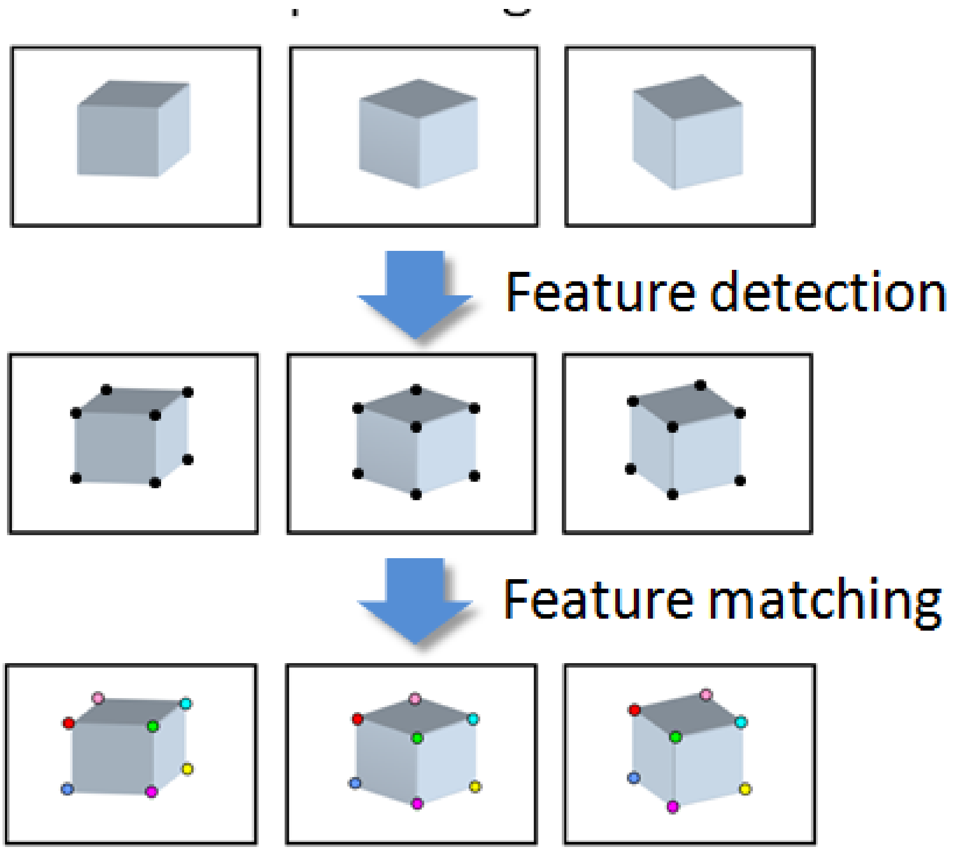
Camera Calibration

COS 429
Princeton University

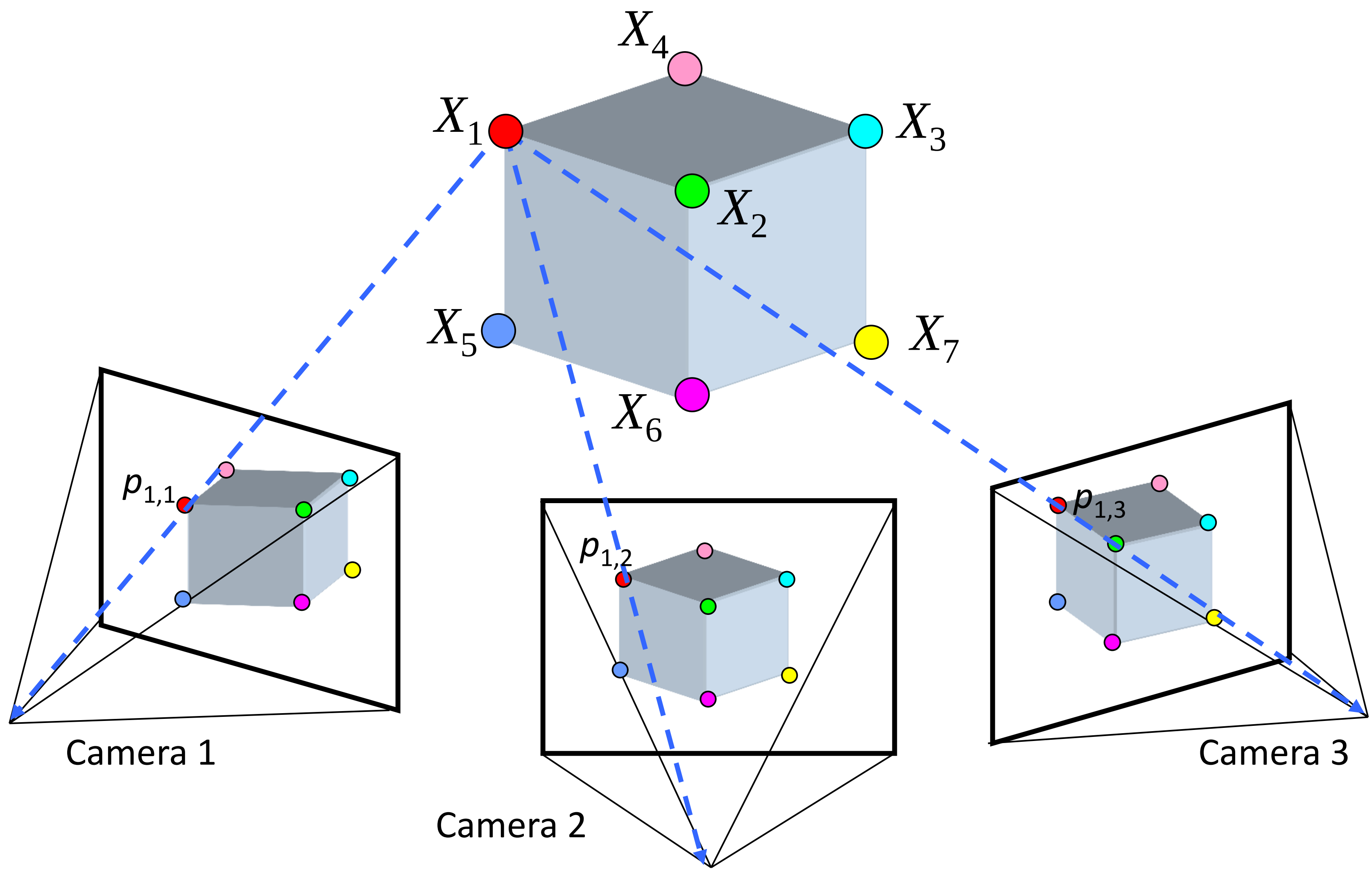
Point Correspondences

What can you figure out from point correspondences?



Noah Snavely

Point Correspondences



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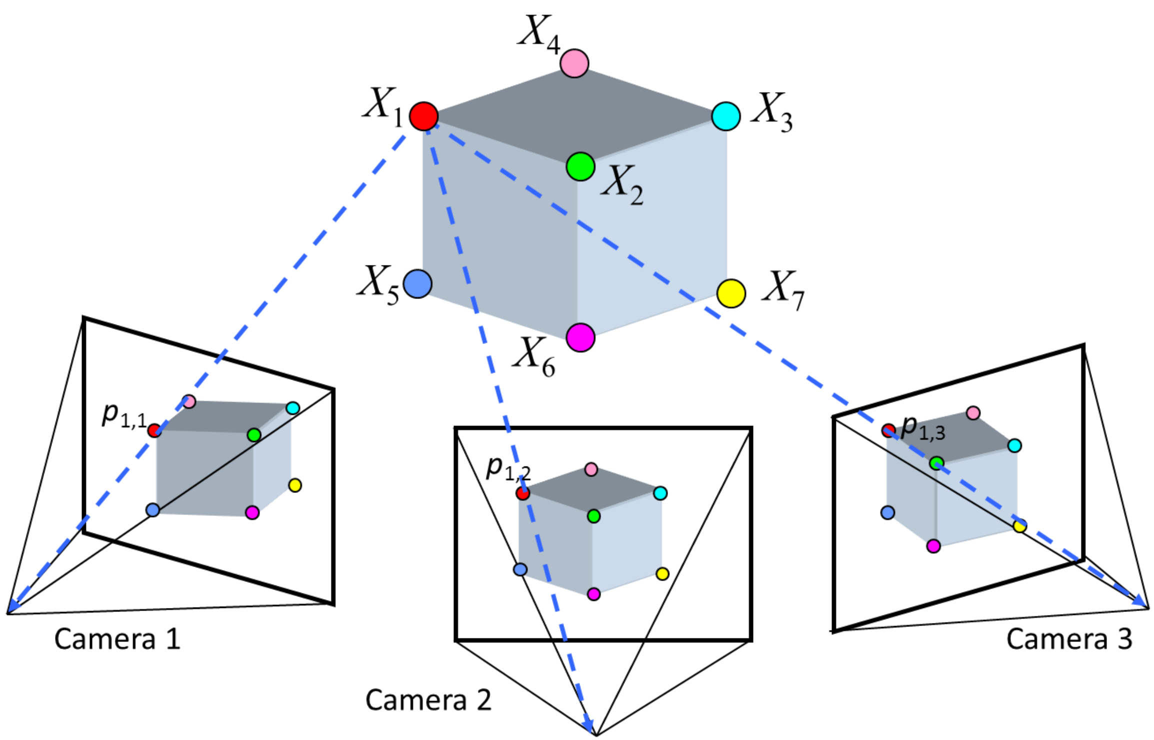
Camera calibration

Suppose we know 3D point positions and correspondences to pixels in every image

- Can we compute the camera parameters?



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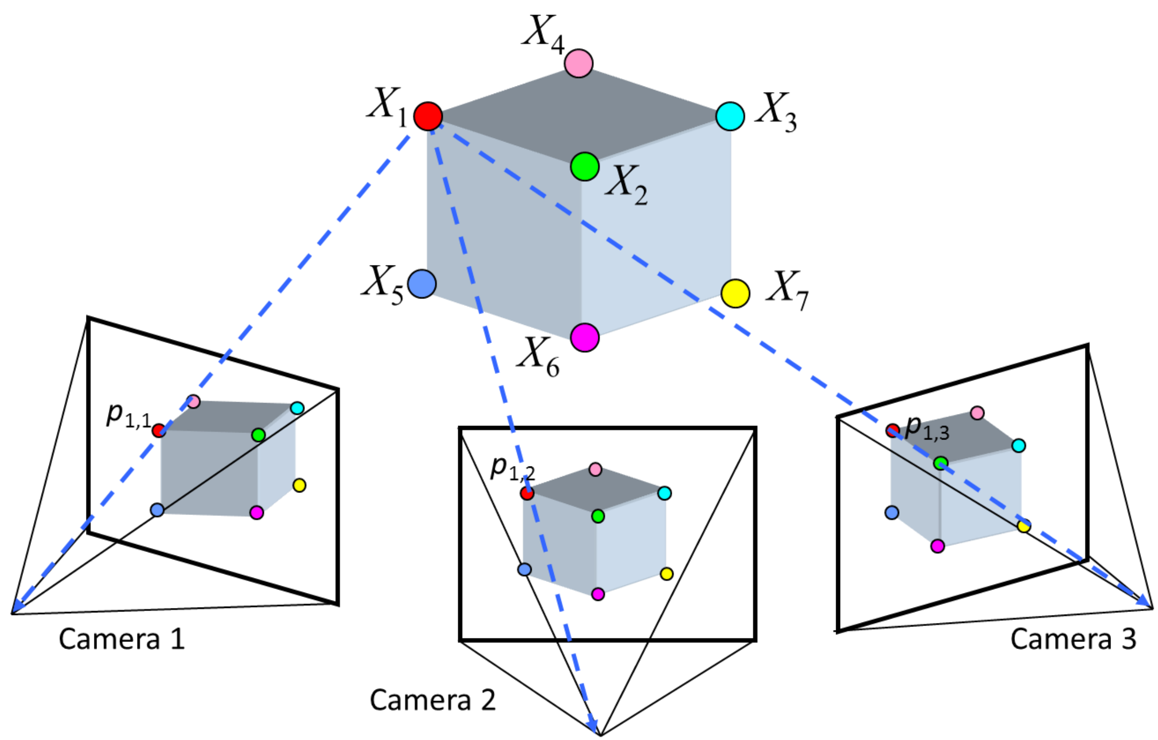
Camera triangulation

Suppose we know the camera parameters and correspondences between points and pixels in every image?



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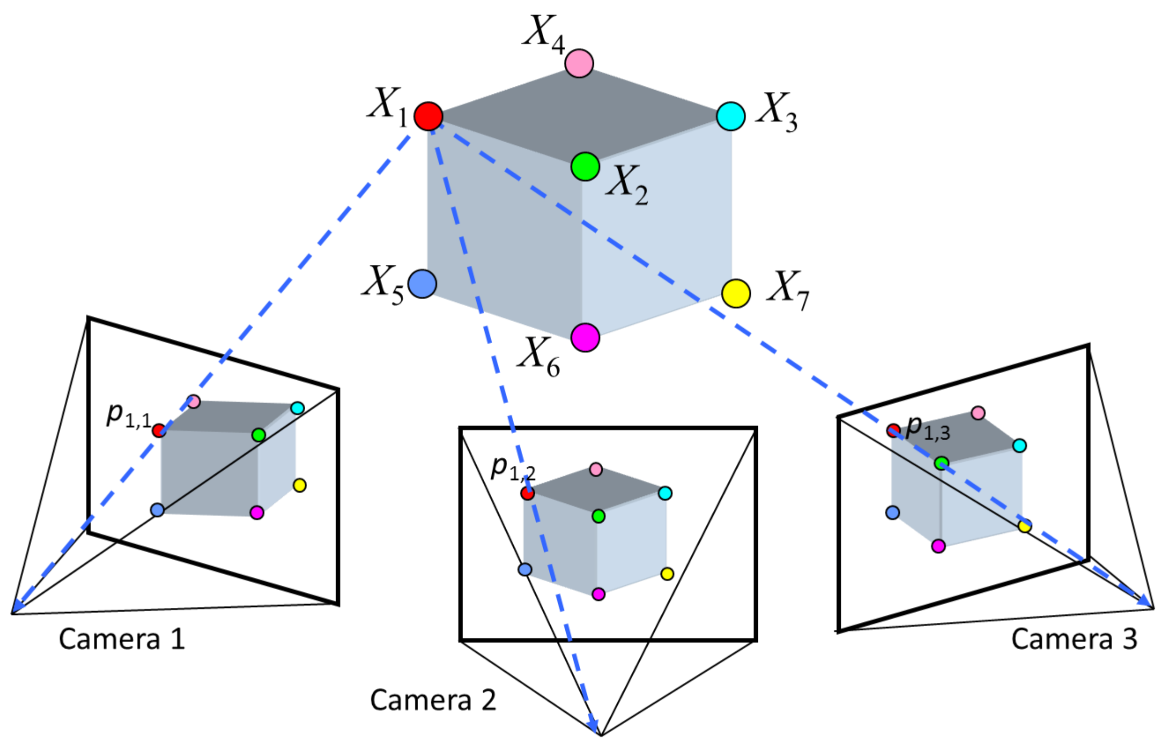
Camera calibration & triangulation

Need to understand how 3D points project into images



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Outline

- Camera model
- Camera calibration
- Camera triangulation
- Structure from motion

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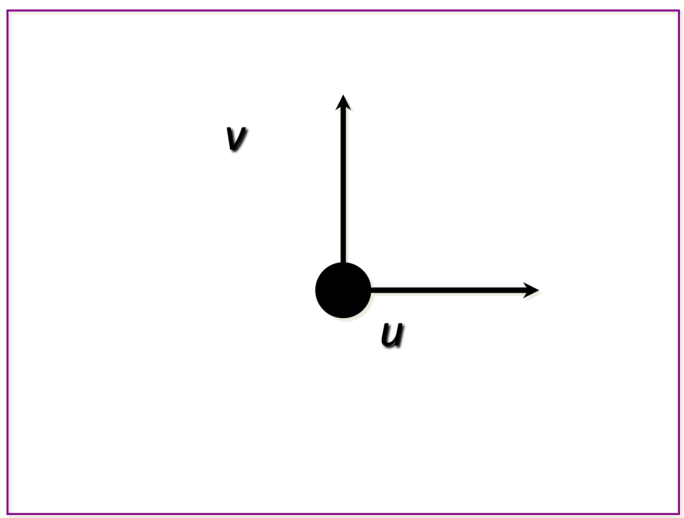
- Camera model
- Camera calibration
- Camera triangulation
- Structure from motion

2D Image Basics

- Origin at center, for now
- Y axis is up
- Will often write (*u*, *v*) for image coordinates



3D Geometry Basics

- 3D points = column vectors

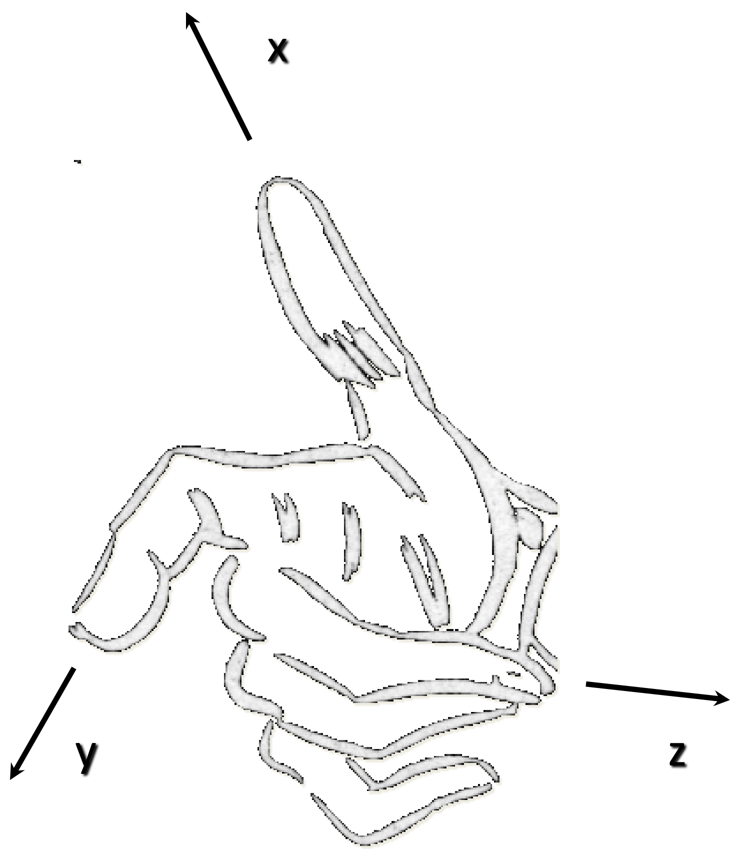
$$\vec{p} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

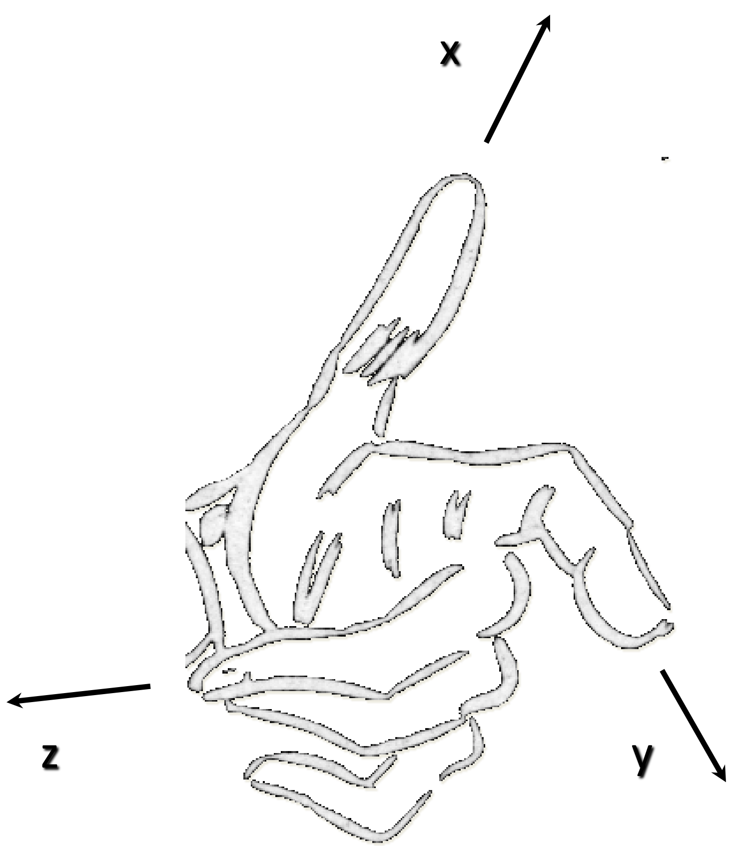
- Transformations = pre-multiplied matrices

$$\mathbf{T}\vec{p} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

3D Geometry Basics

- Right-handed vs. left-handed coordinate systems





Rotation

- Rotation about the *z* axis

$$\mathbf{R}_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Rotation about *x*, *y* axes similar (cyclically permute *x*, *y*, *z*)

Arbitrary Rotation

- Any rotation is a composition of rotations about *x*, *y*, and *z*
- Composition of transformations = matrix multiplication (watch the order!)
- Result: orthonormal matrix
  - Each row, column has unit length
  - Dot product of rows or columns = 0
  - Inverse of matrix = transpose

Arbitrary Rotation

- Rotate around *x*, *y*, then *z*:

$$\mathbf{R} = \begin{pmatrix} \sin\theta_y \cos\theta_z & -\cos\theta_x \sin\theta_z + \sin\theta_x \cos\theta_y \cos\theta_z & \sin\theta_x \sin\theta_z + \cos\theta_x \cos\theta_y \cos\theta_z \\ \sin\theta_y \sin\theta_z & \cos\theta_x \cos\theta_z + \sin\theta_x \cos\theta_y \sin\theta_z & -\sin\theta_x \cos\theta_z + \cos\theta_x \cos\theta_y \sin\theta_z \\ \cos\theta_y & -\sin\theta_x \sin\theta_y & -\cos\theta_x \sin\theta_y \end{pmatrix}$$

- Don’t do this! It’s probably buggy!
  Compute simple matrices and multiply them…

Scale

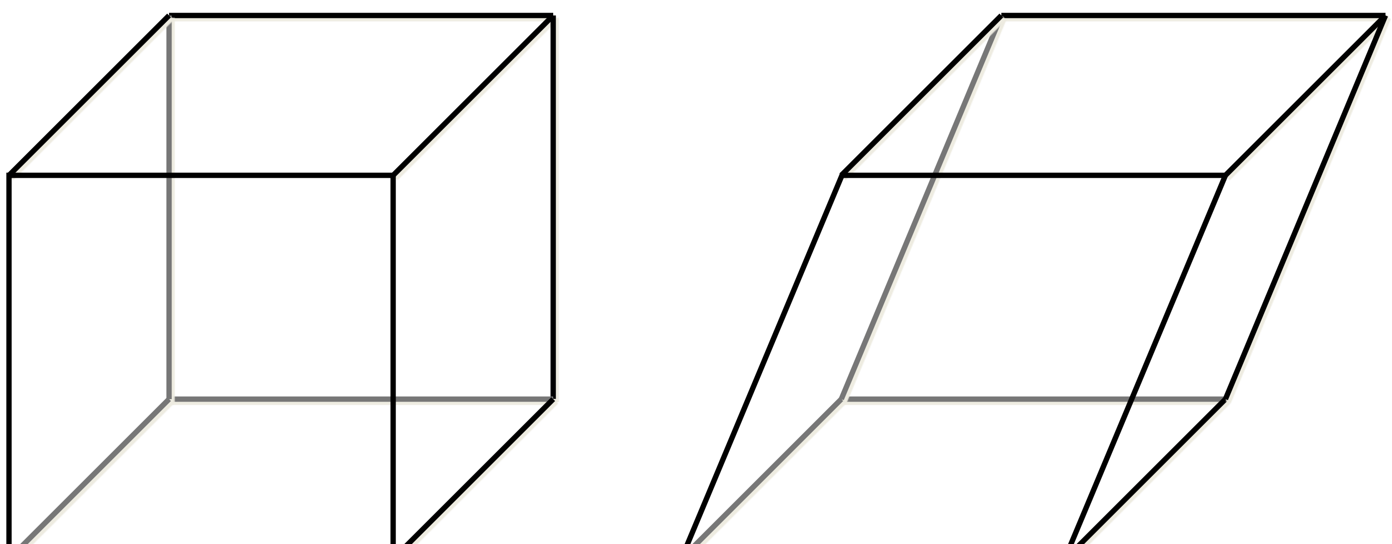
- Scale in *x*, *y*, *z*:

$$\mathbf{S} = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{pmatrix}$$

Shear

- Shear parallel to *xy* plane:

$$\boldsymbol{\sigma}_{xy} = \begin{pmatrix} 1 & 0 & \sigma_x \\ 0 & 1 & \sigma_y \\ 0 & 0 & 1 \end{pmatrix}$$



Translation

- Can translation be represented by multiplying by a 3×3 matrix?
- No.
- Proof?

$$\forall \mathbf{A}: \quad \mathbf{A}\vec{0} = \vec{0}$$

Homogeneous Coordinates

- Add a fourth dimension to each 3D point:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

- To get "real" (3D) coordinates, divide by *w*:

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \rightarrow \begin{pmatrix} x/w \\ y/w \\ z/w \end{pmatrix}$$

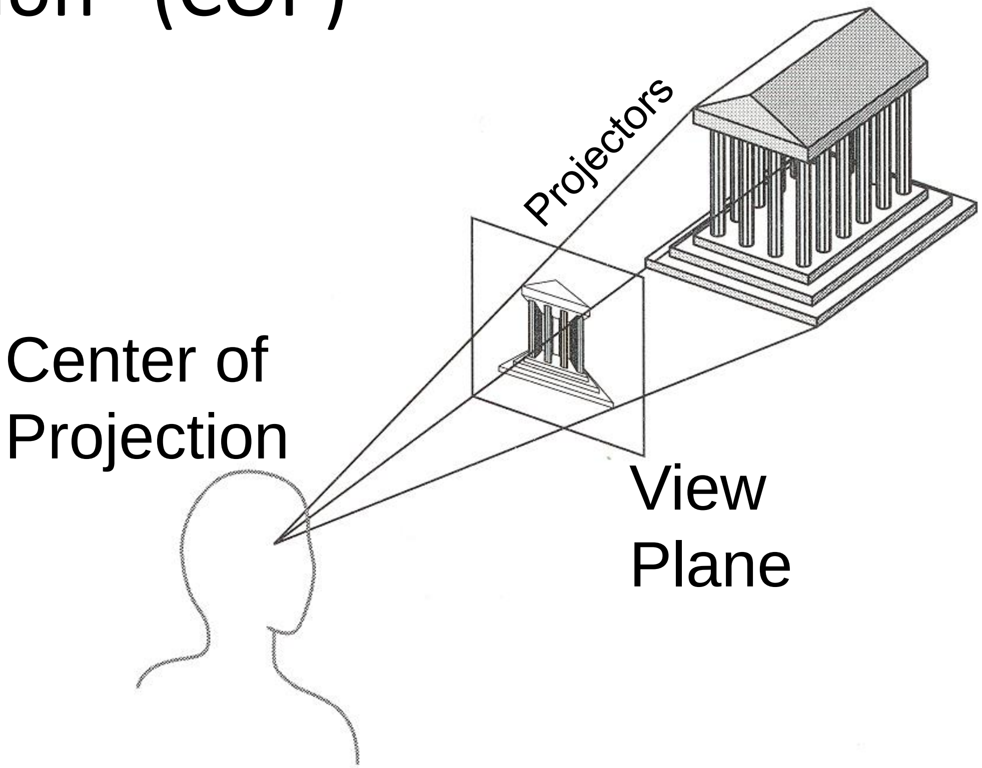
Translation in Homogeneous Coordinates

$$\begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} x + t_x w \\ y + t_y w \\ z + t_z w \\ w \end{pmatrix}$$

- After divide by *w*, this is just a translation by (t_x, t_y, t_z)

Perspective Projection

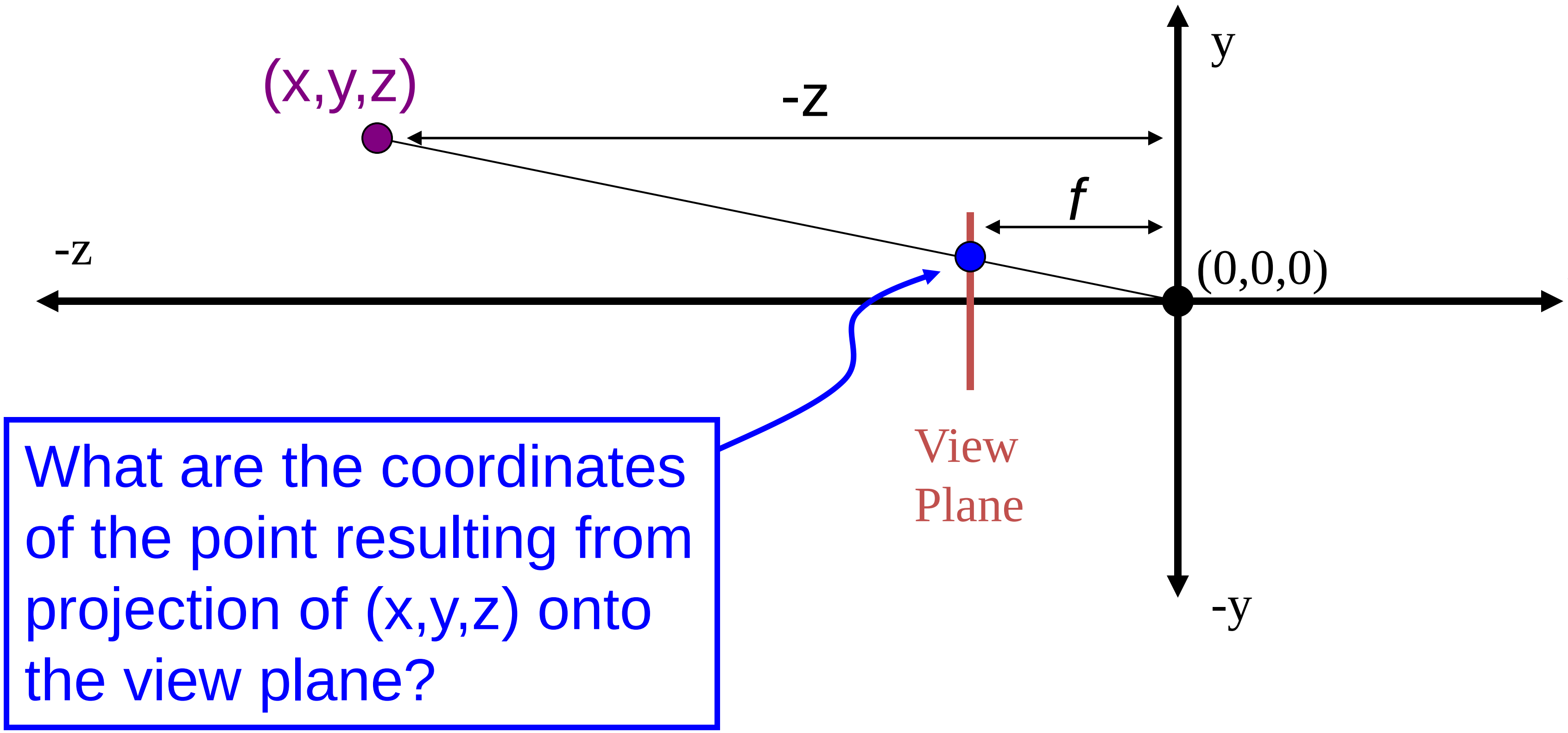
- Map points onto “view plane” along “projectors” emanating from “center of projection” (COP)



Angel Figure 5.9

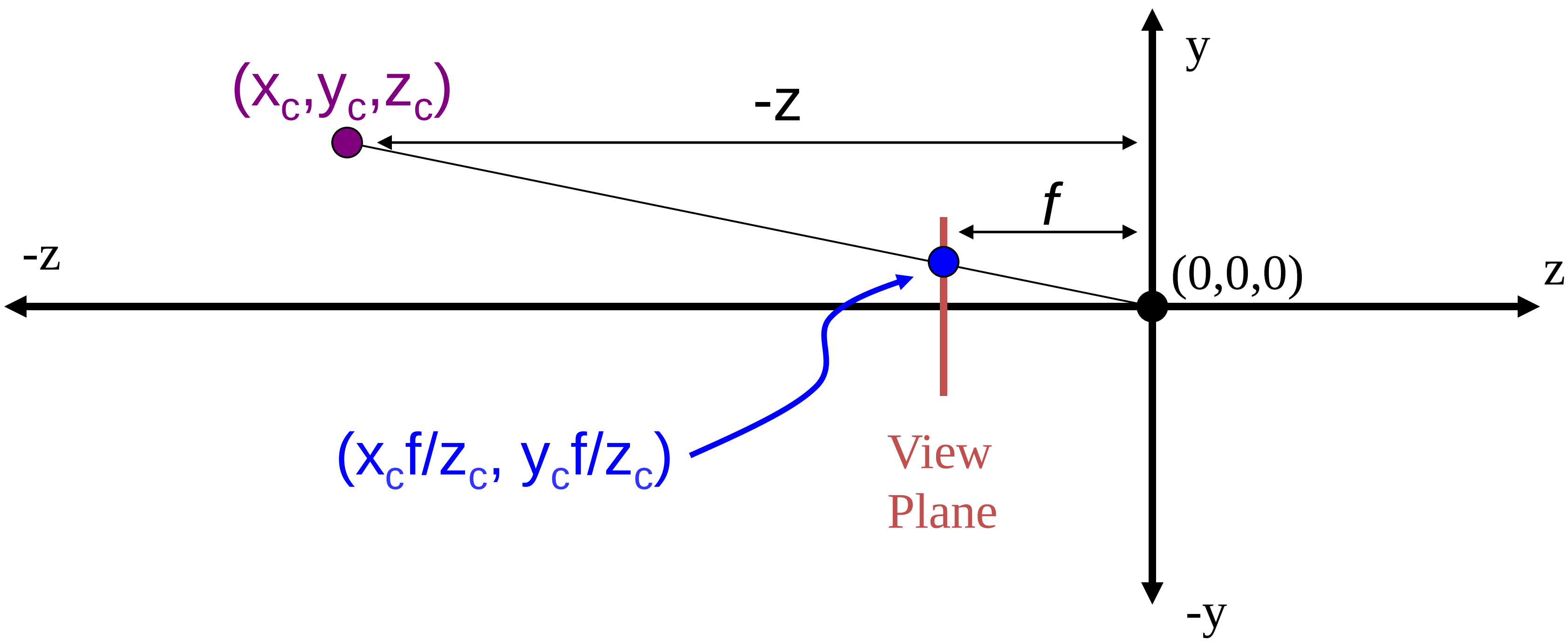
Perspective Projection

- Compute 2D coordinates from 3D coordinates with similar triangles



Perspective Projection

- Compute 2D coordinates from 3D coordinates with similar triangles



Perspective Projection Matrix

- 4x4 matrix representation:

$$x_s = x_c f / z_c \qquad x_s = x'/w' \qquad x' = x_c$$

$$y_s = y_c f / z_c \qquad y_s = y'/w' \qquad y' = y_c$$

$$z_s = f \qquad z_s = z'/w' \qquad z' = z_c$$

$$w_s = 1 \qquad w' = z_c / f$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Perspective Projection Properties

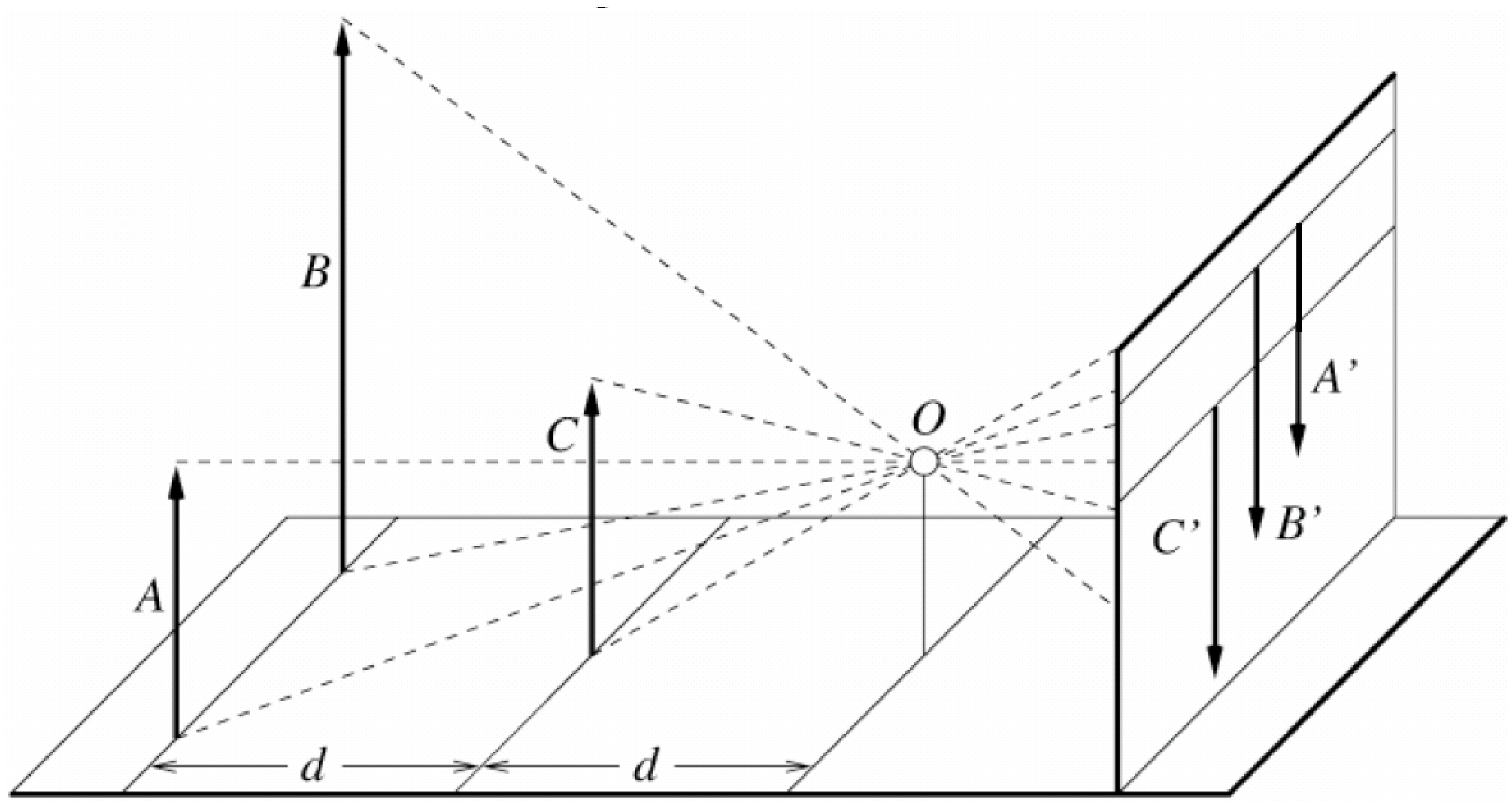
- Points → points
- Lines → lines (collinearity preserved)
- Distances and angles are **not** preserved
- Many points along same ray map to same point in image

- Degenerate cases:
  - Line through focal point projects to a point.
  - Plane through focal point projects to line
  - Plane perpendicular to image plane projects to part of the image.

Slide by Kristen Grauman

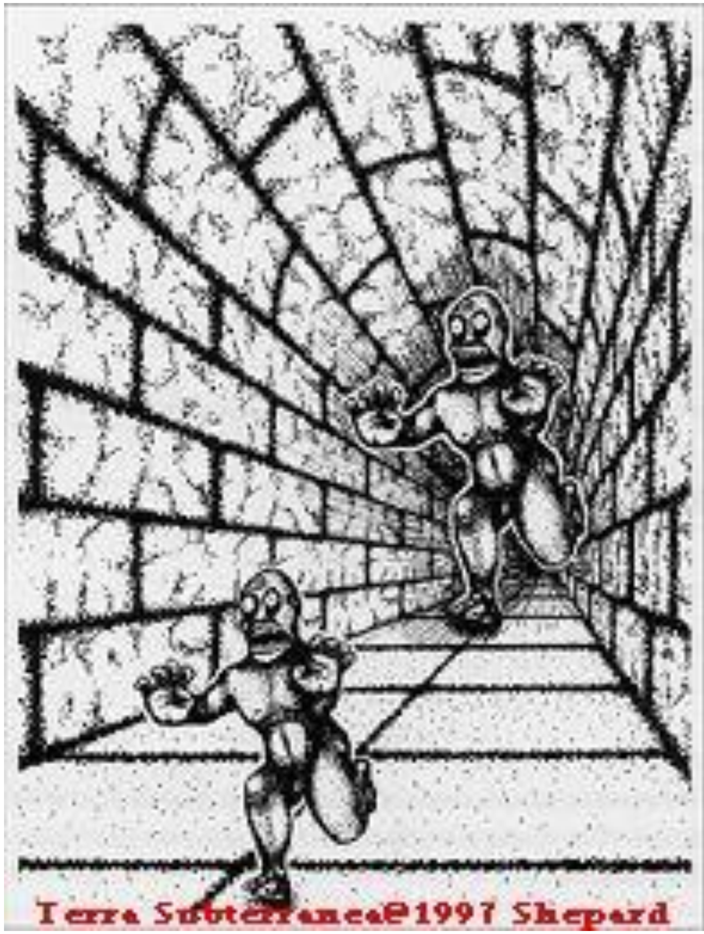
Perspective Effects

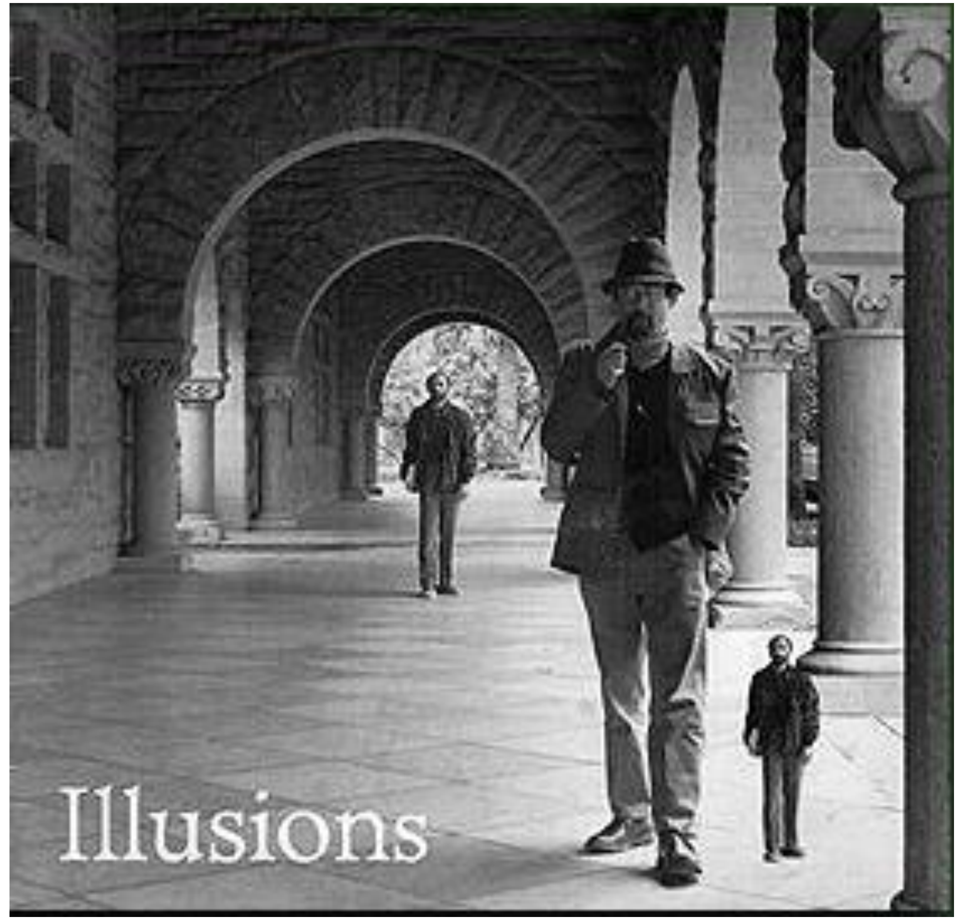
- Far away objects appear smaller



Forsyth and Ponce

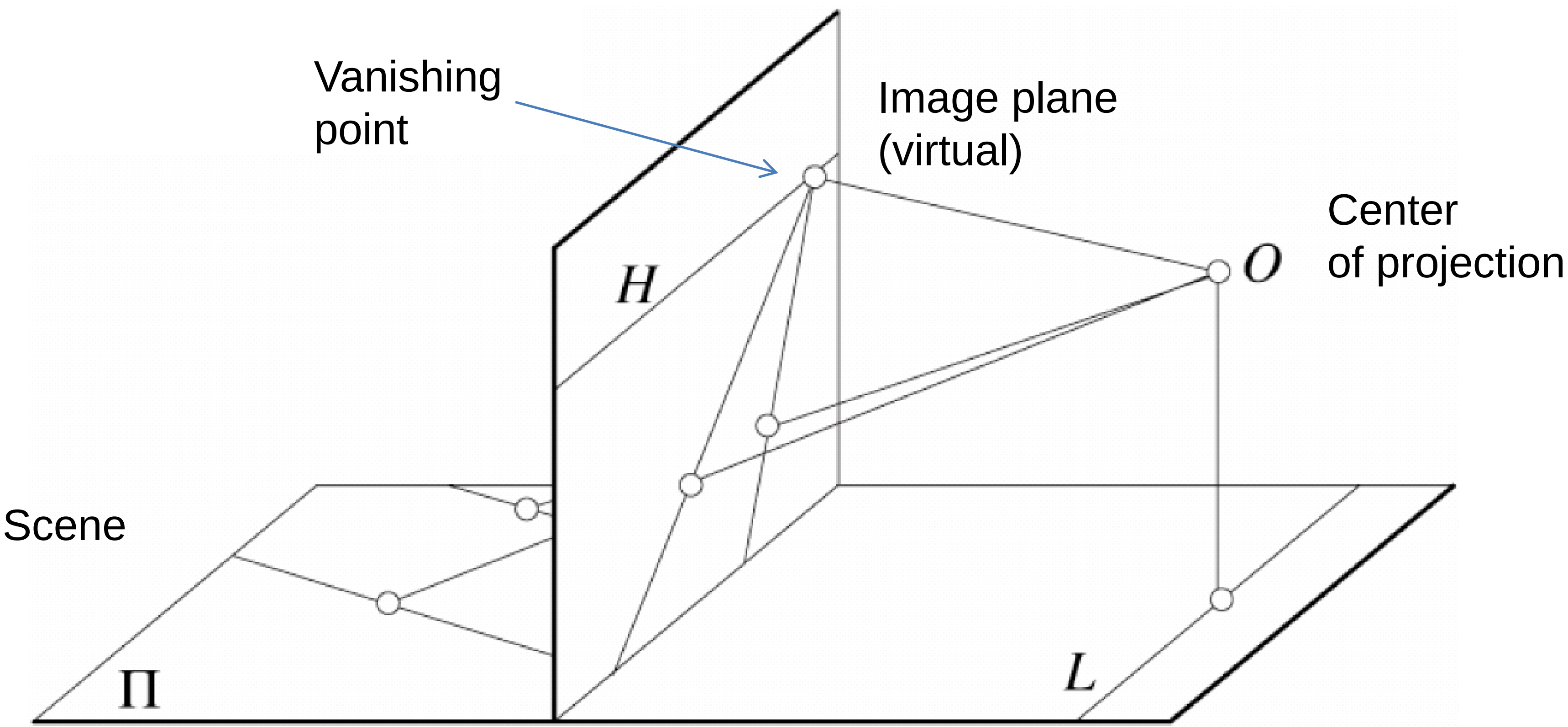
Perspective Effects





Perspective Effects

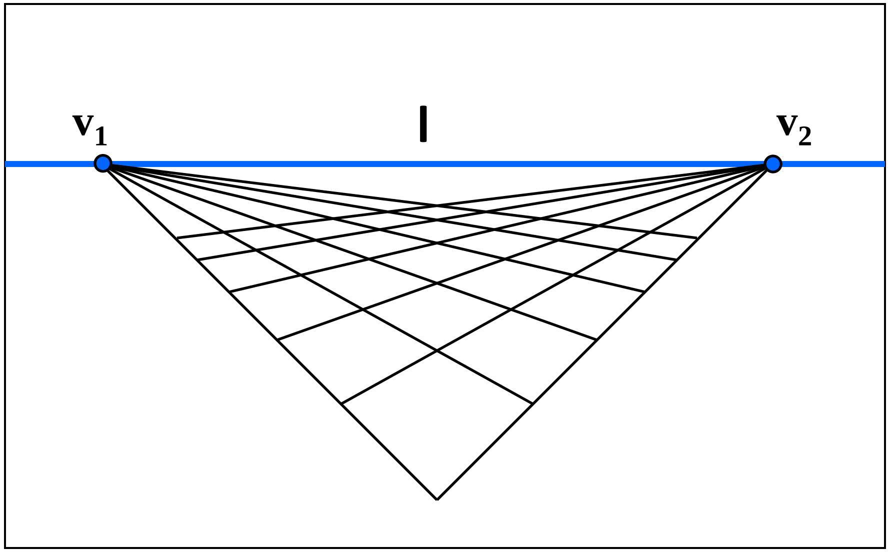
- Parallel lines in the scene intersect at a point after projection onto image plane



Slide by Kristen Grauman

Perspective Effects

- Vanishing Points
  - Any set of parallel lines on a plane define a vanishing point
  - The union of all of these vanishing points is the *horizon line*
    - also called *vanishing line*
  - Different planes (can) define different vanishing lines



A Camera Model

Scale to pixel size

Camera orientation

Camera location

Perspective projection

Translate to image center

3D point (homogeneous coords)

Then unwarp radial distortion

$$\mathbf{T}_{img}\mathbf{S}_{pix}\mathbf{P}_{cam}\mathbf{R}_{cam}\mathbf{T}_{cam}\vec{x}$$

Then perform homogeneous divide, and get (u,v) coords

A Camera Model

Camera coordinates

Normalized device coordinates

Image coordinates
Pixel coordinates

World coordinates

$$\mathbf{T}_{img}\mathbf{S}_{pix}\mathbf{P}_{cam}\mathbf{R}_{cam}\mathbf{T}_{cam}\vec{x}$$

A Camera Model

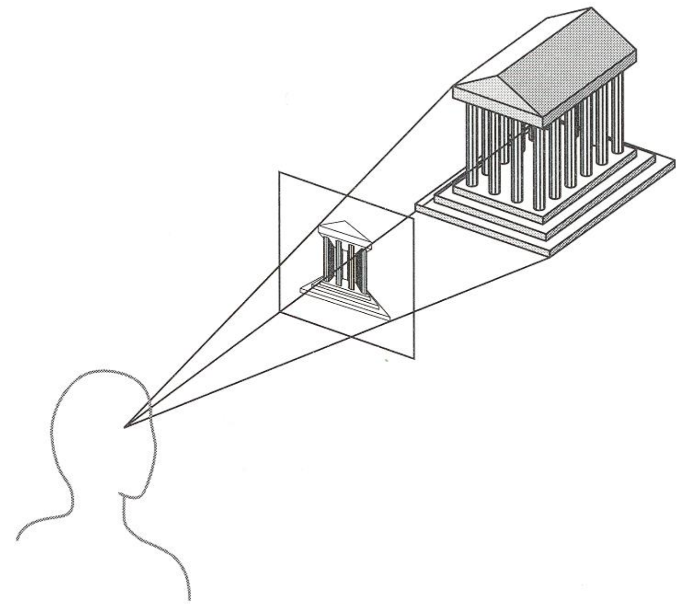
Intrinsics

Extrinsics

$$\mathbf{T}_{img}\mathbf{S}_{pix}\mathbf{P}_{cam}\mathbf{R}_{cam}\mathbf{T}_{cam}\vec{x}$$

Camera parameters

- **Extrinsic:** how to map world coordinates to camera coordinates (translation and rotation)
- **Intrinsic:** how to map camera coordinates to image coordinates (projection, translation, scale)



Camera parameters

- **Extrinsic:** how to map world coordinates to camera coordinates (translation and rotation)

$$\mathbf{P}_c = \mathbf{R}(\mathbf{P}_w - \mathbf{T})$$

Camera coordinates

World coordinates

Camera parameters

- **Intrinsic:** how to map camera coordinates to image coordinates (projection, translation, scale)

$$x = -\left(f\frac{X}{Z} - o_x\right)s_x$$

$$y = -\left(f\frac{Y}{Z} - o_y\right)s_y$$

Coordinates of projected point in image coordinates

Coordinates of image point in camera coordinates

Coordinates of image center in pixel units

Effective size of a pixel (mm)

Camera parameters

- Substituting previous eqns describing intrinsic and extrinsic parameters, can relate *pixels coordinates* to *world points:*

$$-(x_{im} - o_x)s_x = f\,\frac{\mathbf{R}_1 \cdot (\mathbf{P}_w - \mathbf{T})}{\mathbf{R}_3 \cdot (\mathbf{P}_w - \mathbf{T})}$$

$$-(y_{im} - o_y)s_y = f\,\frac{\mathbf{R}_2 \cdot (\mathbf{P}_w - \mathbf{T})}{\mathbf{R}_3 \cdot (\mathbf{P}_w - \mathbf{T})}$$

$\mathbf{R}_i$ = Row i of rotation matrix

Projection matrix

- This can be rewritten as a matrix product using homogeneous coordinates:

$$\begin{bmatrix} wx_{im} \\ wy_{im} \\ w \end{bmatrix} = \mathbf{M}_{\mathrm{int}}\mathbf{M}_{ext}\begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

where:

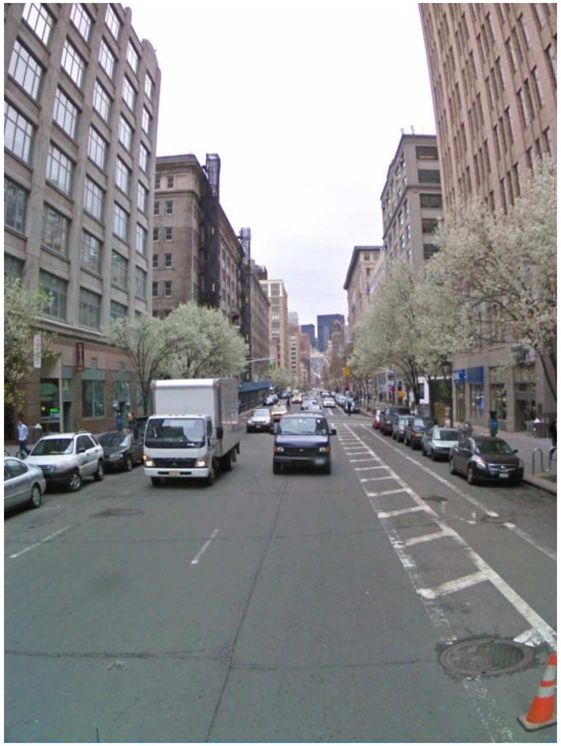
$$\mathbf{M}_{\mathrm{int}} = \begin{bmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M}_{ext} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ r_{21} & r_{22} & r_{23} & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ r_{31} & r_{32} & r_{33} & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{bmatrix}$$

Radial Distortion

- Finally, would like to include transformation to unwarp radial distortion:

$$u_{img} \rightarrow c_u + u^*_{img}\left(1 + \kappa({u^*_{img}}^2 + {v^*_{img}}^2)\right)$$
$$v_{img} \rightarrow c_v + v^*_{img}\left(1 + \kappa({u^*_{img}}^2 + {v^*_{img}}^2)\right)$$



Radial Distortion

- Radial distortion cannot be represented by matrix

$$u_{img} \rightarrow c_u + u^*_{img}\left(1 + \kappa\left({u^*_{img}}^2 + {v^*_{img}}^2\right)\right)$$
$$v_{img} \rightarrow c_v + v^*_{img}\left(1 + \kappa\left({u^*_{img}}^2 + {v^*_{img}}^2\right)\right)$$

- (c_u, c_v) is image center,

$$u^*_{img} = u_{img} - c_u, \quad v^*_{img} = v_{img} - c_v,$$

κ is first-order radial distortion coefficient

A Camera Model

Camera orientation

Scale to pixel size

Camera location

Translate to image center

Perspective projection

3D point (homogeneous coords)

Then unwarp radial distortion

$$\mathbf{T}_{img}\mathbf{S}_{pix}\mathbf{P}_{cam}\mathbf{R}_{cam}\mathbf{T}_{cam}\vec{x}$$

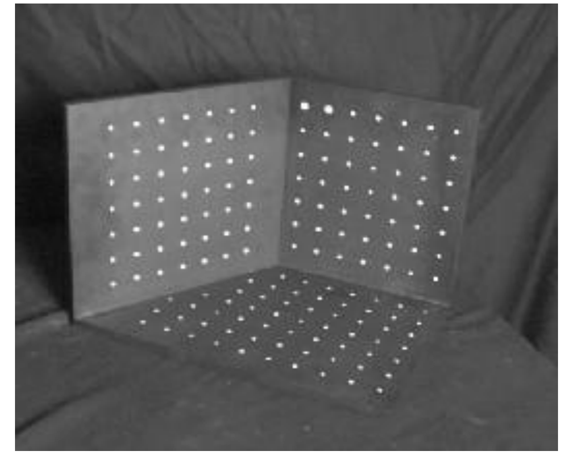
Then perform homogeneous divide, and get (u,v) coords

Outline

- Camera model
- Camera calibration
- Camera triangulation
- Structure from motion

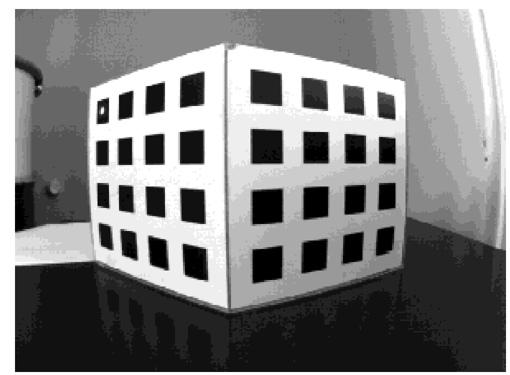
Camera Calibration

- Compute intrinsic and extrinsic parameters using observed camera data



Main idea

- Place “calibration object” with known geometry in the scene
- Get correspondences
- Solve for mapping from scene to image: estimate $\mathbf{M}=\mathbf{M}_{int}\mathbf{M}_{ext}$



The Opti-CAL Calibration Target Image

Camera Calibration

Chromaglyphs
Courtesy of Bruce Culbertson, HP Labs
http://www.hpl.hp.com/personal/Bruce_Culbertson/ibr98/chromagl.htm

Camera Calibration

- Input:
  - 3D $\leftrightarrow$ 2D correspondences
  - General perspective camera model (no radial distortion, for now)

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

- Output:
  - Camera model parameters

Direct linear calibration

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

$$u_i = \frac{m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}}$$

$$v_i = \frac{m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}}$$

$$u_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}) = m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}$$
$$v_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}) = m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}$$

$$\begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -u_iX_i & -u_iY_i & -u_iZ_i & -u_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -v_iX_i & -v_iY_i & -v_iZ_i & -v_i \end{bmatrix} \begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \\ m_{03} \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{20} \\ m_{21} \\ m_{22} \\ m_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Direct linear calibration

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\ & & & & & & \vdots & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n & -u_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n & -v_n \end{bmatrix} \begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \\ m_{03} \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{20} \\ m_{21} \\ m_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Can solve for m_{ij} by linear least squares

Direct linear calibration

- Advantage:
  - Very simple to formulate and solve
- Disadvantages:
  - Doesn’t tell you the camera parameters
  - Doesn’t model radial distortion
  - Hard to impose constraints (e.g., known *f*)
  - Doesn’t minimize the right error function

Why?

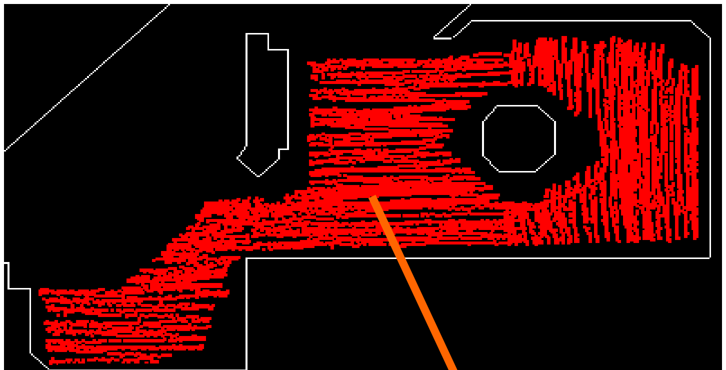
Nonlinear Camera Calibration

- Incorporating additional constraints into camera model
  - No shear, no scale (rigid-body motion)
  - Square pixels
  - Radial distortion
  - etc.
- These impose *nonlinear* constraints on camera parameters

Application: Sea of Images

Aliaga

Application: Sea of Images



Large-scale Dense Capture

Off-line:

- Camera calibration
- Feature correspondence

Real-time:

- Image warping
- Image composition

Interactive

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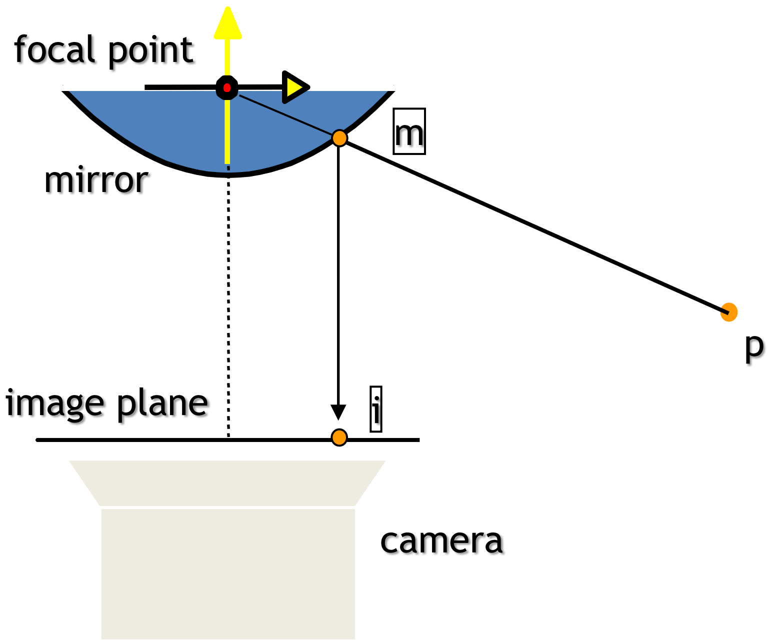
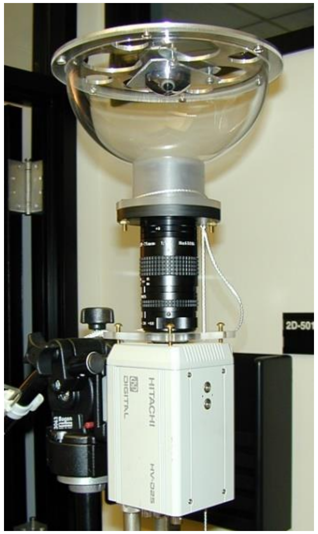
Sea of Images Capture

Hemispherical FOV camera





Paraboloidal Catadioptric Camera
[Nayar97]

Sea of Images Capture

Capture lots of images with hemispherical FOV camera driven on cart

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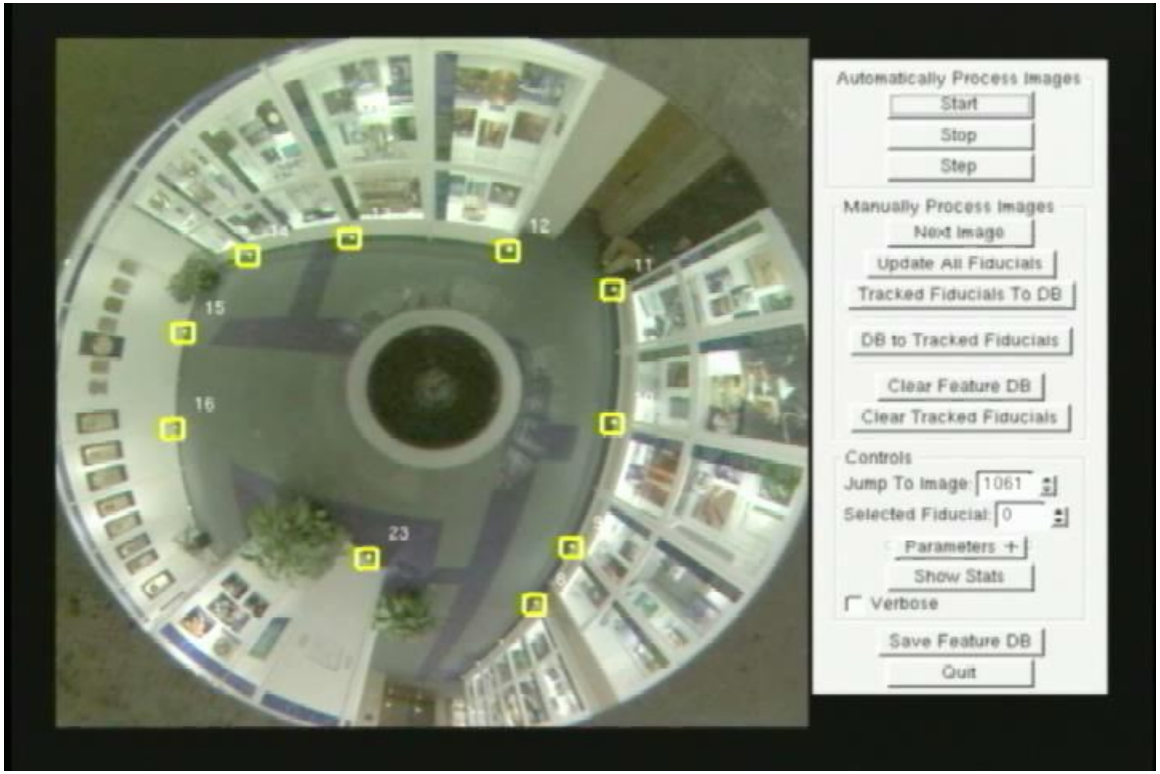
Sea of Images Calibration

Calibrate camera based on positions of fiducials



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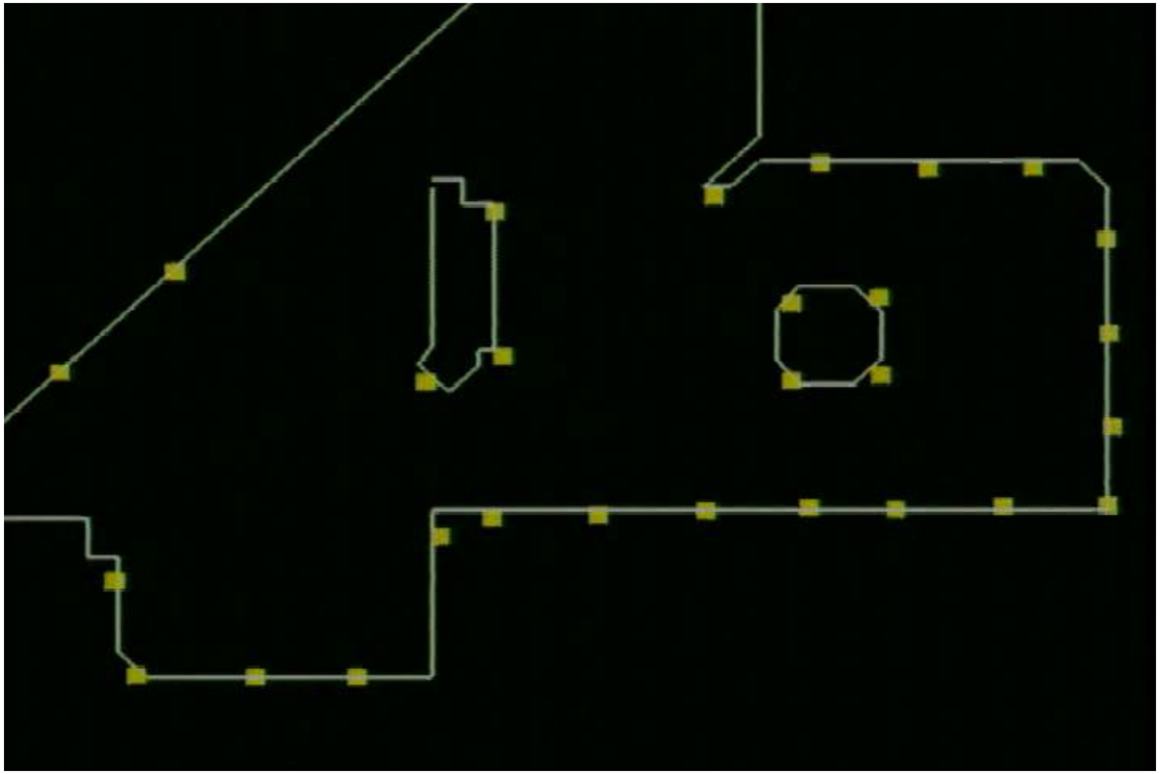
Sea of Images Calibration

Result is a “sea of images” with known camera viewpoints spaced a few inches apart



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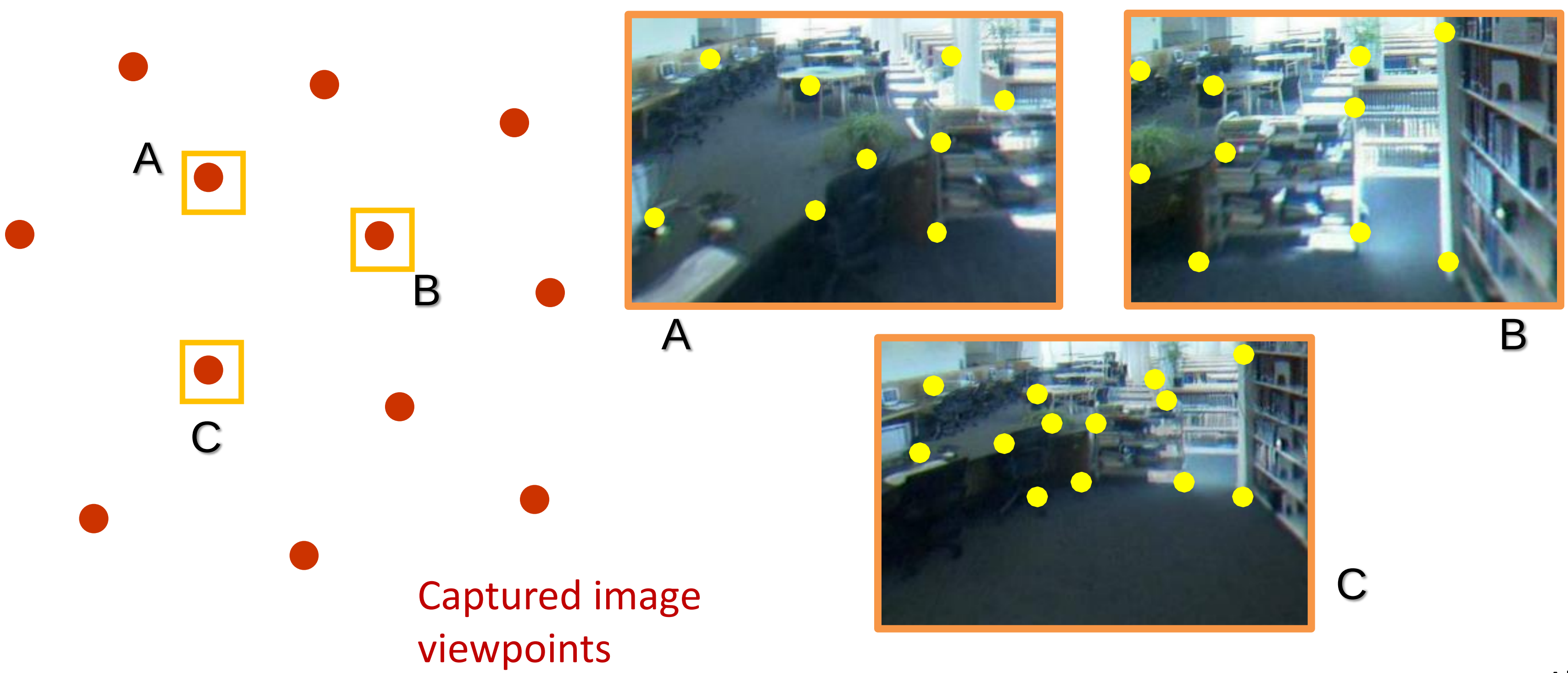
Sea of Images Correspondence

Search for feature correspondences in nearby images



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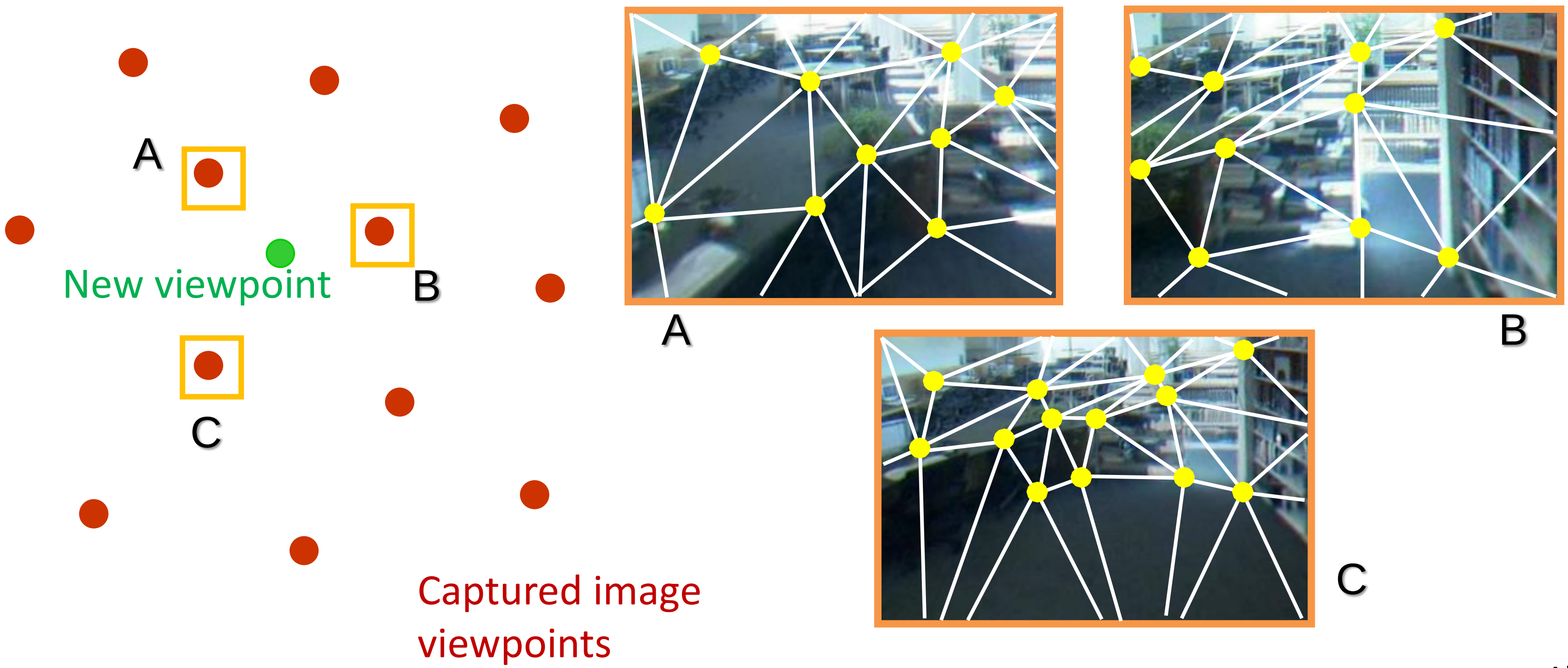
Sea of Images Rendering

Create image from new viewpoint by warping and compositing three nearest images using triangles of feature correspondences



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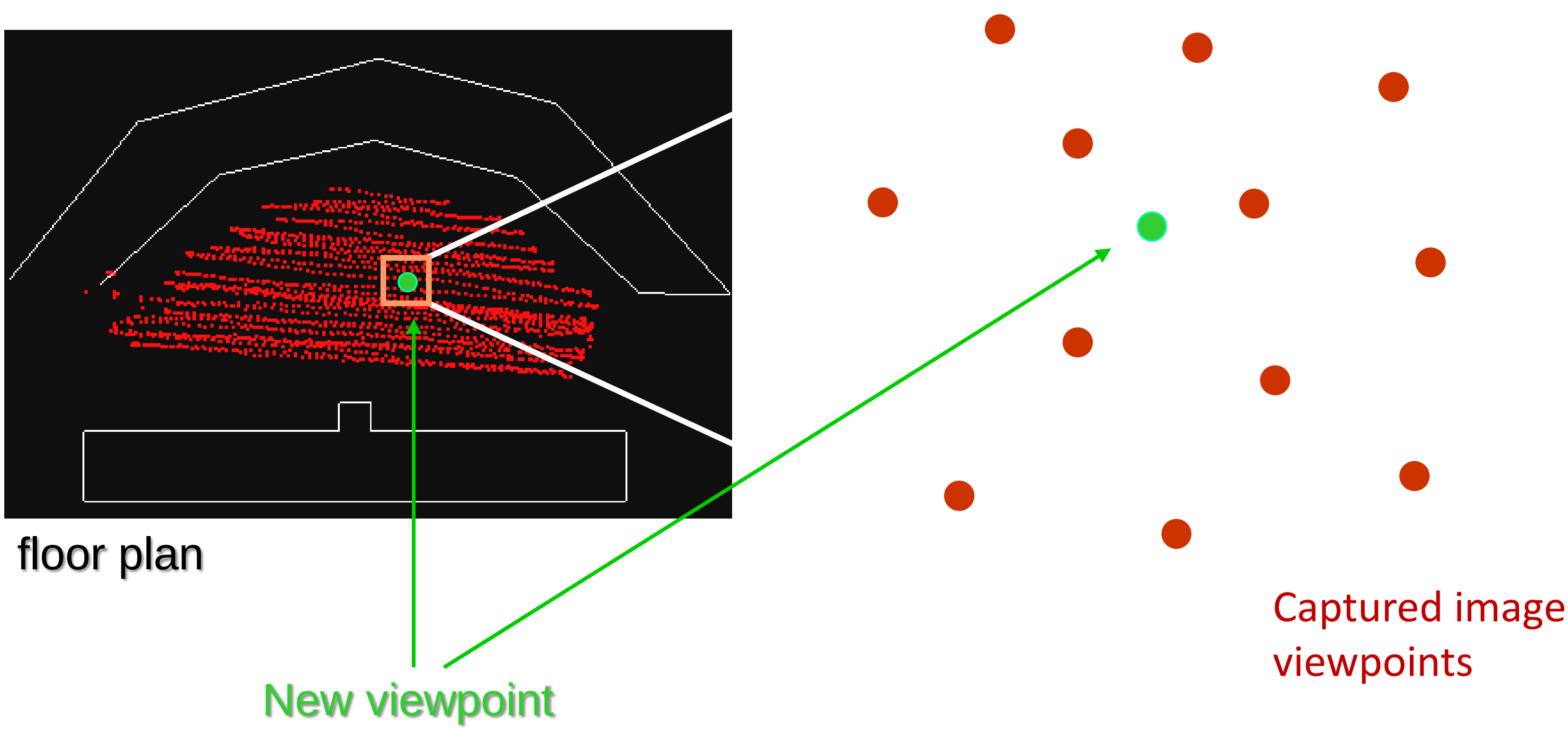
Sea of Images Rendering

Neat data management based on hierarchical encoding to enable interactive walkthrough



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Sea of Images Results

- Bell Labs Museum
  - 900 square ft
  - 9832 images
  - 2.2 inch spacing
- Princeton Library
  - 120 square ft
  - 1947 images
  - 1.6 inches
- Personal Office
  - 30 square feet
  - 3475 images
  - 0.7 inches

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Sea of Images Results

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Sea of Images Results

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Sea of Images Results

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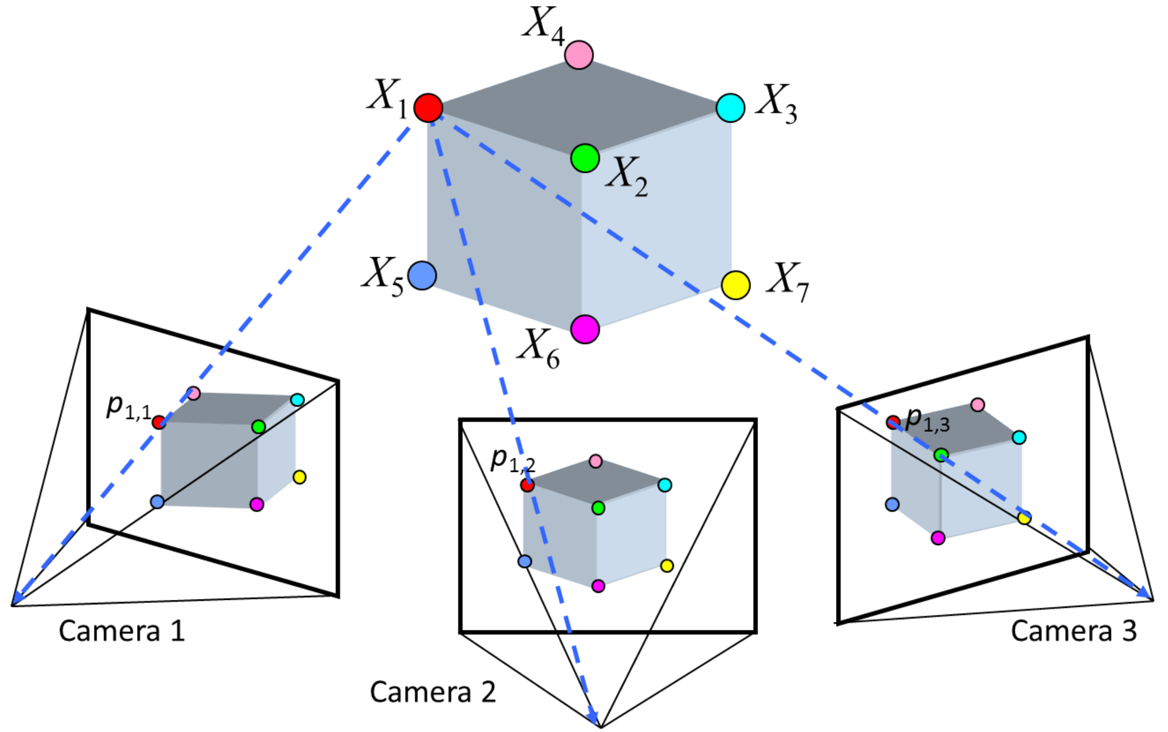
Outline

- Camera model
- Camera calibration
- Camera triangulation
- Structure from motion

Camera calibration summary

Suppose we know 3D point positions and correspondences to pixels in every image

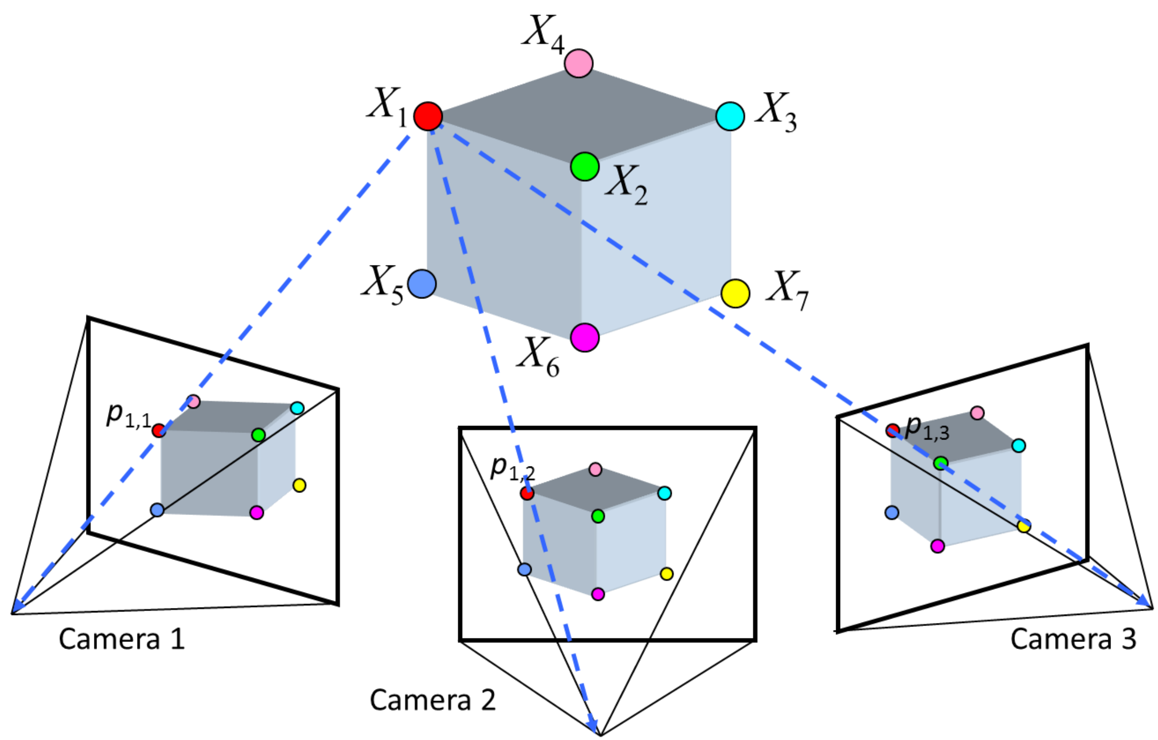
– We can compute the camera parameters



Noah Snavely

Camera triangulation

Suppose we know the camera parameters and correspondences between points and pixels in every image?



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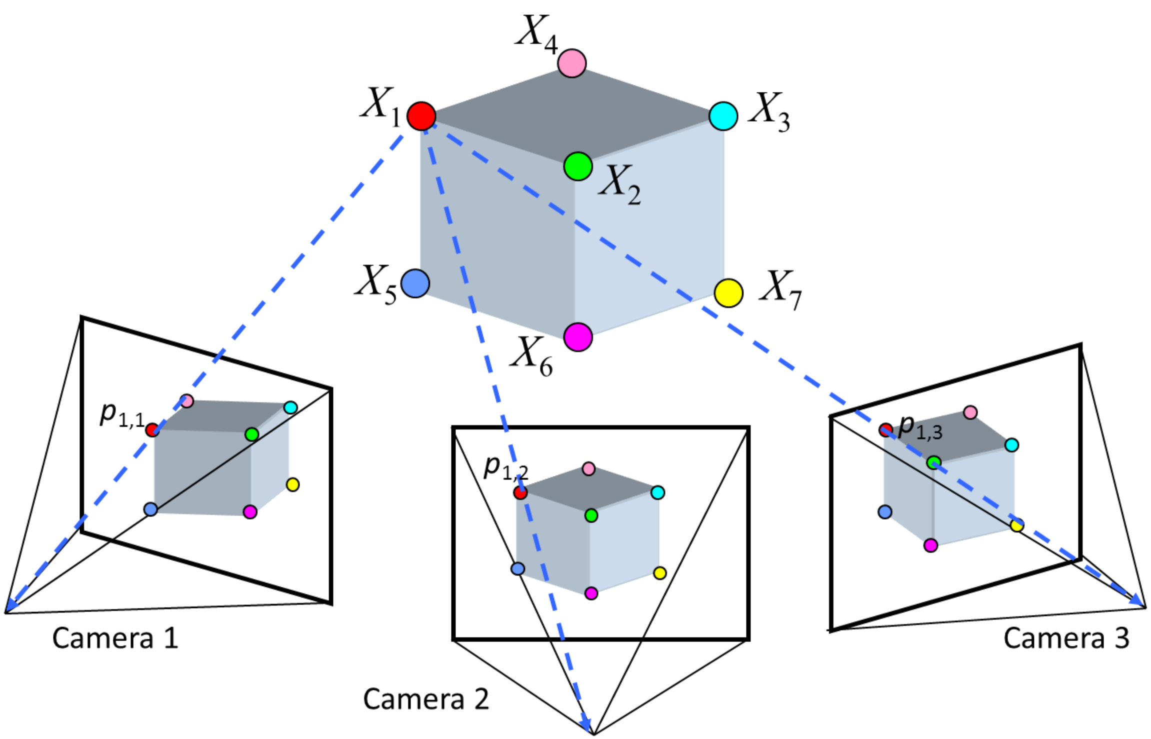
Camera calibration & triangulation

Suppose no camera parameters or point positions

- And have matches between these points and two images
- Can we compute them both together?



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Outline

- Camera model
- Camera calibration
- Camera triangulation
- Structure from motion

Structure from motion

- Given many images
  a) figure out where they were all taken from?
  b) build a 3D model of the scene?

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Structure from motion

- Feature detection
- Feature description
- Feature matching
- Feature correspondence
- Camera calibration and triangulation

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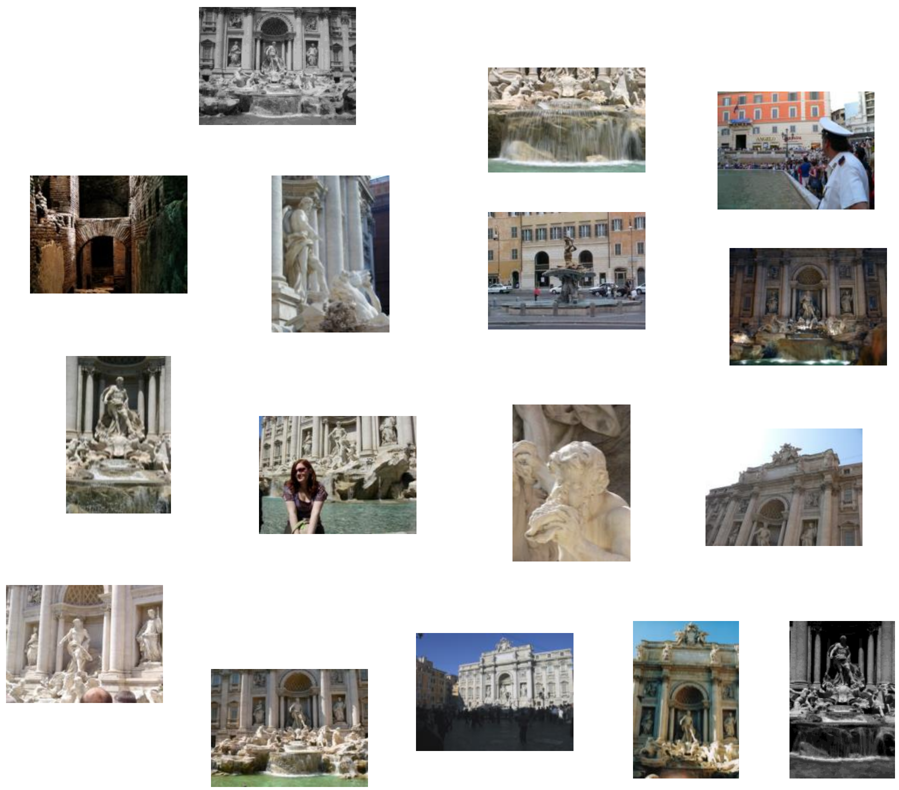
Feature detection

Detect features using SIFT [Lowe, IJCV 2004]



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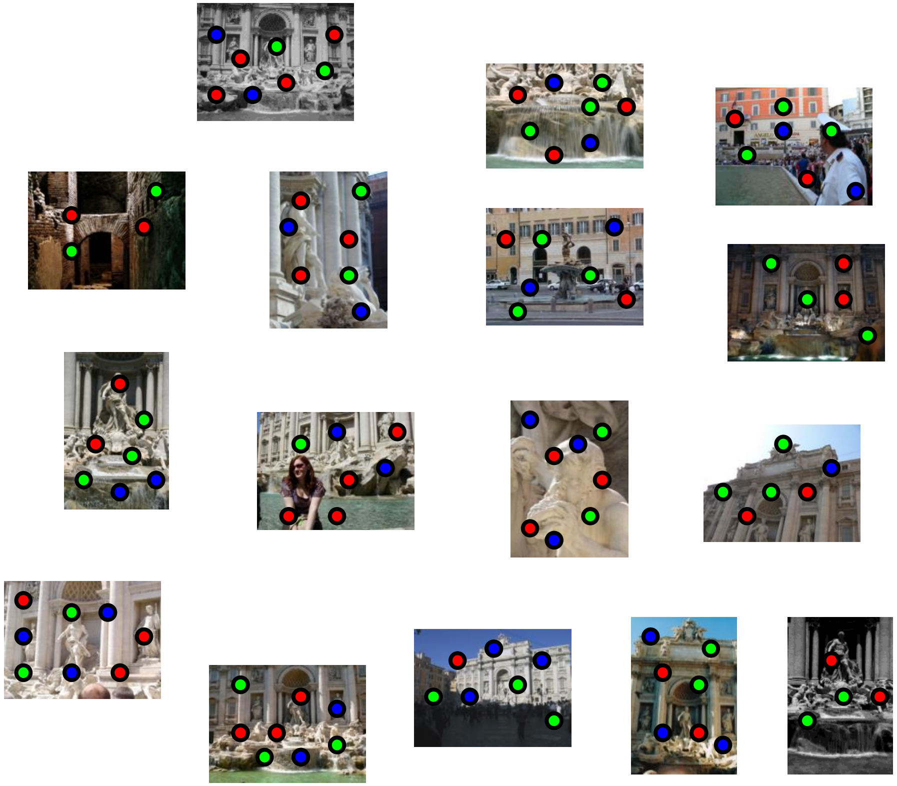
Feature detection

Detect features using SIFT [Lowe, IJCV 2004]



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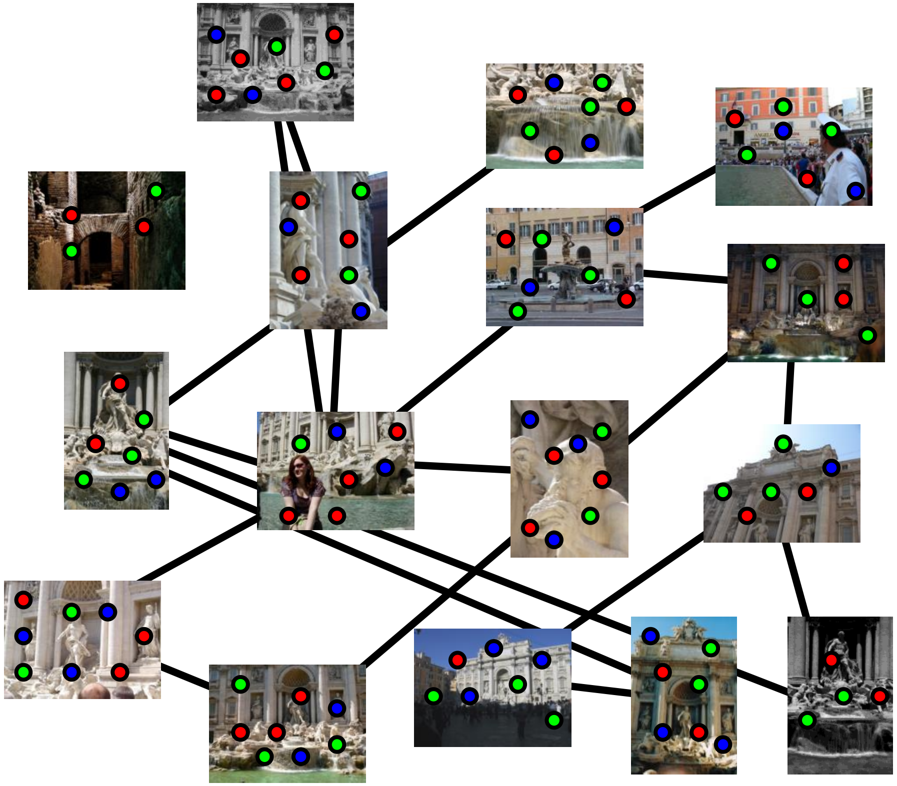
Feature matching

Match features between each pair of images



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Feature matching

Refine matching using RANSAC to correspondences between each pair

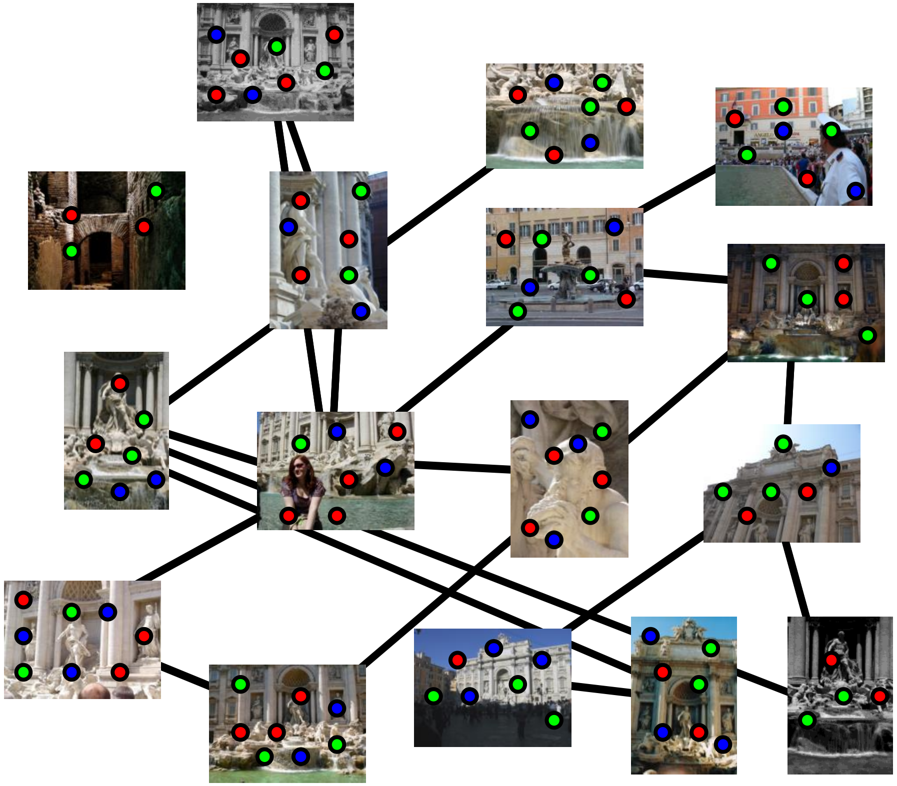

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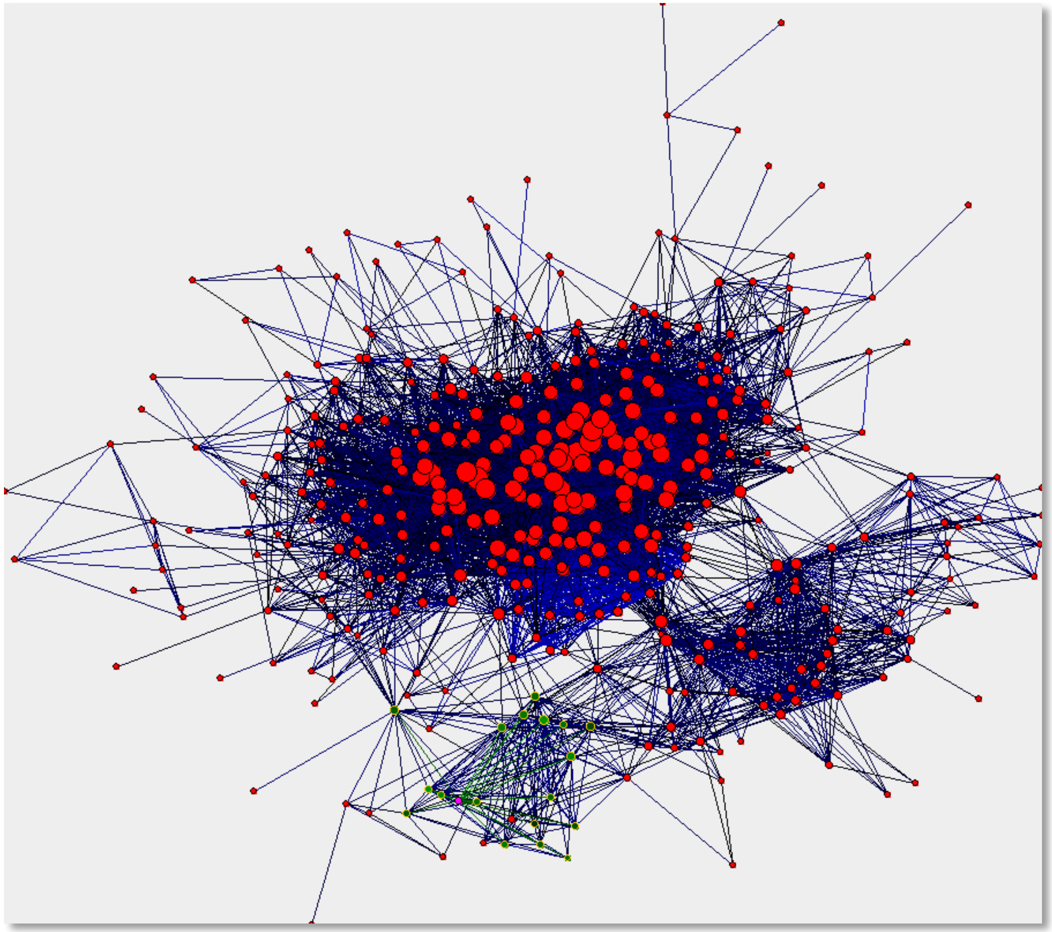
Image connectivity graph



(graph layout produced using the Graphviz toolkit: http://www.graphviz.org/)

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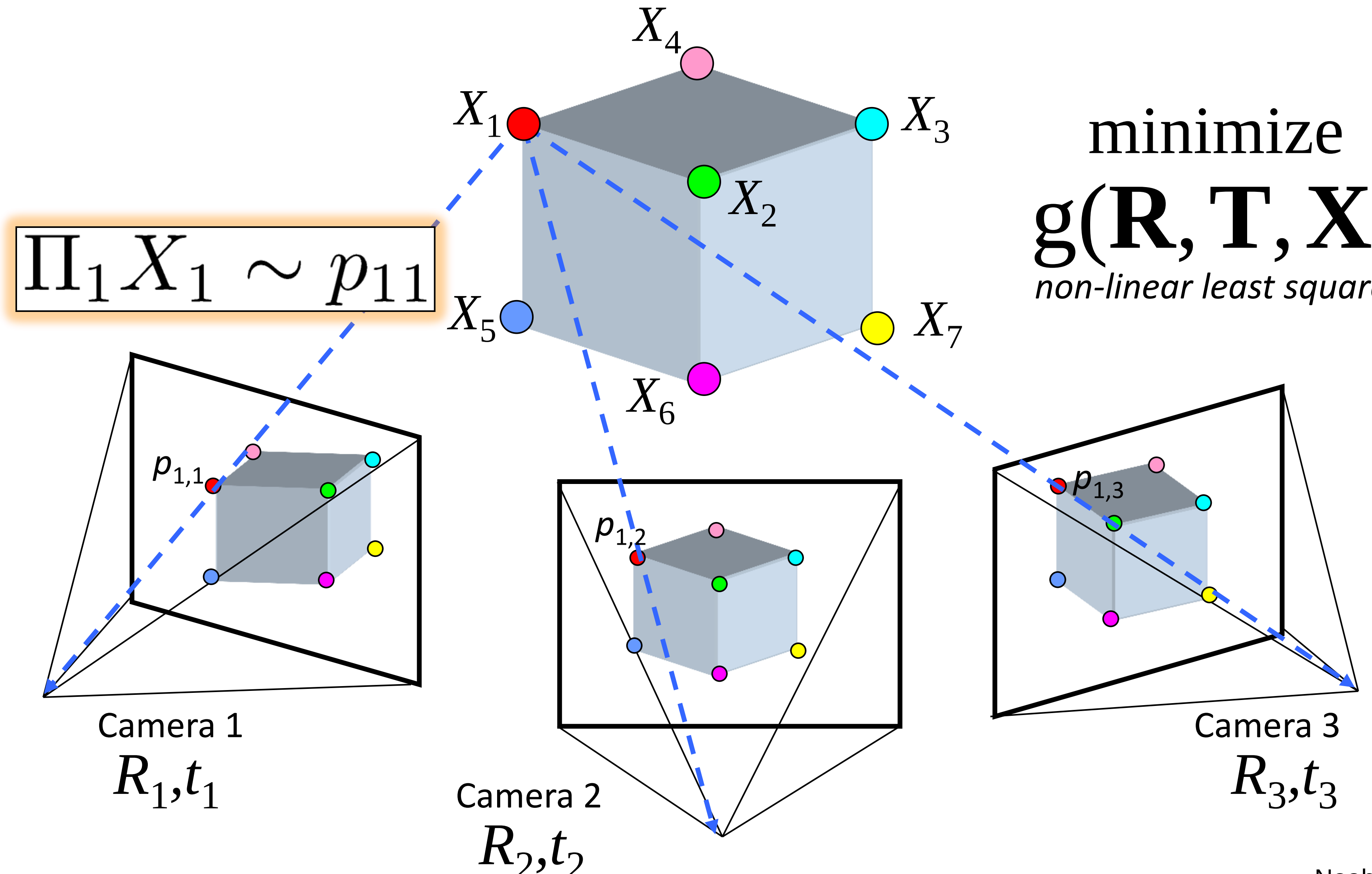
Structure from motion



minimize

$$\mathrm{g}(\mathbf{R}, \mathbf{T}, \mathbf{X})$$

non-linear least squares

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Structure from motion

- Minimize sum of squared reprojection errors:

$$g(\mathbf{X},\mathbf{R},\mathbf{T}) = \sum_{i=1}^{m}\sum_{j=1}^{n} \underbrace{w_{ij}}_{\substack{\textit{indicator variable}: \\ \text{is point } i \text{ visible in image } j\text{ ?}}} \cdot \left\| \underbrace{\mathbf{P}(\mathbf{x}_i,\mathbf{R}_j,\mathbf{t}_j)}_{\substack{\textit{predicted} \\ \text{image location}}} - \underbrace{\begin{bmatrix} u_{i,j} \\ v_{i,j} \end{bmatrix}}_{\substack{\textit{observed} \\ \text{image location}}} \right\|^2$$

- Minimizing this function is called *bundle adjustment*
  - Optimized using non-linear least squares, e.g. Levenberg-Marquardt

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Problem size

- What are the variables?
- How many variables per camera?
- How many variables per point?

- Trevi Fountain collection

    466 input photos

    + > 100,000 3D points

    = very large optimization problem

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Structure from motion

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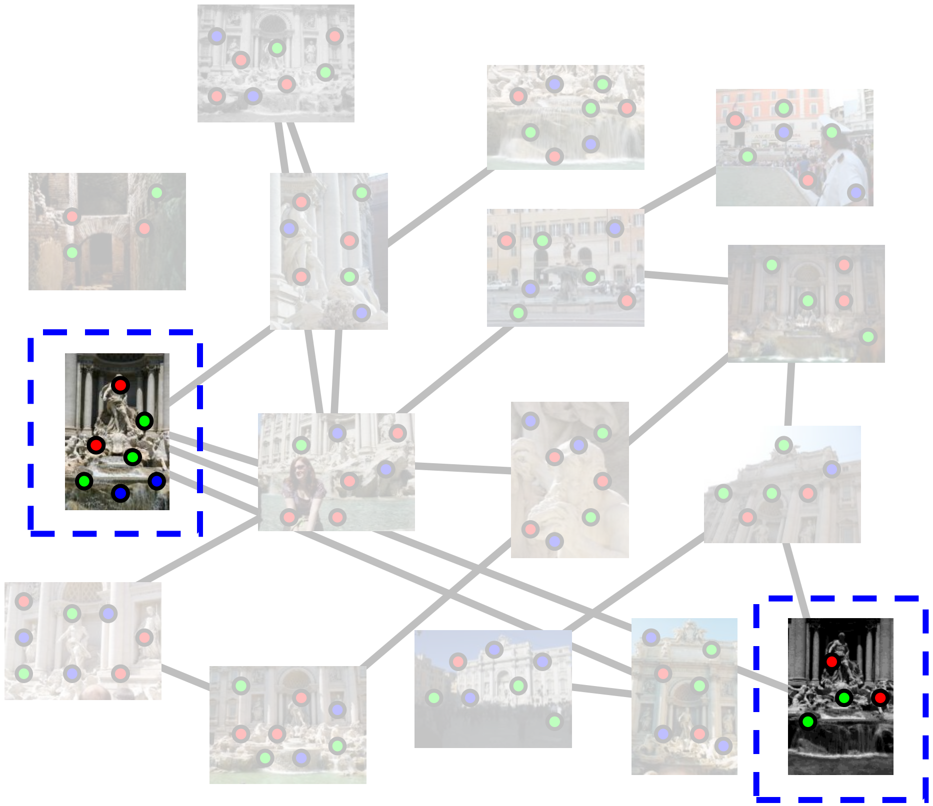
Incremental structure from motion



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Incremental structure from motion

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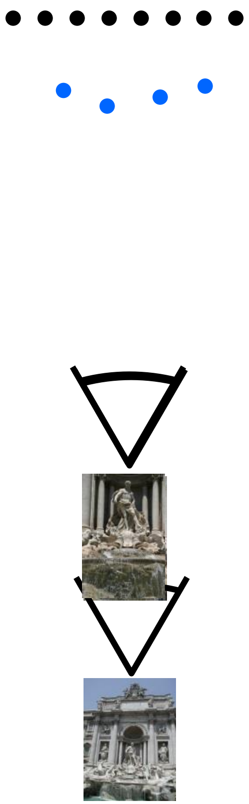
Incremental structure from motion



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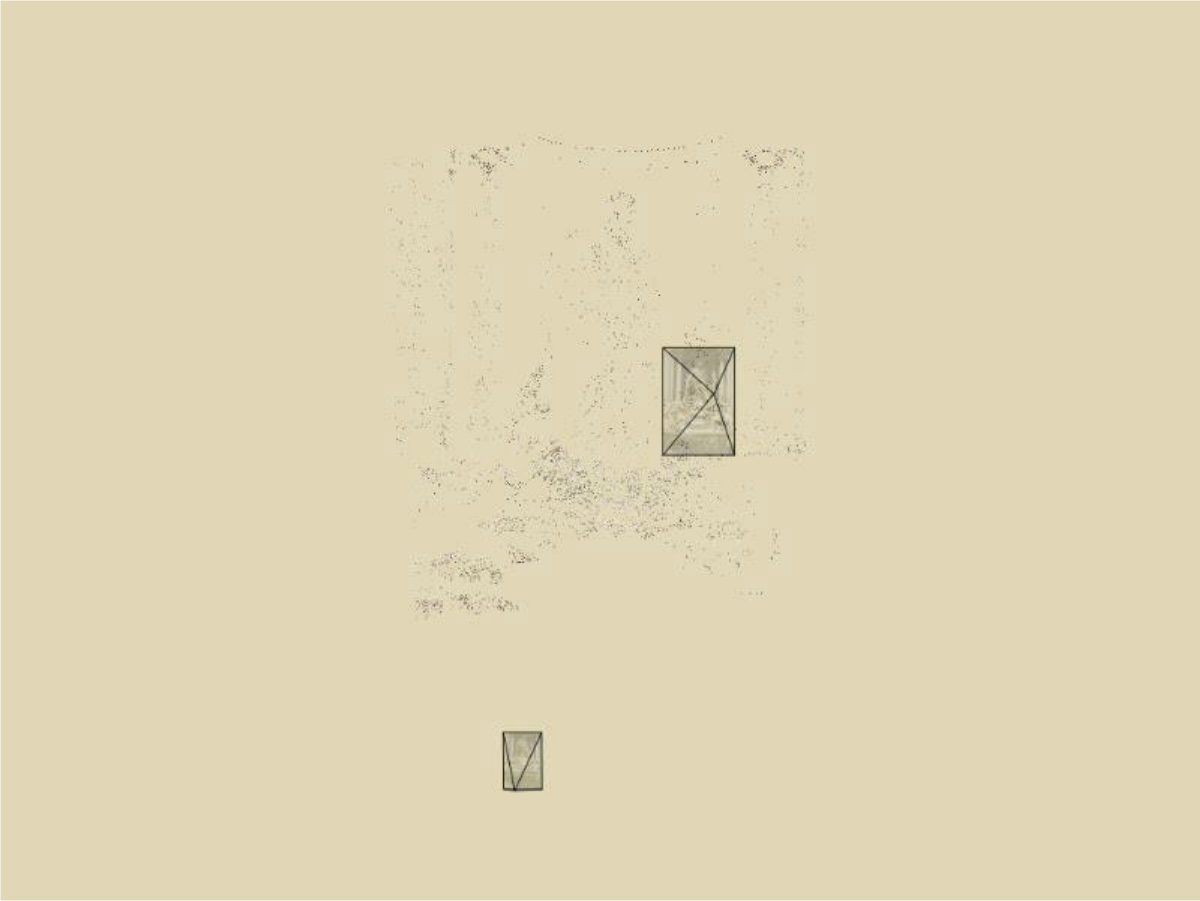
Incremental structure from motion



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Incremental structure from motion

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Structure from Motion Results

- For photo sets from the Internet, 20% to 75% of the photos were registered
- Most unregistered photos belonged to different connected components

> 1 week for 2600 photo

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Structure from Motion Results

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Photo Tourism:
Exploring Photo Collections in 3D

Noah Snavely

Steven M. Seitz

University of Washington

Richard Szeliski

Microsoft Research

15,464

37,383

76,389

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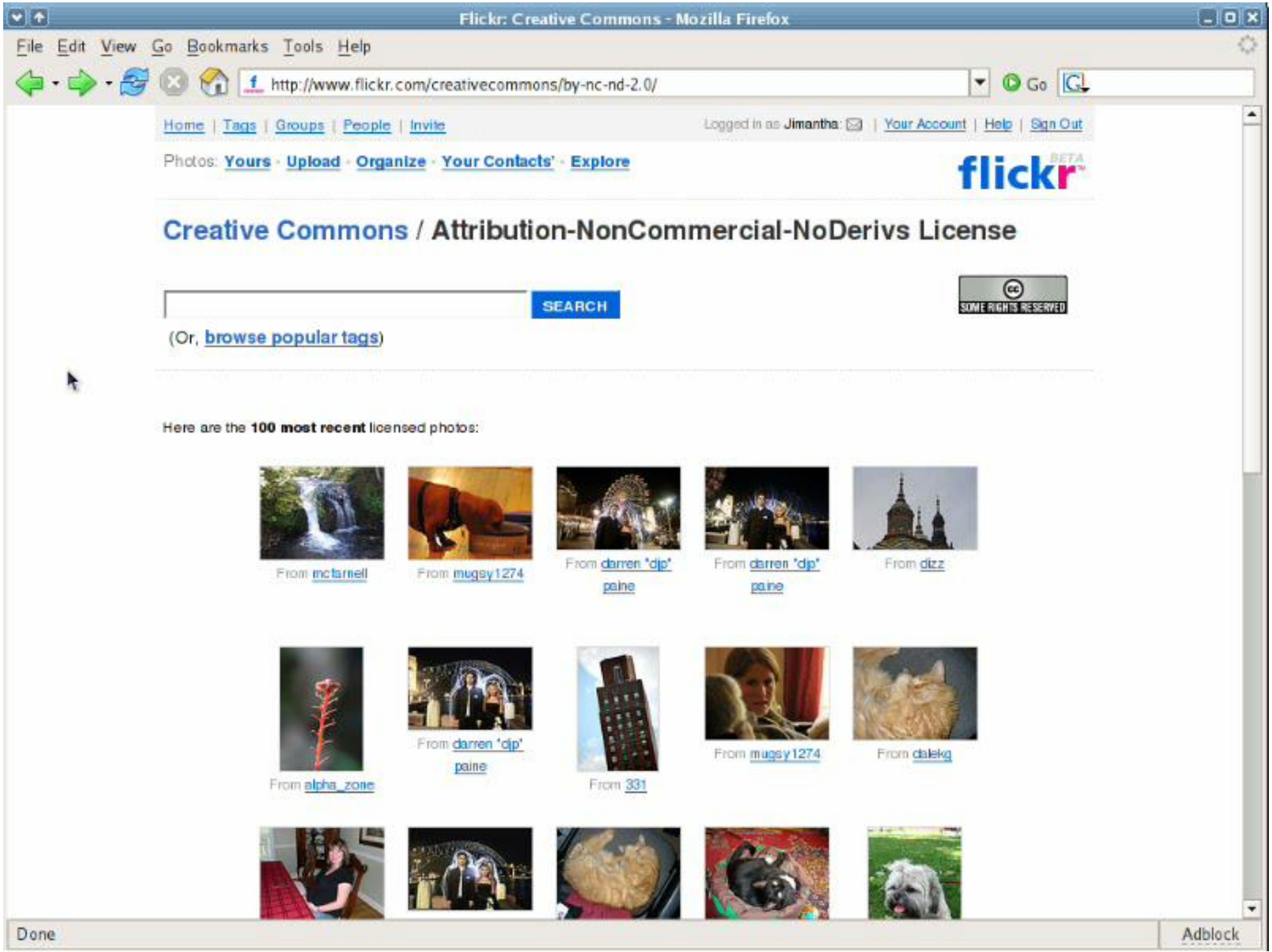

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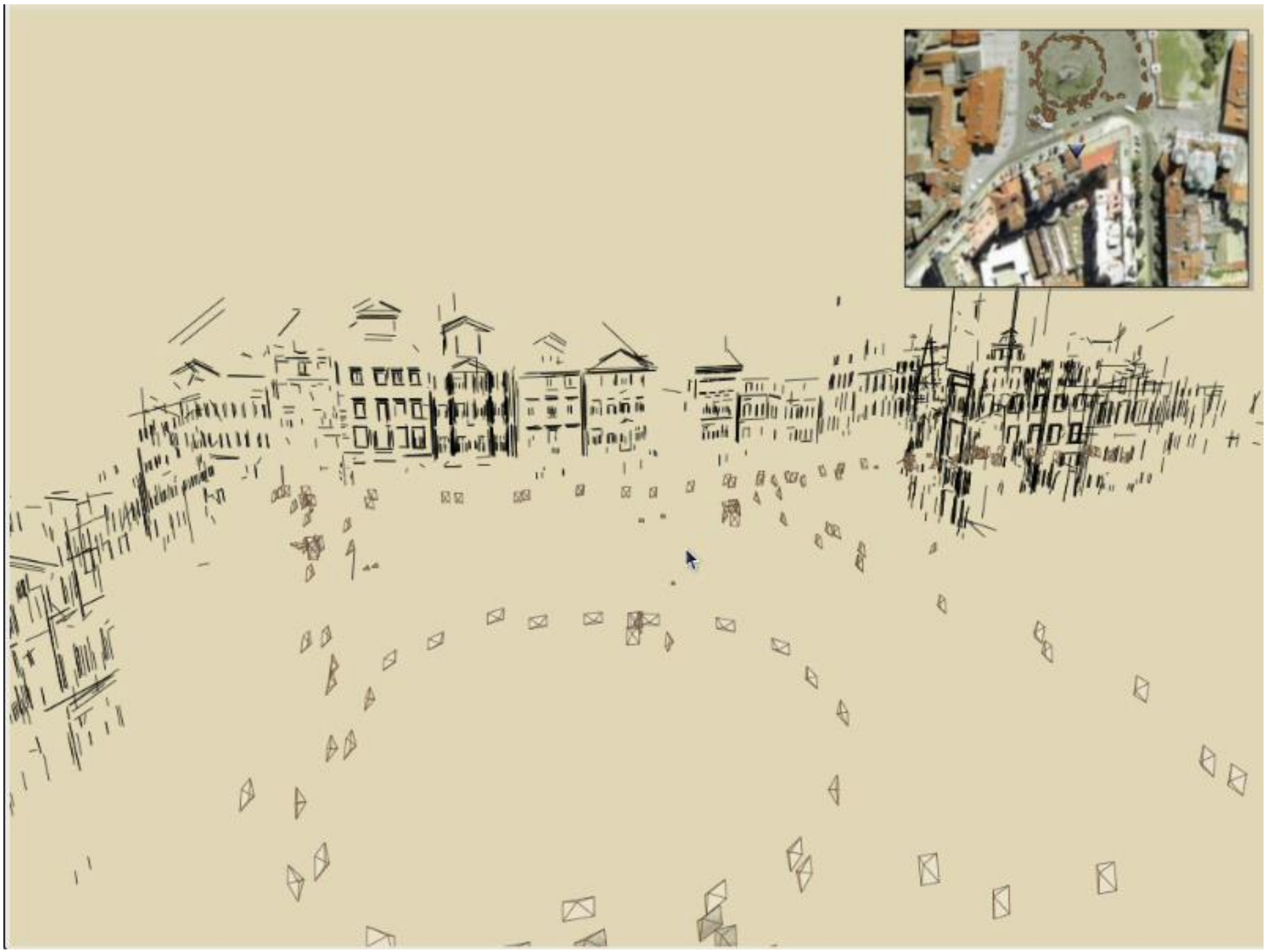
Example: Prague Old Town Square



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Rendering transitions

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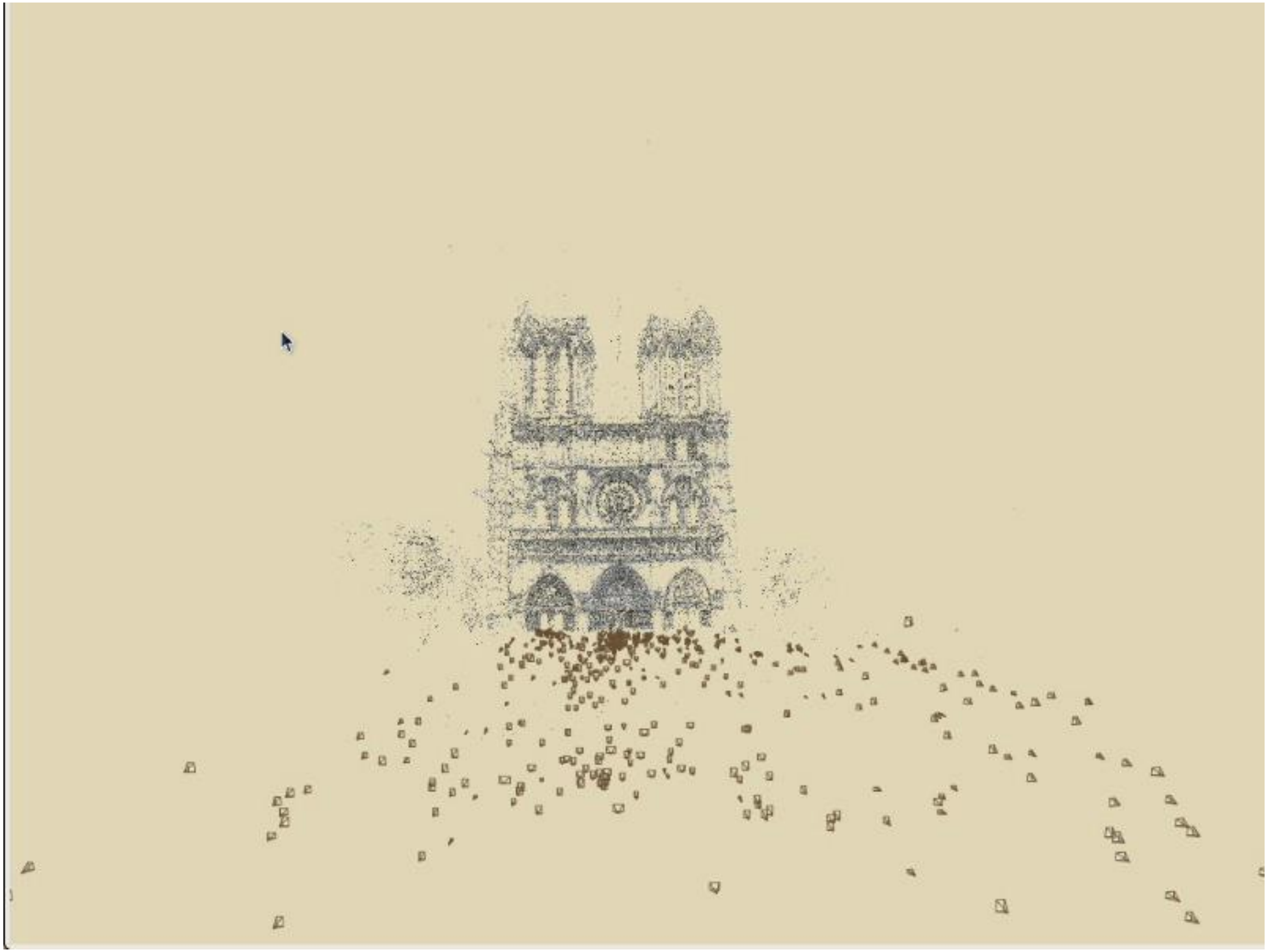
Annotations



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Annotations

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Summary

- Camera model
- Camera calibration
- Camera triangulation
- Structure from motion

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