COS 323, Fall 2012
Exam 1

Name:

NetID:

Honor Code pledge:

Signature:

- This exam consists of 5 questions on 8 pages.
- Do all of your work on these pages, using the back for scratch space and giving the answers in the spaces provided.
- No partial credit will be provided unless you show your work and/or explain your reasoning.
- This is a closed-book, closed-notes exam.
- **Put your NetID on every page (1 point), and write out and sign the Honor Code pledge before turning in the test:**

  "I pledge my honor that I have not violated the Honor Code during this examination."

Question	1	2	3	4	5	NetID on each page	**Total**
Score							

1. Solving linear systems

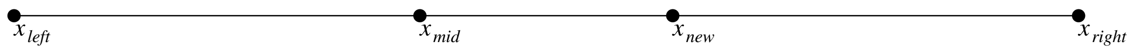
(a) For each of the following algorithms, executed on an $n \times n$ matrix, list the order of growth of its expected running time in big-O notation: (2 pts. each)

Algorithm		Running time
L:	LU decomposition without pivoting	
P:	LU decomposition with partial pivoting	
C:	Cholesky decomposition	
S:	Forward/back substitution (with no other matrix decomposition)	
T:	Tridiagonal solver	

(b) Which of the methods can be used ***successfully*** to solve the following linear systems?
List only the letter(s) corresponding to the algorithm(s), or "N" for none of the above. (2 pts. each)

Matrix $\mathbf{A}$ in $\mathbf{Ax} = \mathbf{b}$	Method(s)
$\begin{pmatrix} 3 & 2 & 3 \\ 5 & 5 & 5 \\ 1 & 2 & 3 \end{pmatrix}$	L, P
$\begin{pmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{pmatrix}$	
$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix}$	
$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 5 \end{pmatrix}$	
$\begin{pmatrix} 3 & 2 & 3 \\ 3 & 2 & 3 \\ 3 & 2 & 3 \end{pmatrix}$	
$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & 3 \\ 0 & 4 & 5 \end{pmatrix}$	
$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	

2. Golden section search (3 pts. each)



(a) You are running golden section search to find a local minimum of some function $f(x)$. At some point during the algorithm, you have a bracket consisting of $(x_{left}, x_{mid}, x_{right})$, as shown above. You find the point $x_{new} > x_{mid}$, evaluate f at that location, and perform the following updates:

$$\begin{aligned} x_{left} &\leftarrow x_{mid} \\ x_{mid} &\leftarrow x_{new} \end{aligned}$$

Given this information, circle the correct statement in each row below, where x_{left}, x_{mid}, x_{new}, and x_{right} refer to the original values, ***before*** the updates.

$f(x_{new}) < f(x_{left})$	$f(x_{new}) > f(x_{left})$	Nothing can be said about the relative values of $f(x_{new})$ and $f(x_{left})$
$f(x_{new}) < f(x_{mid})$	$f(x_{new}) > f(x_{mid})$	Nothing can be said about the relative values of $f(x_{new})$ and $f(x_{mid})$
$f(x_{new}) < f(x_{right})$	$f(x_{new}) > f(x_{right})$	Nothing can be said about the relative values of $f(x_{new})$ and $f(x_{right})$

(b) Suppose, instead, you ended up performing the following update:

$$x_{right} \leftarrow x_{new}$$

Again, circle the correct statement in each row:

$f(x_{new}) < f(x_{left})$	$f(x_{new}) > f(x_{left})$	Nothing can be said about the relative values of $f(x_{new})$ and $f(x_{left})$
$f(x_{new}) < f(x_{mid})$	$f(x_{new}) > f(x_{mid})$	Nothing can be said about the relative values of $f(x_{new})$ and $f(x_{mid})$
$f(x_{new}) < f(x_{right})$	$f(x_{new}) > f(x_{right})$	Nothing can be said about the relative values of $f(x_{new})$ and $f(x_{right})$

(c) Golden Section Search exhibits ______ convergence: (circle the correct completion)

linear quadratic neither linear nor quadratic

3. Reciprocal square root

Consider using Newton's method for root-finding to compute the reciprocal square root

$$x = \frac{1}{\sqrt{a}}$$

of some input a, by solving the equation

$$\frac{1}{x^2} - a = 0.$$

(a) Write down the Newton's-rule update used to compute x_{n+1} from the approximation on the previous iteration, x_n. You should be able to simplify this expression so that it involves only subtraction and multiplication. (10 pts.)

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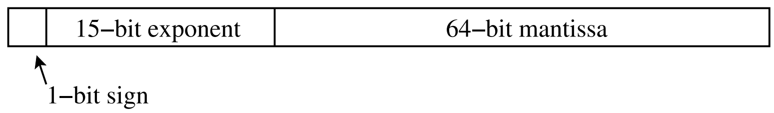
(b) Suppose you are given an initial approximation x_0 that is known to be correct to $\pm 1\%$ (or a relative precision of about 2^{-7}) of the true value. Furthermore, suppose you are performing the computation in ***extended double*** floating-point format, which has the following representation:



How many iterations will you need to perform to have an answer that is correct to machine precision? Explain your reasoning. (10 pts.)

4. Least squares algorithms (4 pts. each)

For ***linear*** least squares estimation, what is the main advantage of

(a) . . . QR decomposition over Cholesky applied to the normal equations?

(b) . . . using SVD to find the pseudoinverse over QR decomposition?

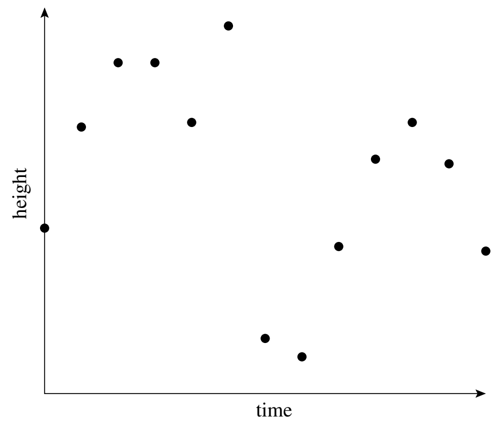
For ***nonlinear*** least squares estimation, what is the main advantage of

(c) . . . Gauss-Newton iteration over Newton's method?

(d) . . . Levenberg-Marquardt over Gauss-Newton?

5. Parameter estimation (20 pts.)

You throw a ball up into the air, and observe it as it bounces. An (obviously imperfect) automated data collection system records the following heights, as a function of time:



Design a strategy for using the data to estimate the ***coefficient of restitution*** of the ball: the ratio of the speed of the ball after the bounce to its speed before the bounce. Describe which of the algorithms we mentioned in class you would use and ***how*** you would use them (i.e., what functions you would fit to which datapoints, and how you would use the results). Your method should be general enough to handle similar datasets from this experiment, without requiring human intervention.

Do not write code, and do not perform any mathematical derivations; just write a few sentences about your general strategy.