| COS 226 | Algorithms and Data Structures | Spring 2008 |
|---|---|---|
| | **Final** | |

This test has 12 questions worth a total of 100 points. You have 180 minutes. The exam is closed book, except that you are allowed to use a one page cheatsheet (8.5-by-11, in your own handwriting). No calculators or other electronic devices are permitted. Give your answers and show your work in the space provided. **Write out and sign the Honor Code pledge before turning in the test.**

*"I pledge my honor that I have not violated the Honor Code during this examination."*

| Problem | Score |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| Sub 1 | |

| Problem | Score |
|---|---|
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| Sub 2 | |

| Total | |
|---|---|

**Name:**

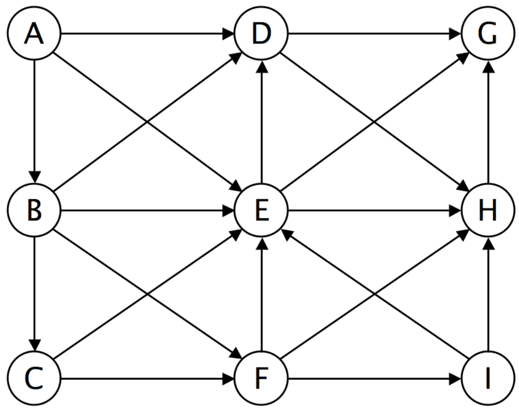
**Login ID:**

**Precept:**

| | | |
|---|---|---|
| P01 | 12:30 | Moses |
| P01A | 12:30 | Szymon |
| P02 | 1:30 | Szymon |
| P02A | 1:30 | Moses |
| P03 | 3:30 | Nadia |

1. **Graph search. (10 points)**

Consider the following directed graph.




(a) Run *depth-first search*, starting at vertex A. Assume the adjacency lists are in lexicographic order, e.g., when exploring vertex E, consider E-D before E-G or E-H. Complete the list of vertices in *preorder* (the order they are first discovered by DFS).

```
A    B    C    ___    ___    ___    ___    ___    ___
```

(b) Run *breadth-first search*, starting at vertex A. Assume the adjacency lists are in lexicographic order. Complete the list of vertices in the order in which they are enqueued.

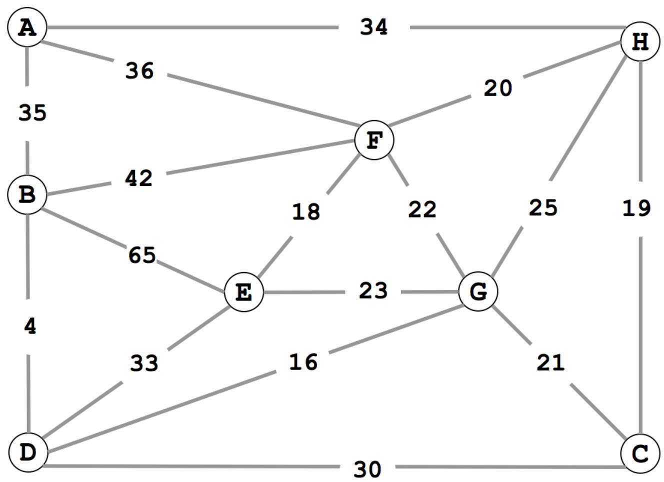
```
A    B    D    E    ___    ___    ___    ___    ___
```

(c) Identify one situation where you would need to use BFS instead of DFS.

(d) Identify one situation where you would need to use DFS instead of BFS.

2. **Minimum spanning tree. (8 points)**

   Consider the following weighted graph.




(a) Complete the list of edges in the MST in the order that *Kruskal's algorithm* includes them. For reference, the edge weights in ascending order are:

```
4   16   18   19   20   21   22   23   25   30   33   34   35   36   42   65
```

```
B-D   ----   ----   ----   ----   ----   ----
```

(b) Complete the list of edges in the MST in the order that *Prim's algorithm* includes them. Start Prim's algorithm from vertex A.

```
A-H   ----   ----   ----   ----   ----   ----
```

3. **Minimum spanning tree. (8 points)**

   Suppose you know the MST of a weighted graph $G$. Now, a new edge $v$-$w$ of weight $c$ is inserted into $G$ to form a weighted graph $G'$. Design an $O(V)$ time algorithm to determine if the MST in $G$ is also an MST in $G'$. You may assume all edge weights are distinct.

   Your answer will be graded for correctness, clarity, and *conciseness*.

   (a) State the algorithm.

   (b) Explain briefly why it takes $O(V)$ time.

4. **Data compression. (4 points)**

   How many bits are in the Huffman encoding of the following message?
   (Do not count the bits to encode the table.)

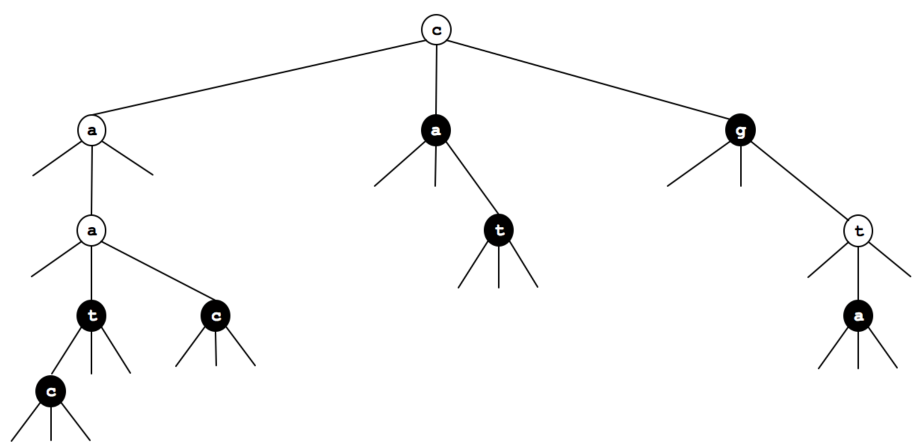
   `a b a a b a c a b a a b a c d a b a a b a c a b a a b a c d e`

   For reference, the frequency of each symbol is given in the table below.

| a | b | c | d | e |
|---|---|---|---|---|
| 16 | 8 | 4 | 2 | 1 |

5. **Ternary search tries. (6 points)**

   Consider the following TST, where the black nodes correspond to strings in the TST.




   (a) Which 7 strings (in alphabetical order) are in the TST?

   (b) Draw the results of adding the following strings into the TST above:

   `cgt     aaca     tt`

6. **String searching. (8 points)**

Complete the following DFA to match precisely those strings (over the two letter alphabet) that contain `bababba` as a substring. State 0 is the start state and state 7 is the accept state.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| a | 0 | 2 | 0 | 4 | 0 | | | 7 |
| b | 1 | 1 | 3 | 1 | 5 | | | 7 |

7. **Algorithm matching. (10 points)**

Match up each application with an algorithm or data structure that we used to solve it in this course. Use each answer exactly once.

___ T9 texting in a cell phone

___ 1D range search

___ 2D range search

___ Document similarity

___ Traveling salesperson problem

___ Sudoku solver

___ Arbitrage detection in currency exchange rates

___ Mark-sweep garbage collector

___ Web crawler

___ Google maps

___ PERT/CPM (Program Evaluation and Review Technique / Critical Path Method).

___ Longest repeated substring

A. Trie

B. Hashing

C. 3-way radix quicksort

D. Binary search tree

E. Kd tree

F. Depth-first search

G. Breadth-first search

H. Dijkstra's algorithm

I. Topological sort

J. Bellman-Ford

K. Enumerate permutations

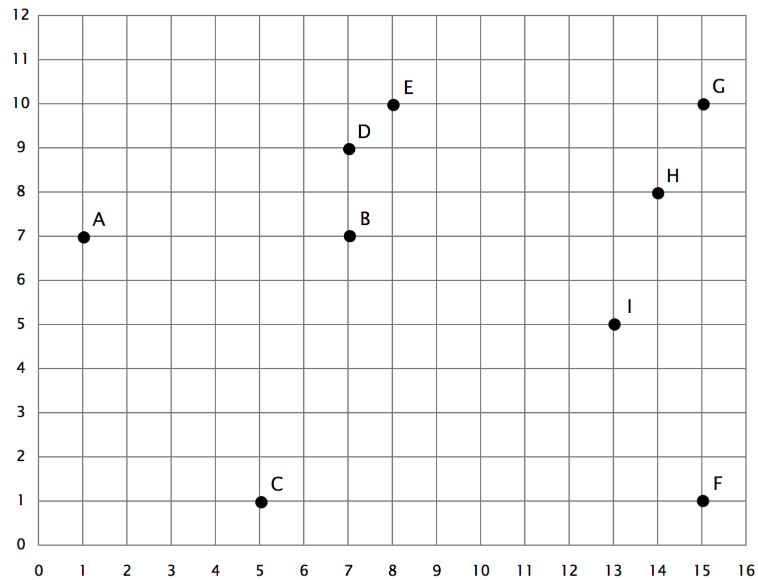
L. Enumerate base-R integers

8. **Regular expressions. (8 points)**

   Draw the NFA that results from the RE-to-NFA conversion algorithm described in lecture when applied to the regular expression `(a | (bc)*) d*`. Label the start state 0, the accept state 1, and remaining states in the order they are created by the RE-to-NFA algorithm.

9. **Convex hull. (8 points)**

Run the Graham scan algorithm to compute the convex hull of the 9 points below, using `F` as the base point, and continuing counterclockwise starting at `G`.




(a) List the points in the order that they are considered for insertion into the convex hull.

```
F   G   H   I   ___   ___   ___   ___   ___
```

(b) Give the points that appear on the trial hull (after each of the remaining iterations).

```
1.   F -> G -> H

2.   F -> G -> H -> I

3.

4.

5.

6.

7.
```

10. **4-sum. (12 points)**

Consider the 4-SUM problem: *Given $N$ integers, do any 4 of them sum up to exactly 0?*

(a) Consider the following brute-force solution (we ignore integer overflow).

```
public static fourSum(int[] a) {
   int N = a.length;
   for (int i = 0; i < N; i++)
      for (int j = i+1; j < N; j++)
         for (int k = j+1; k < N; k++)
            for (int l = k+1; l < N; l++)
               if (a[i] + a[j] + a[k] + a[l] == 0) return true;
   return false;
}
```

What is the order of growth of the worst-case running time? Circle the best answer.

$N$ $\qquad$ $N \log N$ $\qquad$ $N^2$ $\qquad$ $N^3$ $\qquad$ $N^4$ $\qquad$ $2^N$

(b) Design an algorithm for 4-SUM that runs in $O(N^2)$ time and uses $O(N^2)$ memory. Assume that you have access to a hashing-based symbol table that can `put()` and `get()` integer keys in constant time per operation.

11. **Reductions and shortest paths. (10 points)**

Given a digraph $G$, a distinguished vertex $s$, and nonnegative *vertex* weights, the *single-source vertex-weighted shortest path problem* is to to find the shortest path from $s$ to each vertex, where the length of the path is the sum of the weights of the vertices on the path.

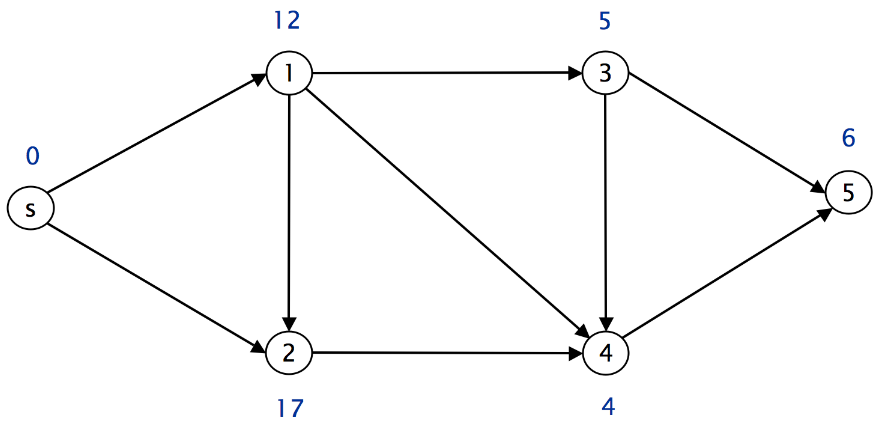



Figure 1: *The shortest path from* $s$ *to* 5 *is* $s - 1 - 4 - 5$ *and has length 22.*

Give a linear-time reduction from the single-source *vertex-weighted* shortest path problem to the classic single-source *edge-weighted* shortest path problem. Demonstrate your reduction by drawing the corresponding digraph $G'$ along with its edge weights.

12. **Suffix sorting. (8 points)**

Your Harvard friend is trying to sort the suffixes of a string `s` consisting of `N` ASCII characters, none of which is `'\0'`. The code calls `RadixQuicksort3way.sort()`, which sorts an array of `'\0'`-terminated strings.

```
// form the N suffixes, appending '\0' to the end of each string
String[] suffixes = new String[N];
for (int i = 0; i < N; i++) {
   suffixes[i] = s.substring(i, N) + "\0";
}

// sort the N strings
RadixQuicksort3way.sort(suffixes);
```

Unfortunately, when your friend uses this code for large `N`, it fails spectacularly, even for non-pathological inputs.

(a) Briefly explain the problem.

(b) Fix it so that it runs efficiently.