Global Illumination Overview

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Princeton University
COS 526, Fall 2012

Overview

- Rendering equation
  - Rendering is integration
- Solution methods
  - Direct illumination
  - Recursive ray tracing
  - Distribution ray tracing
  - Path tracing
  - Photon mapping
  - Radiosity
  - etc.

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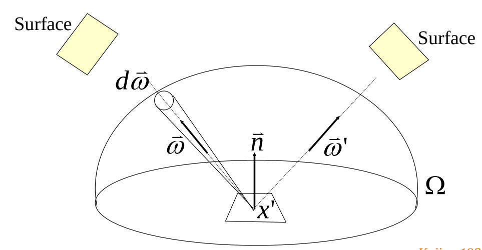
Rendering Equation

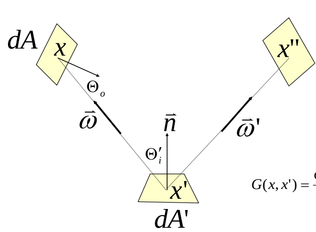
$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}')L_i(x',\vec{\omega})(\vec{\omega}\bullet\vec{n})d\vec{\omega}$$



Kajiya 1986

Rendering Equation (2)

$$L(x'\to x'') = L_e(x'\to x'') + \int_S f_r(x\to x'\to x'')L(x\to x')V(x,x')G(x,x')dA$$



$$G(x,x') = \frac{\cos\Theta_i' \cos\Theta_o}{\|x-x'\|^2}$$

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Rendering Equation

- Rendering = integration
  - Antialiasing
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  - Indirect illumination
  - Caustics

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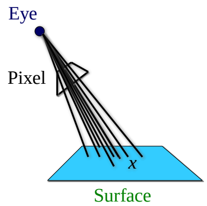

$$L_P = \int_S L(x \rightarrow e)\,dA$$

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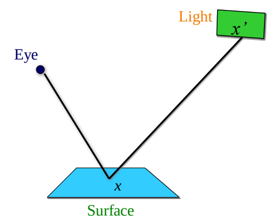

$$L(x,\vec{w}) = L_e(x,x \rightarrow e) + \int_S f_r(x,x' \rightarrow x, x \rightarrow e)\,L(x' \rightarrow x)\,V(x,x')\,G(x,x')\,dA$$

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Herf

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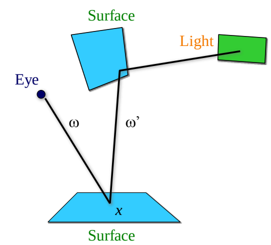

$$L_o(x,\vec{w}) = L_e(x,\vec{w}) + \int_\Omega f_r(x,\vec{w}',\vec{w})\,L_i(x,\vec{w}')\,(\vec{w}' \bullet \vec{n})\,d\vec{w}$$

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Debevec

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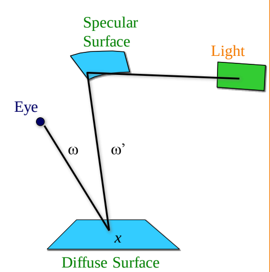

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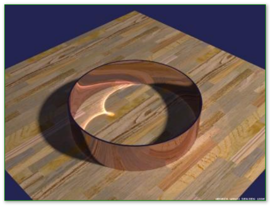
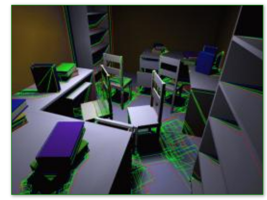


Jensen

$$L_o(x,\vec{w}) = L_e(x,\vec{w}) + \int_{\Omega} f_r(x,\vec{w}',\vec{w}) L_i(x,\vec{w}')(\vec{w}' \bullet \vec{n}) d\vec{w}$$

Challenge

- Rendering integrals are difficult to evaluate
  - Multiple dimensions
  - Discontinuities
    - » Partial occluders
    - » Highlights
    - » Caustics



Drettakis

$$L(x,\vec{w}) = L_e(x,x \to e) + \int_S f_r(x,x' \to x, x \to e) L(x' \to x) V(x,x') G(x,x') dA$$

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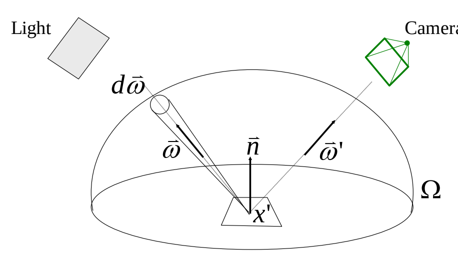
Direct Illumination

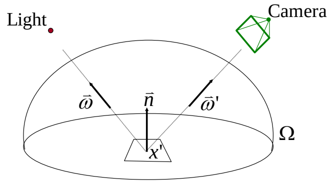
$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega_L} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n}) d\vec{\omega}$$



OpenGL

$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n}) d\vec{\omega}$$

Assume direct illumination from point lights and ignore visibility



$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \sum_{i=1}^{nlights} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n})$$

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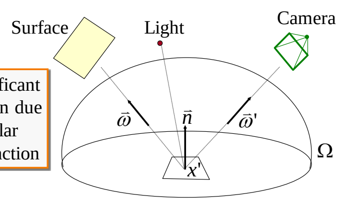
Recursive Ray Tracing

$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n}) d\vec{\omega}$$

Assume only significant indirect illumination due to perfect specular reflection and refraction

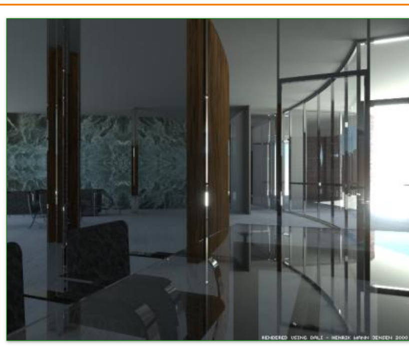


$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \sum_{i=1}^{nlights} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n}) + specular$$

Recursive Ray Tracing?

Paul Debevec

Recursive Ray Tracing?



Jensen

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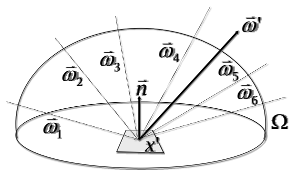
Distribution Ray Tracing

$$L_o(x',\vec{\varpi}') = L_e(x',\vec{\varpi}') + \int_\Omega f_r(x',\vec{\varpi},\vec{\varpi}')L_i(x',\vec{\varpi})(\vec{\varpi} \bullet \vec{n})d\vec{\varpi}$$

Estimate integral for each reflection by random sampling



Also:
- Depth of field
- Motion blur
- etc.

Distribution Ray Tracing?

Henrik Wann Jensen

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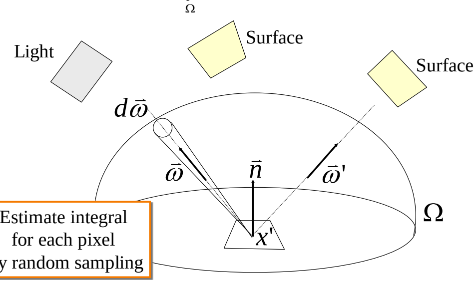
Path Tracing

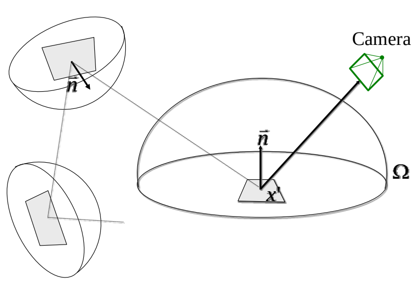
$$L_o(x',\vec{\varpi}') = L_e(x',\vec{\varpi}') + \int_\Omega f_r(x',\vec{\varpi},\vec{\varpi}')L_i(x',\vec{\varpi})(\vec{\varpi} \bullet \vec{n})d\vec{\varpi}$$



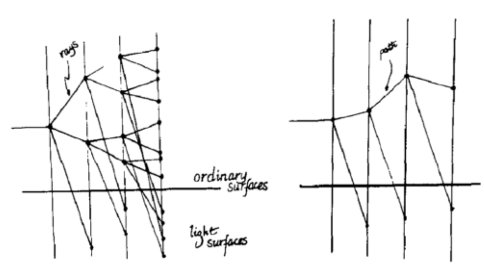
Estimate integral for each pixel by random sampling

Path Tracing

Estimate integral for each pixel by sampling paths from camera



Ray Tracing vs. Path Tracing



Ray tracing

Path tracing

Kajiya

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Photon Mapping

- Two pass method:
  1. Build photon map by tracing paths from lights
  2. Render image by tracing paths from camera

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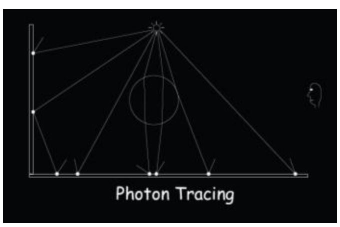

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  1. Build photon map by tracing paths from lights
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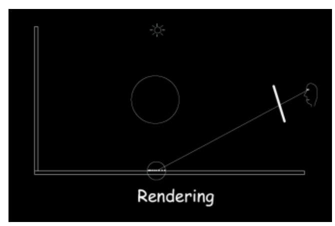
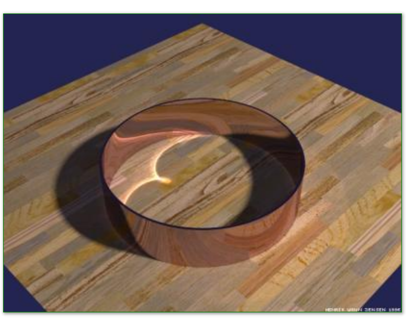


Photon Mapping?



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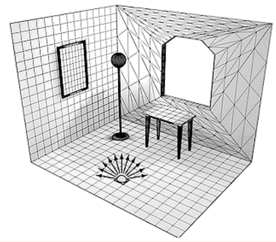
Radiosity

Discretize surfaces into small patches

Baum

Radiosity

Assume simple function (constant) is good approximation for radiosity (sum of energy leaving point in all directions) within patch



Thadani

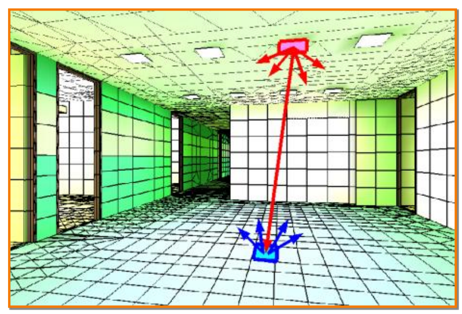
Radiosity

Leads to sparse system of equations

$$B_i A_i = E_i A_i + \rho_i \sum_{j=1}^{N} F_{ji} B_j A_j$$

$$\begin{bmatrix} 1-\rho_1 F_{1,1} & . & . \; . & -\rho_1 F_{1,n} \\ -\rho_2 F_{2,1} & 1-\rho_2 F_{2,2} & . \; . & -\rho_2 F_{2,n} \\ . & . & . \; . & . \\ . & . & . \; . & . \\ -\rho_{n-1} F_{n-1,1} & . & . \; . & -\rho_{n-1} F_{n-1,n} \\ -\rho_n F_{n,1} & . & . \; . & 1-\rho_n F_{n,n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ . \\ . \\ . \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ . \\ . \\ . \\ E_n \end{bmatrix}$$

Radiosity



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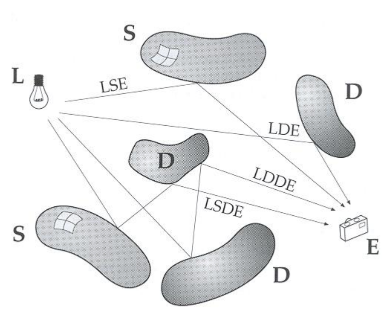
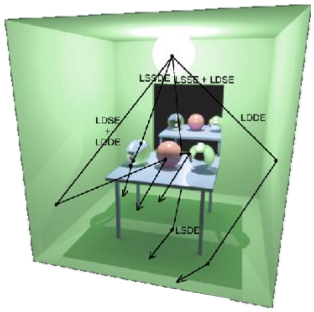
Which method is best?

Path Types



Path Types

- OpenGL
  - LDE
- Ray tracing
  - LDS*E
- Path tracing
  - L(D|S)*E
- Radiosity
  - LD*E



John Hart

Summary

- Rendering is integration
  - Rendering equation
- Different solution methods are best when different path types are important
  - OpenGL - LDE
  - Recursive ray tracing – LDS*E
  - Distribution ray tracing – L(SD)*E
  - Path tracing – L(SD)*E
  - Photon mapping – L(SD)*E (biased)
  - Radiosity - LD*E