Introduction to Shape Analysis

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Motivation
Large repositories of 3D data are becoming available

Lecture Outline
Introduction
Applications
Problems
- Feature detection

Applications
Examples:
- Computer graphics
- Geometric modeling
- Archaeology
- Urban planning
- Paleontology
- Molecular bio
- Medicine
- Art

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Reconstructing Frescoes from Thera
(Weyrich, Brown, Rusinkiewicz, et al.)

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Images courtesy of Delson & Freiss

Images courtesy of Ilya Vakser, GRAMM

Hippocampus-amygdala study in schizophrenia

Image courtesy of Polina Golland, MIT
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Lecture Outline

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Shape Analysis Problems

Examples:
- Feature detection
- Segmentation
- Labeling
- Registration
- Matching
- Recognition
- Classification
- Clustering
- Retrieval

“How can we find significant geometric features robustly?”

Shape Analysis Problems

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“How can we decompose a 3D model into its parts?”

Semantic Labels
(Golovinskiy, Lee, et al.)
### Shape Analysis Problems

**Examples:**
- Feature detection
- Segmentation
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- Registration
  - Matching
  - Retrieval
  - Recognition
- Classification
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**Questions:**
- How can we align features of 3D models?
- How can we compute a measure of geometric similarity?
- How can we find similar 3D shapes in a database?
- How can we find a given 3D model in a large database?
- How can we determine the class of a 3D model?
- How can we learn classes of 3D models automatically?
A Quick Diversion …

Which is harder to analyze?

Images courtesy of Georgia Tech and www.dreamhorse.com

Lecture Outline

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• Feature detection

Point Feature Detection

Goals:
• Invariant to transformations
• Robust to small surface deviations (holes, noise, etc.)
• Common across different surfaces in same class
• Semantic?

Point Features

Applications:
• Maintaining shape features as process mesh
• Matching shape features as align meshes
• Reasoning about part decomposition
• Visualization
• etc.

Features

Definition (Merriam-Webster)’”
• “a prominent characteristic”

Point Features

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• etc.

Scan A  Scan B  Best Alignment
Point Feature Detection
Algorithmic methods to detect feature points?

Point Feature Detection
Some relevant properties
Average geodesic distance
Gauss curvature
Differences of curvature
Shape diameter function
etc.

Point Feature Detection
Multiscale methods
Many methods consider scale-space persistence

Feature Point Study
Ask people on the Amazon Mechanical Turk

Key question
How should we ask people which points are salient?
• "Please select salient points"
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- "Please select salient points"
- Please select a pattern of points from which another person can recognize the object's class by viewing only those points

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Schelling approach
We asked people to:

- Please select points that you think other people will select

Based on the “focal point” theory of [Schelling60]

- A solution that people tend to use in the absence of communication, because it seems natural, special or relevant to them

Schelling Feature Points
Schelling feature points

Schelling Feature Points
Schelling feature point distributions
Relation with geometric properties?

Local properties
- Curvatures
- Mesh Saliency
- HKS at small t

Global properties
- HKS at large t
- SDF [Shapira 08]
- Symmetry
- Segment Center
- AGD
- Etc.

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Relation with geometric properties?

~15% of schelling points are not explained geometrically

Summary

Geometric analysis can yield insights into features and relationships in 3D surface data

Curvature

Curvature $\kappa$ of a curve is reciprocal of radius of circle that best approximates it

Defined at a point $p$ in a direction $w$

Line has $\kappa = 0$
Principal Curvatures

The curvature at a point varies between some minimum and maximum – these are the principal curvatures $\kappa_1$ and $\kappa_2$. They occur in the principal directions $d_1$ and $d_2$ which are perpendicular to each other.

Gaussian and Mean Curvature

Gauss Curvature
$$K = \kappa_1 \kappa_2$$

Mean Curvature
$$H = \frac{1}{2} (\kappa_1 + \kappa_2)$$

What Does Curvature Tell Us?

Planar points:
- Zero Gaussian curvature and zero mean curvature
- Tangent plane intersects surface at infinity points

Parabolic points:
- Zero Gaussian curvature, non-zero mean curvature
- Tangent plane intersects surface along 1 curves

Elliptical points:
- Positive Gaussian curvature
- Convex/concave depending on sign of mean curvature
- Tangent plane intersects surface at 1 point
What Does Curvature Tell Us?

Hyperbolic points:
- Negative Gaussian curvature
- Tangent plane intersects surface along 2 curves

Principal Component Analysis (PCA)

Tensor voting
- Extract points \( \{q_i\} \) in neighborhood
- Compute covariance matrix \( M \)
- Analyze eigenvalues and eigenvectors of \( M \) (via SVD)
- Eigenvectors are "Principal Axes of Inertia"
- Eigenvalues are variances of the point distribution in those directions

Principal Component Analysis (PCA)

Mesh Saliency:
- Motivated by models of perceptual salience
- Difference between mean curvature blurred with \( \sigma \) and blurred with \( 2\sigma \)

Eigenvectors are "Principal Axes of Inertia"
Eigenvalues are variances of the point distribution in those directions
What Does PCA Tell Us?

Provides estimate of normal direction
- Eigenvector (principal axis) associated with smallest eigenvalue

What Does PCA Tell Us?

Helps us construct a local coordinate frame for every point
- Map $\hat{e}_1$ to X axis
- Map $\hat{e}_2$ to Y axis
- Map $\hat{e}_3$ to Z axis

What Does PCA Tell Us?

Helps differentiate nearly plane-like, from stick-like, from sphere-like, etc.

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$\lambda_2 / (\lambda_1 + \lambda_2 + \lambda_3)$