FLIPPED LECTURE 6

- *Shortest paths*
- *Max flow*

Algorithms
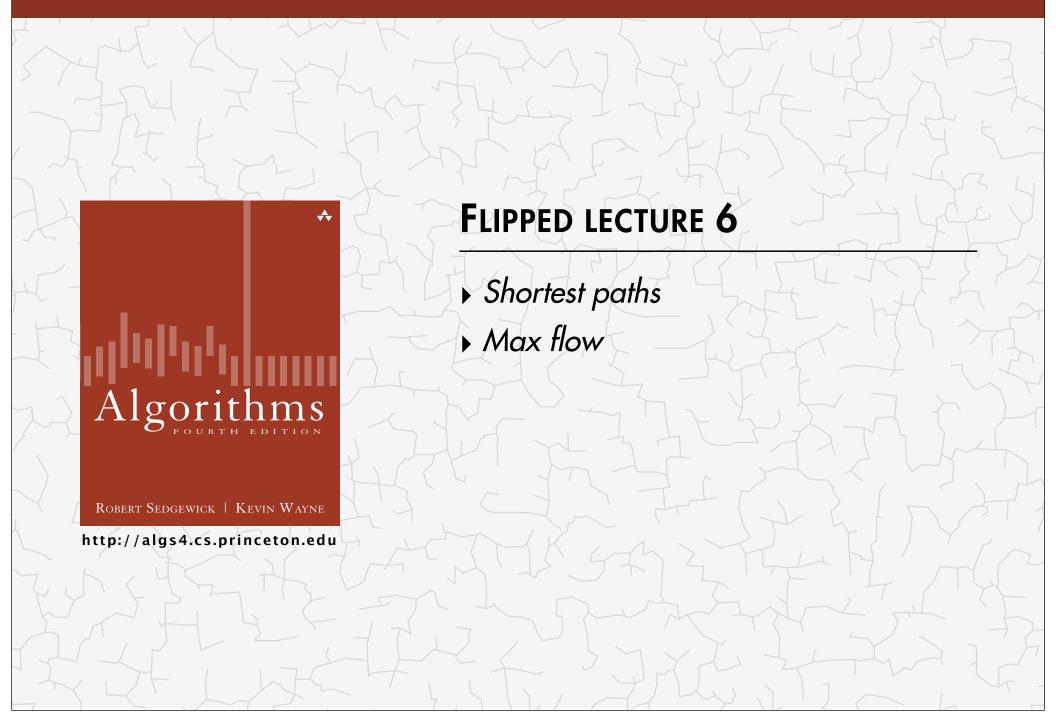FOURTH EDITION

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SPTs and MSTs

Dijkstra's Algorithm:

- Give a simple example of a graph where Dijkstra's algorithm will fail.
- Run Dijkstra's on this graph and explain why the SPT is wrong.

MST on directed graphs:

- Will Dijkstra's algorithm compute the MST on a undirected graph?

SPT on undirected graphs:

- See week8-precept worksheet.

More MSTs and SPTs

MST vs. SPT

- T/F: Adding c to every weight can change the undirected MST.
- T/F: Adding c to every weight can change the directed SPT.

Prim's vs. Dijkstra's:

- Lazy Prim's algorithm:
  - Maintain a PQ of edges seen so far.
  - Remove the edge of minimum weight from the PQ, and **if it does not create a cycle, add that edge** to the MST.
- Dijkstra's algorithm
  - Maintain a PQ of vertices, ordered by distance from the source.
  - Remove the min distance vertex from the PQ. For each outgoing edge, **if it gives a shorter path from source, add that edge** to the SPT.
- Why do we have to check for a cycle in Lazy Prim's before adding an edge, but not Dijkstra's?

Dijkstra's algorithm

edge	weight
0 → 2	3.0
0 → 3	1.0
1 → 3	1.0
1 → 6	5.0
2 → 1	2.0
3 → 1	17.0
3 → 2	13.0
3 → 5	3.0
3 → 7	8.0
4 → 7	1.0
5 → 1	4.0
6 → 4	-11.0 (Negative!!)
7 → 4	2.0

v	distTo[]	edgeTo[]
0	∞	*null*
1	7.0	5
2	13.0	3
3	0.0	*null*
4	10.0	7
5	3.0	3
6	12.0	1
7	8.0	3

The table above gives the SPT immediately after vertex 4 has been relaxed.

- Give the order in which the first 5 vertices were relaxed.
- Which vertex will be relaxed next and how will the SPT table change?
- Suppose we create a new graph that is the same except that the weight for the edge 3→7 is different. **Give a new weight for 3→7 such that Dijkstra's will work correctly.**

Design problem: Longest paths on DAGs

Longest paths on a graph:

- Design an algorithm for finding the longest path in a DAG.
- Followup: Why is this problem so hard for general graphs?

Design problem: Cycle detection as Max-flow

Ford-Fulkerson

- Does Ford-Fulkerson work if our FlowNetwork graph contains cycles?

Specific cycle-detection

- How do we detect a cycle in an undirected graph?
- Suppose we want to find a cycle that involves two specific vertices s and t. Design a linear time algorithm to find ANY cycle involving s and t.

Heteronormative dystopian mate assignment problem

Bottleneck assignment problem (tough!)

- Suppose we have N men and N women.
- Each person has M binary attributes (tall vs. short, hirsute vs. hairless, etc.)
- Each person specifies their desire for each attribute.
- Design an algorithm to find a perfect matching such that the most unlucky person is matched with the fewest number of attributes.