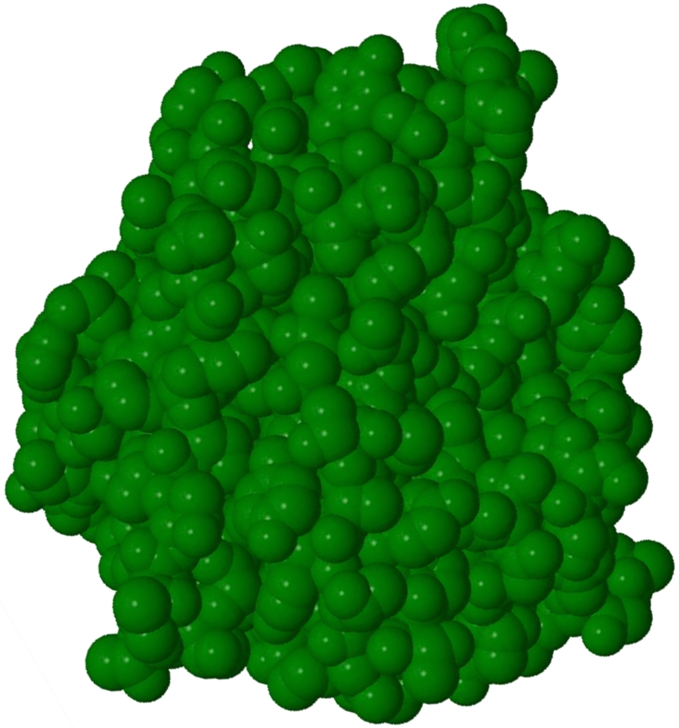
Cultural Heritage



Site Monitoring

Problem

Most 3D data lacks structural and functional annotations



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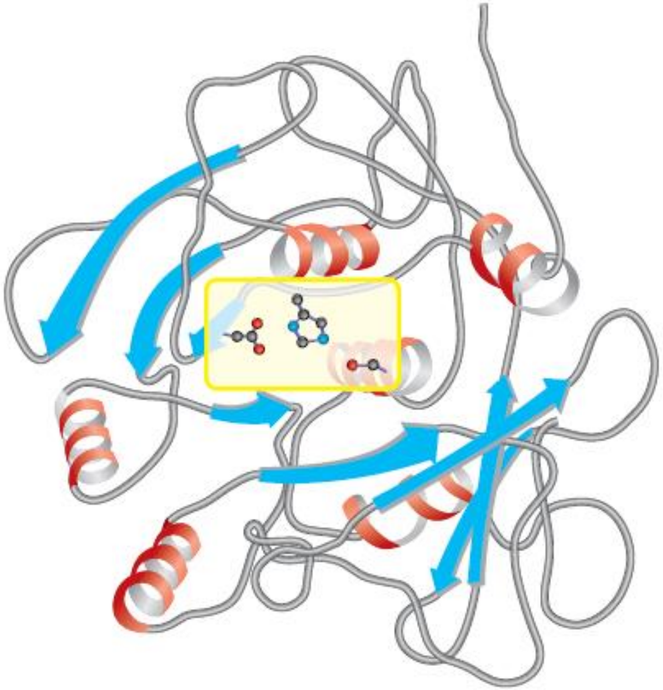
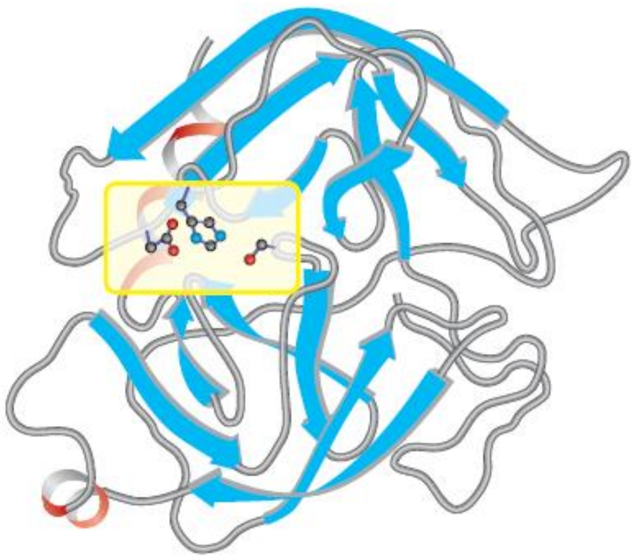
Goal

Infer structures and relationships automatically from 3D data (shape analysis)



Subtilisin
(bacterial serine protease)



Chymotrypsin
(mammalian serine protease)

Applications

Application domains:

- Molecular biology
- Paleontology
- Archaeology
- Urban planning
- Geometric modeling

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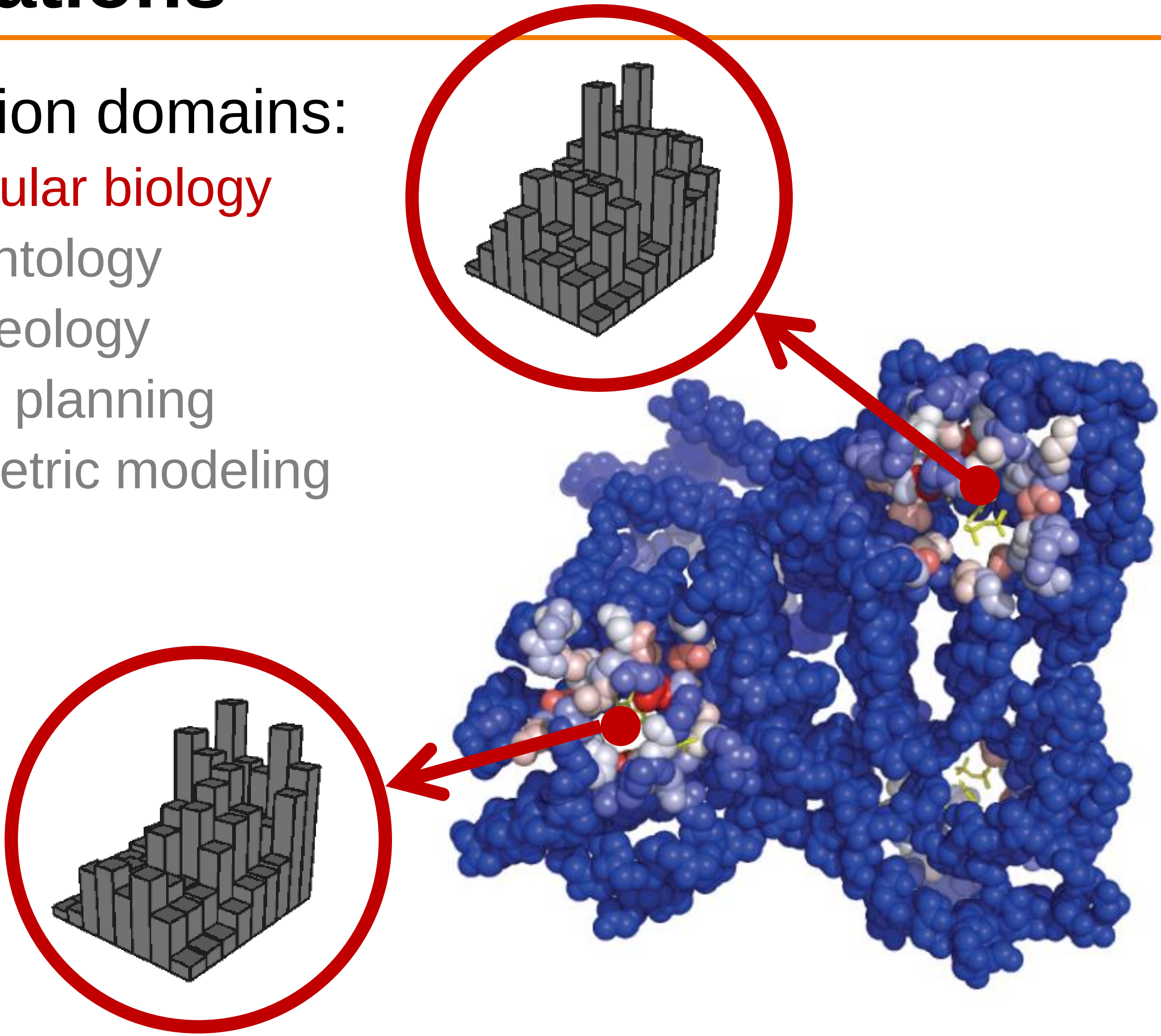

Applications

Application domains:

- ◦ Molecular biology
- ➢ Paleontology
- ◦ Archaeology
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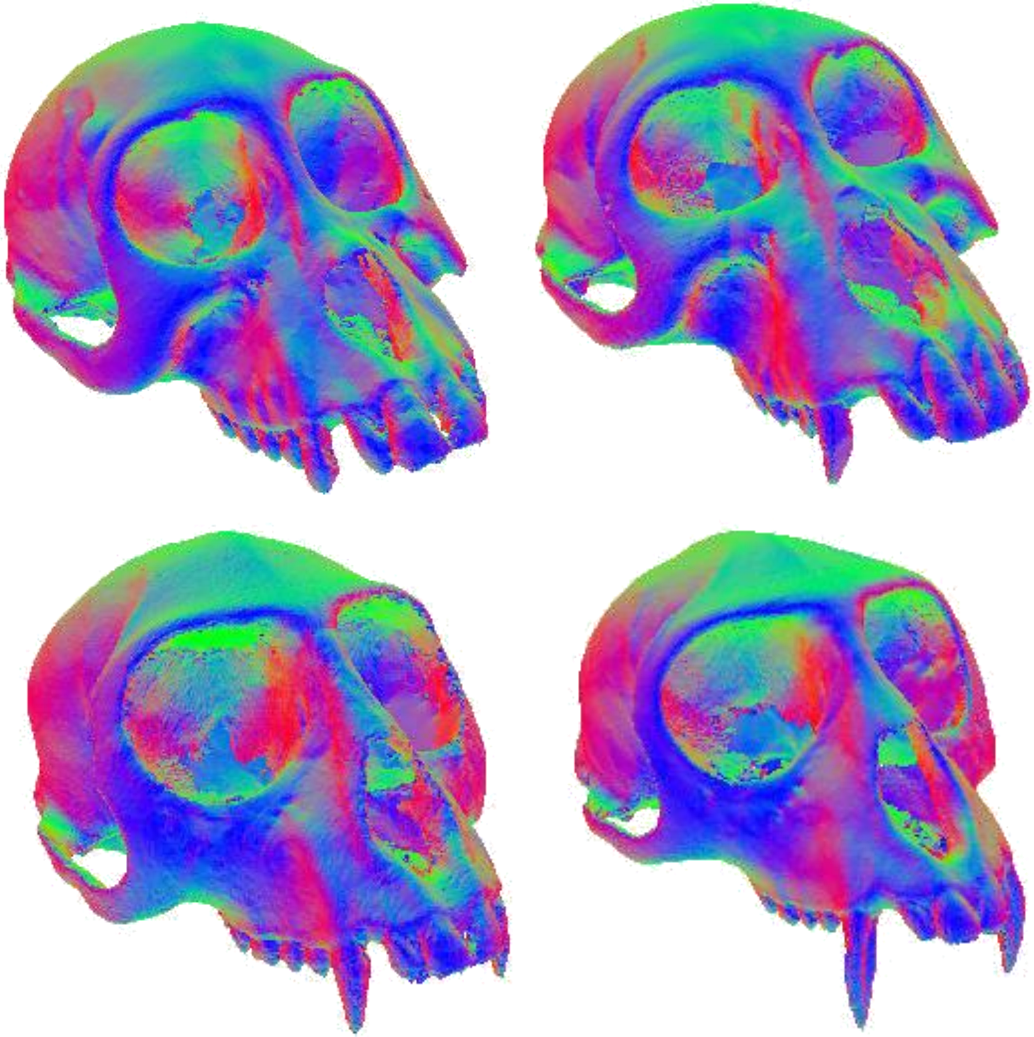
- ◦ Geometric modeling



Delsen et al.

Applications

Application domains:

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Applications

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Applications

Application domains:

- Molecular biology
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Challenges

Research problems …

- Detect features
- Find correspondences
- Detect symmetries
- Define distances
- Infer part structures
- Transfer properties
- Recognize objects
- Etc.

Challenges

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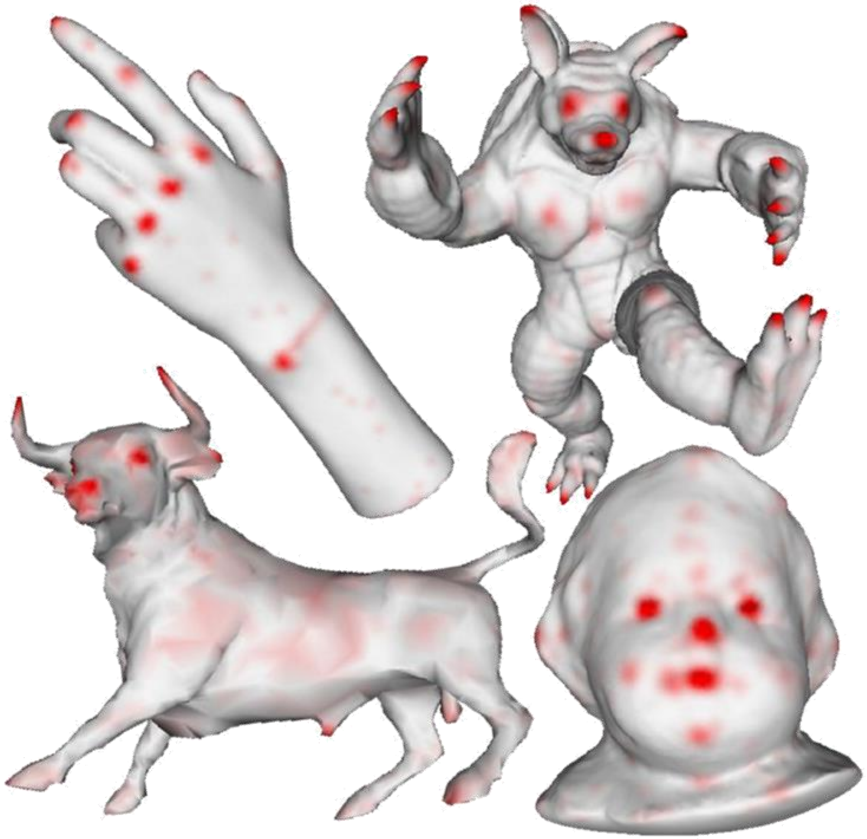

Schelling Points
(Chen, in preparation)

Challenges

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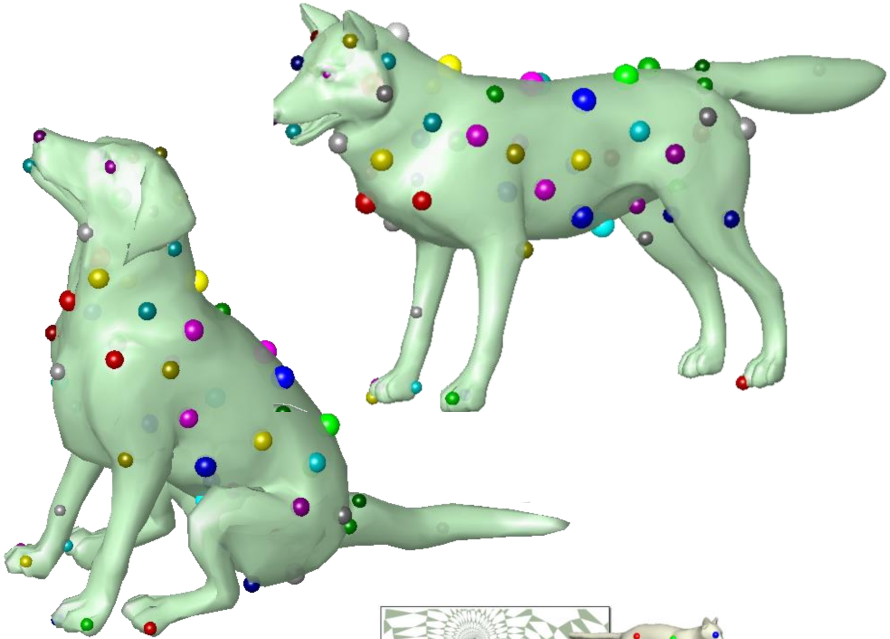

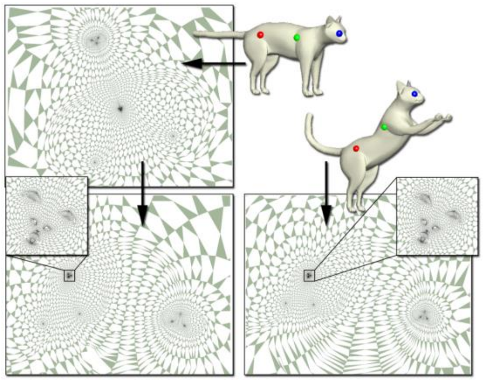

Mobius Voting
(Lipman, 2009)

Challenges

Research problems …

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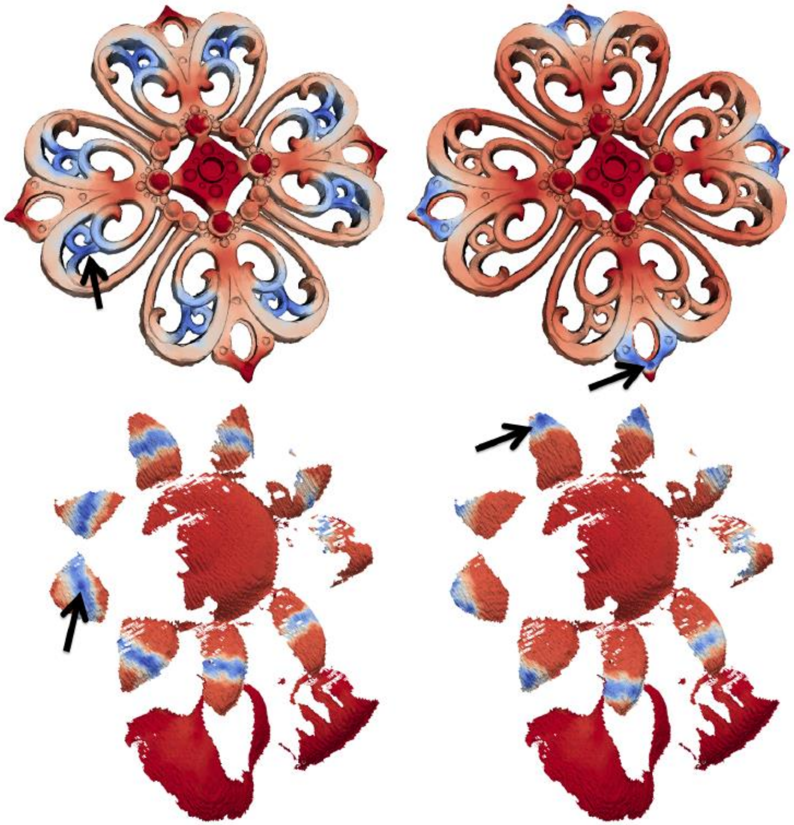

Symmetry Factored Distance
(Lipman, 2010)

Challenges

Research problems …

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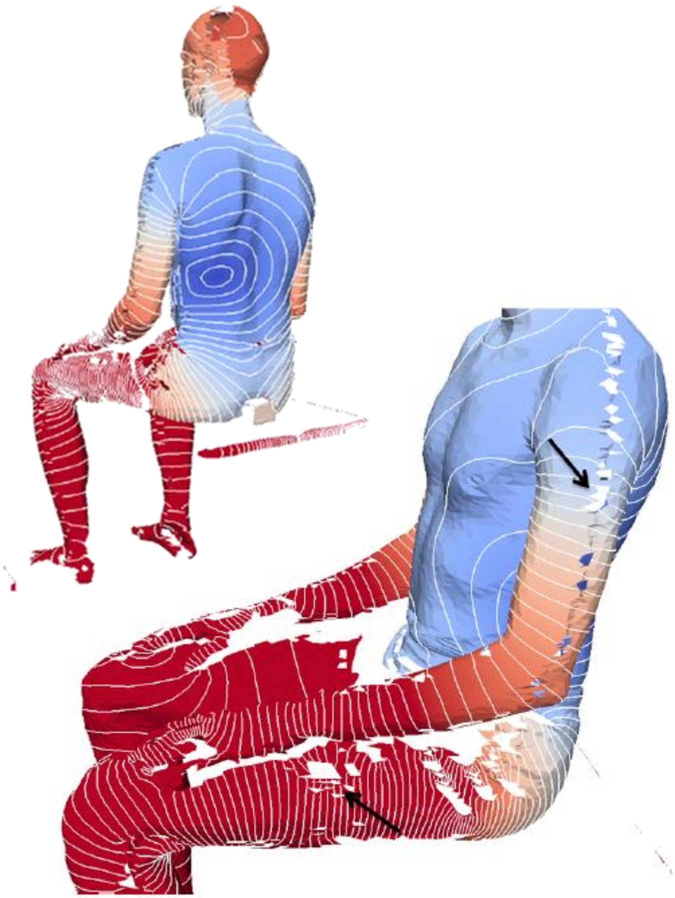

Biharmonic Distance
(Lipman, 2010)

Challenges

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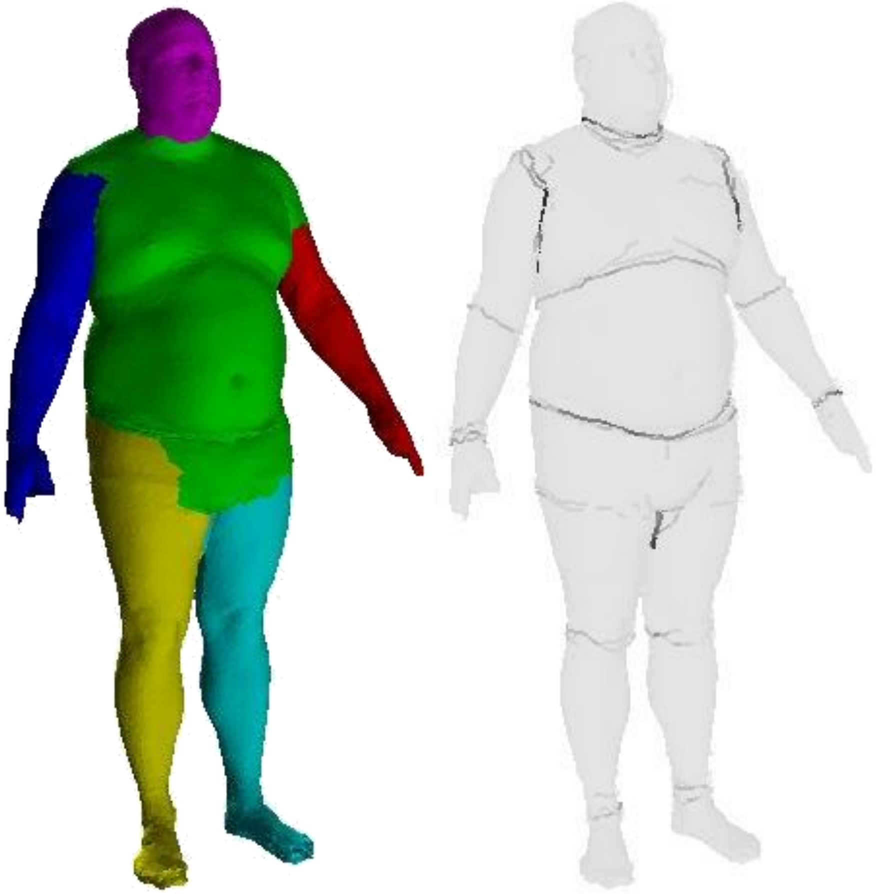

Randomized Cuts
(Golovinskiy 2008, Chen, 2009)

Challenges

Research problems …

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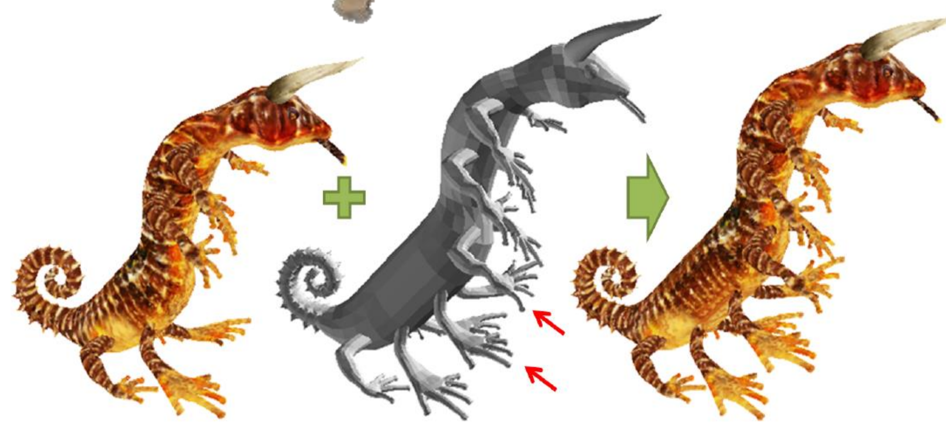

MeshMatch
(Chen, in preparation)

Challenges

Research problems …

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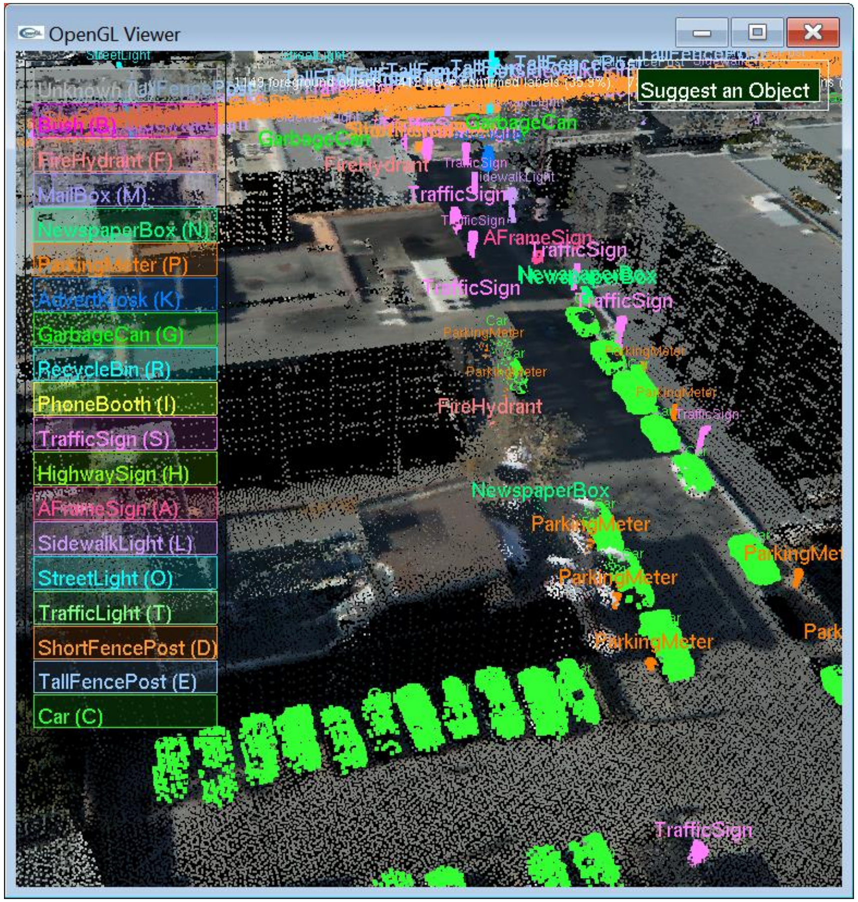
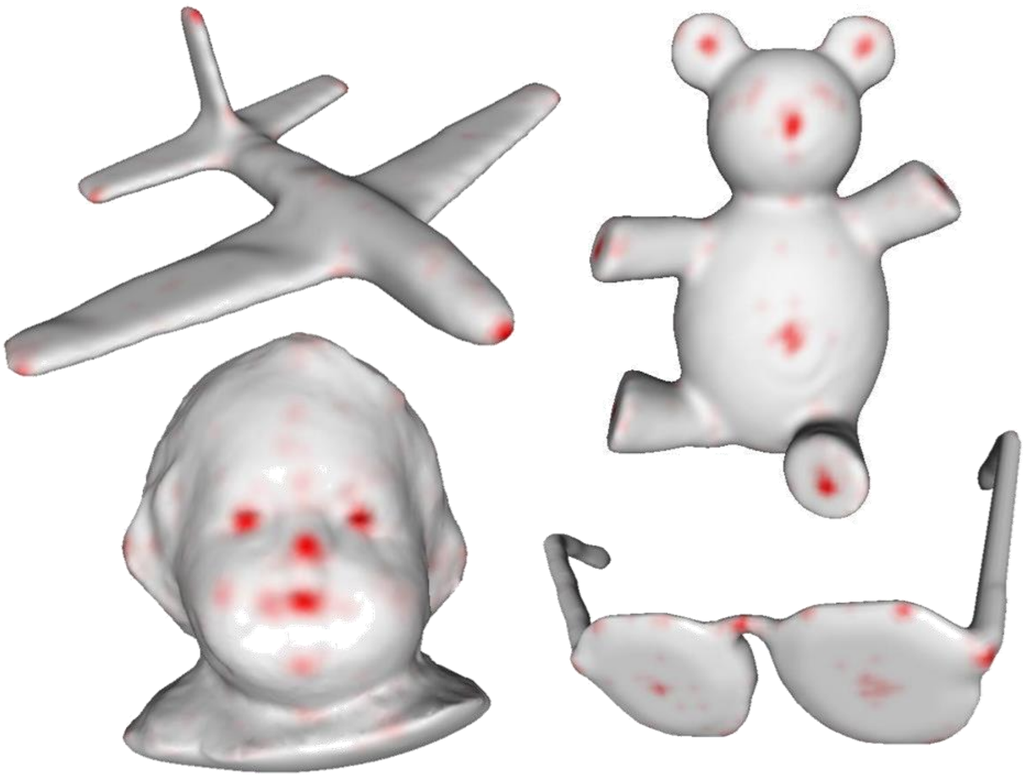


User-Driven Learning
(Boyko, in preparation)

Feature Detection

Feature Detection

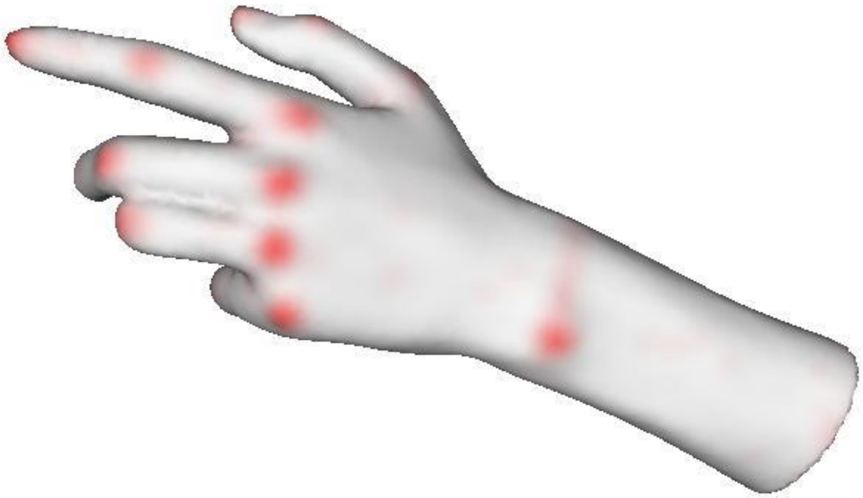
Goal: predictive model for salient 3D feature points



Feature Detection

Goals:

- Invariant to transformations
- Robust to small surface deviations (holes, noise, etc.)
- Common across different surfaces in same class
- Salient



Salience: the quality or fact of being more prominent in a person's awareness or in his memory of past experience" [Oxford English Dictionary]

Feature Detection

Methods based on geometric surface properties ...

- Minimum curvature
- Gauss curvature [Lipman09]
- Multiscale persistence [Li07]
- Differences of curvature [Lee05]
- Shape descriptor likelihood [Chua96]
- Shape descriptor distinction [Shilane07]
- Heat Kernel Signature [Sun09]
- Average geodesic distance [Zhang08]
- Distance to convex hull [Katz05]
- Iterative furthest point
- etc.

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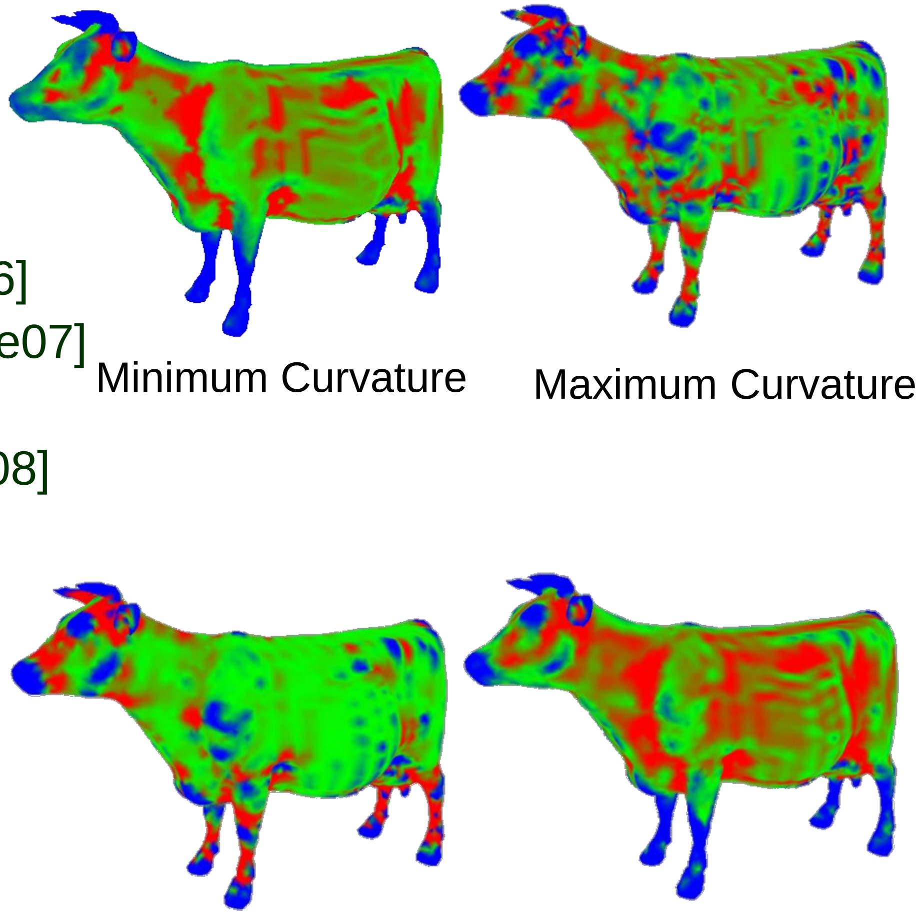
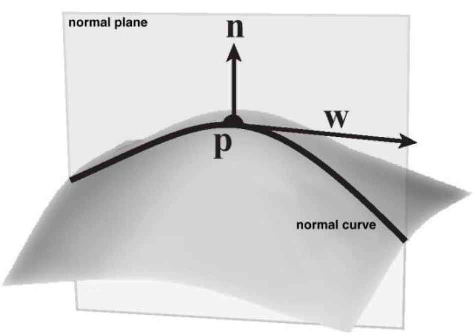


Minimum Curvature

Maximum Curvature

Gauss Curvature

Mean Curvature



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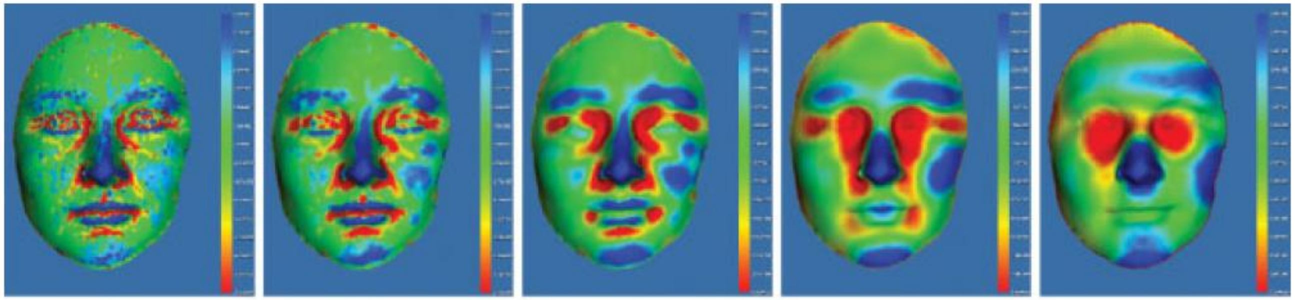

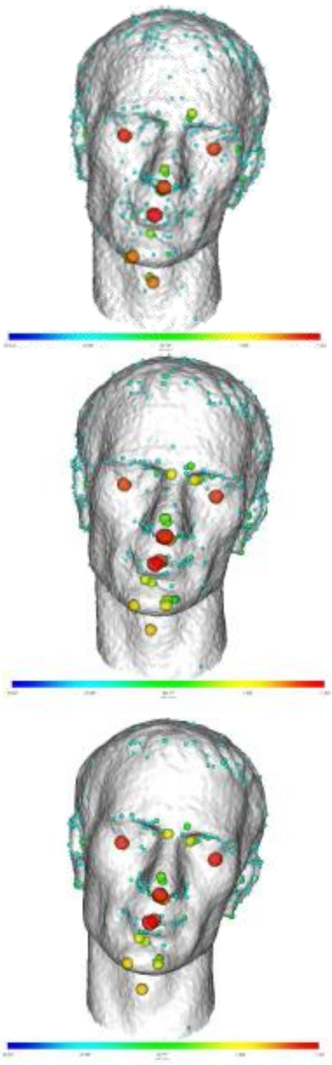

Zou

Feature Detection

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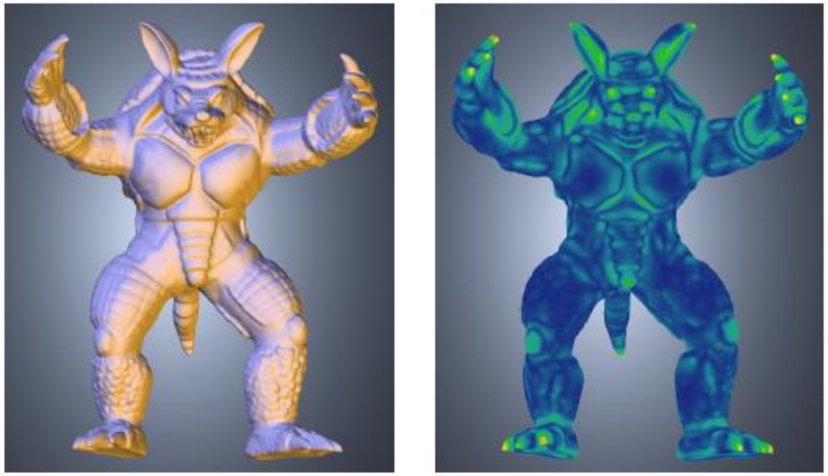

Scale-Space Differences of Curvature

Lee

Feature Detection

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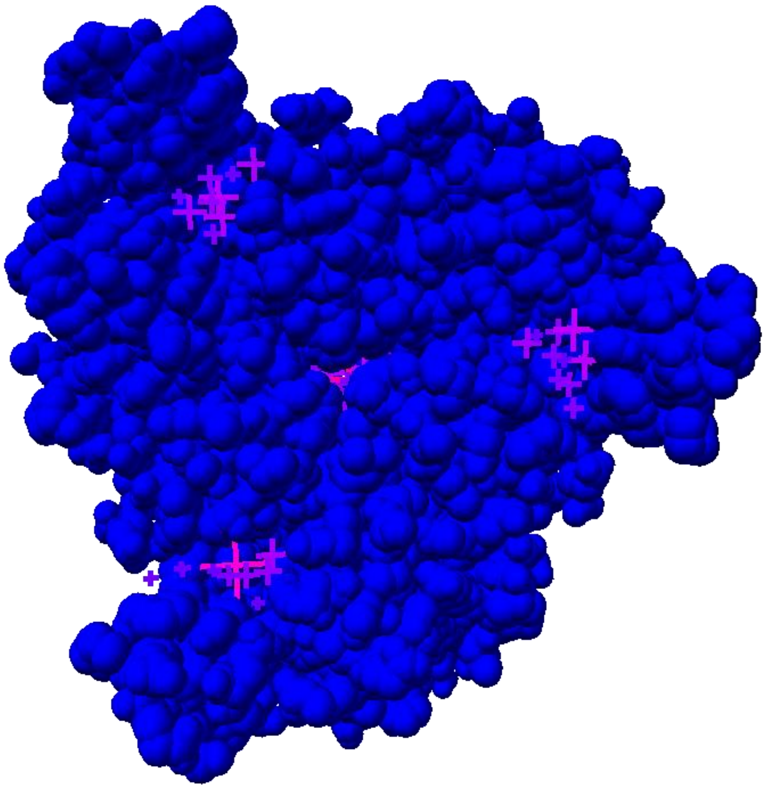

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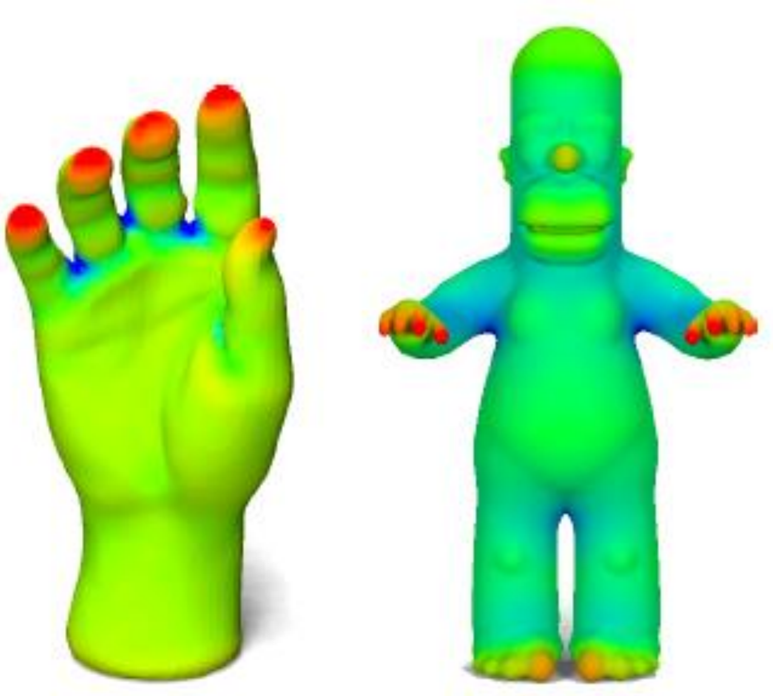

Sun

Feature Detection

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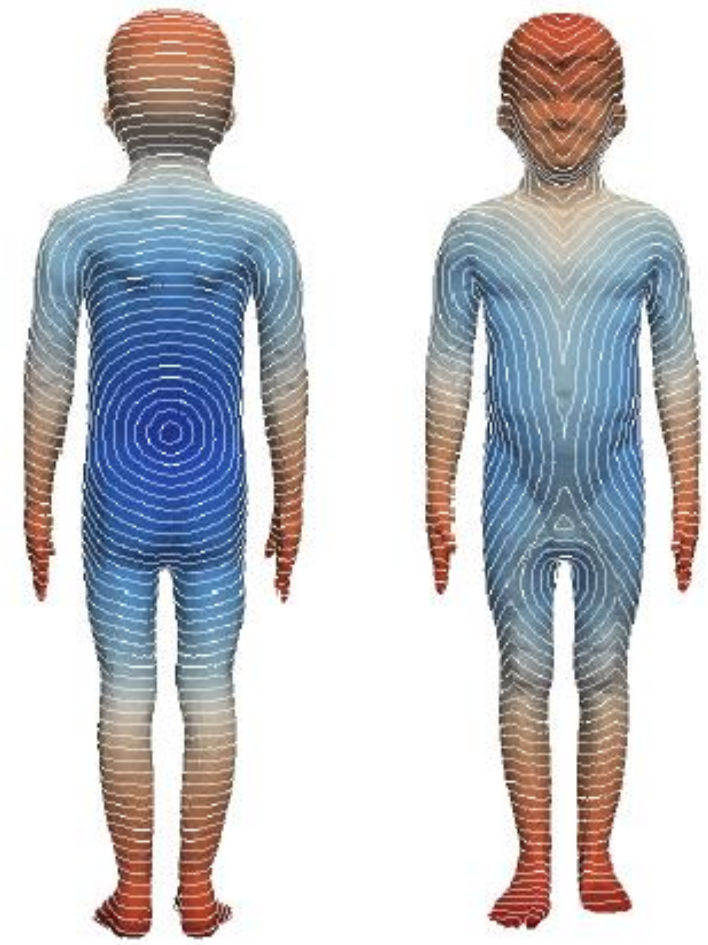

Geodesic Distance

Feature Detection

Methods based on geometric surface properties ...

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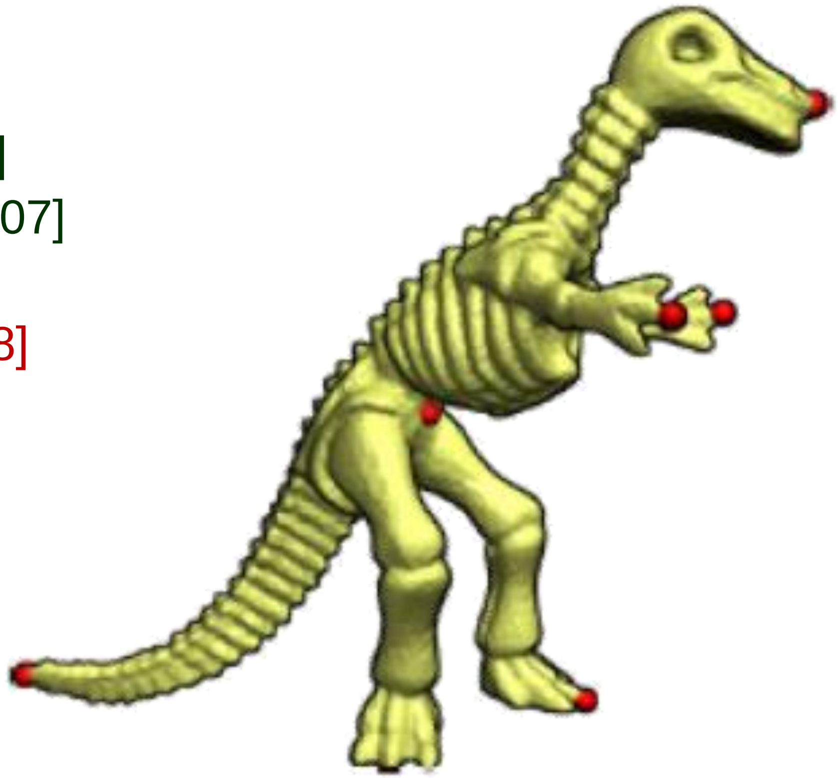

Extrema of Average Geodesic Distance

Feature Detection

Methods based on geometric surface properties ...

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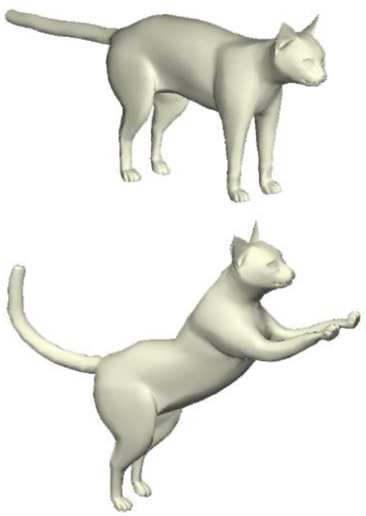

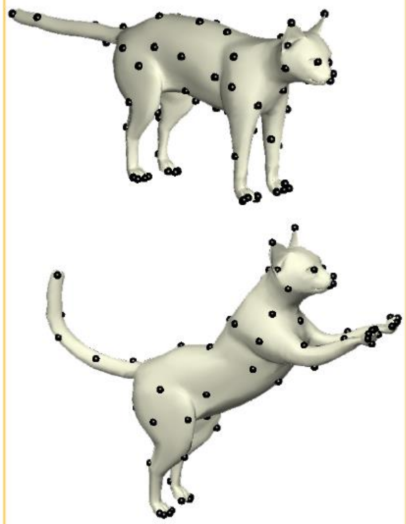

Iterative Furthest Point distribution

Learning Features From People

Key Question

How should we ask people which points are salient?

- "Please select salient points"

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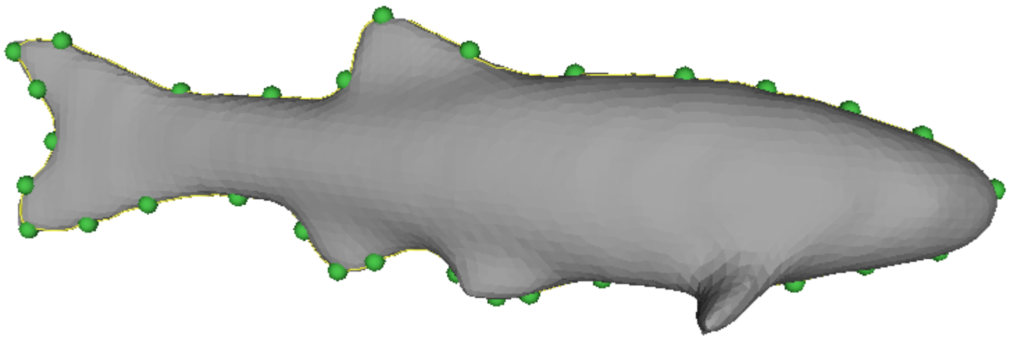
- "Please select a pattern of points from which another person can recognize the object's class by viewing only those points as they spin in 3D"



Key Question

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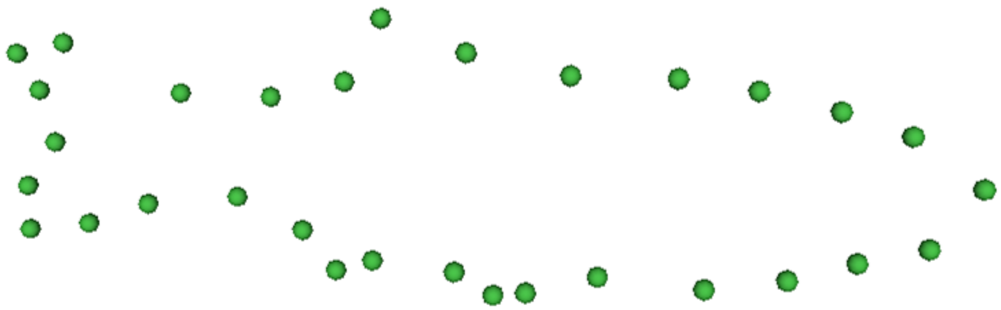


Key Question

We asked people to:

- "Please select points that you think other people will select"

Based on the "focal point" theory of Schelling [1960]

- Nash equilibrium solutions to a pure-coordination game
- "A solution that people will tend to use in the absence of communication, because it seems natural, special or relevant to them"

Example Schelling Experiments I

Write down any amount of money, imagining that if everyone writes down the same amount, then each receives that amount as a prize; otherwise nobody receives anything [Schelling60]

Example Schelling Experiments I

Write down any amount of money, imagining that if everyone writes down the same amount, then each receives that amount as a prize; otherwise nobody receives anything [Schelling60]

30% choose 1 million

93% choose 1, 10, 100, …, 10^X

Example Schelling Experiments II

Select a time and place in New York City to meet someone without any prior communication [Schelling60]

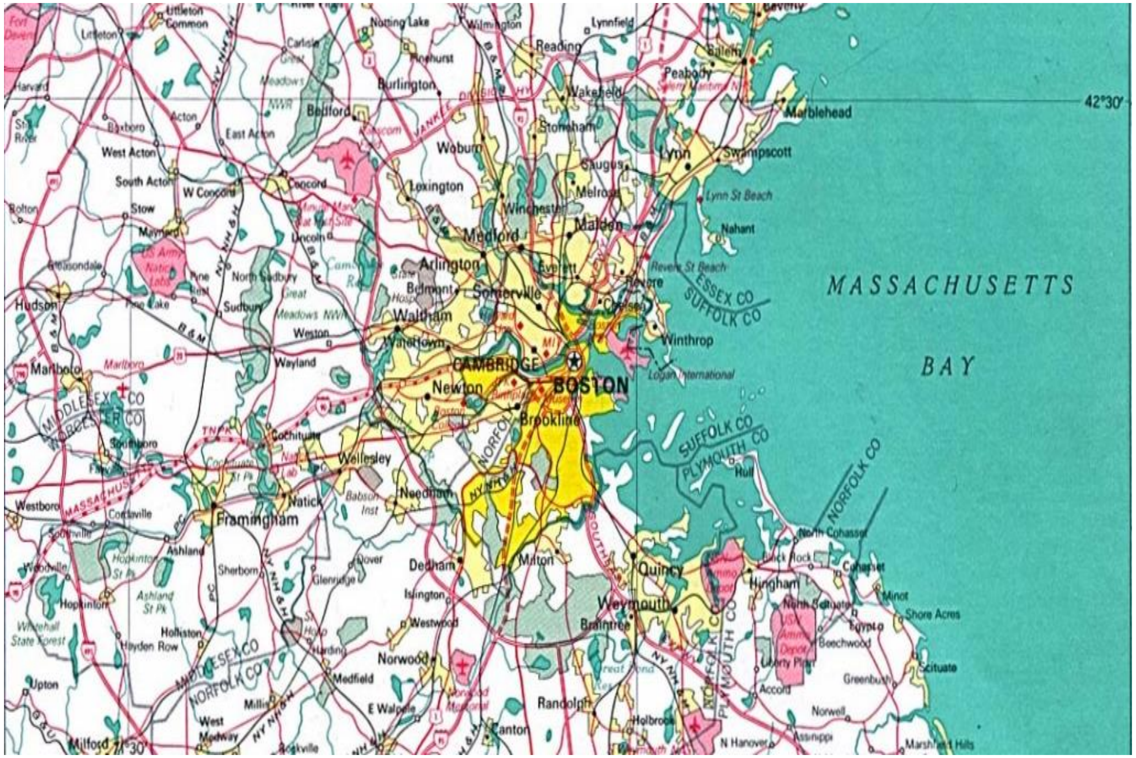
Example Schelling Experiments II

Select a time and place in New York City to meet someone without any prior communication [Schelling60]

Grand Central Terminal at noon

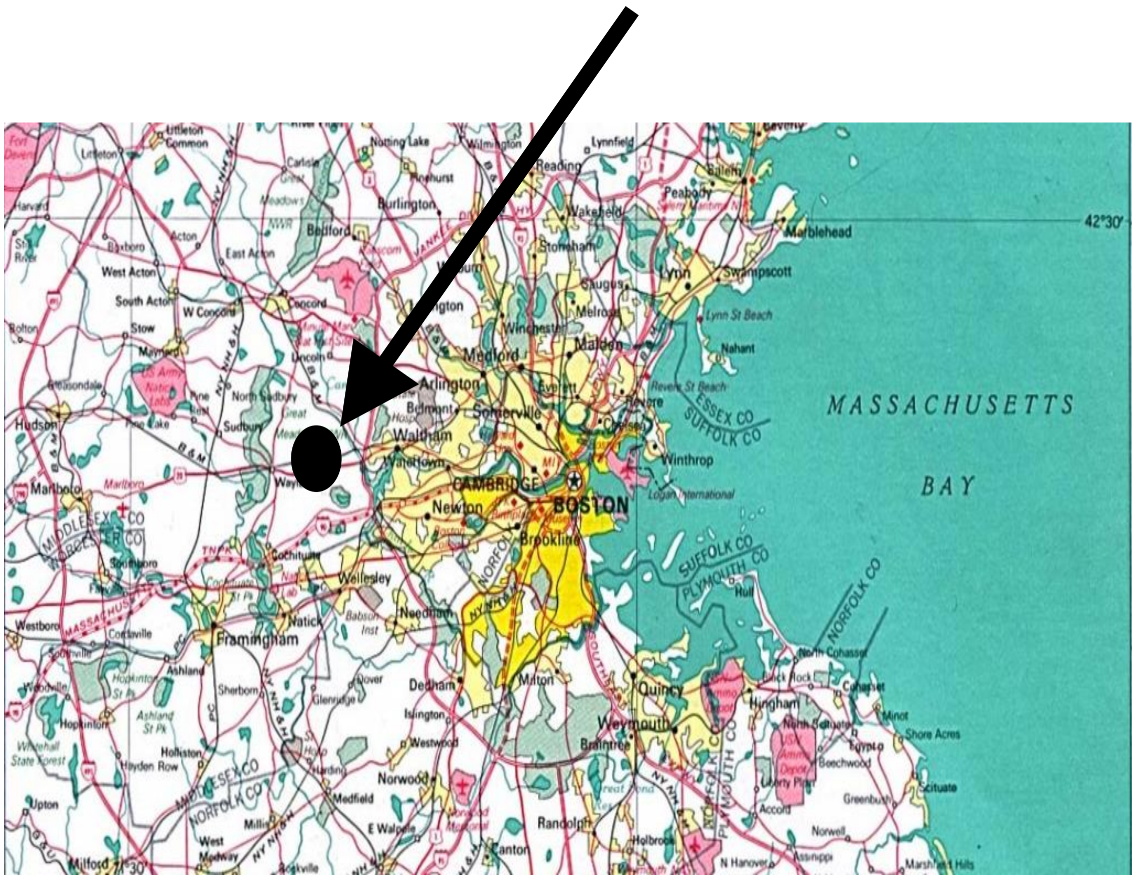
Example Schelling Experiments III

Please pick the point in this image that you think is most likely to be picked by other people in the room



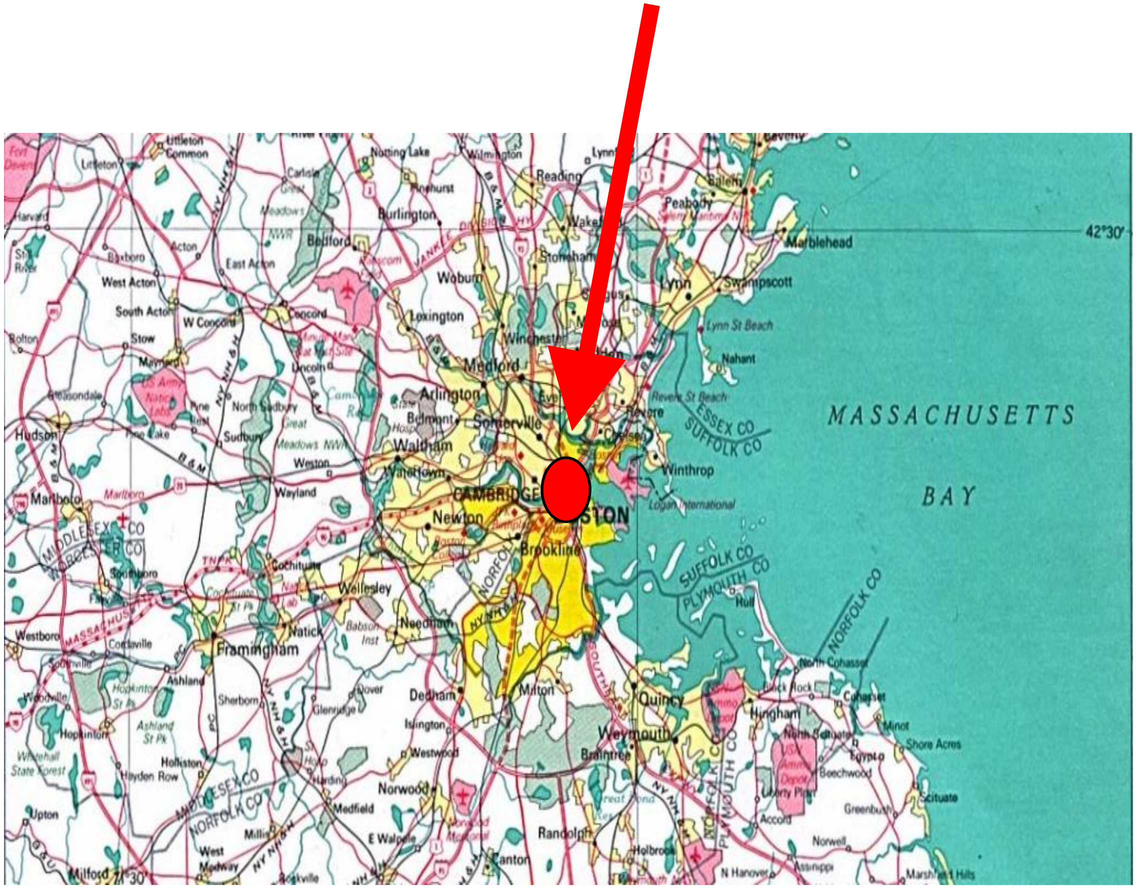
Example schelling Experiments III

Raise your hand if you selected the **black point**



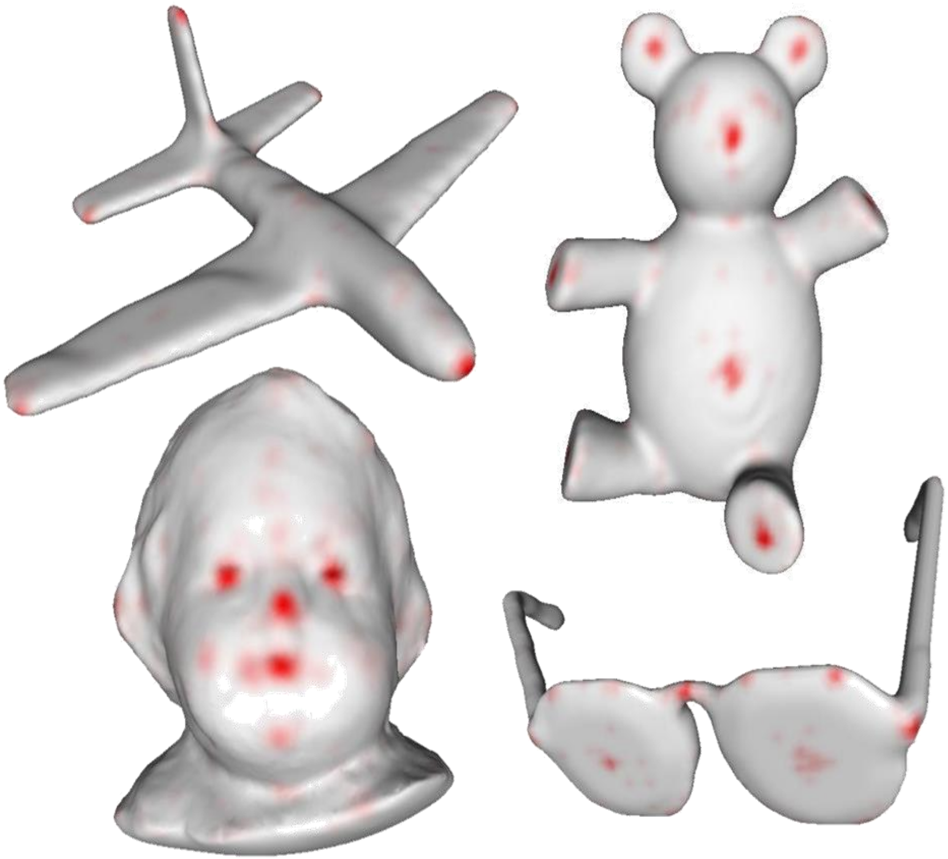
Example schelling Experiments III

Raise your hand if you selected the **red point**

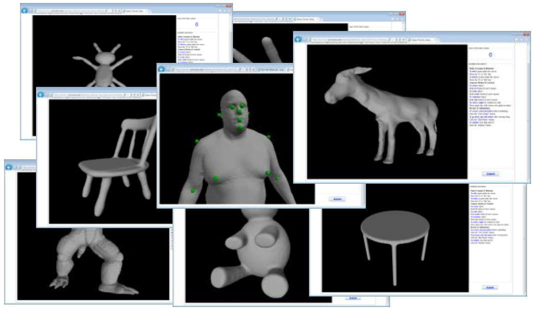


Schelling Points on 3D Surfaces

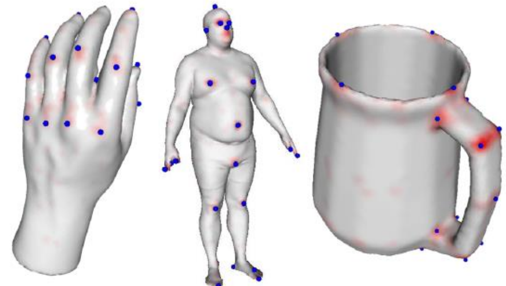
Use focal points to study salience on 3D surface meshes



Study Methodology



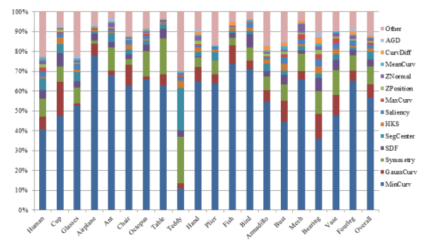
Data Collection on Mechanical Turk



Schelling Point Distributions

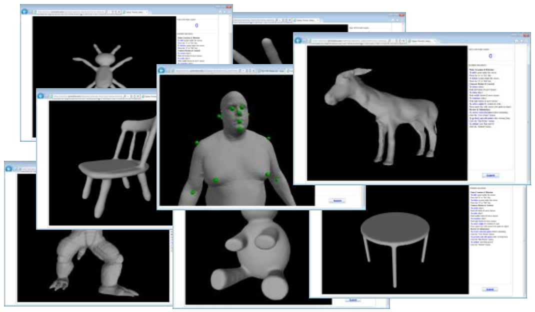
Geometric Analysis

Property (p)	Best Filter (f)	Info Gain G(f)	G(%(f))	Importance I(f)	I(%(f))
MinCurv	$\|(B(p,1))\|$	195	203	216	801
GaussCurv	$\|p\|$	201	211	59	267
Symmetry	$B(p,4)$	12	25	18	84
SDF	p	[illegible]	[illegible]	31	54
SegCenter	[illegible]	[illegible]	[illegible]	15	33
HKS(101)	p	104	141	15	31
Saliency(0.3)	p	[illegible]	[illegible]	26	30
MaxCurv	$B(\|p\|,4)$	[illegible]	[illegible]	19	30
ZPosition	p	20	51	29	27
ZNormal	p	11	14	24	25
MeanCurv	$B(p,4)$	43	98	11	23
CurvDiff	p	29	46	22	21
AGD	p	25	85	18	18



Descriptive Statistics

Study Methodology



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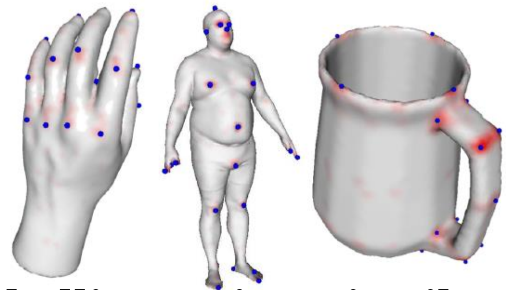


Schelling Point Distributions

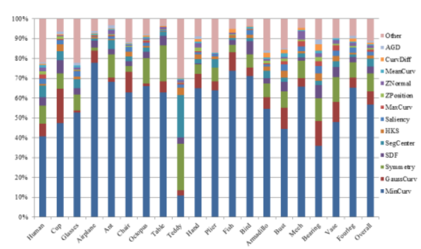
New Mesh

Geometric Analysis

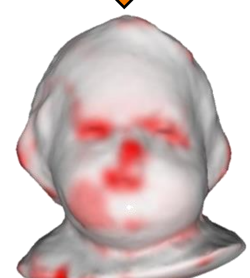
Property (p)	Best Filter (f)	Info Gain G(f)	Info Gain G(%(f))	Importance I(f)	Importance I(%(f))
MinCurv	$\lvert(B(p,1))\rvert$	195	203	216	801
GaussCurv	$\lvert p\rvert$	201	211	59	267
Symmetry	$B(p,4)$	12	25	18	84
SDF	[illegible]	[illegible]	[illegible]	[illegible]	54
SegCenter	[illegible]	[illegible]	[illegible]	15	33
HKS(101)	p	104	141	15	31
Saliency(0.3)	[illegible]	[illegible]	[illegible]	26	30
MaxCurv	$B(\lvert p\rvert,4)$	[illegible]	[illegible]	19	30
ZPosition	p	20	51	29	27
ZNormal	p	11	14	24	25
MeanCurv	$B(p,4)$	43	98	11	23
CurvDiff	p	29	46	22	21
AGD	p	25	85	18	18

Predictive Model

```
%(Blur(MinCurv,4) <= 65.56
|   Symmetry >  35.669
|   |   SDF >  0.528
|   |   |   Symmetry <= 101.366
|   |   |   |   %(GeodesicTenPercentile) <= 44.399
|   |   |   Symmetry >  101.366
|   |   |   |   ZNormal <= 0.287
%(Blur(MinC[illegible]
|   SegCent[illegible]
|   |   %(MaxCurv) <= 37.769
|   |   |   %(Sa[illegible]
|   |   |   |   [illegible] <= 33.576
|   |   |   %(Sa[illegible]
|   |   |   |   %(Blur(|MaxCurv|),3) <= 77.394
|   |   %(MaxCurv) >  37.769
|   |   |   %(SDF) <= 43.417
|   |   |   |   Blur(Symmetry,4) <= 59.815
|   SegCenter >  0.772
|   |   MeanCurv >  5.973
|   |   |   %(ZNormal) <= 92.829
|   |   |   |   Blur(Symmetry,4) <= 63.682
```



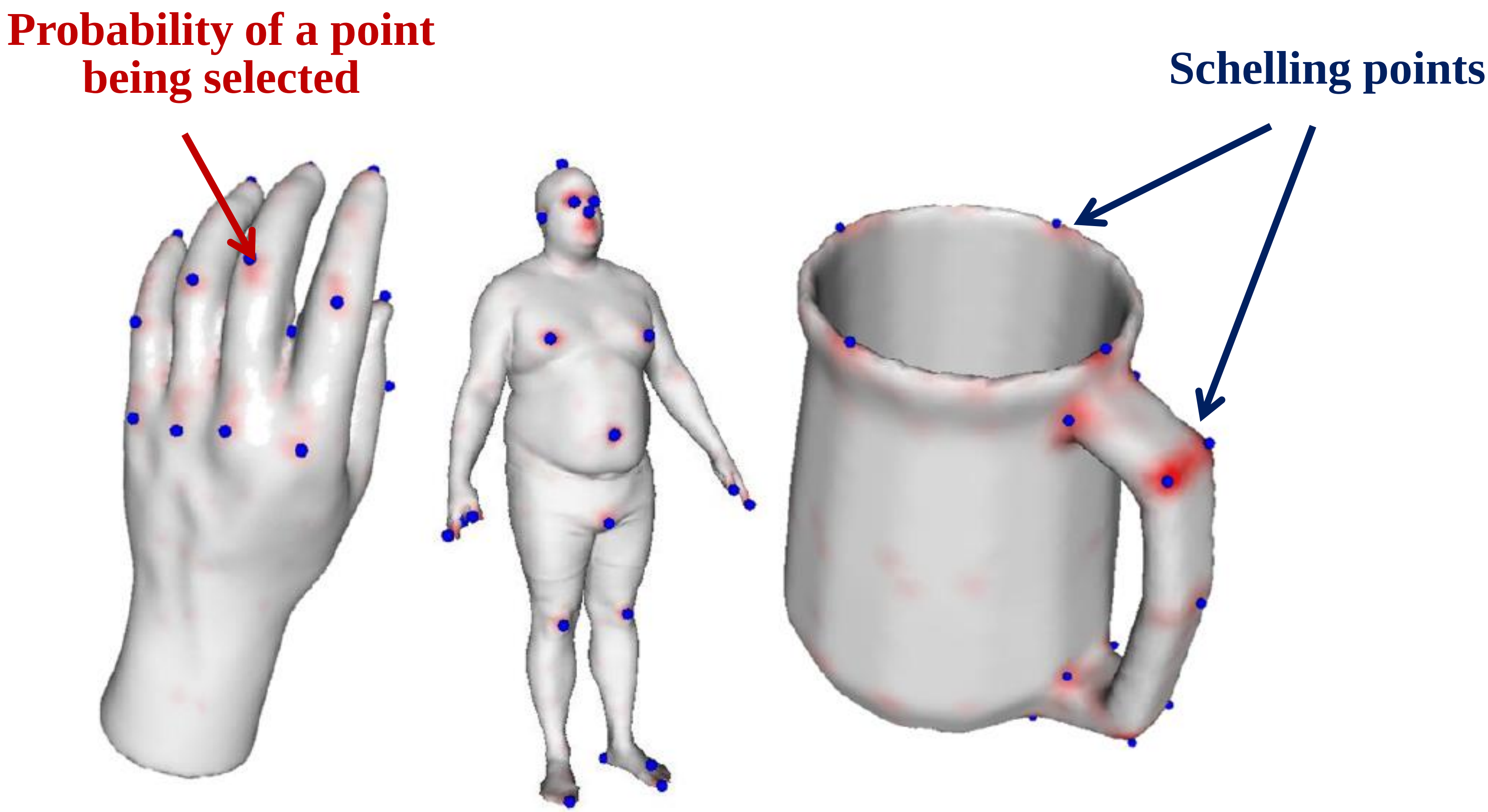
Descriptive Statistics



Predicted Salience

Main Conclusions I

Small sets of points are indeed selected consistently

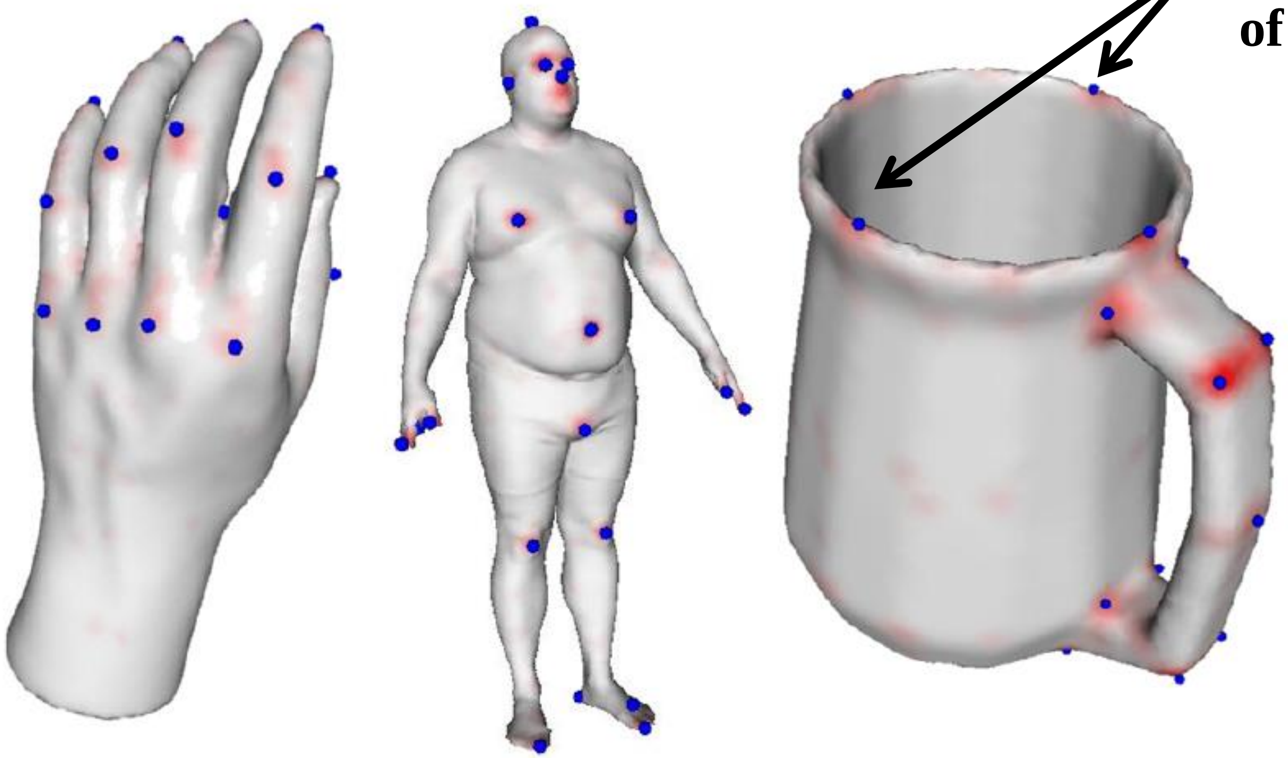


Main Conclusions II

Schelling points are not randomly distributed

Schelling points tend to be distributed evenly on surface

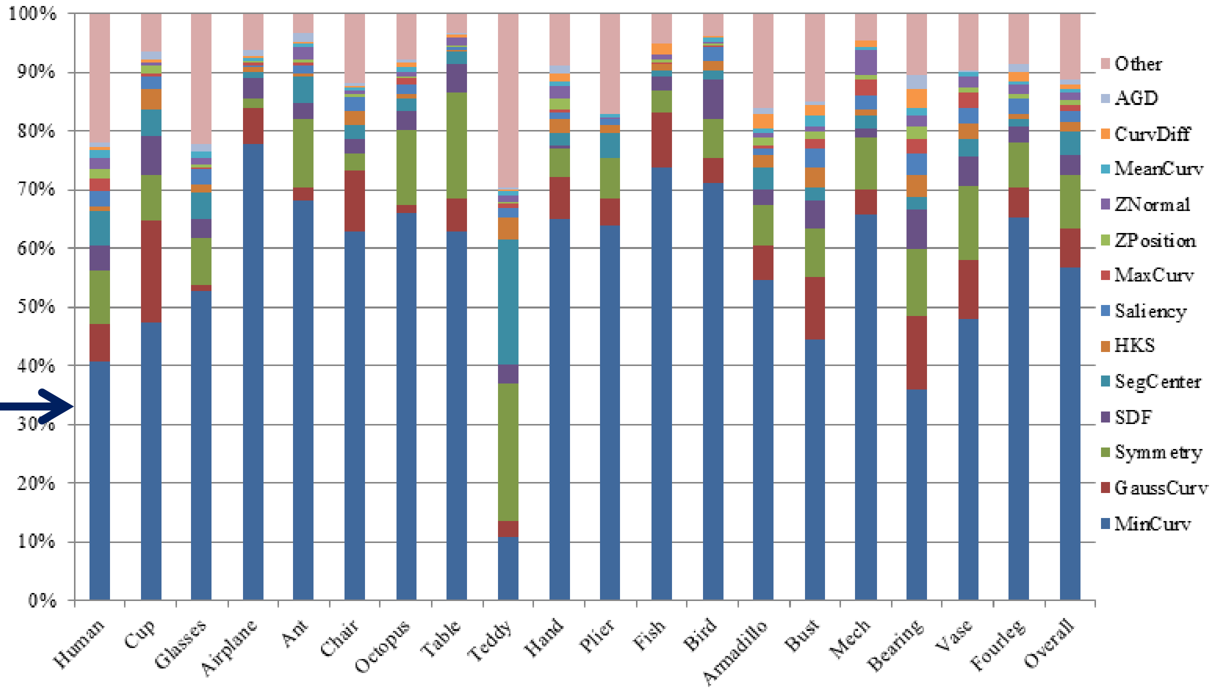
Schelling points tend to adhere to symmetries of the surface

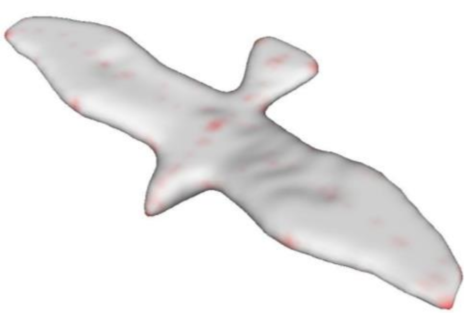


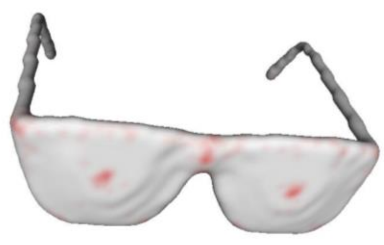
Main Conclusions III

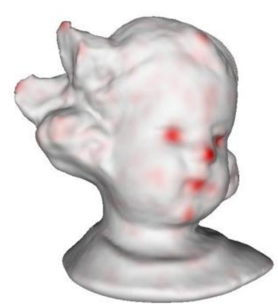
Local curvature features explain ~65% of Schelling points

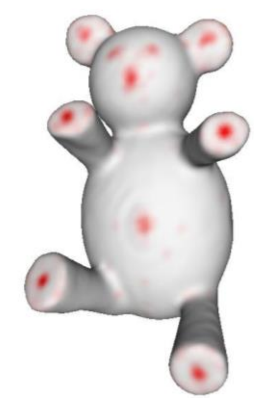
Minimum Curvature





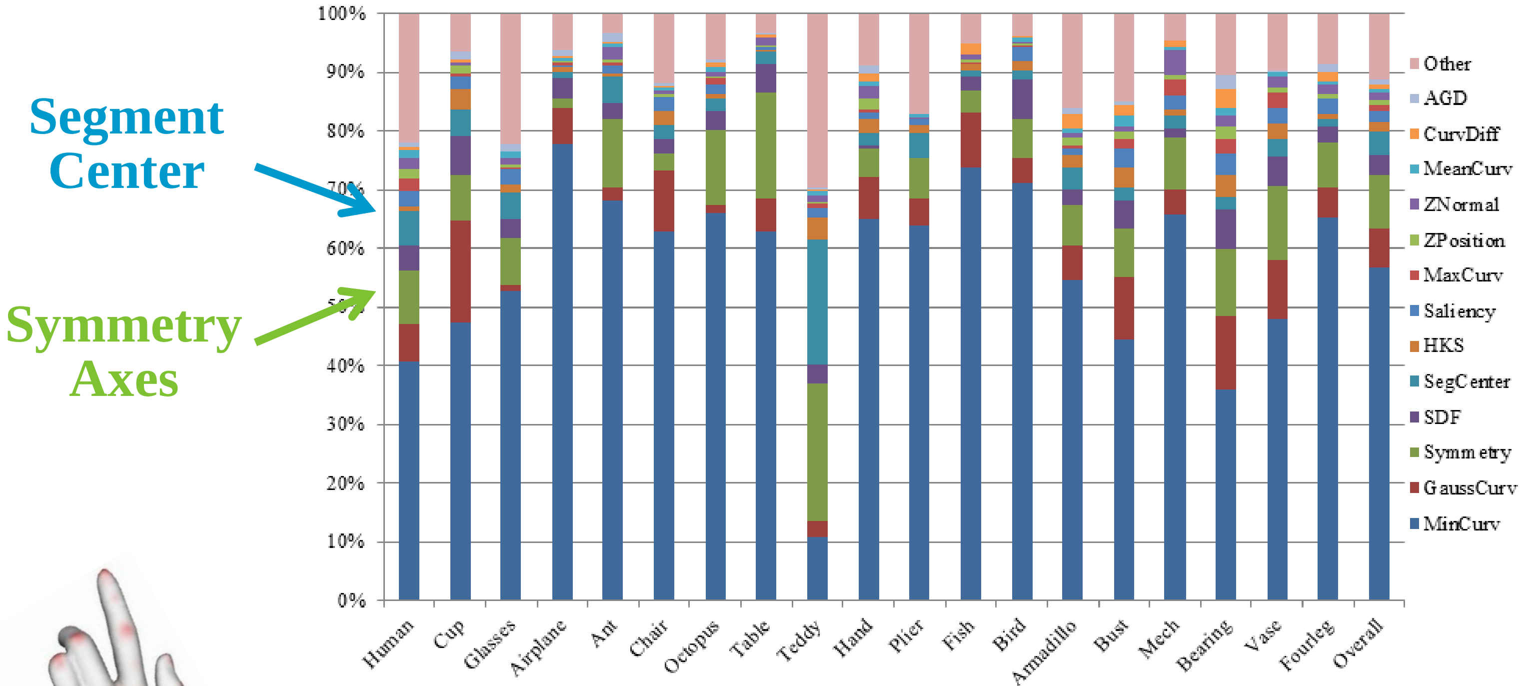


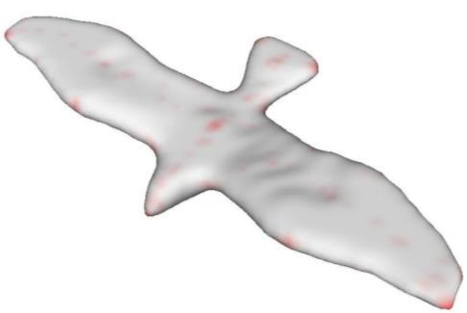


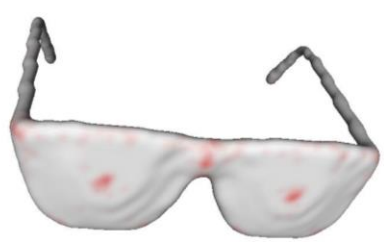


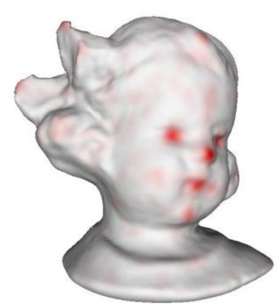
Main Conclusions III

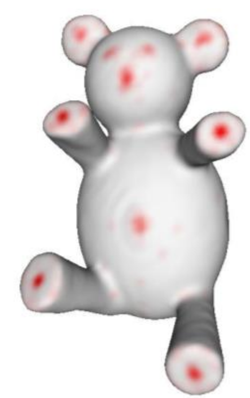
Global shape features explain ~20% of Schelling points







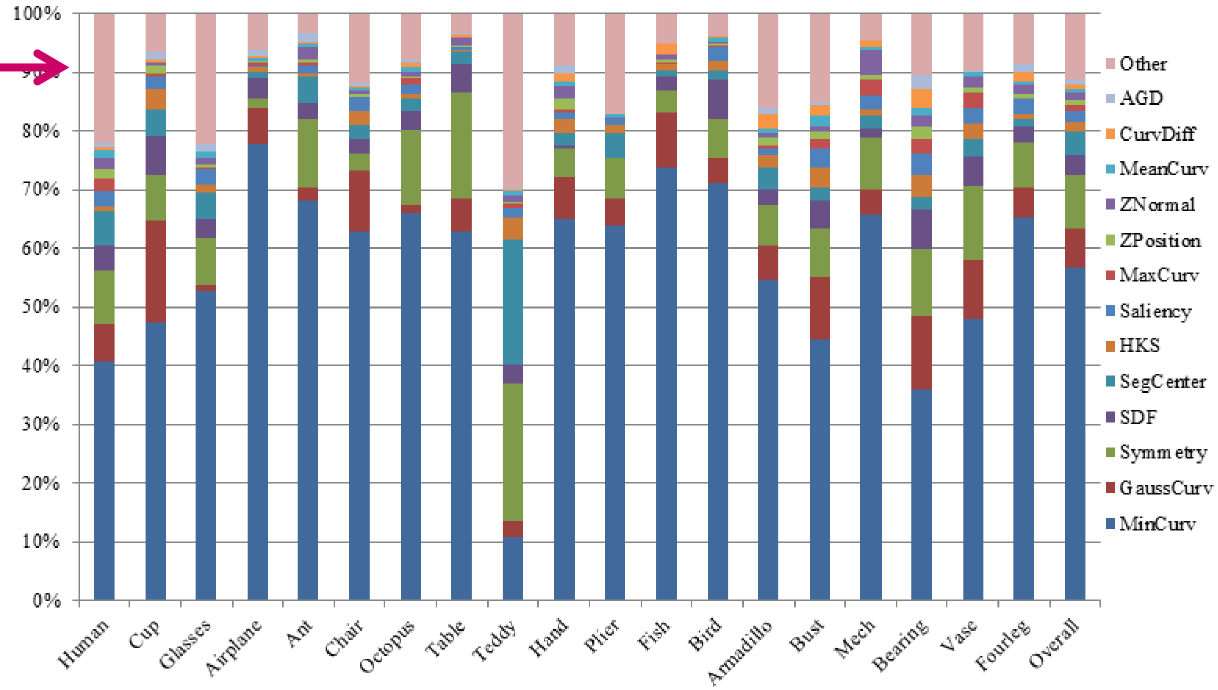


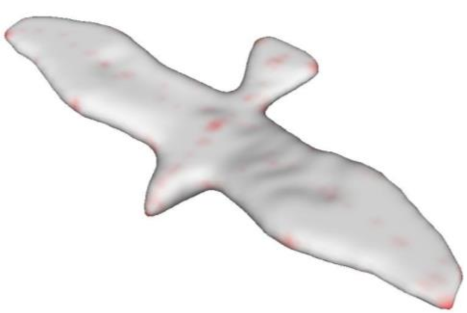


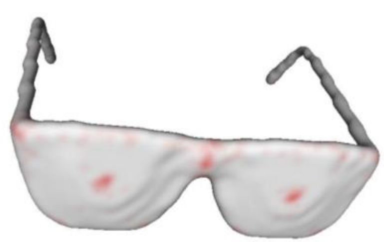
Main Conclusions III

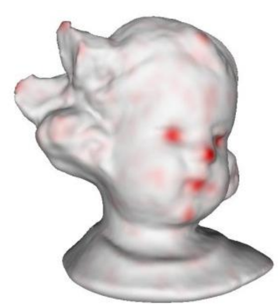
~15% of Schelling points are not explained geometrically

???





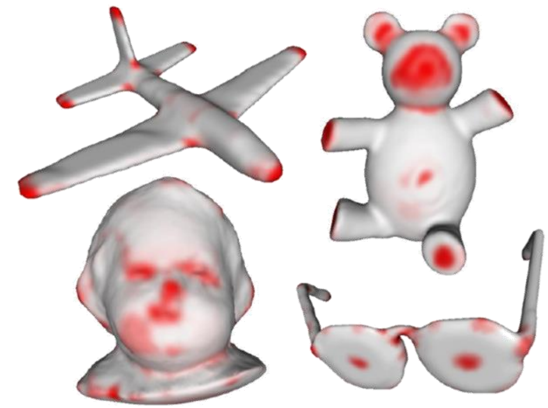


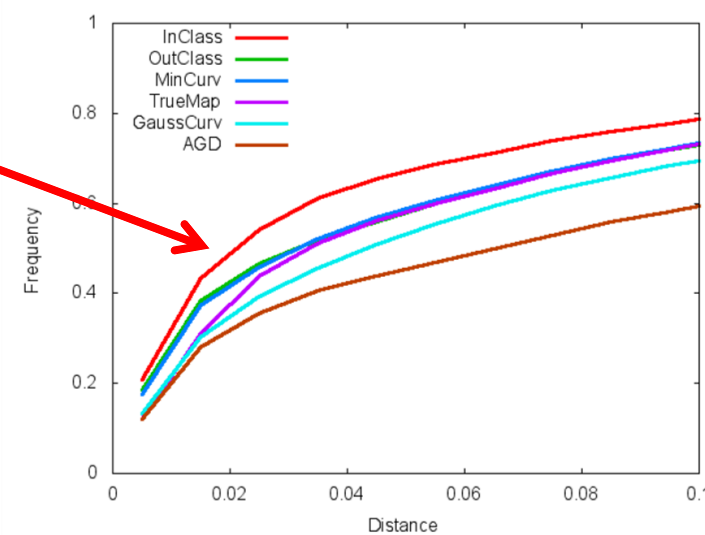


Main Conclusions IV

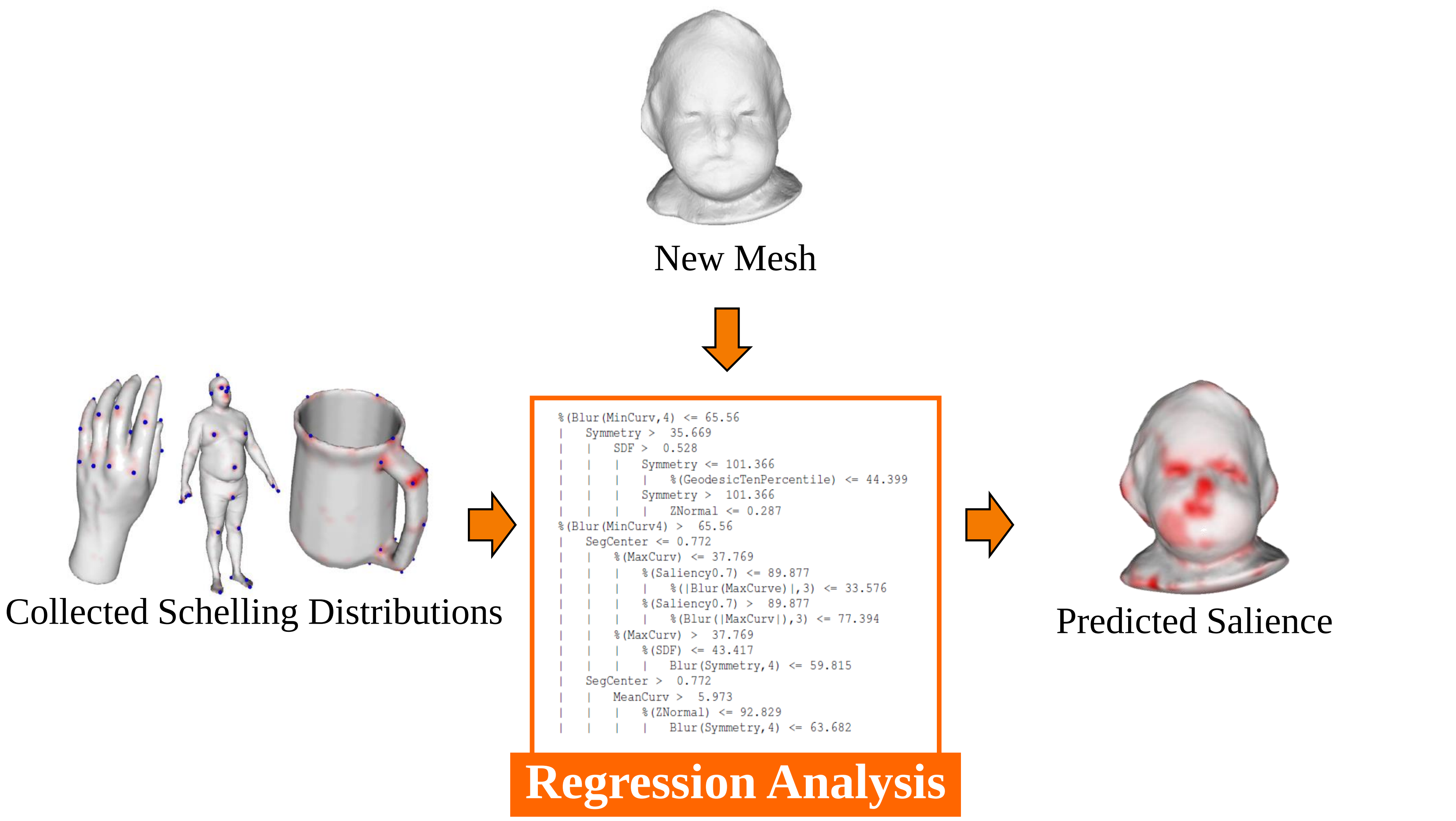
Predictive models combining many geometric properties outperform ones based on any single property

```
%(Blur(MinCurv,4) <= 65.56
|   Symmetry >  35.669
|   |   SDF >  0.528
|   |   |   Symmetry <= 101.366
|   |   |   |   %(GeodesicTenPercentile) <= 44.399
|   |   |   Symmetry >  101.366
|   |   |   |   ZNormal <= 0.287
%(Blur(MinCurv4) >  65.56
|   SegCenter <= 0.772
|   |   %(MaxCurv) <= 37.769
|   |   |   %(Saliency0.7) <= 89.877
|   |   |   |   %(|Blur(MaxCurve)|,3) <= 33.576
|   |   |   %(Saliency0.7) >  89.877
|   |   |   |   %(Blur(|MaxCurv|),3) <= 77.394
|   |   %(MaxCurv) >  37.769
|   |   |   %(SDF) <= 43.417
|   |   |   |   Blur(Symmetry,4) <= 59.815
|   SegCenter >  0.772
|   |   MeanCurv >  5.973
|   |   |   %(ZNormal) <= 92.829
|   |   |   |   Blur(Symmetry,4) <= 63.682
```



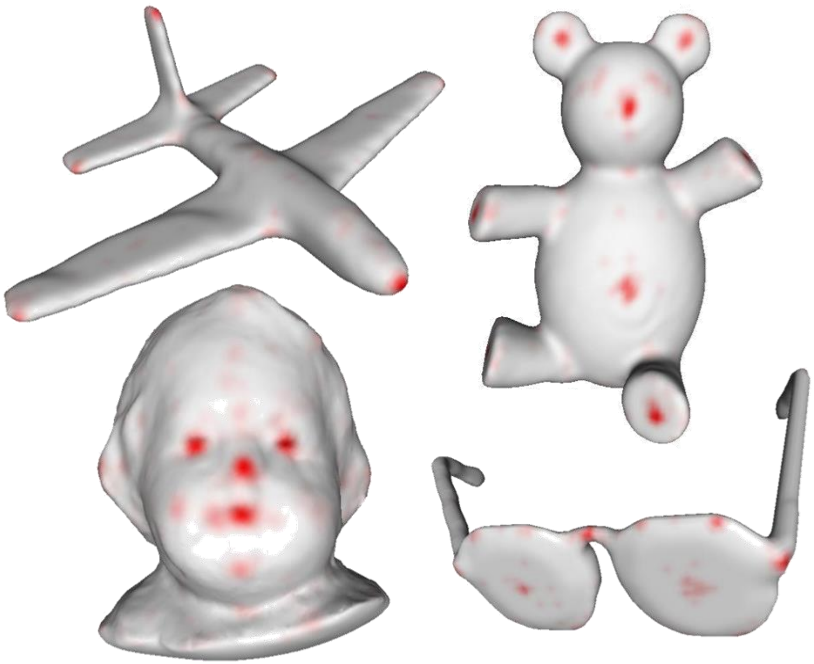


Predicting schelling distribution

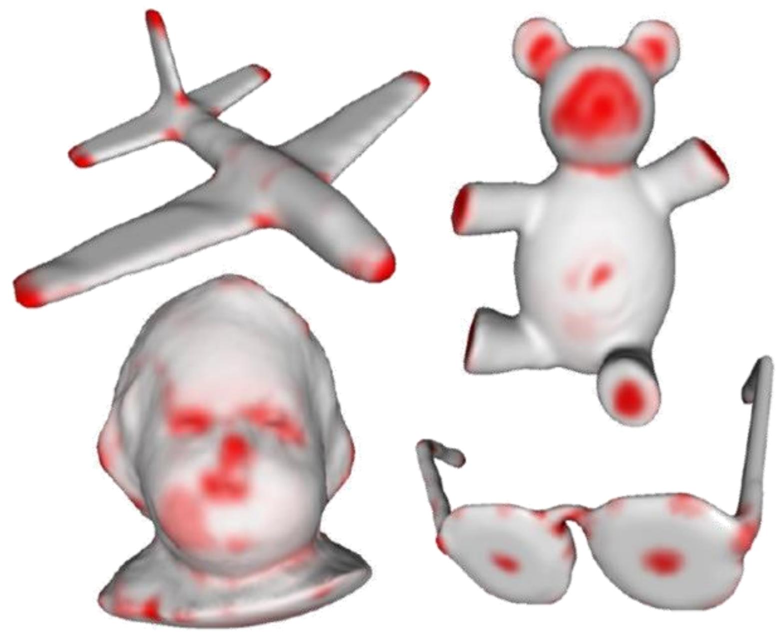


Predicting schelling distribution

Results



Ground Truth



Predicted

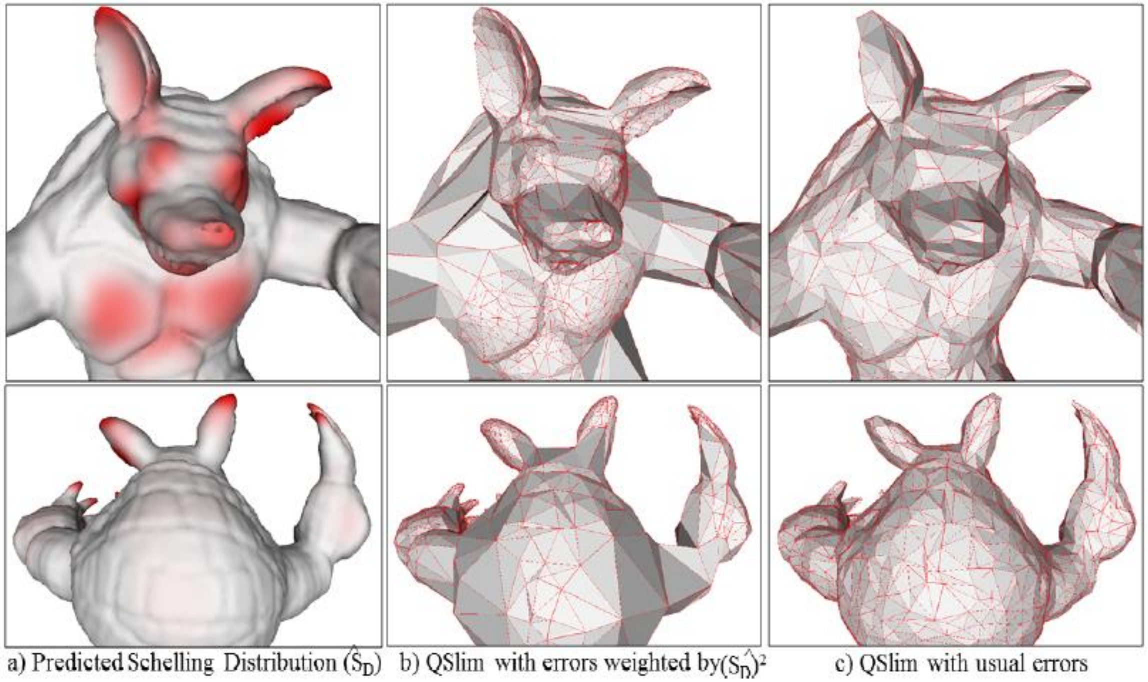
Applications

Mesh saliency:

- Simplification
- Segmentation
- View selection
- etc.

Feature points:

- Recognition
- Matching
- Morphing
- etc.

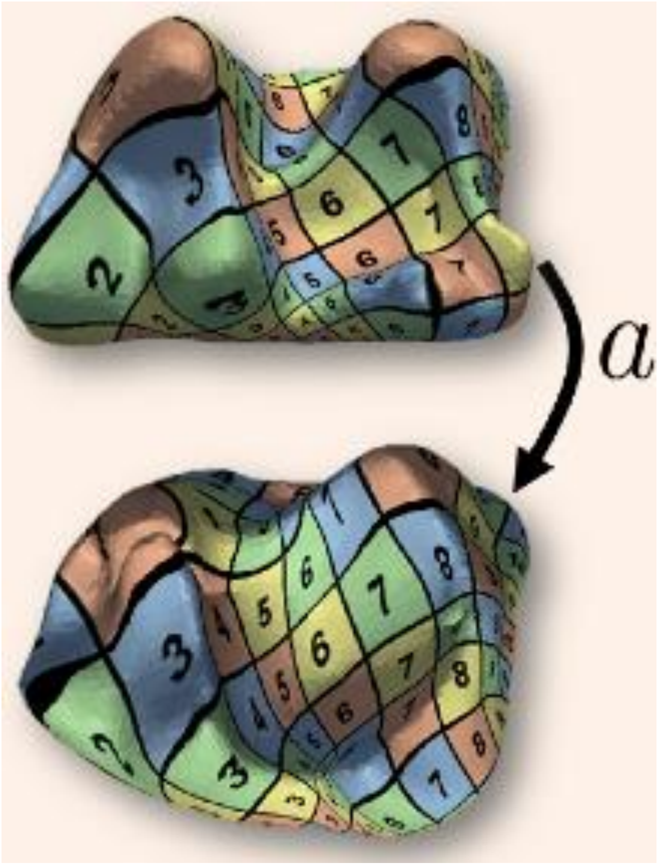


Simplification guided by saliency estimates

Surface Matching

Surface Matching

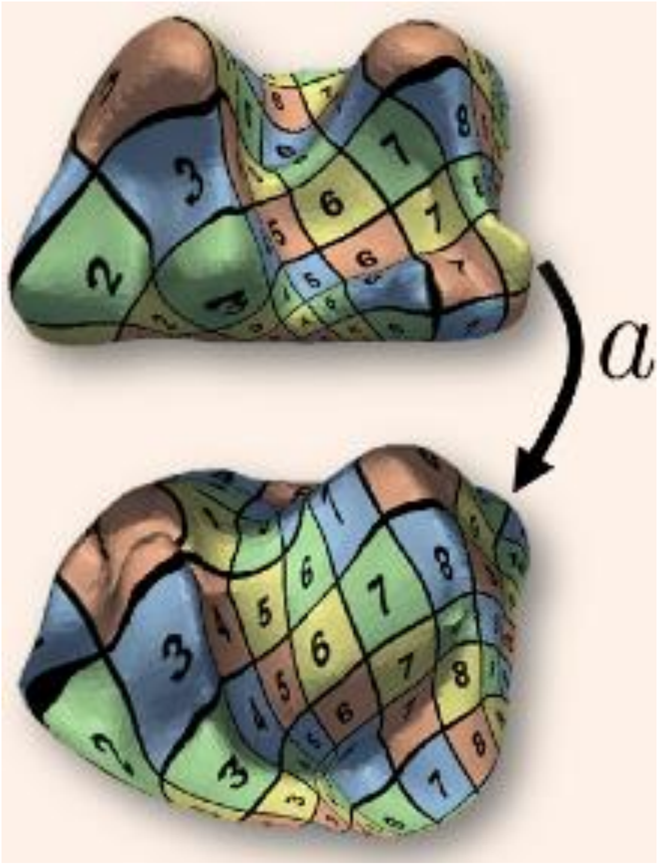
Goal: find map between surfaces



Surface Matching

Goal: find map between surfaces

- Non-rigid
- Bijective
- Smooth
- Shape preserving
- Automatic
- Efficient computation
- Provide metric
- Semantic alignment



Applications

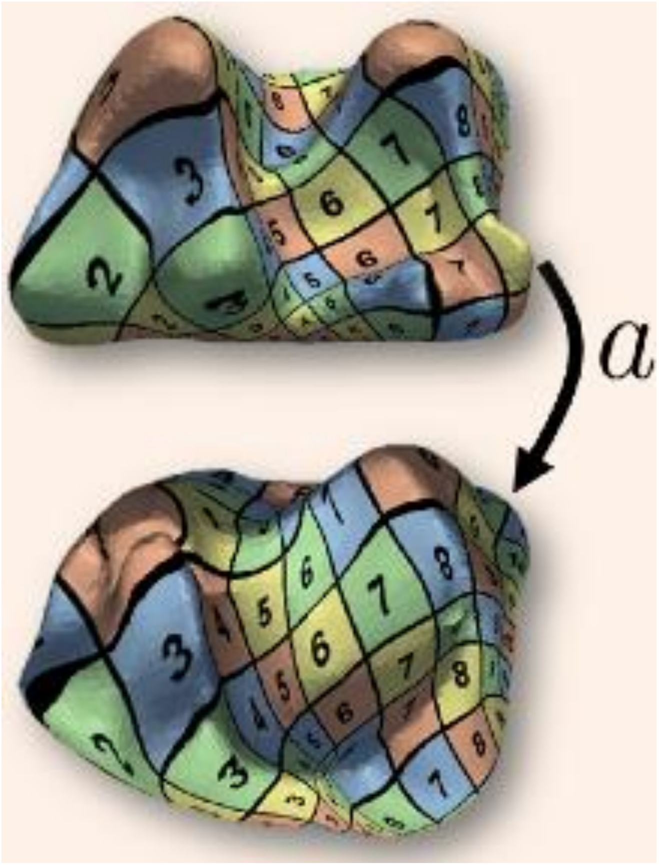
Registration

Comparison

Property transfer

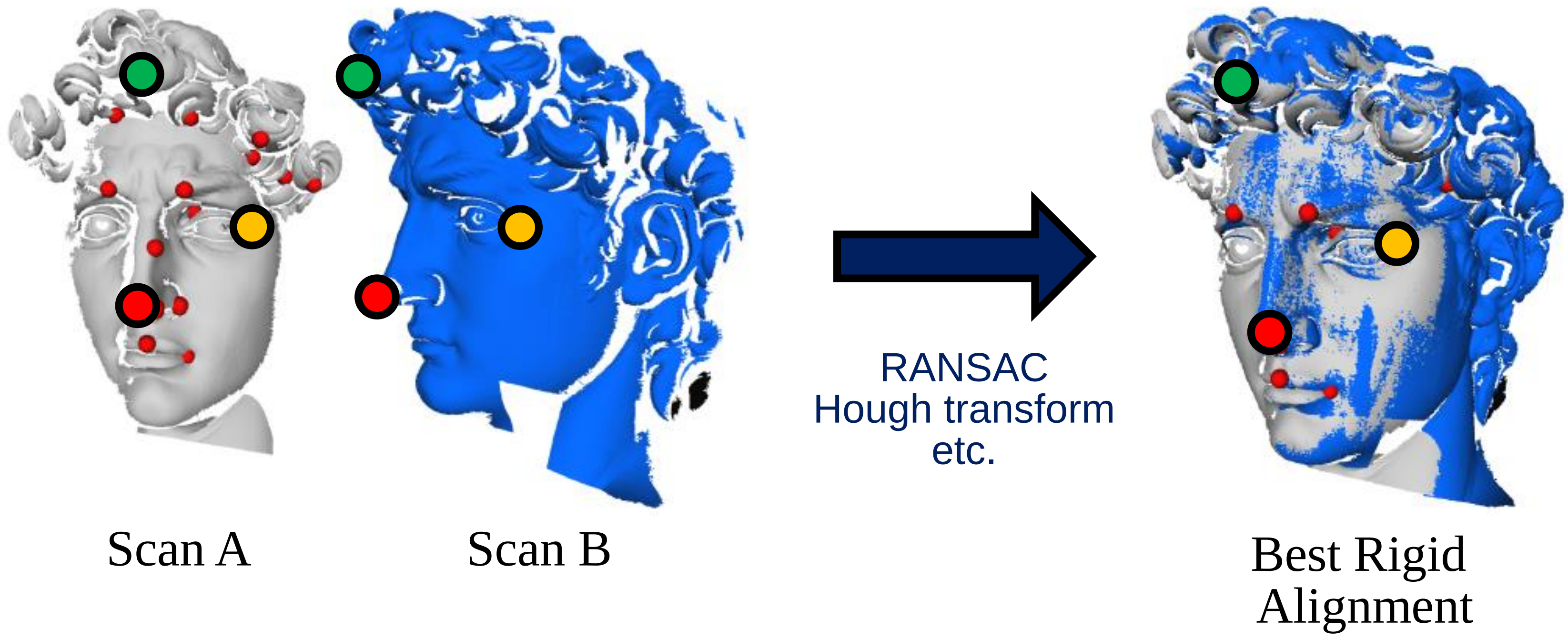
Morphing

etc.



Possible Approach

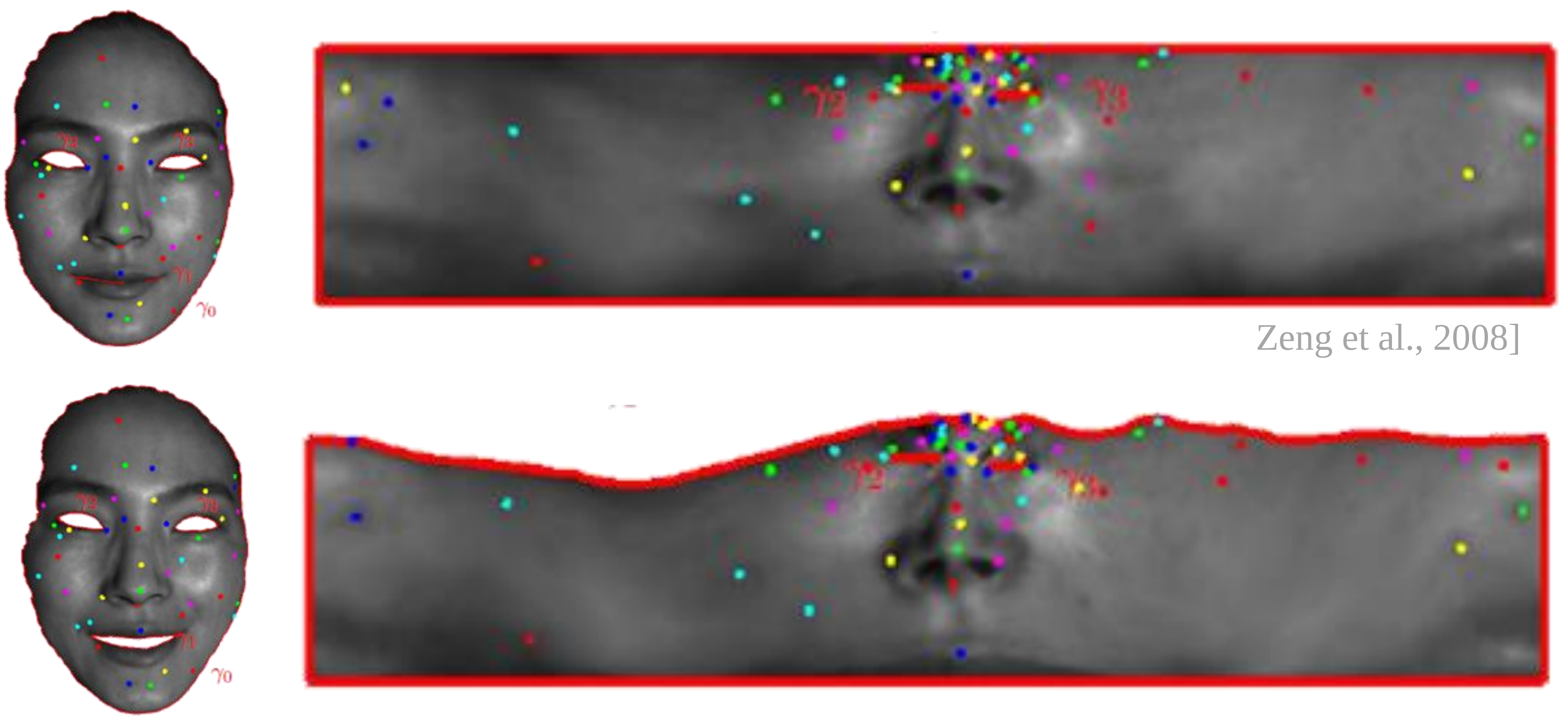
Find feature correspondences and solve for map that best aligns them



Suitable only for "low-dimensional" maps

Challenge

Many feature points are needed for most maps between surfaces

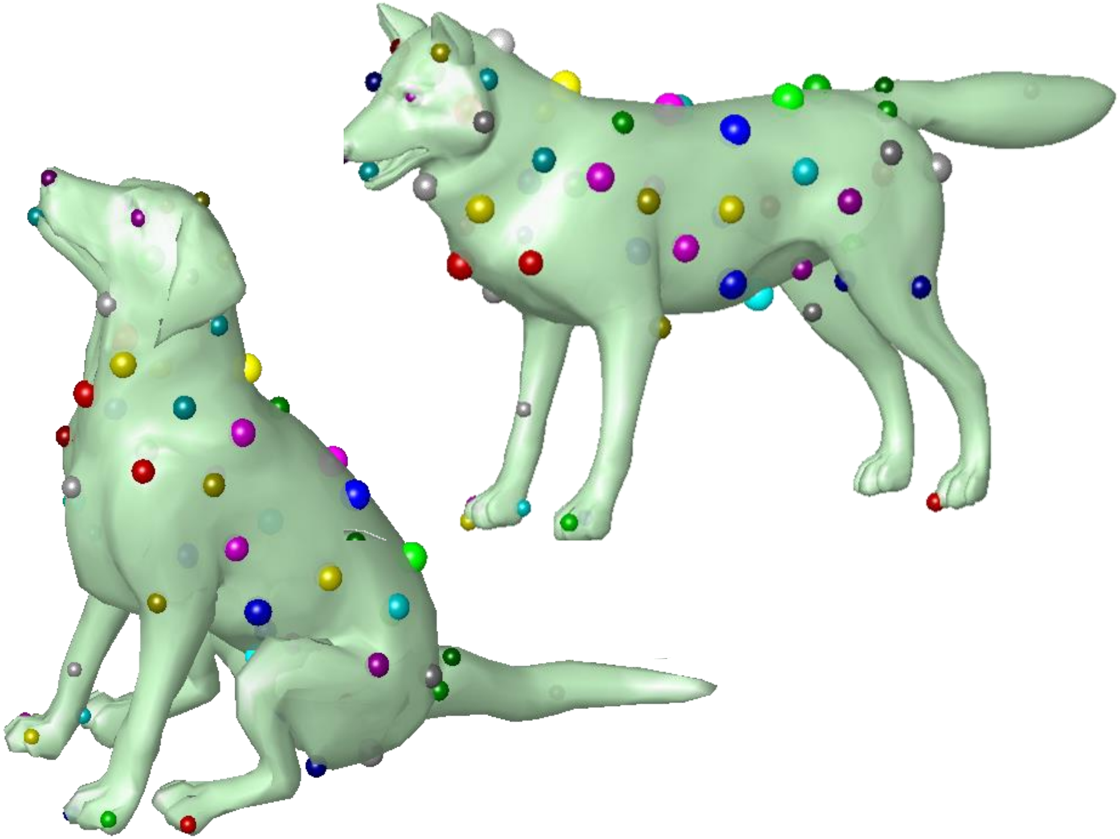


Zeng et al., 2008]

Least Squares Conformal Map
(preserve angles as best as possible)

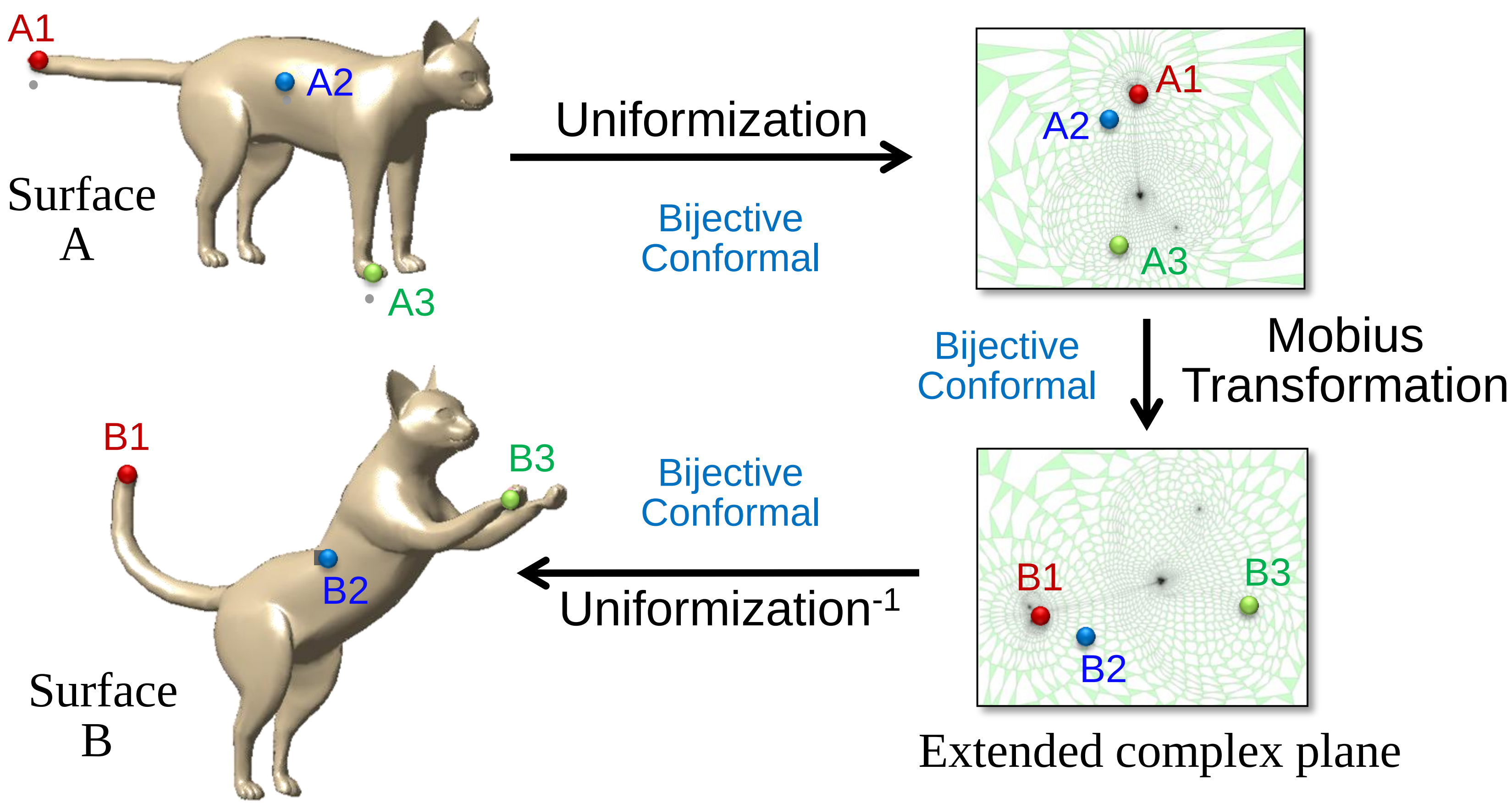
Problem

Automatically finding many correspondences is difficult for surfaces



Key Observation

Any three point correspondences define a bijective, conformal map between genus zero surfaces



Key Observation

We can search for the “lowest distortion” bijective, conformal map between genus zero surfaces using algorithms that sample triplets of correspondences (e.g., RANSAC, Hough transform, etc.)

Polynomial-time algorithm
for non-rigid surface mapping

Surface Mapping Algorithm

Example: RANSAC algorithm

For i = 1 to ~N^3

Sample three points (A1,A2,A3) on surface A

Sample three points (B1,B2,B3) on surface B

Compute conformal map M: (A1,A2,A3)→(B1,B2,B3)

Remember M if distortion is smallest

Surface Mapping Algorithm

Example: RANSAC algorithm

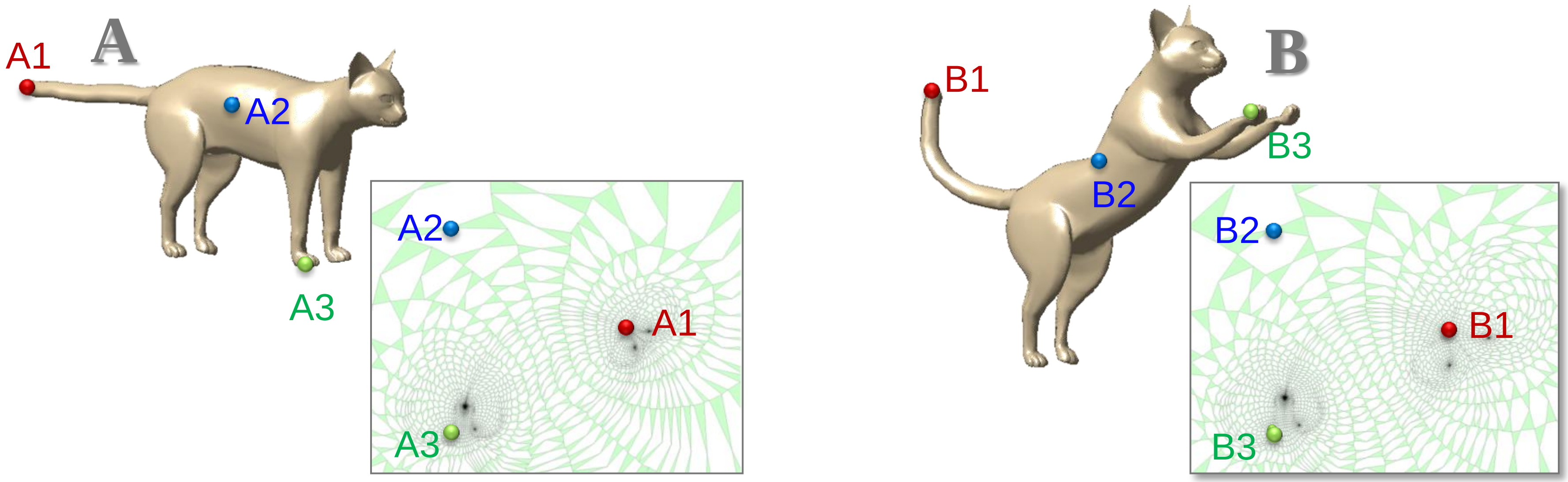
For i = 1 to ~N^3

Sample three points (A1,A2,A3) on surface A

Sample three points (B1,B2,B3) on surface B

Compute conformal map M: (A1,A2,A3)→(B1,B2,B3)

Remember M if distortion is smallest



Measure distortion by relative change of area
(deviation from isometry)

Surface Mapping Algorithm

Example: RANSAC algorithm

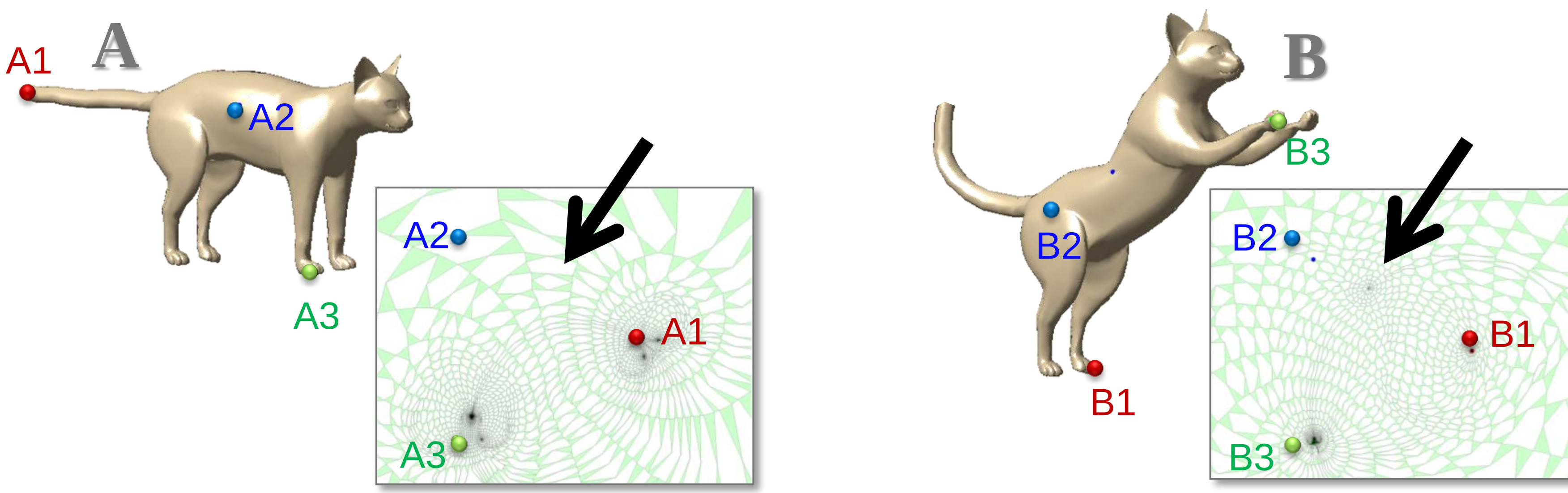
For i = 1 to $\sim N^3$

Sample three points (A1,A2,A3) on surface A

Sample three points (B1,B2,B3) on surface B

Compute conformal map M: (A1,A2,A3)→(B1,B2,B3)

Remember M if distortion is smallest



Measure distortion by relative change of area
(deviation from isometry)

Surface Mapping Algorithm

RANSAC algorithm properties:

- Non-rigid
- Bijective
- Smooth
- Shape preserving
- Automatic
- Efficient computation
- Provides metric
- Semantic alignment?

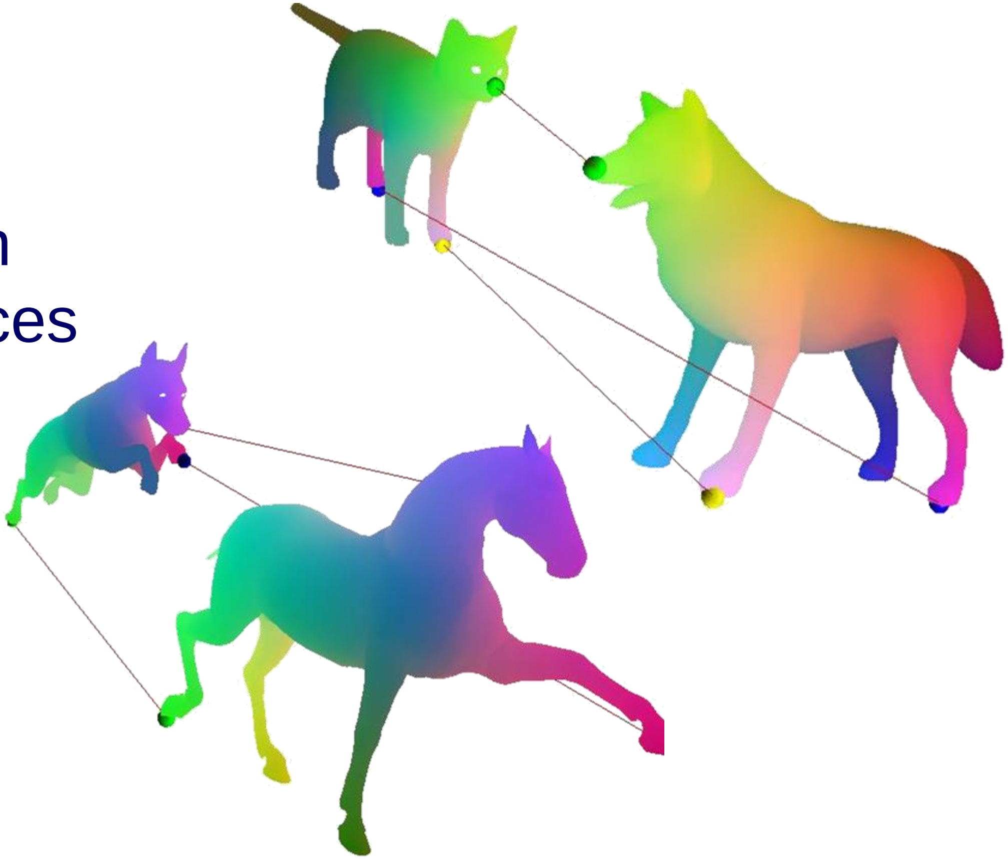
Experimental Results

Data:

- 51 pairs of meshes representing animals from TOSCA and SHREC Watertight data sets

Methodology:

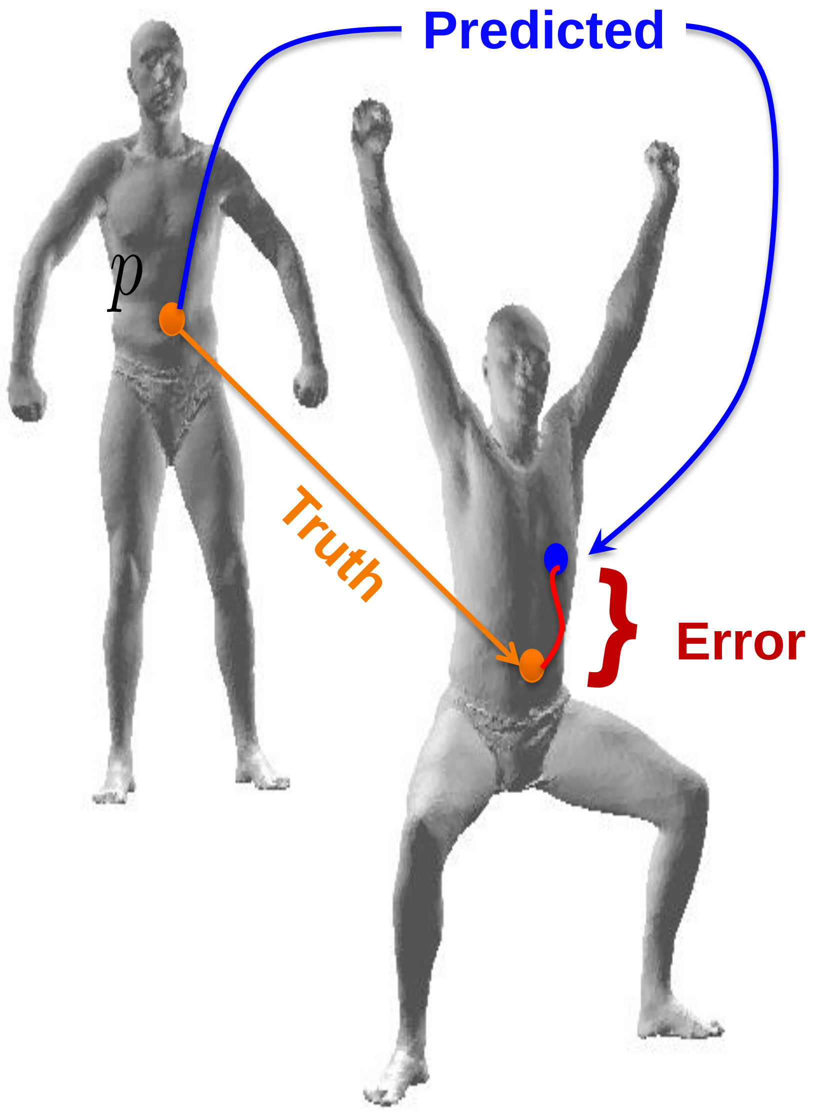
- Predict surface maps
- Compare to ground truth semantic correspondences



Experimental Results

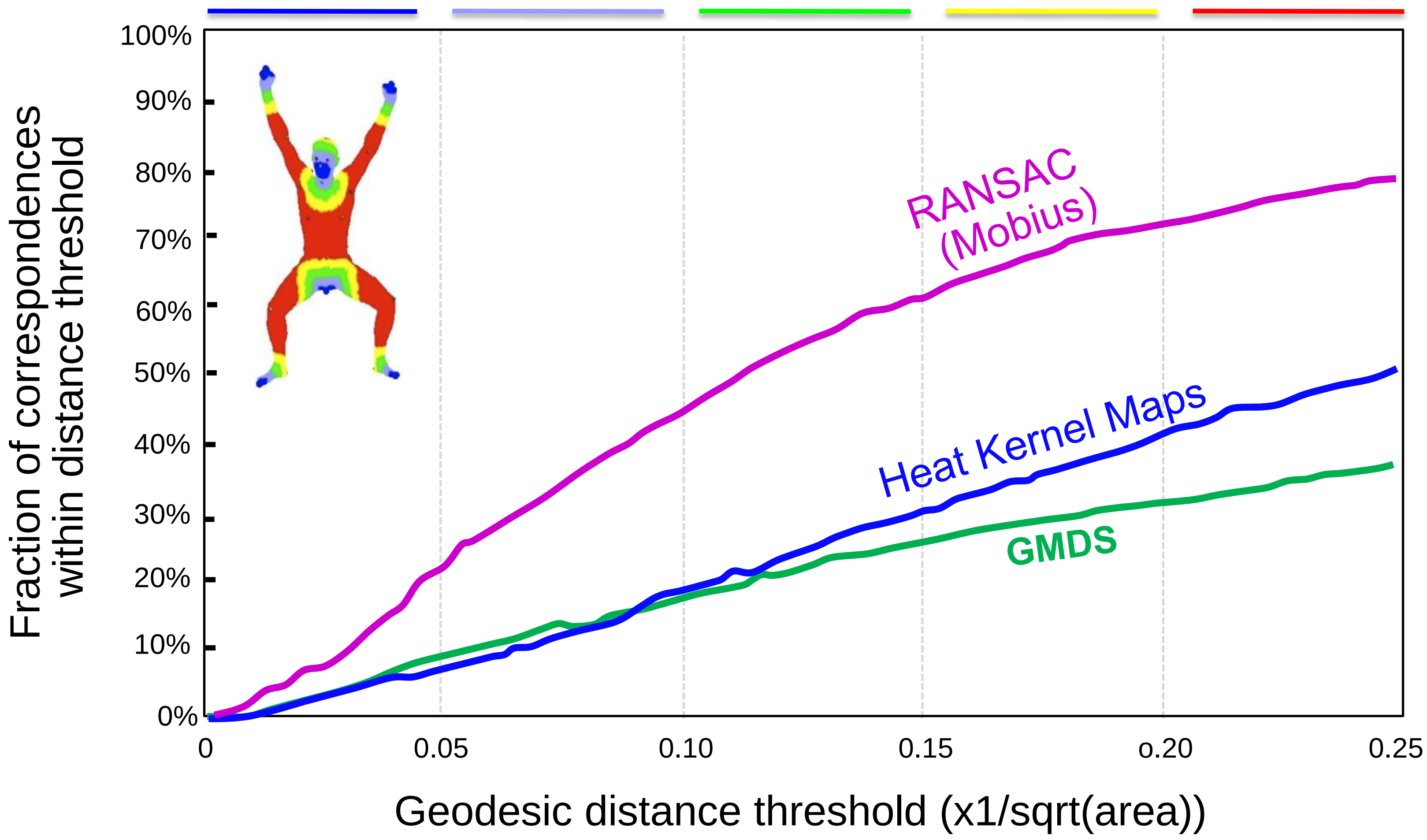
Evaluation:

1. For every point with a ground truth correspondence, measure geodesic distance between predicted correspondence and ground truth correspondence
2. Plot fraction of points within geodesic error threshold



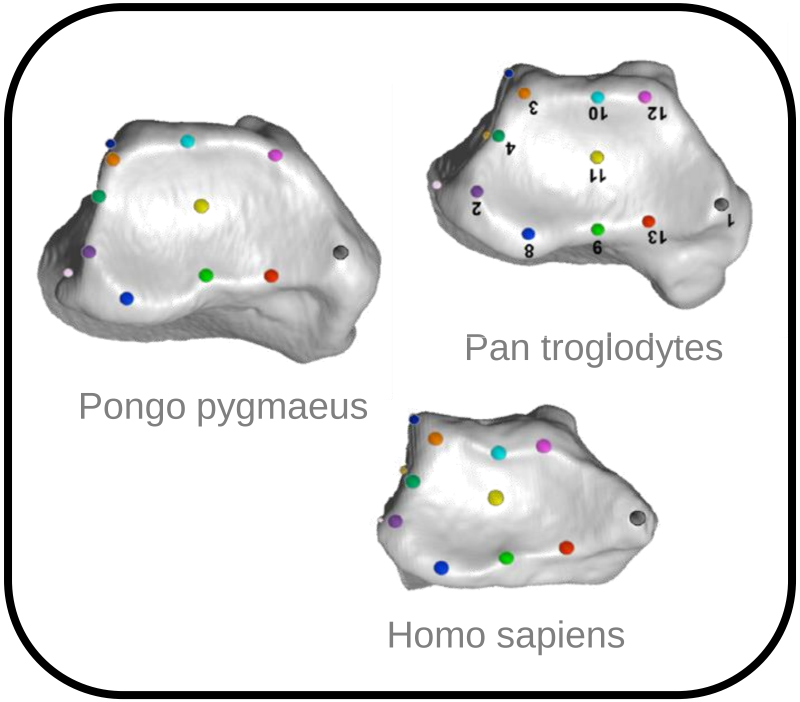
Experimental Results

Results:

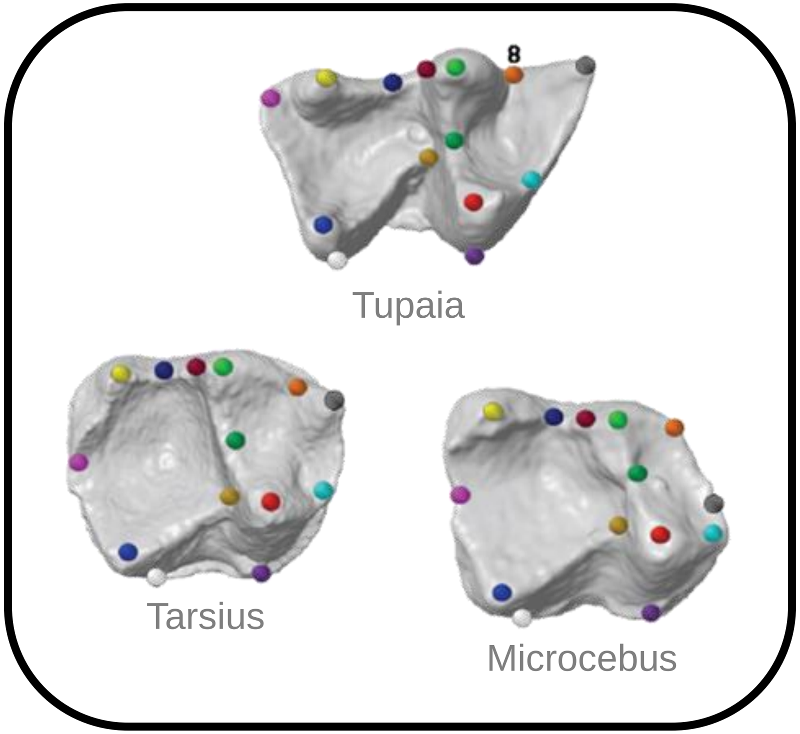


Application

Automatically quantify the geometric similarity of anatomical surfaces



Distal Radius



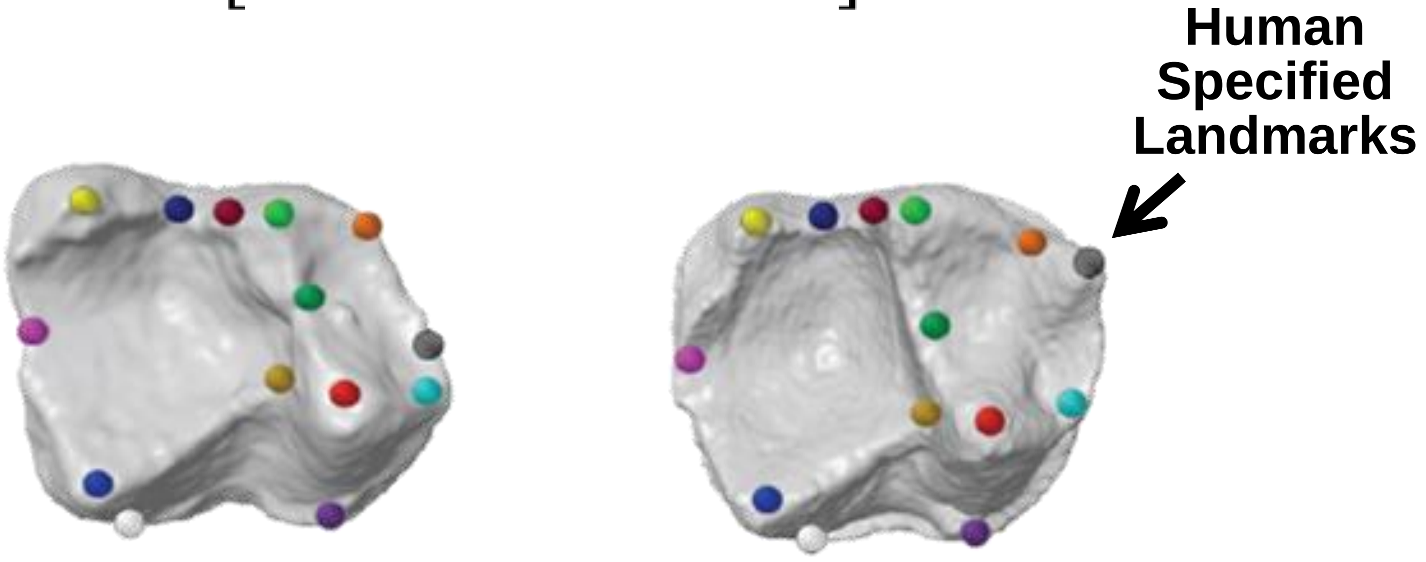
Mandibular Molar

[Boyer, Lipman, St. Clair, Puente, Patel, Funkhouser, Jernvall, and Daubechies, 2011]

Application

Traditional Procrustes distance:

$$d(X,Y) = min_R\left[\left(\sum_{i=1}^{N}\|R(X_i)\ -Y_i\|^2\right)^{1/2}\right]$$

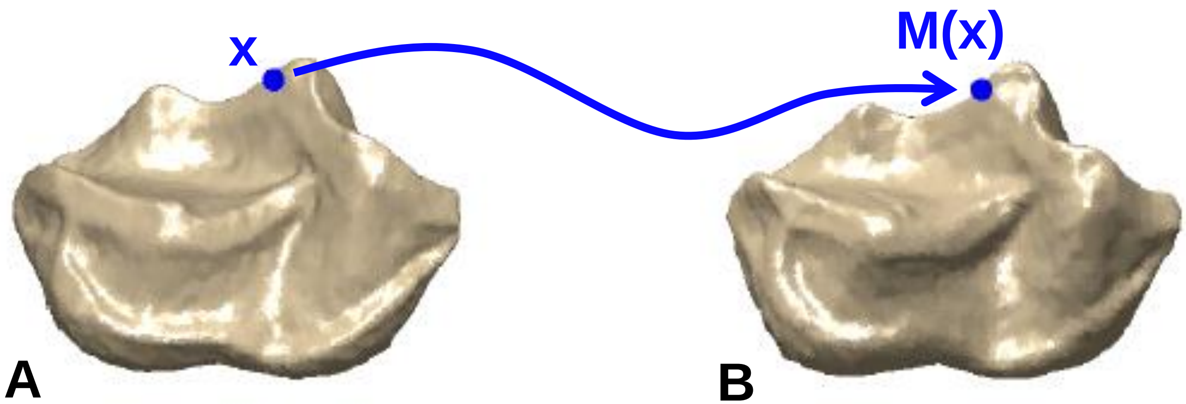


$X = \{ X_i \}$ $Y = \{ Y_i \}$

Application

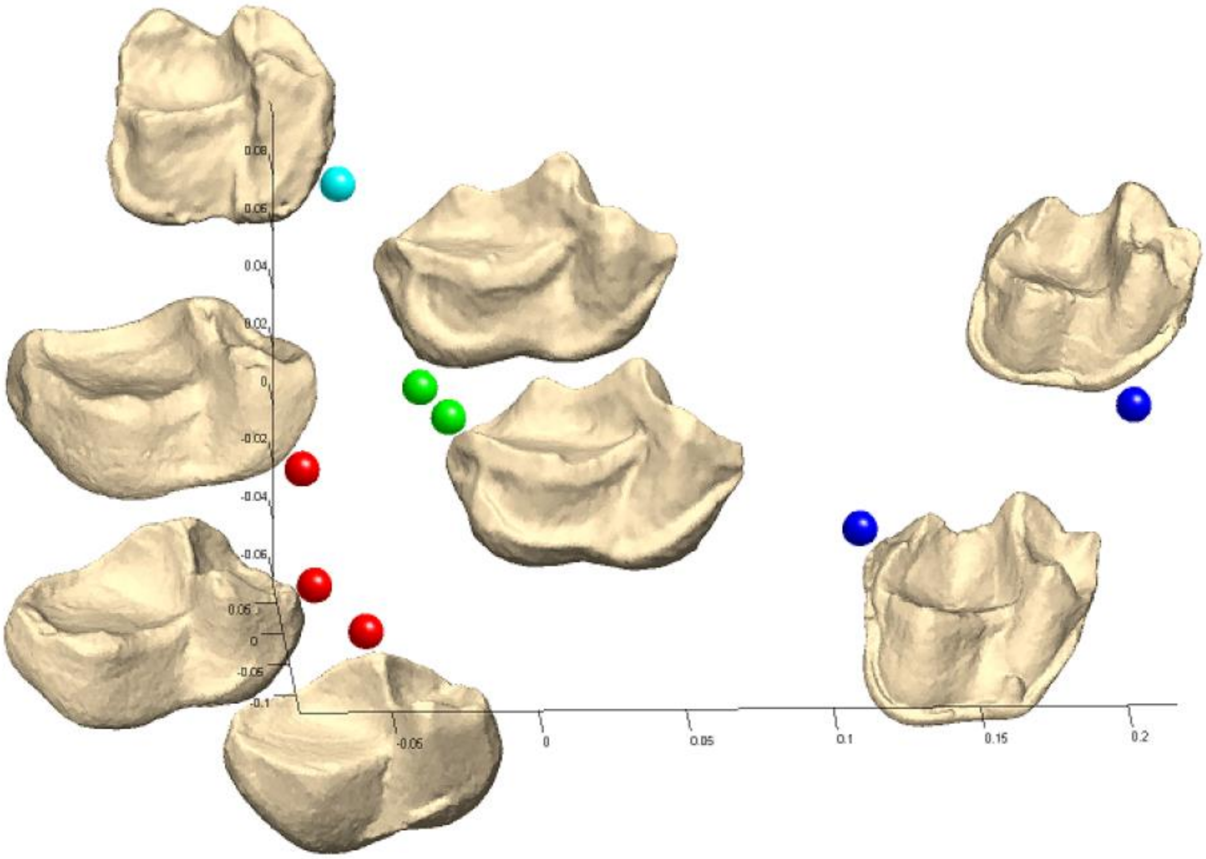
New continuous Procrustes distance:

$$d(A,B) = min_{R,M}\left[\left(\int_A \|R(x) - M(x)\|^2\, dx\right)^{1/2}\right]$$



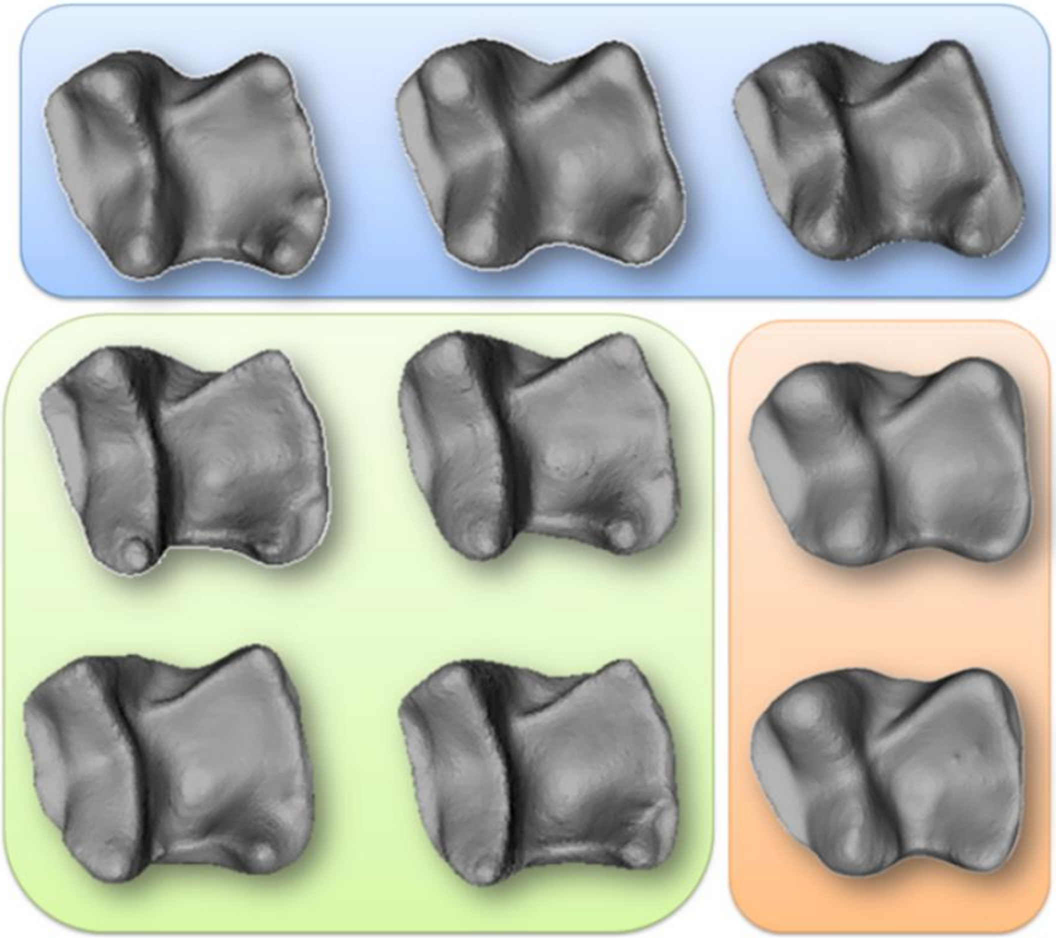
Application

Embedding based on new distance



Application

Clustering based on new distance



Species Groups of Galaga Genus

Application

Classification based on nearest-neighbors

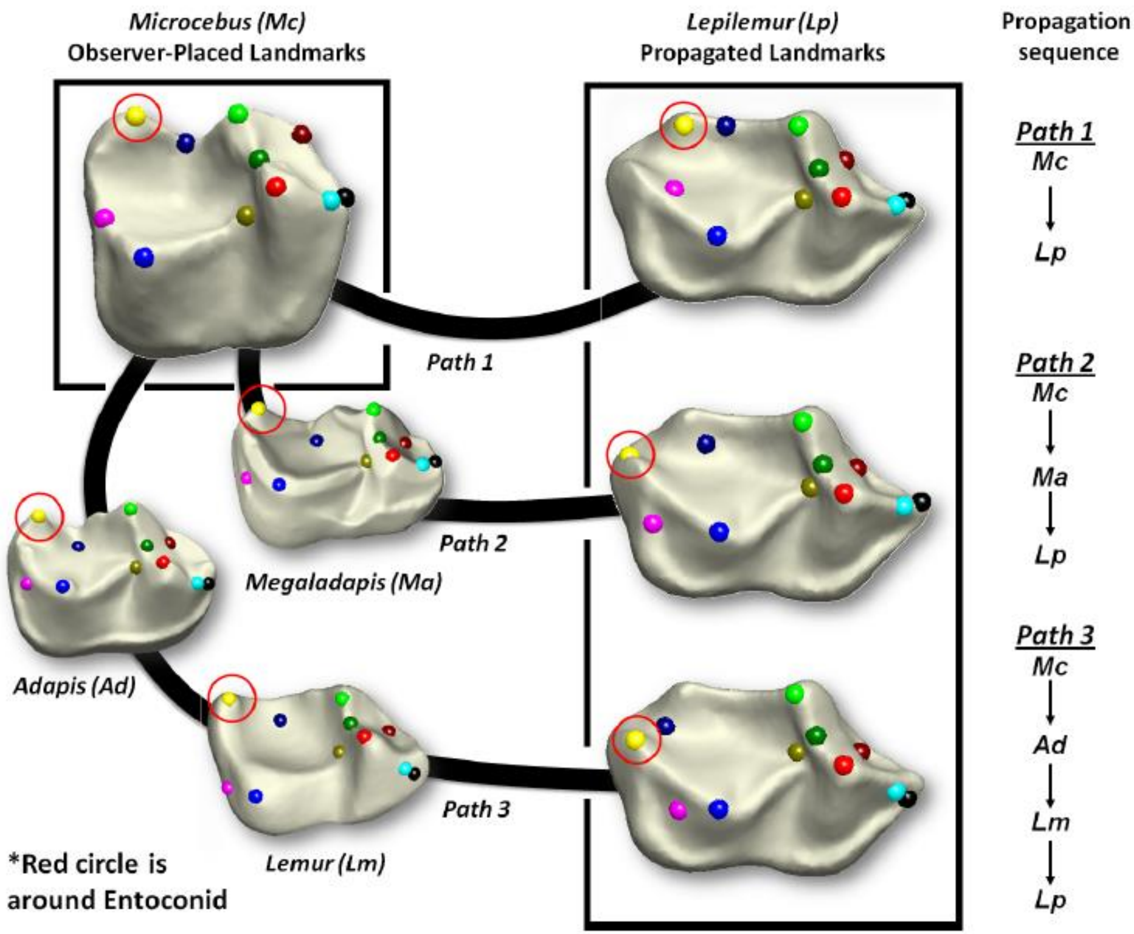
Mandibular Molar	# Groups	# Objects	New Distance	Human Landmarks
Genus	24	99	90.9%	91.9%
Family	17	106	92.5%	94.3%
Order	5	116	94.8%	95.7%

First Metatarsal	# Groups	# Objects	New Distance	Human1 Landmarks	Human2 Landmarks
Genus	13	59	79.9%	76.3%	88.1%
Family	9	61	91.8%	83.6%	93.4%
Superfamily	2	61	100%	100%	100%

Distal Radius	# Groups	# Objects	New Distance	Human Landmarks
Genus	4	45	84.4%	77.7%

Application

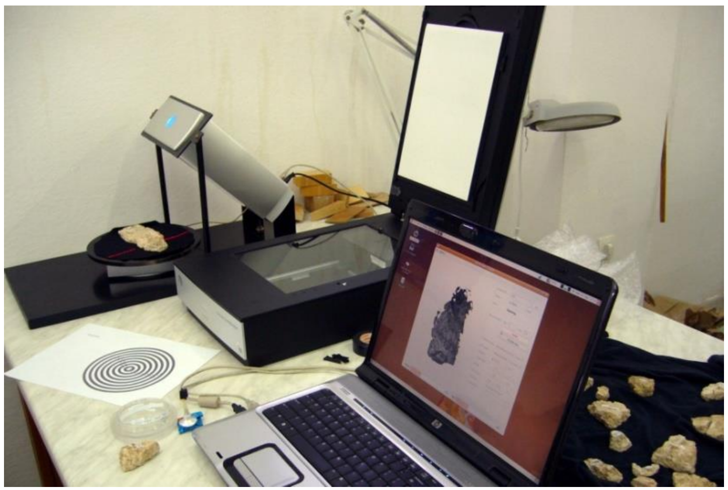
Propagating correspondences

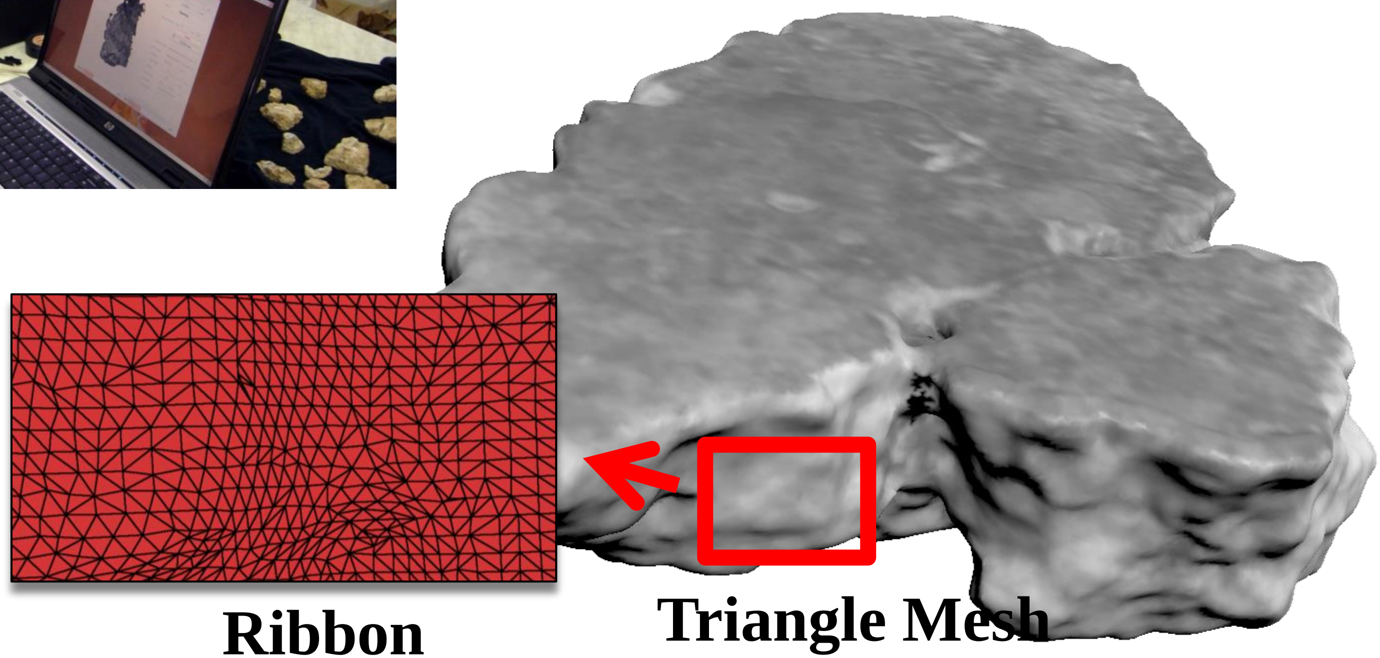


Reconstruction

Computer-Assisted Reconstruction

1) Scan digital representations of fragments

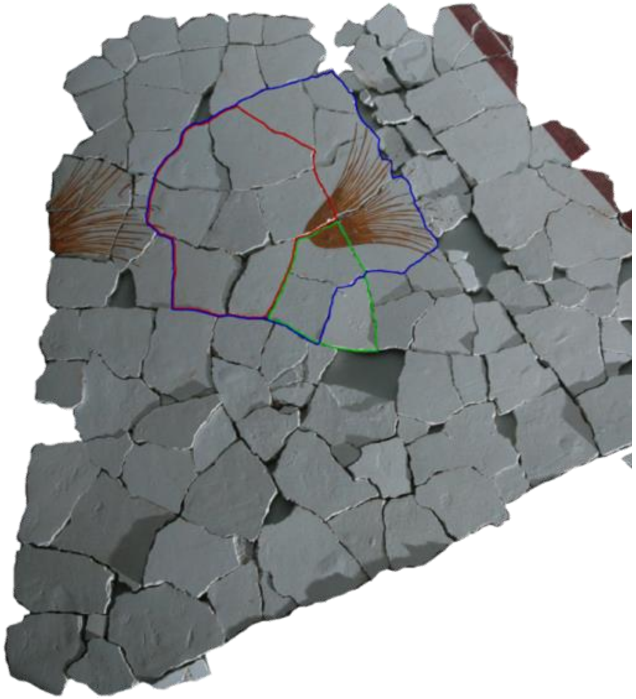




Computer-Assisted Reconstruction

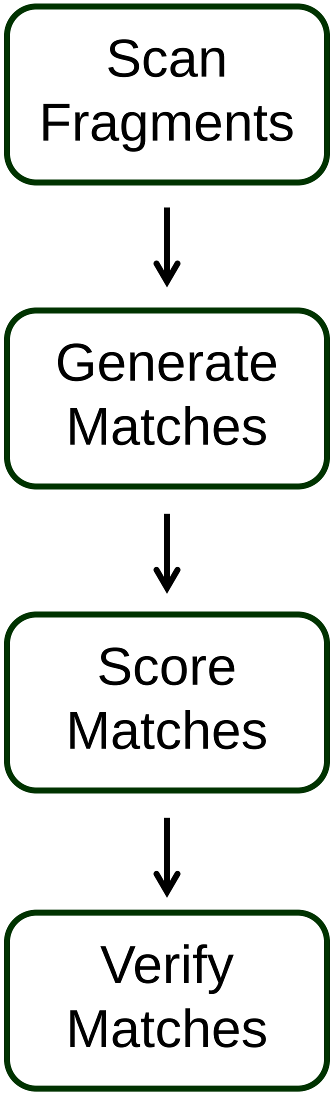
2) Reconstruct frescoes with computer algorithms

Scanned Fragments



Reconstructed Fresco

Computer-Assisted Reconstruction



Computer-Assisted Matching

Scan Fragments

↓

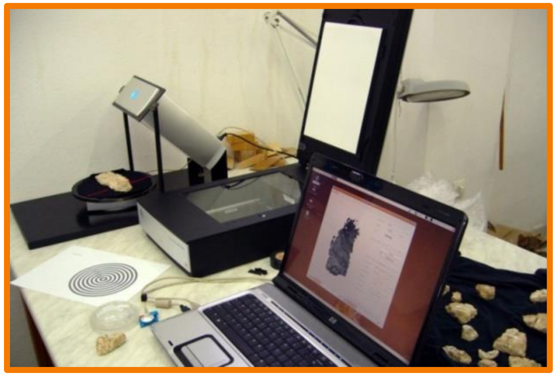
Generate Matches

↓

Score Matches

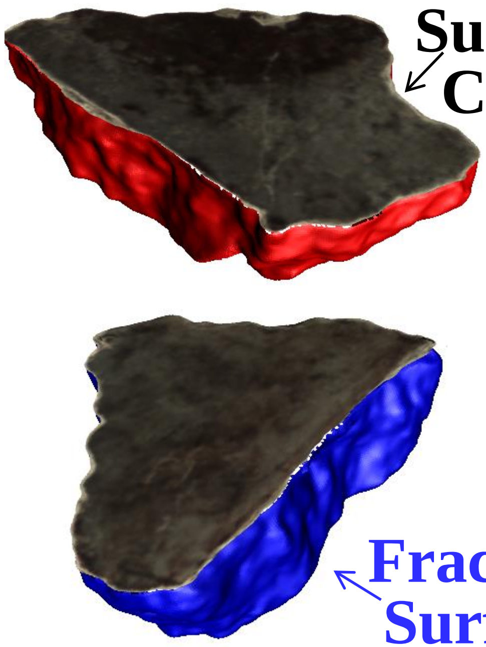
↓

Verify Matches



Scanning System

Brown et al., SIGGRAPH 2008



Scanned Fragments

Computer-Assisted Matching

Scan Fragments

↓

Generate Matches

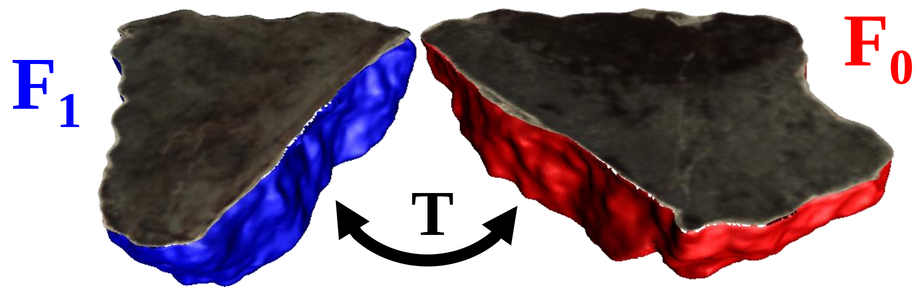
↓

Score Matches

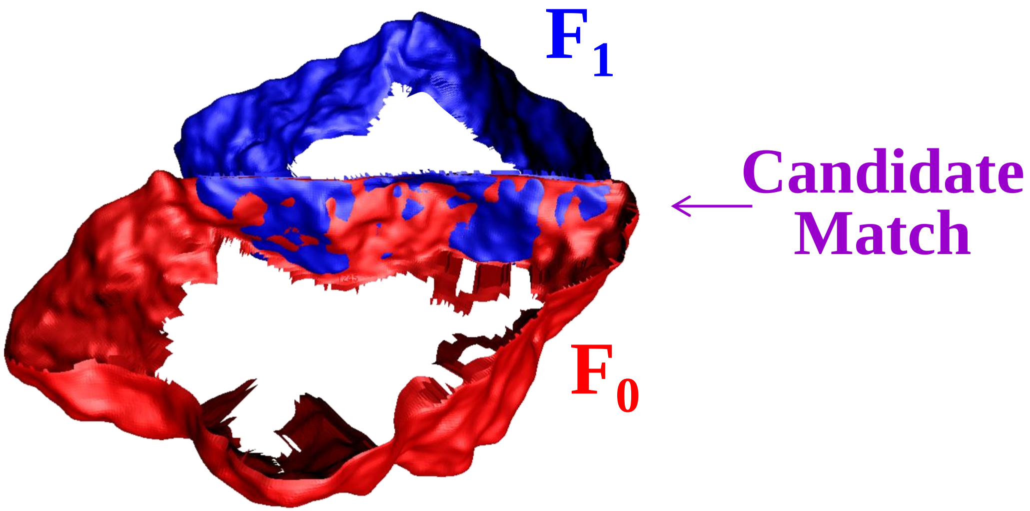
↓

Verify Matches

For every pair of fragments F_0 and F_1 …



Generate candidate match for every possible aligning transformation T



Computer-Assisted Matching

Scan Fragments

↓

Generate Matches

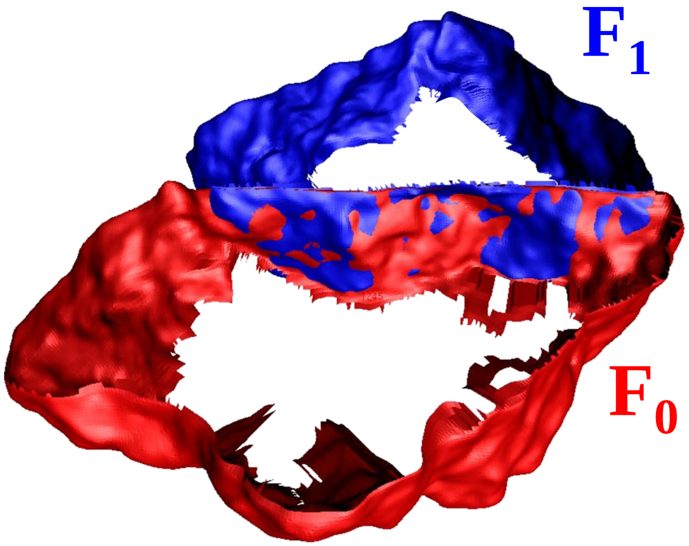
↓

Score Matches

↓

Verify Matches

For every candidate match, compute a score representing “how good it is”



Candidate Match

$$S(F_0,F_1,T)$$

Score

Computer-Assisted Matching

Scan Fragments

↓

Generate Matches

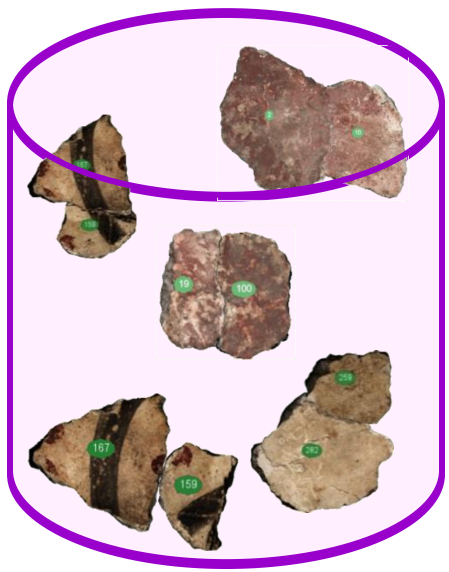
↓

Score Matches

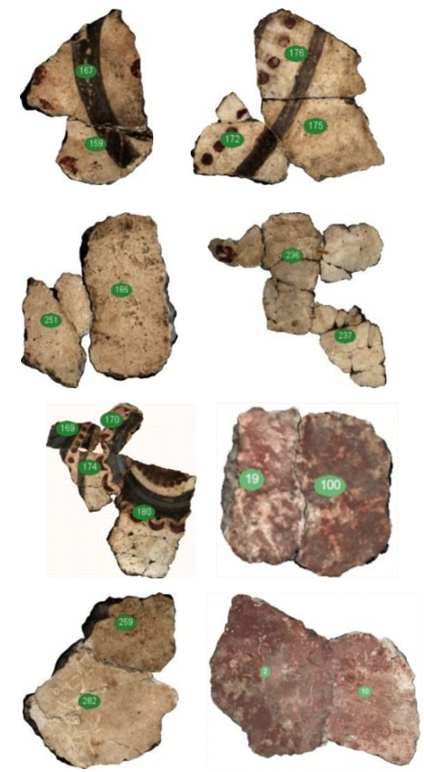
↓

Verify Matches

Sort the candidate matches by score, and check top ones to see if they are correct

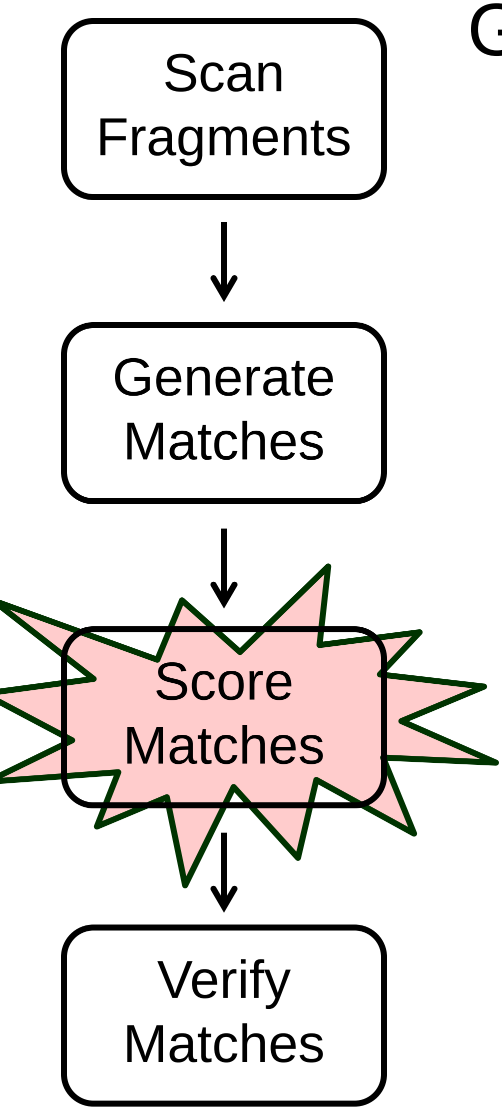


Candidate Matches (millions)



Verified Correct Matches (tens or hundreds)

Focus of This Talk



Goal: Develop a scoring method that accurately estimates the probability that a candidate match is correct

Previous Methods

Most prior systems scored matches using functions combining a few match properties with weights

- McBride et al., 2003

$$\lambda_1 \cdot C_{\text{distance}} + \lambda_2 \cdot \sqrt{C_{\text{length}}} + \lambda_3 \cdot \sqrt{C_{\text{diagnostic}}}$$

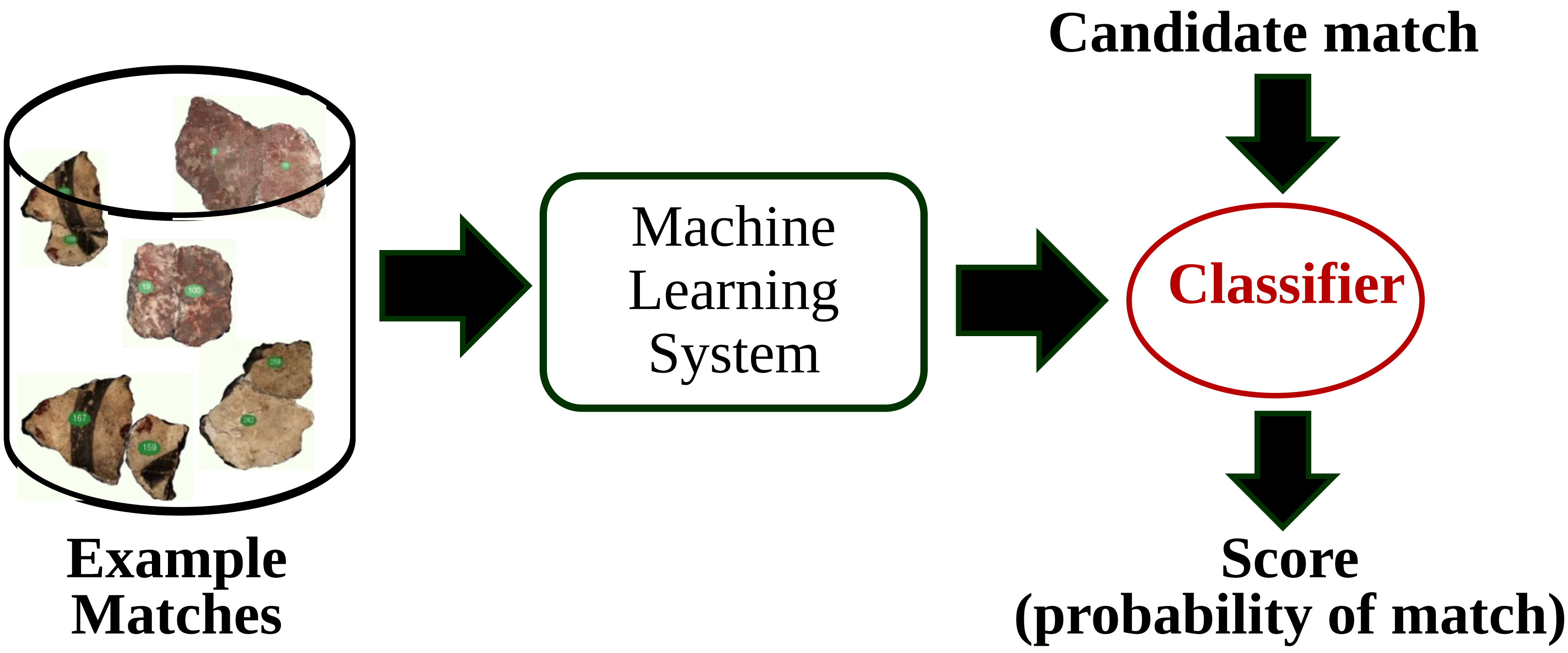
- Brown et al., 2008 (Ribbonmatcher Error)

$$\lambda_1 \cdot C_{\text{WindowRMSD}} + \lambda_2 \cdot C_{\text{Thickness}}$$

Our Approach

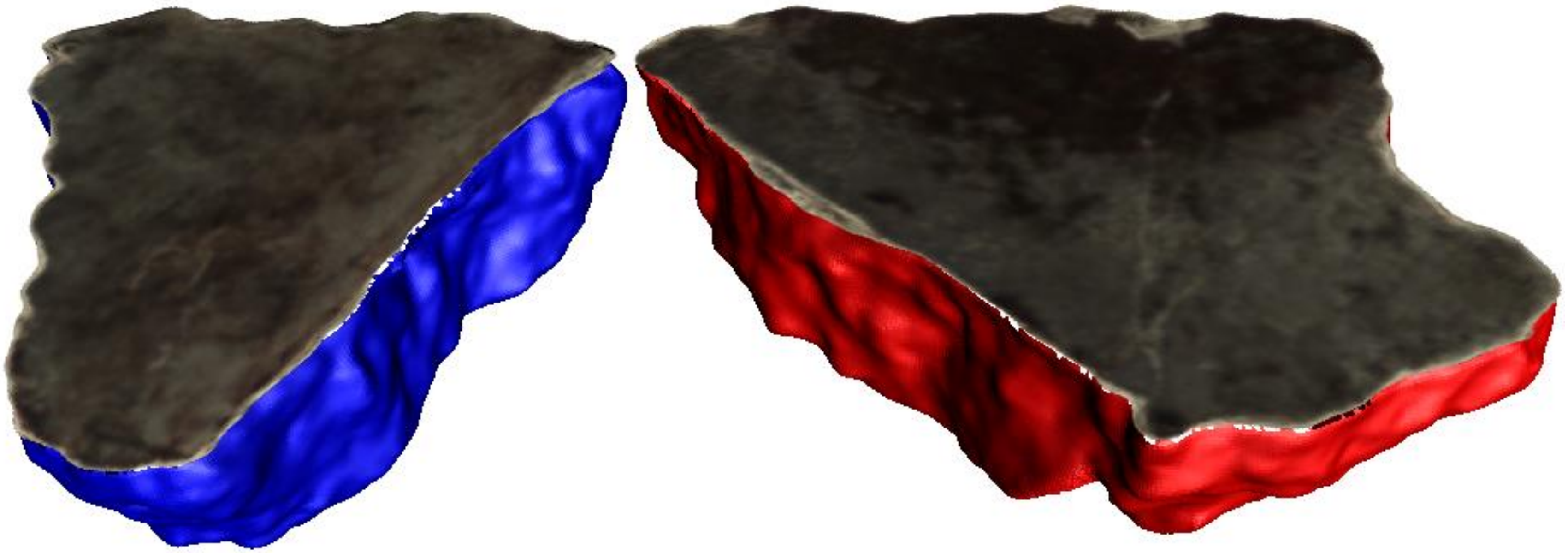
Machine learning

- User provides example correct and incorrect matches
- System learns classifier to predict correctness of new candidate matches based on their properties



Computing Match Properties

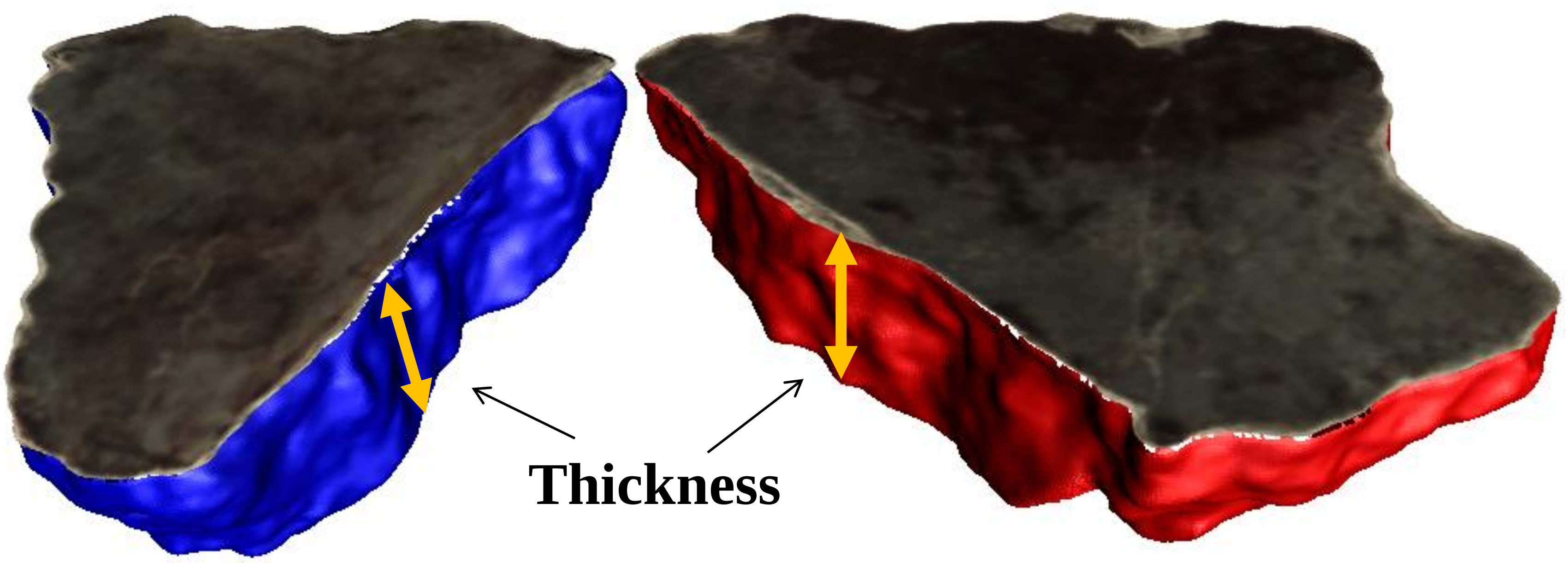
Measure compatibility of fragments



Computing Match Properties

Measure compatibility of fragments

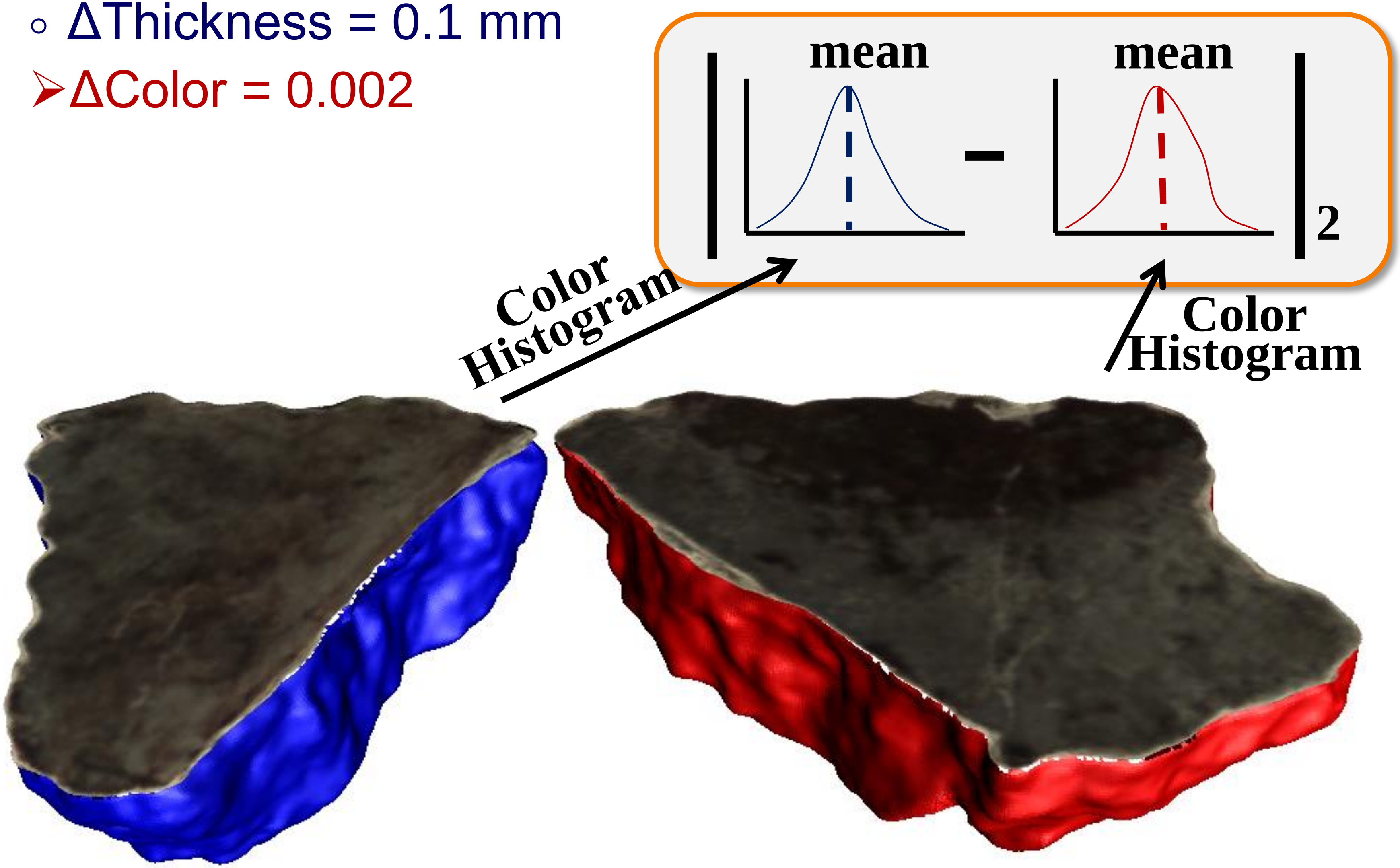
- ΔThickness = 0.1 mm



Computing Match Properties

Measure compatibility of fragments

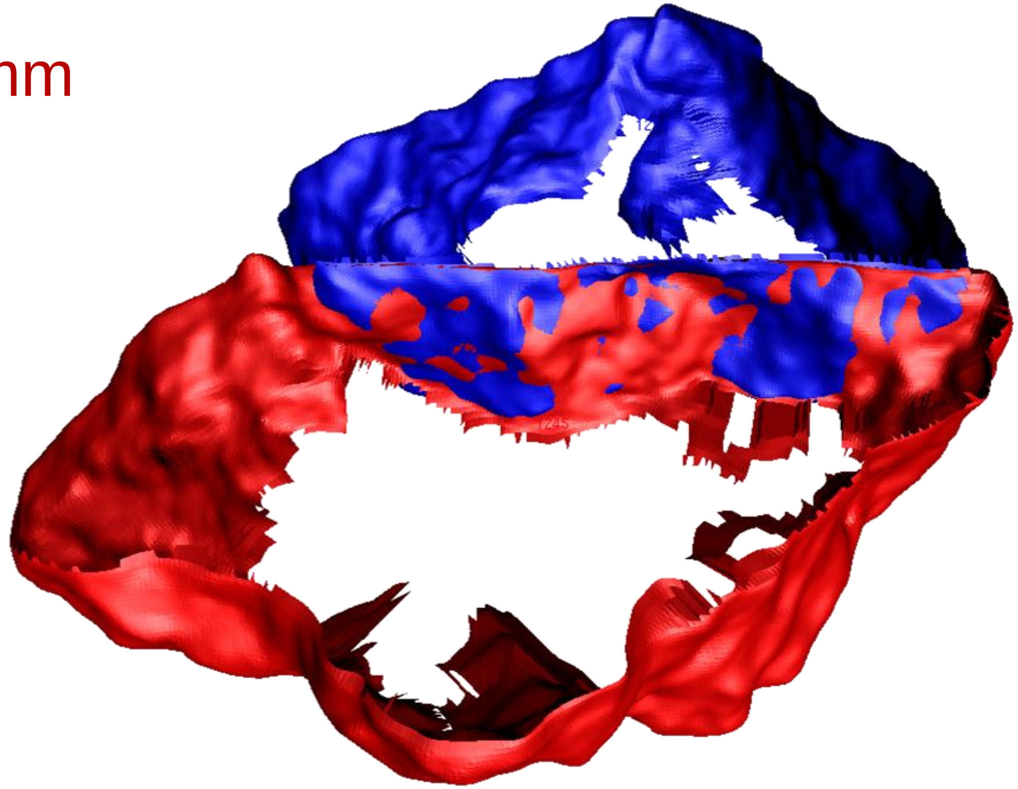
- ΔThickness = 0.1 mm
- ➢ΔColor = 0.002



Computing Match Properties

Measure alignment of fragments

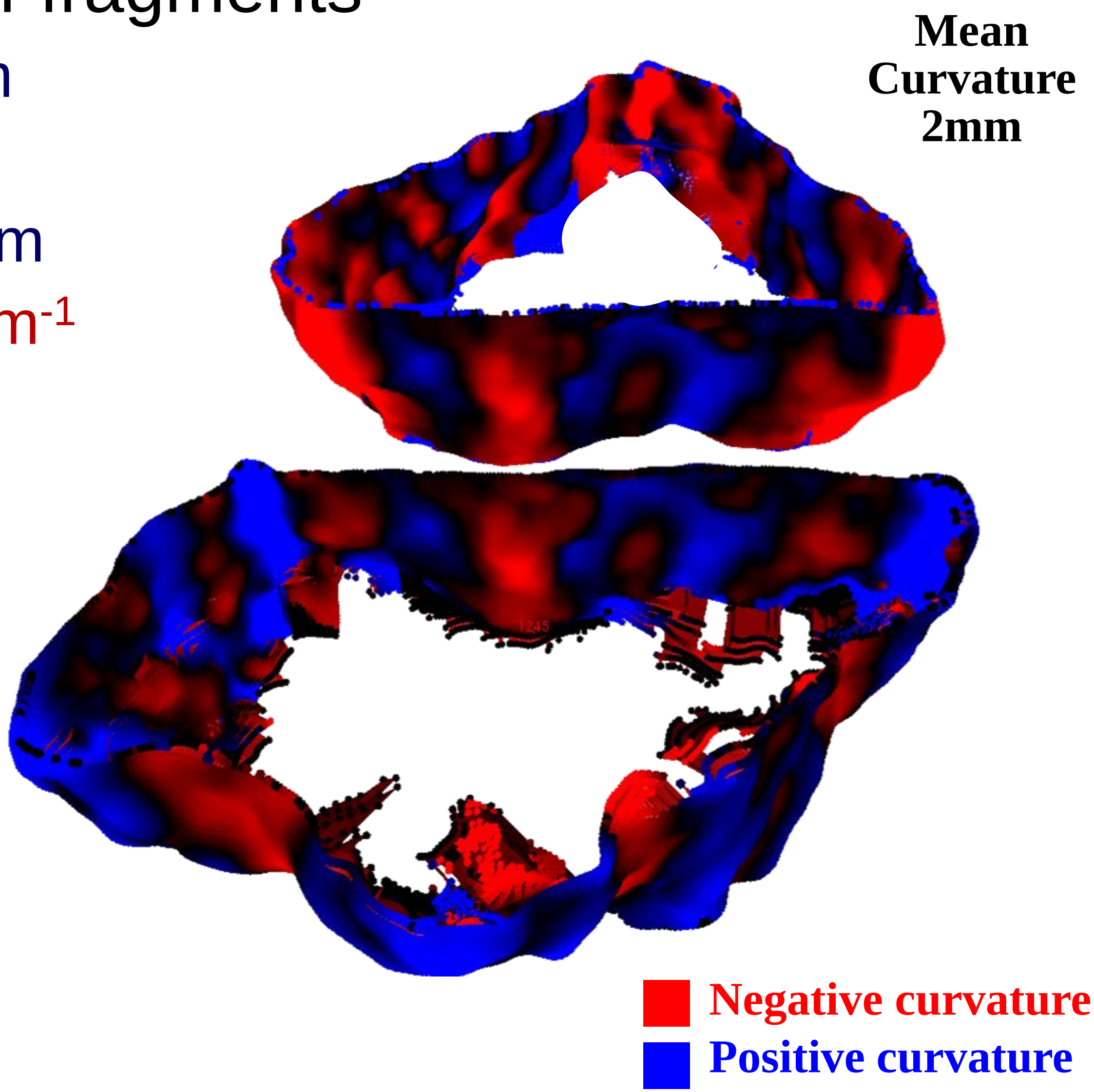
- ΔThickness = 0.1 mm
- ΔColor = 0.002
- ➢ΔAlignment = 0.24 mm



Computing Match Properties

Measure alignment of fragments

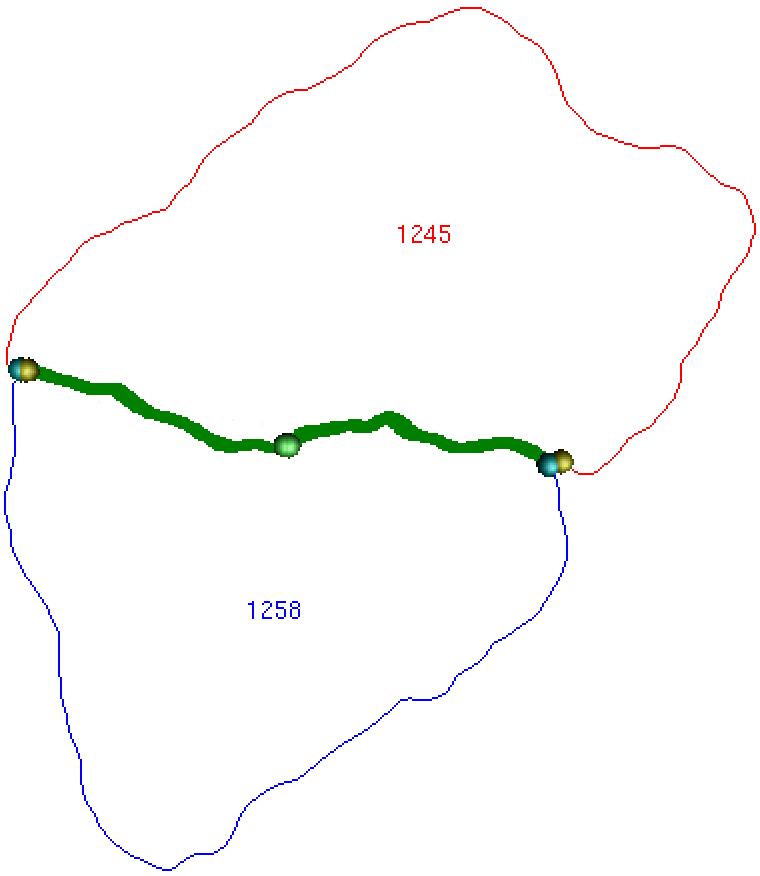
- ΔThickness = 0.1 mm
- ΔColor = 0.002
- ΔAlignment = 0.24 mm
- ➢ ΔCurvature = 0.06 mm^{-1}



Computing Match Properties

Measure alignment of fragments

- ΔThickness = 0.1 mm
- ΔColor = 0.002
- ΔAlignment = 0.24 mm
- ΔCurvature = 0.06 mm^{-1}
- ➢ Length = 43.6 mm
- Overlap = 0.7 mm
- Min int. angle = 88 °
- Max ext. angle = 191 °
- Etc.



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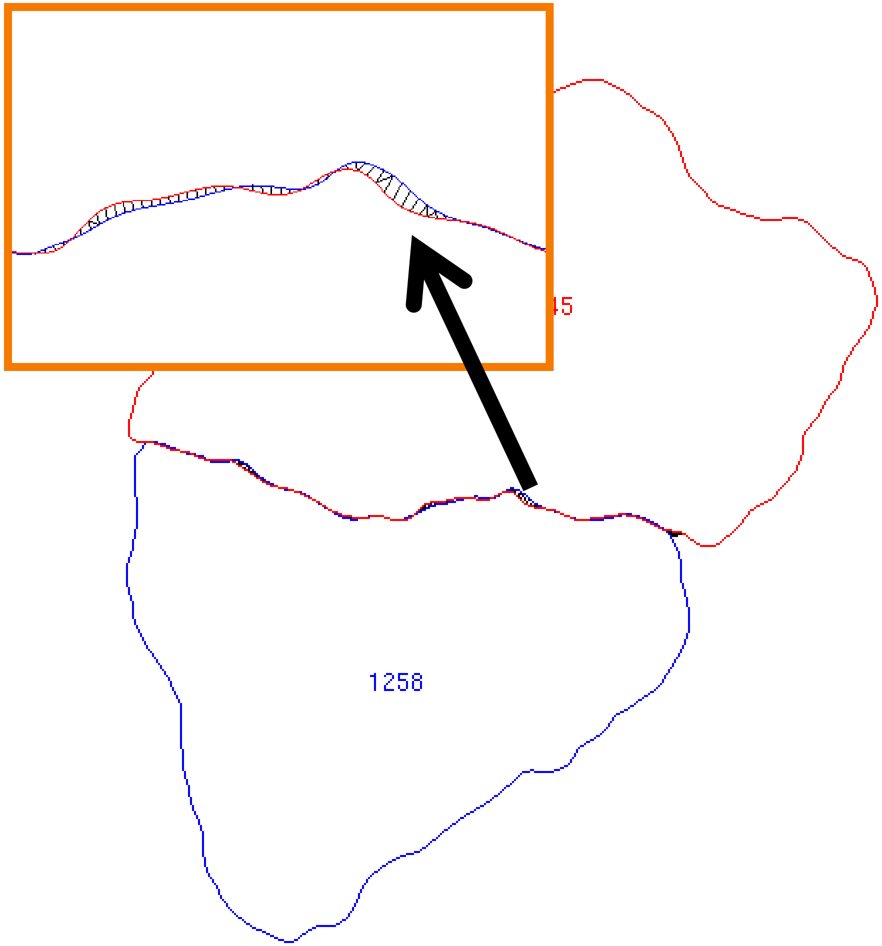

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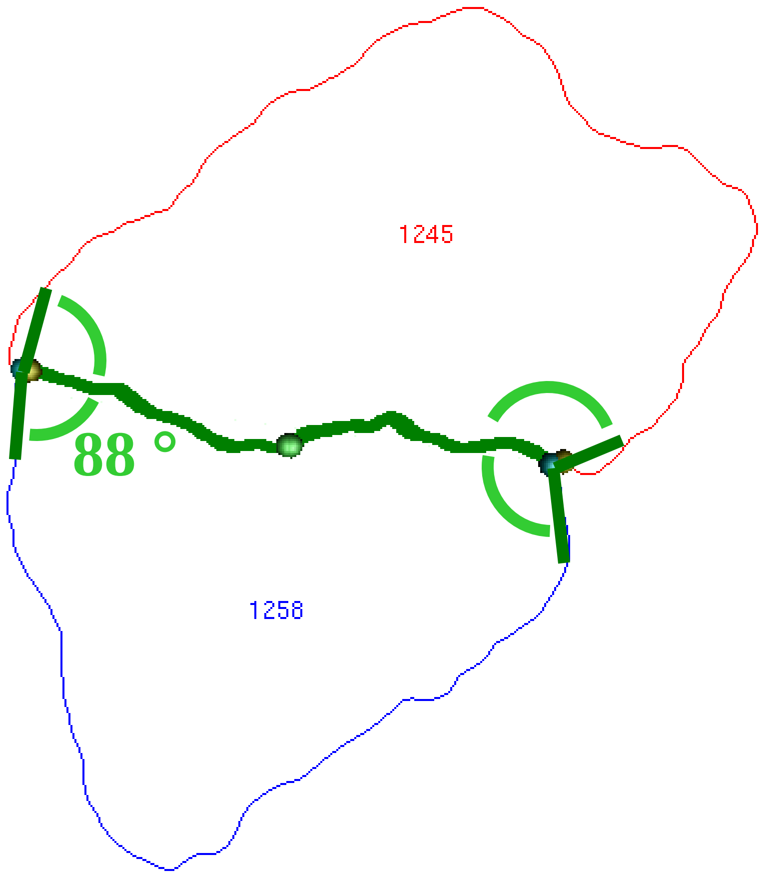

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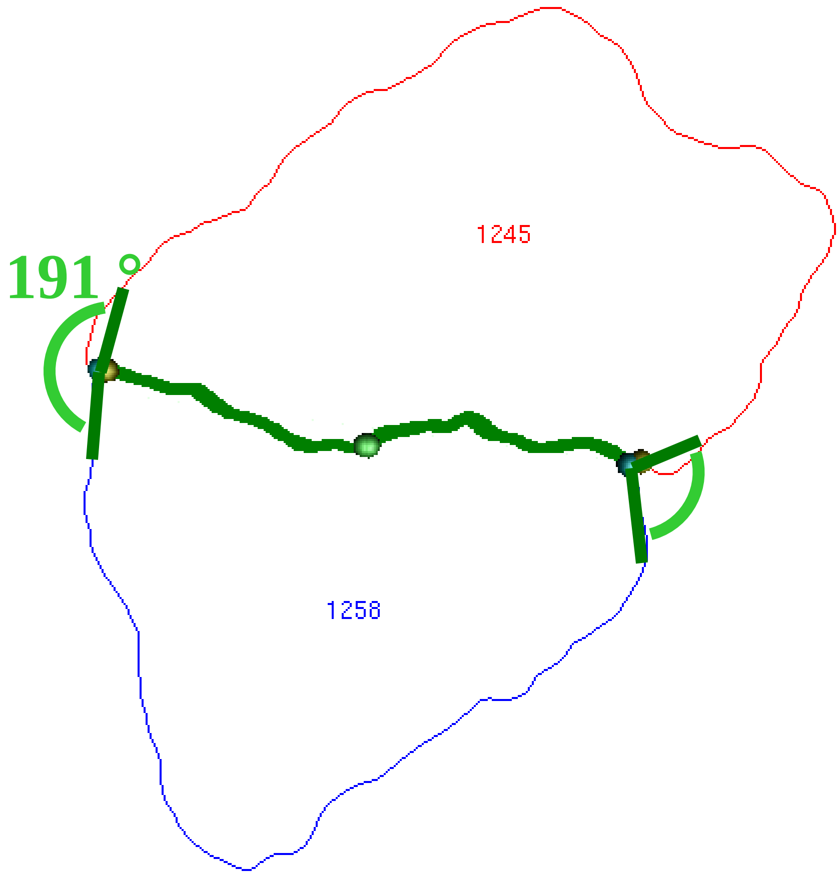


Computing Match Properties

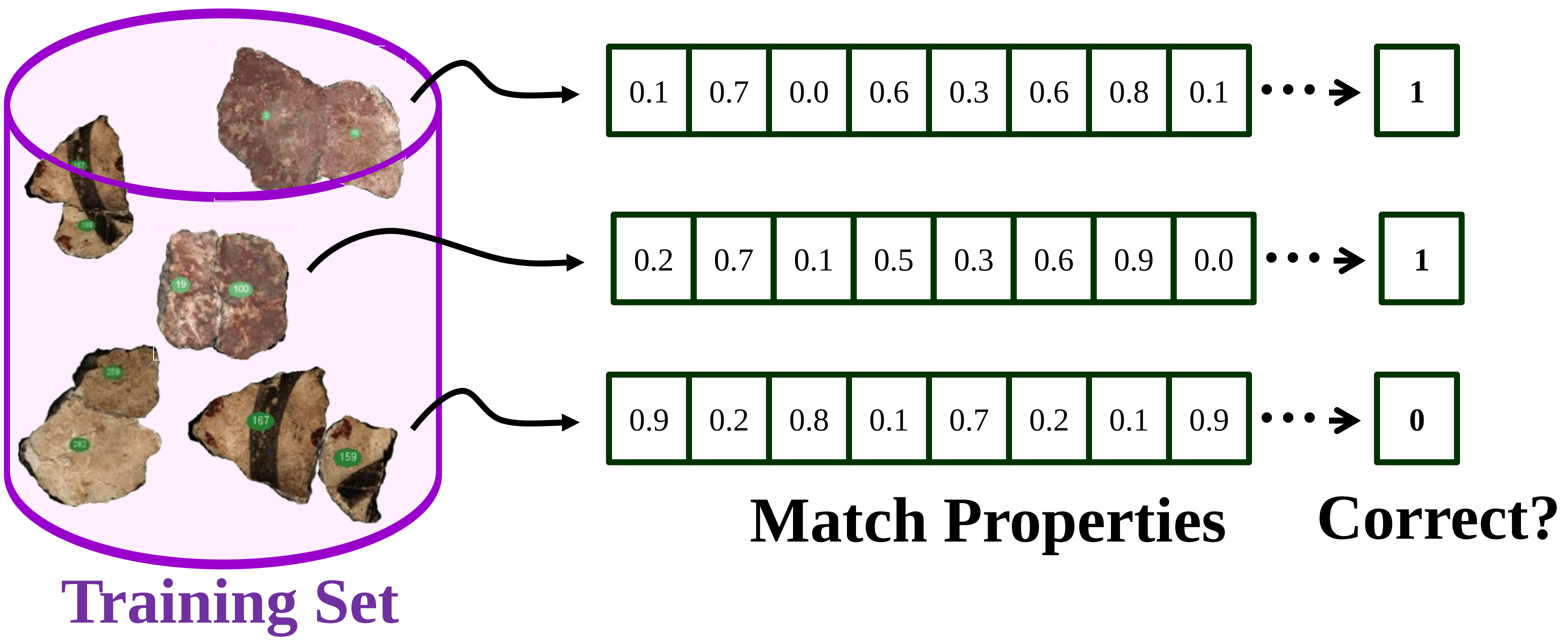
In all, 64 properties per match

ContourContactLength	$0.5 \cdot (\|CR_0\| + \|CR_1\|)$
ContourContactDensity	$0.5 \cdot (\|CC_0\|/\|CR_0\| + \|CC_1\|/\|CR_1\|)$
ContourContactRMSD	$\sqrt{\sum_{i,j}(C_{i,i}[j] - C_{i,1-i}[j])^2}$, where $(C_{i,i}[j], C_{i,1-i}[j]) \in CC_i, i \in \{0,1\}, j \in \{0,..,\|CC_i\|\}$
ContourContactLinearity	$\sqrt{\sum_{i,j}(C_{i,i}[j] - L_i)^2}$, where $C_{i,i}[j] \in CC_i, i \in \{0,1\}, j \in \{0,..,\|CC_i\|\}$, and L_i is the minimizing line
ContourContactCurvL2 (4 properties)	$\sqrt{\sum_{i,j}(\mathrm{Curv}(C_{i,i}[j],t,s) - \mathrm{Curv}(C_{i,1-i}[j],t,s))^2}$, where $(C_{,ii}[j], C_{i,1-i}[j]) \in CC_i, i \in \{0,1\}, j \in \{0,..,\|CC_i\|\}$, $t \in$ { Horizontal }, and $s \in$ { 1mm, 2mm, 4mm, 8mm }
ContourContactLengthFraction (4 properties)	Stat($\|CR_i\|$)/Measurement(C_i)), where Stat $\in$ { Min, Max }, and Measurement $\in$ { Perimeter, $\sqrt{Area}$ }
ContourWindowRMSD (3 properties)	$\sqrt{\sum_{i,j}(C_{i,i}[j] - C_{i,1-i}[j])^2}$, where $(C_{i,i}[j], C_{i,1-i}[j]) \in CW(s), j \in \{0,..,\|CW(s)\|\}$, and $s \in$ { 4mm, 8mm, 16mm }
ContourMergeConvexity	Convexity($C_0 \cup C_1$)
ContourMergeConvexityFraction (2 properties)	Stat(Convexity(C_0) / Convexity($C_0 \cup C_1$), Convexity(C_1) / Convexity($C_0 \cup C_1$)), where Stat $\in$ { Min, Max }
ContourOverlapArea	$\|C_0 \cap C_1\|$
ContourOverlapDepth (2 properties)	Stat(Depth($C_{i,i}[j]$)), where $C_{i,i}[j] \in CC_i, i \in \{0,1\}, j \in \{0,..,\|CC_i\|\}$, and Stat $\in$ { Avg, Max }
ContourJunctionAngle (4 properties)	Stat(Angle(CJ_i, t)), where Stat $\in$ { Min, Max }, and $t \in$ { Exterior, Interior }

RibbonContactArea	$0.5 \cdot (\|RR_0\| + \|RR_1\|)$
RibbonContactDensity	$0.5 \cdot (\|RC_0\|/\|RR_0\| + \|RC_1\|/\|RR_1\|)$
RibbonContactLength	$0.5 \cdot ((\|RR_0 \rightarrow C_0\| + \|RR_1 \rightarrow C_1\|)$, where $RR_i \rightarrow C_i$ is the projection of RR_i onto C_i
RibbonContactRMSD	$\sqrt{\sum_{i,j}(R_{i,i}[j] - R_{i,1-i}[j])^2}$, where $(R_{i,i}[j], R_{i,1-i}[j]) \in RC_i, i \in \{0,1\}, j \in \{0,..,\|RC_i\|\}$
RibbonContactPlanarity	$\sqrt{\sum_{i,j}(R_{i,i}[j] - P_i)^2}$, where $R_{i,i}[j] \in RC_i, i \in \{0,1\}, j \in \{0,..,\|RC_i\|\}$, and P_i is the minimizing vertical plane
RibbonContactHCurvL2 (4 properties)	$\sqrt{\sum_{i,j}(\mathrm{Curv}(R_{i,i}[j],t,s) - \mathrm{Curv}(R_{i,1-i}[j],t,s))^2}$, where $(R_{i,i}[j], R_{i,1-i}[j]) \in RC_i, i \in \{0,1\}, j \in \{0,..,\|RC_i\|\}$, $t \in$ { Horizontal }, and $s \in$ { 1mm, 2mm, 4mm, 8mm }
RibbonContactCurvL2 (4 properties)	$\sqrt{\sum_{i,j}(\mathrm{Curv}(R_{i,i}[j],t,s) - \mathrm{Curv}(R_{i,1-i}[j],t,s))^2}$, where $(R_{i,i}[j], R_{i,1-i}[j]) \in RC_i, i \in \{0,1\}, j \in \{0,..,\|RC_i\|\}$, $t \in$ { Vertical, Mean }, and $s \in$ { 1mm, 2mm }
RibbonWindowRMSD (3 properties)	$\sqrt{\sum_{i,j}(R_{i,i}[j] - R_{i,1-i}[j])^2}$, where $(R_{i,i}[j], R_{i,1-i}[j]) \in RW(s), j \in \{0,..,\|RW(s)\|\}$, and $s \in$ { 4mm, 8mm, 16mm }
RibbonJunctionAngle (4 properties)	Stat(Angle(RJ_i, t)), where Stat $\in$ { Min, Max }, and $t \in$ { Exterior, Interior }
FragmentThicknessL2	(Thickness(F_0) - Thickness(F_1))^2, where Thickness(F_i) is the average number of columns with scanned vertex positions in each row of R_i
FragmentFrontColorL2 (12 properties)	(Stat(I_0, c) - Stat(I_1, c))^2, where Stat $\in$ { Mean, Median, Variance }, and $c \in$ { Red, Green, Blue, Luminance }
FragmentAreaFraction	min($\|C_0\|/\|C_1\|, \|C_1\|/\|C_0\|$)

Learning a Scoring Function

Learn a classifier that predicts the probability that a match is correct based on its properties



Learning a Scoring Function

Our classifier

- Decision Tree
  - Each branch checks the value of a property
  - Each leaf has linear regression model
  - Produces score “roughl modeling probability
  - Selects good features automatically

```
RibbonContactRMSD <= 0.429 :
  RibbonContactRMSD <= 0.375 :
    RibbonContactPlanarity <= 0.517 :
      ContourContactRMSD <= 0.286 :
        ContourContact4mmHorizCurvL2 <= 0.009 : LM1 (29)
        ContourContact4mmHorizCurvL2 > 0.009 : LM2 (112)
      ContourContactRMSD > 0.286 : LM3 (560)
    RibbonContactPlanarity > 0.517 :
      RibbonContactArea <= 446.36 :
        RibbonContactRMSD <= 0.36 :
          RibbonJunctionMinInteriorAngle <= 2.232 :
            ContourContactRMSD <= 0.217 : LM4 (17)
            ContourContactRMSD > 0.217 :
              ContourContactMinLenAreaFract <= 0.309 : LM5 (20)
              ContourContactMinLenAreaFract > 0.309 :
                RibbonContactRMSD <= 0.331 : LM6 (12)
                RibbonContactRMSD > 0.331 : LM7 (20)
          RibbonJunctionMinInteriorAngle > 2.232 : LM8 (29)
        RibbonContactRMSD > 0.36 : LM9 (91)
      RibbonContactArea > 446.36 : LM10 (53)
  RibbonContactRMSD > 0.375 :
    RibbonContactArea <= 235.969 : LM11 (3015)
    RibbonContactArea > 235.969 :
      RibbonContact1mmMeanCurvL2 <= 0.121 : LM12 (603)
      RibbonContact1mmMeanCurvL2 > 0.121 : LM13 (151)
RibbonContactRMSD > 0.429 : LM14 (7416)
```

Decision tree learned on Synthetic

Learning a Scoring Function

Our classifier

- Decision Tree
  - Each branch checks the value of a property
  - Each leaf has

del

ghl

s

Truth =
-0.0013 * RibbonContactRMSD
+ 0 * RibbonContactArea
+ 0.0001 * RibbonContactPlanarity
+ 0.0005 * RibbonContact1mmMeanCurvatureL2
+ 0 * RibbonJointMinInteriorAngle
+ 0 * RibbonJointMaxExteriorAngle
- 0.0001 * ContourContactRMSD
- 0.0007 * ContourContact4mmHorizontalCurvatureL2
+ 0.0002

“Matches with large ContactRMSD are unlikely”
(score is near zero)

```
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  RibbonContactRMSD <= 0.375 :
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      ContourContactRMSD <= 0.286 :
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```

Decision tree learned on Synthetic

Learning a Scoring Function

Our classifier

- Decision Tree
  - Each branch checks the value of a property
  - Each leaf has

del

ghl

s

Truth =
-5.1265 * RibbonContactRMSD
+ 0 * RibbonContactArea
+ 0.0138 * RibbonContactPlanarity
+ 0.012 * RibbonContact1mmMeanCurvatureL2
- 0.0286 * RibbonJointMinInteriorAngle
+ 0.0006 * RibbonJointMaxExteriorAngle
- 0.0011 * ContourContactRMSD
- 0.1677 * ContourContact4mmHorizontalCurvatureL2
+ 0.6273 * ContourContactMinLengthAreaFraction
+ 1.9331

"Matches with small ContactRMSD, high Planarity, a small interior angle at least at one junction, and a large relative contact length are likely to be correct" (score is large)

```
RibbonContactRMSD <= 0.429 :
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    RibbonContactPlanarity <= 0.517 :
      ContourContactRMSD <= 0.286 :
        ContourContact4mmHorizCurvL2 <= 0.009 : LM1 (29)
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RibbonContactRMSD > 0.429 : LM14 (7416)
```

Decision tree learned on Synthetic

Experimental Data Sets

Synthetic Fresco

- Made specifically for this project
- Made in the style of Akrotiri wall paintings
- Destroyed purposely in 2007 A.D.

Tongeren Vrijthof

- Tongeren, Belgium
- Roman building
- Destroyed by fire between 1 A.D. – 300 A.D.

Akrotiri

- Thera (Santorini, Greece)
- Late Bronze Age settlement
- Destroyed by earthquake around 1650 B.C.

Experiment Design

Train on Fresco X

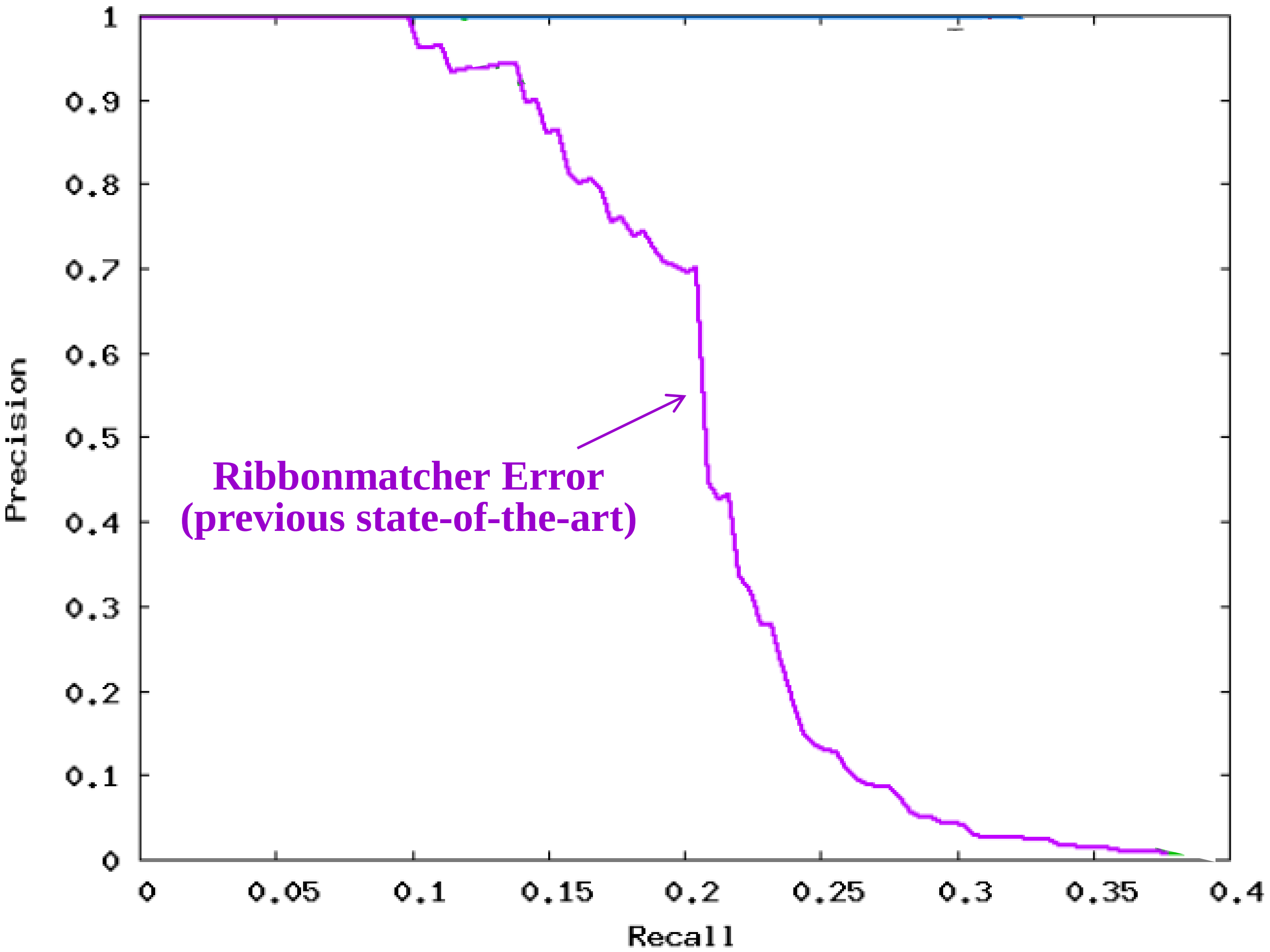
- Use ribbonmatcher to generate candidate matches
- Compute properties of candidate matches
- Mark candidate matches that are correct
- Learn classifier to predict correctness of matches

Test on Fresco Y

- Use ribbonmatcher to generate candidate matches
- Compute properties of candidate matches
- Mark candidate matches that are correct
- Apply classifier to predict correctness of (score) matches
- Sort matches by score, and plot precision vs. recall

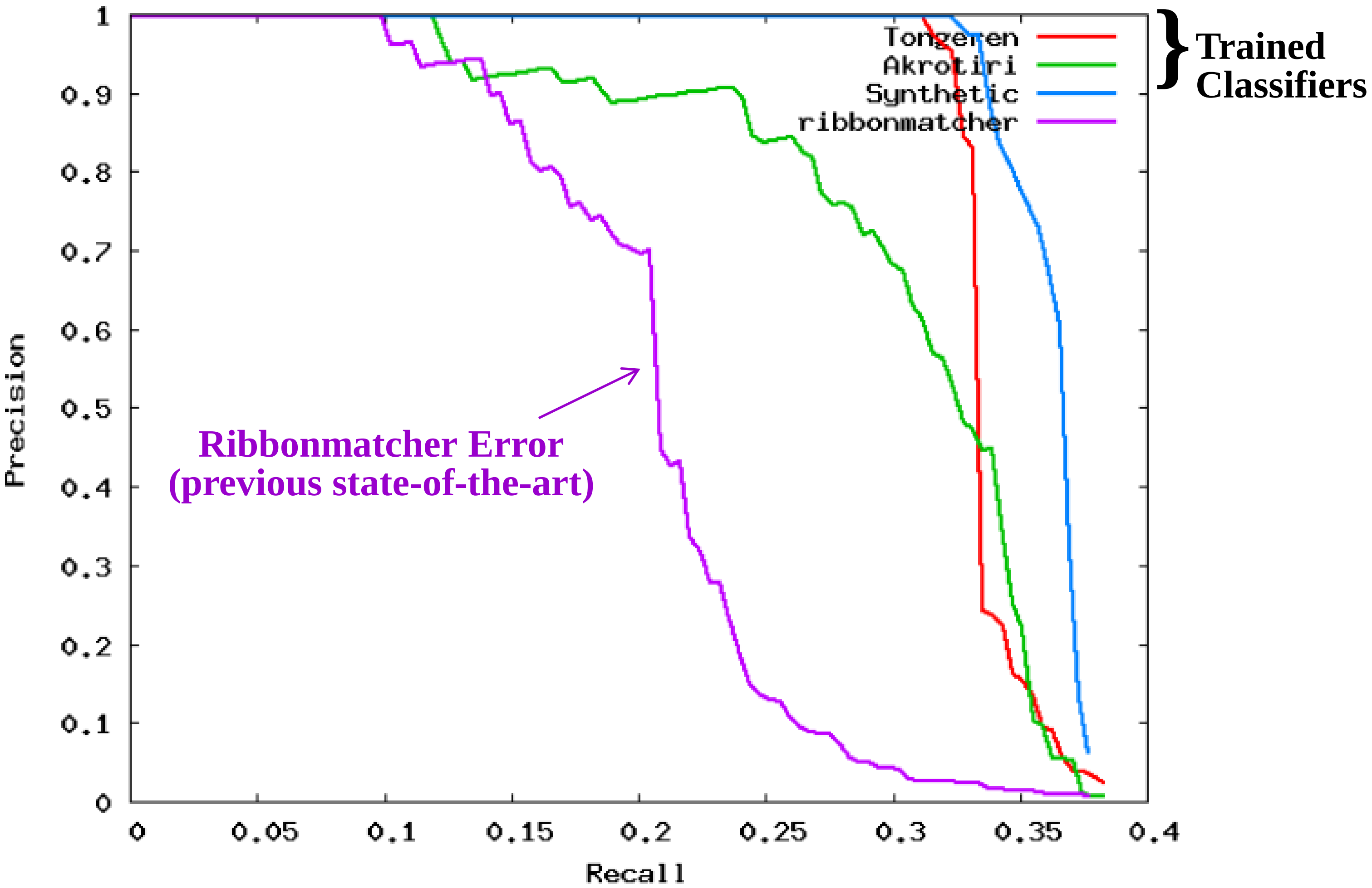
Experiment Results

Testing on Synthetic Fresco:



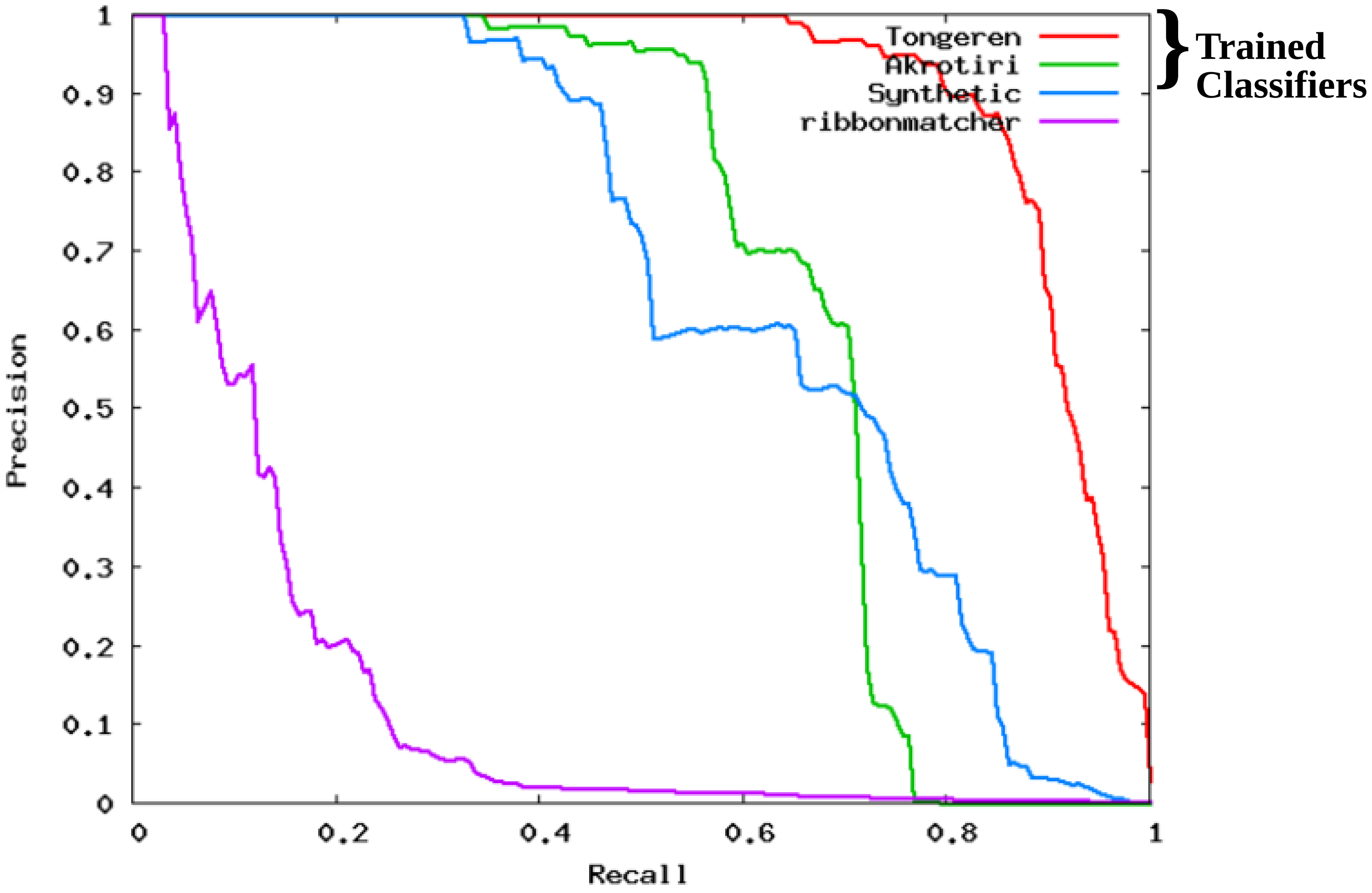
Experiment Results

Testing on Synthetic Fresco:



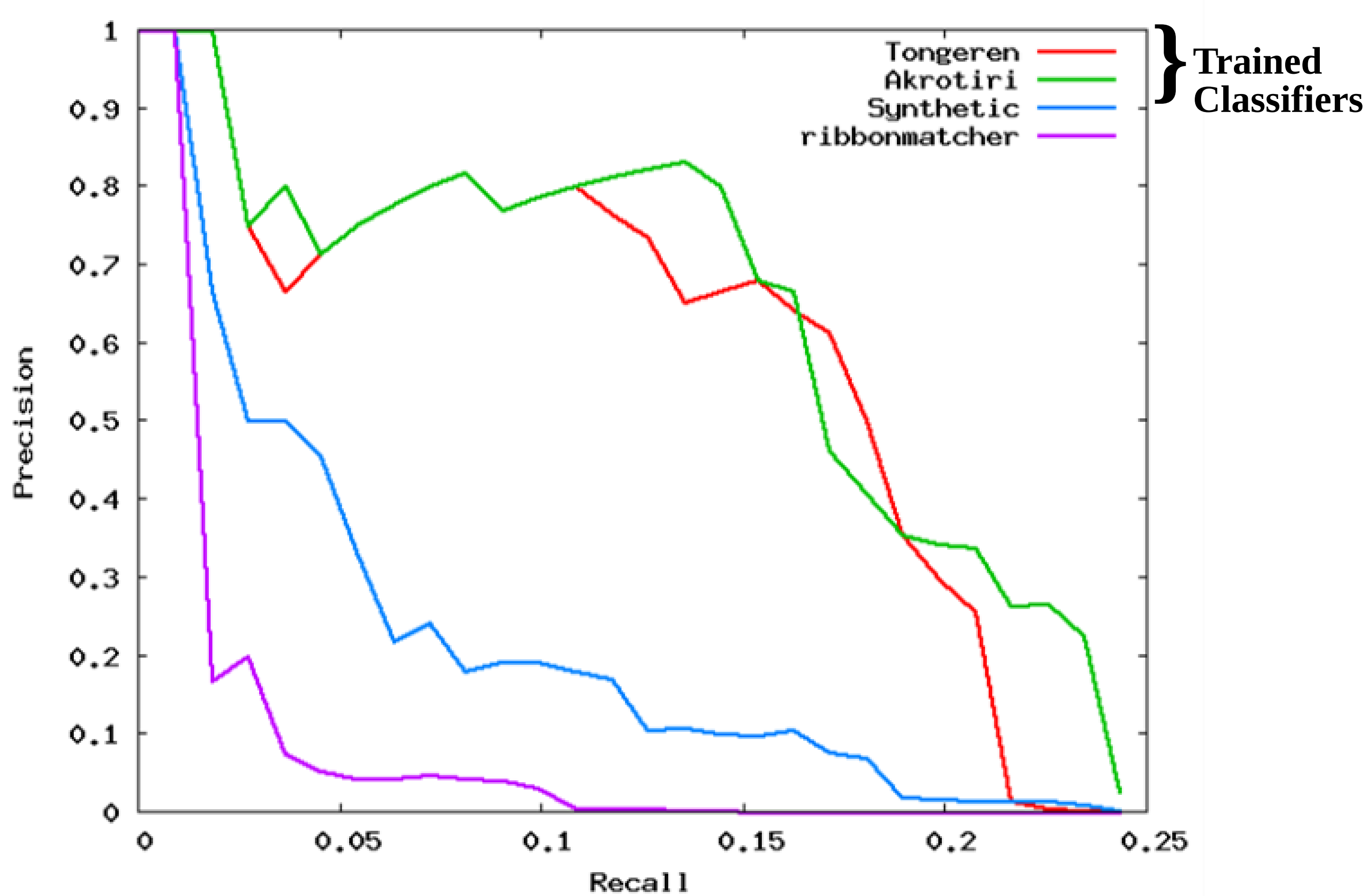
Experiment Results

Testing on Tongeren Vrijthof Fresco:



Experiment Results

Testing on Akrotiri Fresco:



Results of Predictions Sent to Akrotiri

Totals of all predictions:

- Likely: 48 correct, 1 incorrect, 1 uncheckable
- Probable: 7 correct, 0 incorrect
- Possible: 25 correct, 19 incorrect, 1 uncheckable
- Maybe: 5 correct, 10 incorrect
- Remote: 2 correct, 15 incorrect
- Longshot: 0 incorrect, 14 incorrect

Summary:

- **87 correct matches**
- **36 missed (found by conservators)**
- **43 new (not found by conservators)**

Summary

3D surface analysis uses many of the same techniques as 2D image analysis, except …

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3D surface analysis uses many of the same techniques as 2D image analysis, except …

- More complex topology
- Irregular sampling
- One more dimension
- Fewer high-frequency features
- etc.