

10. Creating Data Types

10. Creating Data Types

- Overview
- Point charges
- Turtle graphics
- Complex numbers

Object-oriented programming (OOP)

Object-oriented programming (OOP).

- Create your own data types (sets of values and ops on them).
- Use them in your programs (manipulate *objects*).

An *object* holds a data type value.
Variable names refer to objects.

data type	set of values	examples of operations
Color	three 8-bit integers	get red component, brighten
Picture	2D array of colors	get/set color of pixel (i, j)
String	sequence of characters	length, substring, compare

An **abstract data type** is a data type whose representation is *not specified*.

Impact: We can use ADTs without knowing implementation details.

- Previous lecture: how to write client programs for several useful ADTs
- This lecture: how to implement your own ADTs

Implementing a data type

To **create** a data type, you need provide code that

- Defines the set of values (**instance variables**).
- Implements operations on those values (**methods**).
- Creates and initialize new objects (**constructors**).

In Java, a data-type implementation is known as a **class**.

A Java class



Instance variables

- Declarations associate variable names with types.
- Set of type values is "set of values".

Methods

- Like static methods.
- Can refer to instance variables.

Constructors

- Methods with the same name as the type.
- No return type declaration.
- Invoked by new, returns object of the type.

Anatomy of a Class

```

text file named Charge.java
public class Charge
{
    private double rx, ry; // position
    private double q; // charge

    public Charge(double x0, double y0, double q0)
    {
        rx = x0;
        ry = y0;
        q = q0;
    }

    public double potentialAt(double x, double y)
    {
        double k = 8.99e09;
        double dx = x - rx;
        double dy = y - ry;
        return k * q / Math.sqrt(dx*dx + dy*dy);
    }

    public String toString()
    {
        return q + " at (" + rx + ", " + ry + ")";
    }

    public static void main(String[] args)
    {
        Charge c = new Charge(.72, .31, 20.1);
        StdOut.println(c);
        StdOut.printf("%6.2e\n", c.potentialAt(.42, .71));
    }
}

```

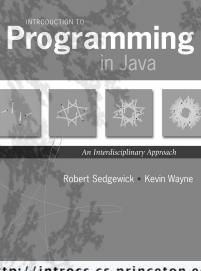
Annotations:

- instance variables**: `rx, ry`, `q`
- constructor**: `Charge(double x0, double y0, double q0)`
- methods**: `potentialAt(double x, double y)`, `toString()`
- test client**: `main(String[] args)`
- static method (familiar)**: `toString()` (labeled "not static")

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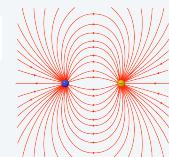
10. Creating Data Types

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ADT for point charges

A **point charge** is an idealized model of a particle that has an electric charge.

An **ADT** allows us to write Java programs that manipulate point charges.



Values	examples		
	position (x, y)	(.53, .63)	(.13, .94)
electrical charge	21.3	81.9	

API (operations)	public class Charge	
	Charge(double x0, double y0, double q0)	
	double potentialAt(double x, double y)	electric potential at (x, y) due to charge
	String toString()	string representation of this charge

Point charge implementation: Methods

Methods define data-type operations (implement APIs).

```
API
public class Charge
{
    Charge(double x0, double y0, double q0)
    double potentialAt(double x, double y) electric potential at (x, y) due to charge
    String toString() string representation of this charge

public class Charge
{
    ...
    public double potentialAt(double x, double y)
    {
        double k = 8.99e09; Vc(x,y) = k  $\frac{q}{r}$ 
        double dx = x - rx;
        double dy = y - ry;
        return k * q / Math.sqrt(dx*dx + dy*dy);
    }

    public String toString()
    { return q + " at " + "(" + rx + ", " + ry + ")"; }
    ...
}
```



Key to OOP. An instance variable reference in a class method refers to the value for the object that was used to invoke the method.

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Point charge implementation

```
text file named Charge.java
public class Charge
{
    private double rx, ry; // position
    private double q; // charge

    public Charge(double x0, double y0, double q0)
    {
        rx = x0;
        ry = y0;
        q = q0;
    }

    public double potentialAt(double x, double y)
    {
        double k = 8.99e09;
        double dx = x - rx;
        double dy = y - ry;
        return k * q / Math.sqrt(dx*dx + dy*dy);
    }

    public String toString()
    { return q + " at " + "(" + rx + ", " + ry + ")"; }

    public static void main(String[] args)
    {
        Charge c = new Charge(.72, .31, 20.1);
        System.out.println(c);
        StdOut.printf("%.2e\\n", c.potentialAt(.42, .71));
    }
}

% java Charge
21.3 at (0.72, 0.31)
3.61e+11
```

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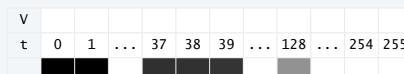
Point charge client: Potential visualization (helper methods)

Read point charges from StdIn.
 • Uses Charge like any other type.
 • Returns an array of Charges

```
public static Charge[] readCharges()
{
    int N = StdIn.readInt();
    Charge[] a = new Charge[N];
    for (int i = 0; i < N; k++)
    {
        double x0 = StdIn.readDouble();
        double y0 = StdIn.readDouble();
        double q0 = StdIn.readDouble();
        a[i] = new Charge(x0, y0, q0);
    }
    return a;
}
```

Convert potential values to a color.
 • Convert V to an 8-bit integer.
 • Use grayscale.

```
public static Color toColor(double V)
{
    V = 128 + V / 2.0e10;
    int t = 0;
    if (V > 255) t = 255;
    else if (V >= 0) t = (int) V;
    return new Color(t, t, t);
}
```

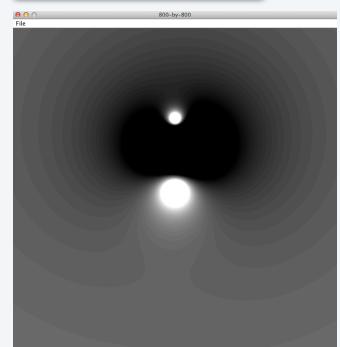


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Point charge client: Potential visualization

```
import java.awt.Color;
public class Potential
{
    public static Charge[] readCharges()
    { // See previous slide. }
    public static Color toColor()
    { // See previous slide. }
    public static void main(String[] args)
    {
        Charge[] a = readCharges();
        int SIZE = 800;
        Picture pic = new Picture(SIZE, SIZE);
        for (int col = 0; col < SIZE; col++)
            for (int row = 0; row < SIZE; row++)
            {
                double V = 0.0;
                for (int k = 0; k < a.length; k++)
                {
                    double x = 1.0 * col / SIZE;
                    double y = 1.0 * row / SIZE;
                    V += a[k].potentialAt(x, y);
                }
                pic.set(col, SIZE-1-row, toColor(V));
            }
        pic.show();
    }
}
```

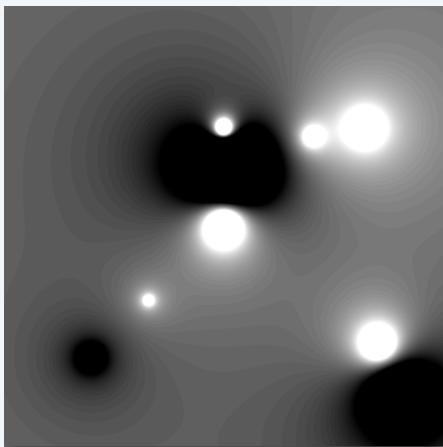
```
% more charges3.txt
3
.51 .63 -100
.50 .50 40
.50 .72 20
% java Potential < charges3.txt
```



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Potential visualization I

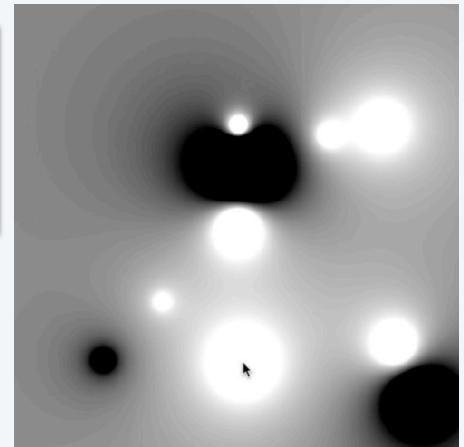
```
% more charges9.txt  
9  
.51 .63 -100  
.50 .50 40  
.50 .72 20  
.33 .33 5  
.20 .20 -10  
.70 .70 10  
.82 .72 20  
.85 .23 30  
.90 .12 -50  
  
% java Potential < charges9.txt
```



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Potential visualization II: A moving charge

```
% more charges9.txt  
9  
.51 .63 -100  
.50 .50 40  
.50 .72 20  
.33 .33 5  
.20 .20 -10  
.70 .70 10  
.82 .72 20  
.85 .23 30  
.90 .12 -50  
  
% java PotentialWithMovingCharge < charges9.txt
```

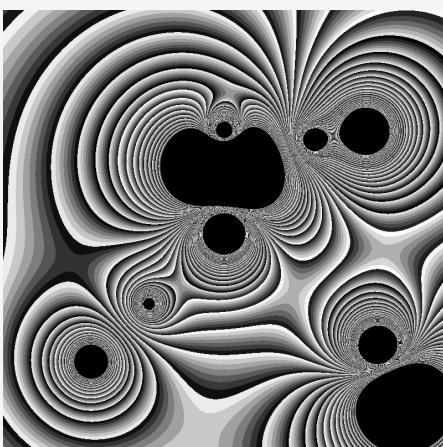


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Potential visualization III: Discontinuous color map

```
public static Color toColor(double V)  
{  
    V = 128 + V / 2.0e10;  
    int t = 0;  
    if (V > 255) t = 255;  
    else if (V >= 0) t = (int) V;  
    t = t*37 % 255  
    return new Color(t, t, t);  
}
```

V	0	1	2	3	4	5	6	7	8	9	...



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Potential visualization IV: Arbitrary discontinuous color map (a bug!)

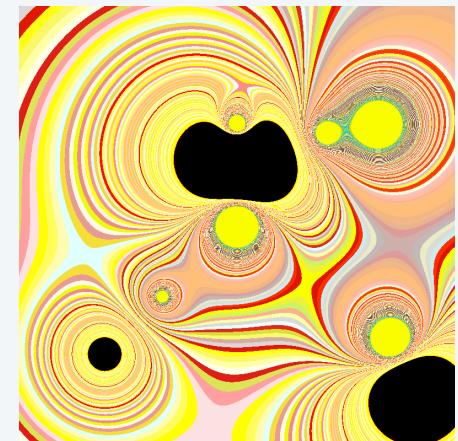
If you are an *artist*

- Choose 255 beautiful colors.
- Put them in an array.
- Index with t to pick a color.

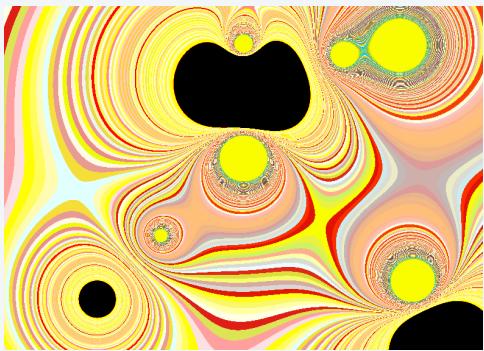
If you are an *computer scientist*

- Play with colors.
- Maybe you'll hit on something...

```
public static Color toColor(double V)  
{  
    V = 128 + V / 2.0e10;  
    int t = 0;  
    if (V > 255) t = 255;  
    else if (V >= 0) t = (int) V;  
    return Color.getHSBColor(t, t, t);  
    return new Color(t, t, t);  
}
```



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TEQ 1 on OOP

Q. Fix the serious bug in this code:

```
public class Charge
{
    private double rx, ry;
    private double q;
    public Charge (double x0, double y0, double q0)
    {
        double rx = x0;
        double ry = y0;
        double q = q0;
    }
}
```

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TEQ 1 on OOP

Q. Fix the serious bug in this code:

```
public class Charge
{
    private double rx, ry;
    private double q;
    public Charge (double x0, double y0, double q0)
    {
        double rx = x0;
        double ry = y0;
        double q = q0;
    }
}
```

A. Remove type declarations.
They create local variables,
giving *no way* for the method
to access the instance variables!

Object-oriented programmers pledge. "I will not shadow instance variables"

Every programmer makes this mistake,
and it is a difficult one to detect.

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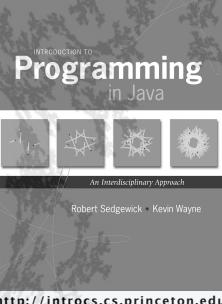


COMPUTER SCIENCE
SEDGEWICK / WAYNE

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- Overview
- Point charges
- Turtle graphics
- Complex numbers

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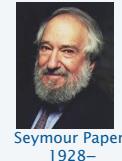
10. Creating Data Types

- Overview
- Point charges
- **Turtle graphics**
- Complex numbers

ADT for turtle graphics

A **turtle** is an idealized model of a plotting device.

An **ADT** allows us to write Java programs that manipulate turtles.



Seymour Papert
1928–

Values	position (x, y)	(.5, .5)	(.75, .75)	(.22, .12)
	orientation	90°	135°	10°



API (operations)	public class Turtle		
	Turtle(double x0, double y0, double q0)	rotate delta degrees counterclockwise	
	void turnLeft(double delta)	move distance step, drawing a line	

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Turtle graphics implementation: Test client

Best practice. Begin by implementing a simple test client.

```
public static void main(String[] args)
{
    Turtle turtle = new Turtle(0.0, 0.0, 0.0);
    turtle.goForward(1.0);
    turtle.turnLeft(120.0);
    turtle.goForward(1.0);
    turtle.turnLeft(120.0);
    turtle.goForward(1.0);
    turtle.turnLeft(120.0);
}
```



% java Turtle

Note: Client drew triangle without computing $\sqrt{3}$



What we expect, once the implementation is done.

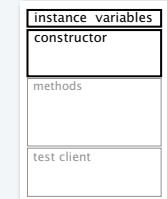
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Turtle implementation: Instance variables and constructor

Instance variables define data-type values.

Constructors create and initialize new objects.

```
public class Turtle
{
    private double x, y;           ← instance variables
    private double angle;          ← are not final
    public Turtle(double x0, double y0, double a0)
    {
        x = x0;
        y = y0;
        angle = a0;
    }
    ...
}
```



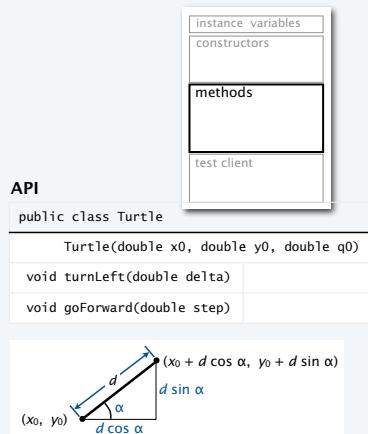
Values	position (x, y)	(.5, .5)	(.75, .75)	(.22, .12)
	orientation	90°	135°	10°

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Turtle implementation: Methods

Methods define data-type operations (implement APIs).

```
public class Turtle
{
    ...
    public void turnLeft(double delta)
    { angle += delta; }
    public void goForward(double d)
    {
        double oldx = x;
        double oldy = y;
        x += d * Math.cos(Math.toRadians(angle));
        y += d * Math.sin(Math.toRadians(angle));
        StdDraw.line(oldx, oldy, x, y);
    }
    ...
}
```



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Turtle implementation

```
text file named Turtle.java
public class Turtle
{
    private double x, y;
    private double angle;

    public Turtle(double x0, double y0, double a0)
    {
        x = x0;
        y = y0;
        angle = a0;
    }

    public void turnLeft(double delta)
    { angle += delta; }
    public void goForward(double d)
    {
        double oldx = x;
        double oldy = y;
        x += d * Math.cos(Math.toRadians(angle));
        y += d * Math.sin(Math.toRadians(angle));
        StdDraw.line(oldx, oldy, x, y);
    }

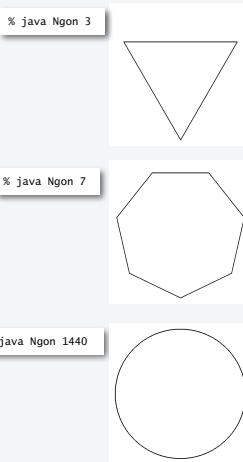
    public static void main(String[] args)
    {
        Turtle turtle = new Turtle(0.0, 0.0, 0.0);
        turtle.goForward(1.0); turtle.turnLeft(120.0);
        turtle.goForward(1.0); turtle.turnLeft(120.0);
        turtle.goForward(1.0); turtle.turnLeft(120.0);
    }
}
```

The diagram illustrates the implementation of the `Turtle` class. It shows the class structure with instance variables, a constructor, and methods. A test client is shown running the `main` method, which creates a `Turtle` object and performs three iterations of forward and left turns. A triangle is drawn on the right.

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Turtle client: N-gon

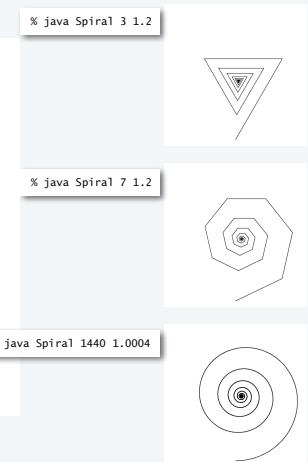
```
public class Ngon
{
    public static void main(String[] args)
    {
        int N      = Integer.parseInt(args[0]);
        double angle = 360.0 / N;
        double step  = Math.sin(Math.toRadians(angle/2.0));
        Turtle turtle = new Turtle(0.5, 0, angle/2.0);
        for (int i = 0; i < N; i++)
        {
            turtle.goForward(step);
            turtle.turnLeft(angle);
        }
    }
}
```



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Turtle client: Spira Mirabilis

```
public class Spiral
{
    public static void main(String[] args)
    {
        int N      = Integer.parseInt(args[0]);
        double decay = Integer.parseInt(args[1]);
        double angle = 360.0 / N;
        double step  = Math.sin(Math.toRadians(angle/2.0));
        Turtle turtle = new Turtle(0.5, 0, angle/2.0);
        for (int i = 0; i < (10 * N); i++)
        {
            step /= decay;
            turtle.goForward(step);
            turtle.turnLeft(angle);
        }
    }
}
```



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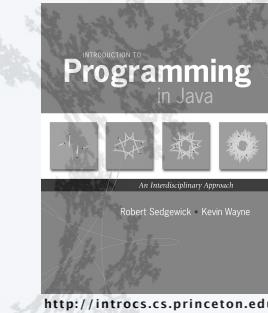
Spira Mirabilis in the wild



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10. Creating Data Types

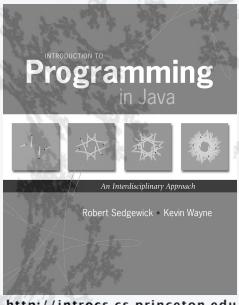
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- Complex numbers



<http://introcs.cs.princeton.edu>

10. Creating Data Types

- Overview
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- **Complex numbers**



Crash course in complex numbers

A **complex number** is a number of the form $a + bi$ where a and b are real and $i \equiv \sqrt{-1}$.

Complex numbers are a *quintessential mathematical abstraction* that have been used for centuries to give insight into real-world problems not easily addressed otherwise.

To perform **algebraic operations** on complex numbers, use real algebra, replace i^2 by -1 and collect terms.

- Addition example: $(3 + 4i) + (-2 + 3i) = 1 + 7i$.
- Multiplication example: $(3 + 4i) + (-2 + 3i) = -18 + i$.



Leonhard Euler
1707–1783 A. L. Cauchy
1789–1857

Example: $|3 + 4i| = 5$

The **magnitude** or **absolute value** of a complex number $a + bi$ is $|a + bi| = \sqrt{a^2 + b^2}$.

Applications: Signal processing, control theory, quantum mechanics, analysis of algorithms...

ADT for complex numbers

A **complex number** is a number of the form $a + bi$ where a and b are real and $i \equiv \sqrt{-1}$.

An **ADT** allows us to write Java programs that manipulate complex numbers.

Values	complex number	$3 + 4i$	$-2 + 2i$
	real part	3.0	-2.0
	imaginary part	4.0	2.0
<pre>public class Complex</pre>			
API (operations)	Complex(double real, double imag)		
	Complex plus(Complex b)	<i>sum of this number and b</i>	
	Complex times(Complex b)	<i>product of this number and b</i>	
	double abs()	<i>magnitude</i>	
	String toString()	<i>string representation</i>	

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Complex number data type implementation: Test client

Best practice. Begin by implementing a simple test client.

```
public static void main(String[] args)
{
    Complex a = new Complex( 3.0, 4.0);
    Complex b = new Complex(-2.0, 3.0);
    StdOut.println("a = " + a);
    StdOut.println("b = " + b);
    StdOut.println("a * b = " + a.times(b));
}
```

$$\begin{aligned} a &= v + wi \\ b &= x + yi \\ a \times b &= vx + vyi + wxi + wyi^2 \\ &= vx - wy + (vy + wx)i \end{aligned}$$

```
% java Complex
a = 3.0 + 4.0i
b = -2.0 + 3.0i
a * b = -18.0 + 1.0i
```

What we expect, once the implementation is done.

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Complex number data type implementation: Instance variables and constructor

Instance variables define data-type values.

Constructors create and initialize new objects.

```
public class Complex
{
    private final double re; instance variables
    private final double im; are final
    public Complex(double real, double imag)
    {
        re = real;
        im = imag;
    }
    ...
}
```



Values	complex number	$3 + 4i$	$-2 + 2i$
	real part	3.0	-2.0
	imaginary part	4.0	2.0

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Complex number data type implementation: Methods

Methods define data-type operations (implement APIs).

```
public class Complex
{
    ...
    public Complex plus(Complex b)
    {
        double real = re + b.re;
        double imag = im + b.im;
        return new Complex(real, imag);
    }
    public Complex times(Complex b)
    {
        double real = a.re * b.re - a.im * b.im;
        double imag = a.re * b.im + a.im * b.re;
        return new Complex(real, imag);
    }
    public double abs()
    {
        return Math.sqrt(re*re + im*im);
    }
    public String toString()
    {
        return re + " + " + im + "i";
    }
    ...
}
```

$$\begin{aligned} a &= v + wi \\ b &= x + yi \\ a \times b &= vx + vyi + wxi + wyi^2 \\ &= vx - wy + (vy + wx)i \end{aligned}$$

API

public class Complex	
Complex(double real, double imag)	
Complex plus(Complex b)	<i>sum of this number and b</i>
Complex times(Complex b)	<i>product of this number and b</i>
double abs()	<i>magnitude</i>
String toString()	<i>string representation</i>

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Complex number data type implementation

```

text file named  
Complex.java
public class Complex
{
    private double re;
    private double im;

    public Complex(double real, double imag)
    { re = real; im = imag; }

    public Complex plus(Complex b)
    {
        double real = re + b.re;
        double imag = im + b.im;
        return new Complex(real, imag);
    }

    public Complex times(Complex b)
    {
        double real = a.re * b.re - a.im * b.im;
        double imag = a.re * b.im + a.im * b.re;
        return new Complex(real, imag);
    }

    public double abs()
    {
        return Math.sqrt(re*re + im*im);
    }

    public String toString()
    {
        return re + " + " + im + "i";
    }
}

public static void main(String[] args)
{
    Complex a = new Complex(3.0, 4.0);
    Complex b = new Complex(-2.0, 3.0);
    StdOut.println("a = " + a);
    StdOut.println("b = " + b);
    StdOut.println("a * b = " + a.times(b));
}

```

instance variables

constructor

methods

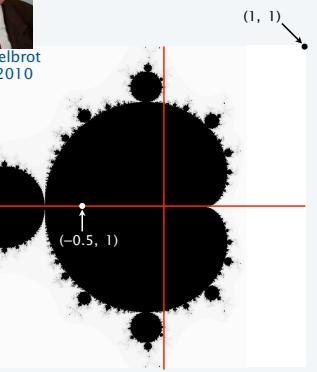
test client

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The Mandelbrot set

The **Mandelbrot set** is a set of complex numbers.

- Represent each complex number $x + yi$ by a point (x, y) in the plane.
- If a point is *in* the set, we color it BLACK.
- If a point is *not* in the set, we color it WHITE.



Examples

- In* the set: $-0.5 + 0i$.
- Not* in the set: $1 + i$.

Challenge

- No simple formula exists for testing whether a number is in the set.
- Instead, the set is defined by an *algorithm*.

Determining whether a point is in the Mandelbrot set

Is a complex number z_0 in the set?

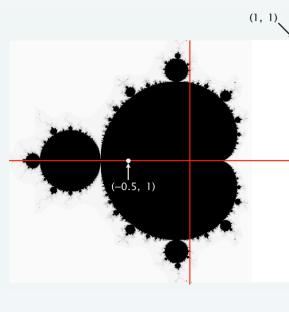
- Iterate $z_{t+1} = (z_t)^2 + z_0$.
- If $|z_t|$ diverges to infinity, z_0 is *in* the set.
- If not, z_0 is *not* in the set.

t	z_t
0	$-1/2 + 0i$
1	$-1/4 + 0i$
2	$-7/16 + 0i$
3	$-79/256 + 0i$
4	$-26527/65536 + 0i$

↑ converges to 0
 $z = -1/2 + 0i$ is *in* the set

t	z_t
0	$1 + i$
1	$1 + 3i$
2	$-7 + 7i$
3	$1 - 97i$
4	$-9407 - 193i$

↑ diverges to infinity
 $z = 1 + i$ is *not* in the set

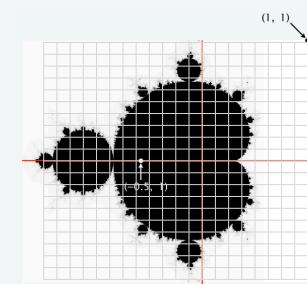


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Plotting the Mandelbrot set

Practical issues

- Cannot plot infinitely many points.
- Cannot iterate infinitely many times.



Approximate solution for first issue

- Sample from an N -by- N grid of points in the plane.
- Zoom in to see more detail (stay tuned!).

Approximate solution for second issue

- Fact: if $|z_t| > 2$ for any t , then z is *not* in the set.
- Pseudo-fact: if $|z_{55}| \leq 2$ then z is "likely" in the set.

Important note: Solutions imply significant computation.

Complex number client: Mandelbrot set visualization (helper method)

Mandlebrot function of a complex number.

- Returns WHITE if the number is not in the set.
- Returns BLACK if the number is (probably) in the set.

```
public static Color mand(Complex z0)
{
    Complex z = z0;
    for (int t = 0; t < 255; t++)
    {
        if (z.abs() > 2.0) return Color.WHITE; ←
        z = z.times(z);
        z = z.plus(z0);
    }
    return Color.BLACK;
}
```

For a more dramatic picture,
return new Color(255-t, 255-t, 255-t)
or colors picked from a color table.

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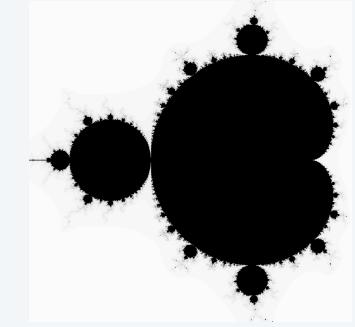
Complex number client: Mandelbrot set visualization

```
import java.awt.Color;
public class Mandelbrot
{
    public static Color mand(Complex z0)
    { // See previous slide. }
    public static void main(String[] args)
    {
        double xc = Double.parseDouble(args[0]);
        double yc = Double.parseDouble(args[1]);
        double size = Double.parseDouble(args[2]);
        int N = 512;
        Picture pic = new Picture(N, N);

        for (int col = 0; col < N; col++)
            for (int row = 0; row < N; row++) ← scale to screen
            { coordinates
                double x0 = xc - size/2 + size*col/N;
                double y0 = yc - size/2 + size*row/N;
                Complex z0 = new Complex(x0, y0);
                Color color = mand(z0);
                pic.set(col, N-1-row, color);
            }
        pic.show();
    }
}
```

(0, 0) is upper
left corner

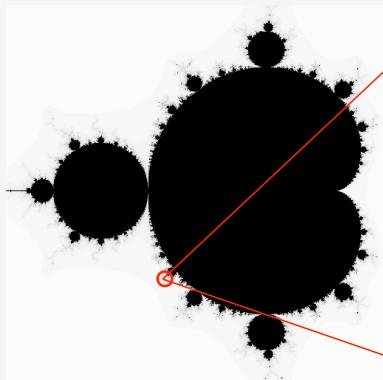
% java Mandelbrot -.5 0 2



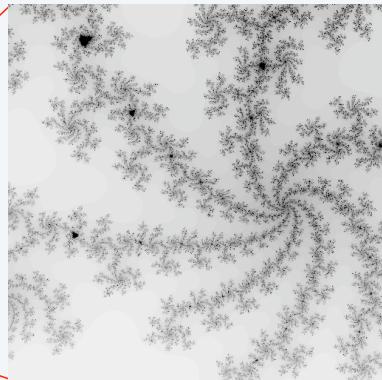
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Mandelbrot Set

% java Mandelbrot -.5 0 2



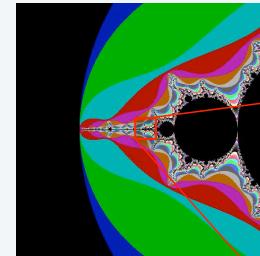
% java Mandelbrot .1045 -.637 .01



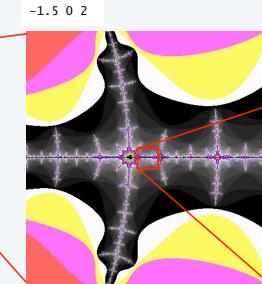
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Mandelbrot Set

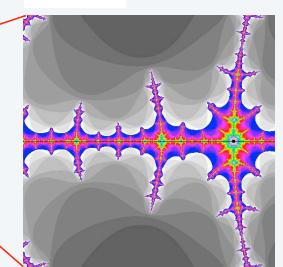
% java ColorMandelbrot -.5 0 2 < mandel.txt



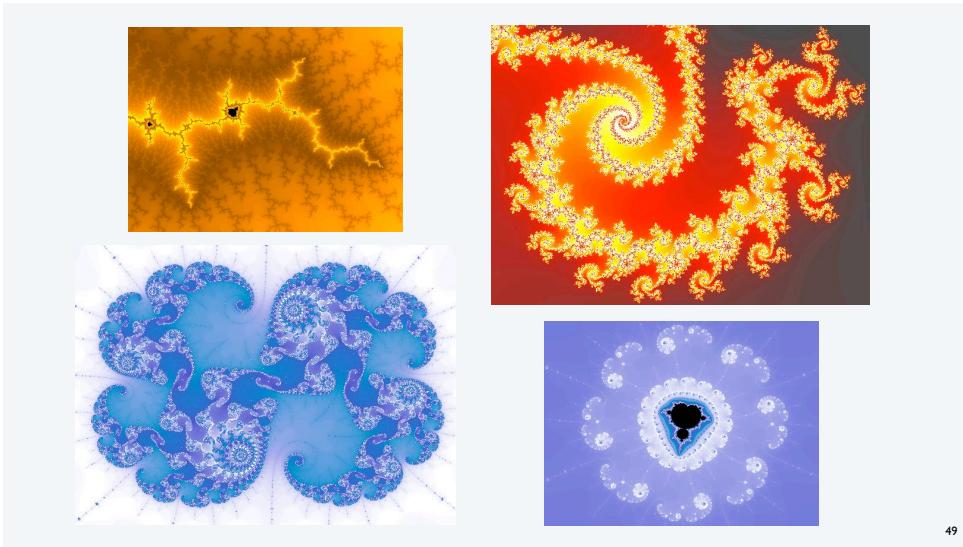
-1.5 0 2



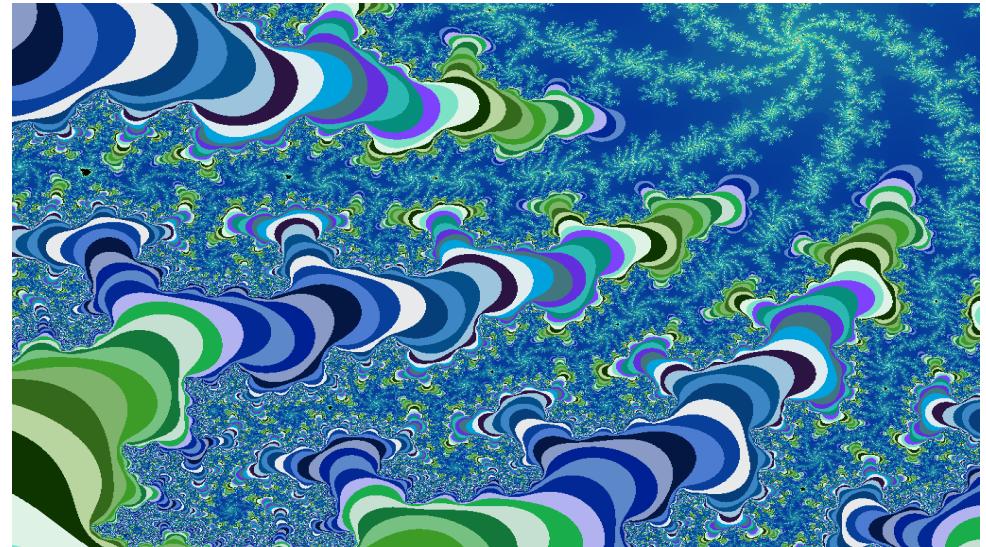
-1.5 0 .002



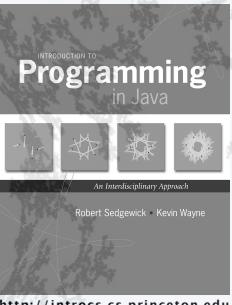
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COMPUTER SCIENCE
SEGEWICK / WAYNE



10. Creating Data Types

- Overview
- Point charges
- Turtle graphics
- Complex numbers

<http://introcs.cs.princeton.edu>

OOP summary

Object-oriented programming (OOP).

- Create your own data types (sets of values and ops on them).
- Use them in your programs (manipulate *objects*).



OOP helps us simulate the physical world

- Java objects model real-world objects.
- Not always easy to make model reflect reality.
- Examples: charged particle, color, sound, genome....



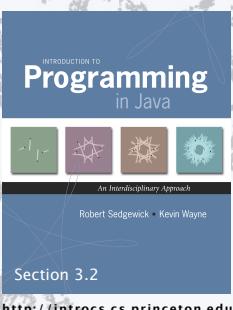
OOP helps us extend the Java language

- Java doesn't have a data type for every possible application.
- Data types enable us to add our own abstractions.
- Examples: complex, vector, polynomial, matrix, picture....



T A G A T G T G C T A G C

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10. Creating Data Types