4.3 Stacks and Queues

Dr. Dmitry Debugalov
Finds the bug on top
of the Stack.

Data Structures and Data Types

Data types

- Set of values.
- Set of operations on those values.
- Some are built in to Java: `int`, `double`, `char`, `String`, . . .
- Most are not: `Complex`, `Picture`, `Charge`, `Stack`, `Queue`, `Graph`, . . .

↑ this lecture

Data structures.

- Represent data.
- Represent relationships among data.
- Some are built in to Java: arrays, . . .
- Most are not: linked list, circular list, tree, sparse array, graph, . . .

↑ this lecture ↑ TSP (assignment 8) ↑ next lecture

Design challenge for every data type: What data structure to use?

- Requirement 1: Space usage.
- Requirement 2: Time usage for data-type methods

Collections

Fundamental data types.

- Set of operations (add, remove, test if empty) on generic data.
- Intent is clear when we insert.
- Which item do we remove?

LIFO = "last in first out"

Stack. (this lecture)

- Remove the item most recently added.
- Ex: cafeteria trays, Web surfing.

FIFO = "first in first out"

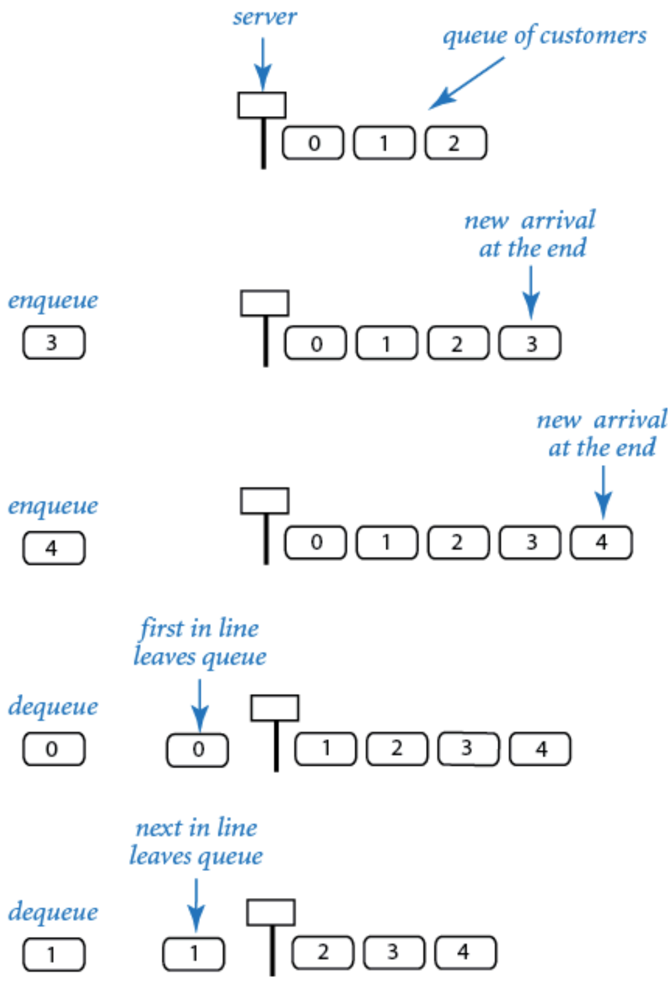
Queue. (see text)

- Remove the item least recently added.
- Ex: Registrar's line.

Symbol Table. (next lecture)

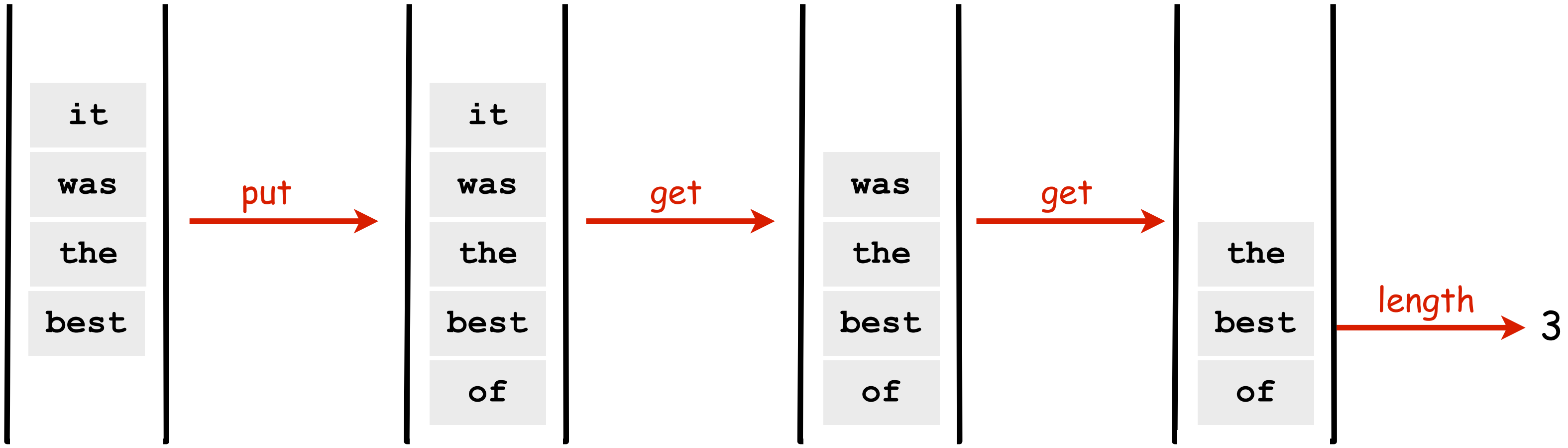
- Remove item with a given key.
- Ex: Phone book

FIFO Queues


server
queue of customers
0
1
2
new arrival
at the end
enqueue
3
0
1
2
3
new arrival
at the end
enqueue
4
0
1
2
3
4
first in line
leaves queue
dequeue
0
0
1
2
3
4
next in line
leaves queue
dequeue
1
1
2
3
4

FIFO Queue API

public class	QueueOfStrings	
	`QueueOfStrings()`	*create an empty queue*
`int`	`length()`	*size of the queue*
`void`	`put(String item)`	*put a string onto the queue*
`String`	`get()`	*get a string from the queue*



Queue Client Code Example: Read from input stream into an array

from previous lecture

```
public class WhiteFilter
{
   public static void main(String[] args)
   {
   In in = new In(args[0]);
   String[] words;
   // Fill words[] with strings from In (stay tuned).
   while (!StdIn.isEmpty())
   {
      String key = StdIn.readString();
      if (search(key, words) != -1)
         StdOut.println(key);
      }
   }
}
```

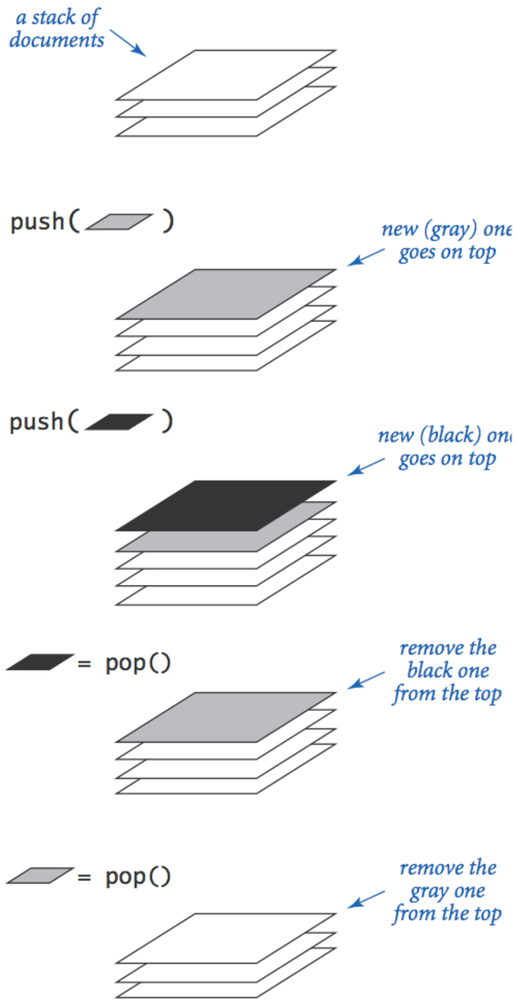
```
QueueOfStrings q = new QueueOfStrings();
while (!in.isEmpty())
   q.put(in.readString());
int N = q.length();
words = new String[N];
for (int i = 0; i < N; i++)
   words[i] = q.get();
```

Solves basic problem

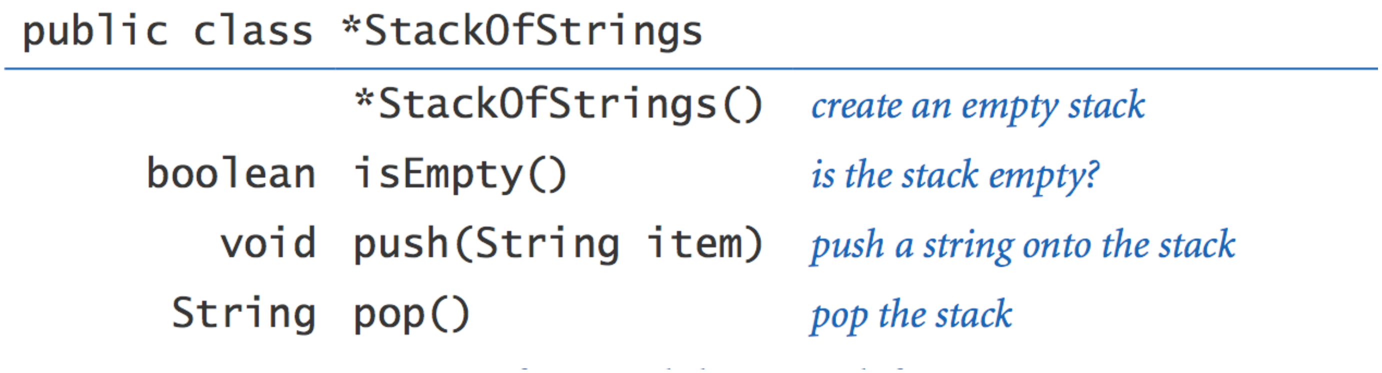
- Can't store strings in array until it is created.
- Can't create array without knowing how many strings in input stream.
- Can't know how many strings in input stream without reading them all.
- Solution: keep them in a Queue

See text for implementation/applications (after learning about Stacks).

Pushdown Stacks

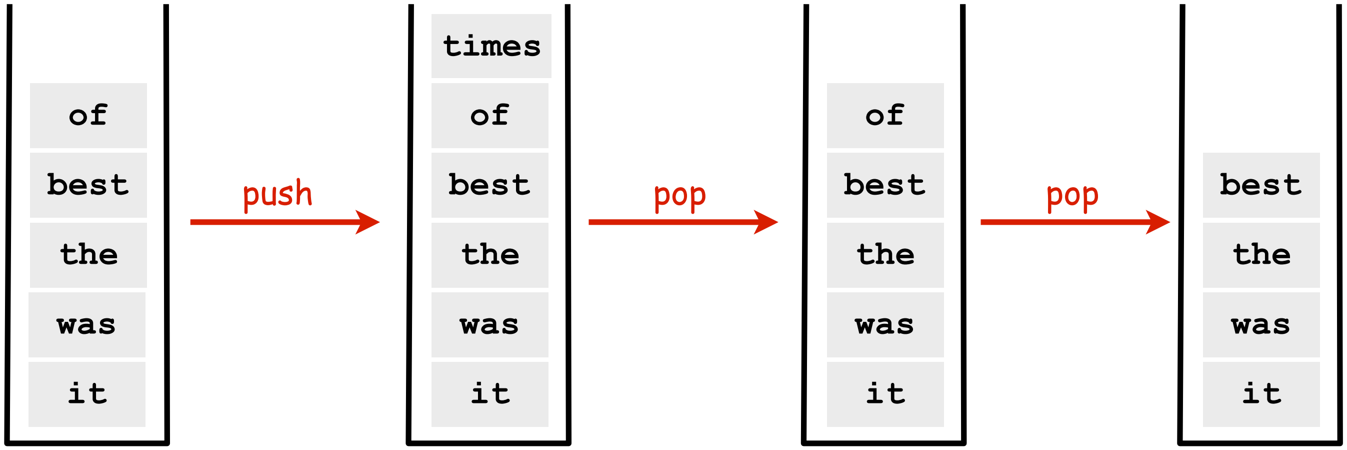

a stack of documents
push( )
new (gray) one goes on top
push( )
new (black) on goes on top
= pop()
remove the black one from the top
= pop()
remove the gray one from the top

Stack API



public class	*StackOfStrings	
	*StackOfStrings()	*create an empty stack*
boolean	isEmpty()	*is the stack empty?*
void	push(String item)	*push a string onto the stack*
String	pop()	*pop the stack*

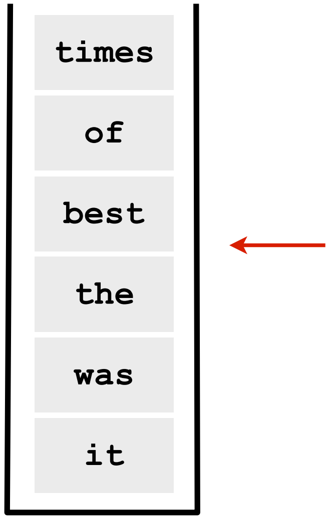
*: we will consider more than one implementation



Stack Client Example 1: Reverse

```
public class Reverse
{
   public static void main(String[] args)
   {
      StackOfStrings stack = new StackOfStrings();
      while (!StdIn.isEmpty())
         stack.push(StdIn.readString());
      while (!stack.isEmpty())
         StdOut.print(stack.pop());
      StdOut.println();
   }
}
```

```
% more tiny.txt
it was the best of times

% java Reverse tiny.txt
times of best the was it
```

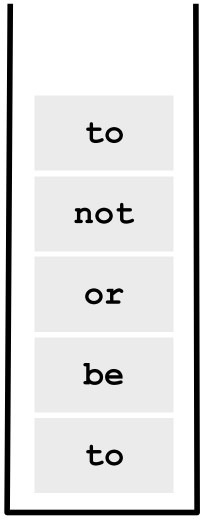


stack contents when StdIn is empty

Stack Client Example 2: Test Client

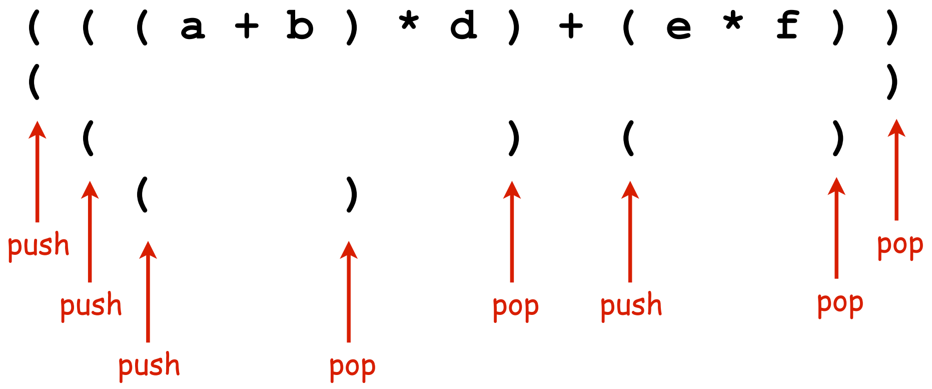
```
public static void main(String[] args)
{
   StackOfStrings stack = new StackOfStrings();
   while (!StdIn.isEmpty())
   {
      String item = StdIn.readString();
      if (item.compareTo("-") != 0)
         stack.push(item);
      else
         System.out.print(stack.pop());
   }
   System.out.println();
}
```

```
% more test.txt
to be or not to - be - - that - - - is

% java StackOfStrings  < test.txt
to be not that or be
```



stack contents just before first pop() operation

Stack Client Example 3: Balanced Parentheses


```
( ( ( a + b ) * d ) + ( e * f ) )
```

Stack Client Example 3: Balanced Parentheses

```
public class Balanced
{
   public static void main(String[] args)
   {
      StackOfStrings stack = new StackOfStrings();
      while (!StdIn.isEmpty())
      {
         String item = StdIn.readString();
         if (item.compareTo("(") == 0)
            stack.push(item);
         if (item.compareTo(")") == 0)
         {
            if (stack.isEmpty())
            {  StdOut.println(“Not balanced”); return; }
            stack.pop();
         }
      }
      if (!stack.isEmpty()) StdOut.println(“Not balanced”);
      else                  StdOut.println(“Balanced”);
   }
}
```

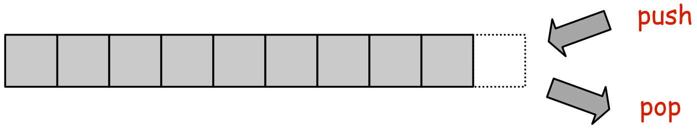
```
% java Balanced
( ( ( a + b ) * d ) + ( e * f ) )
Balanced

% java Balanced
( ( a + b ) * d ) + ( e * f ) )
Not balanced
```

Stack: Array Implementation

Array implementation of a stack.

- Use array `a[]` to store `N` items on stack.
- `push()` add new item at `a[N]`.
- `pop()` remove item from `a[N-1]`.

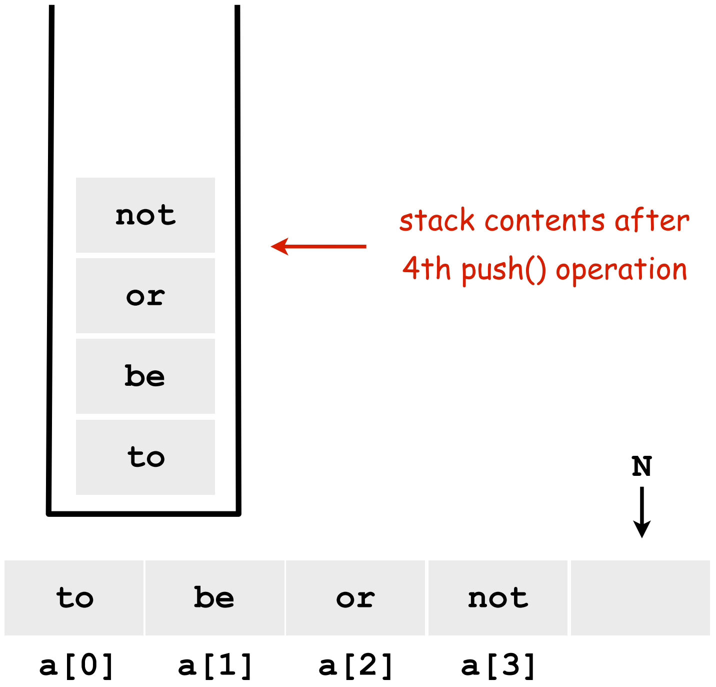
PROBLEM: How big to make array? (Stay tuned.)



```
public class ArrayStackOfStrings
{
   private String[] a;
   private int N = 0;

   public ArrayStackOfStrings(int max)
   {  a = new String[max];  }

   public boolean isEmpty()
   {  return (N == 0);  }

   public void push(String item)
   {  a[N++] = item;  }

   public String pop()
   {  return a[--N];  }
}
```

Strawman solution: Make client provide capacity.

NOTE: This 'solution' violates the API!



stack contents after 4th push() operation

Array Stack: Trace

	StdIn	StdOut	N	a[]				
				0	1	2	3	4
			0					
push	to		1	to				
	be		2	to	be			
	or		3	to	be	or		
	not		4	to	be	or	not	
	to		5	to	be	or	not	to
pop	-	to	4	to	be	or	not	to
	be		5	to	be	or	not	be
	-	be	4	to	be	or	not	be
	-	not	3	to	be	or	not	be
	that		4	to	be	or	that	be
	-	that	3	to	be	or	that	be
	-	or	2	to	be	or	that	be
	-	be	1	to	be	or	that	be
	is		2	to	is	or	not	to

TEQ on Stacks

Q. Can we always insert pop commands (-) to make strings come out sorted?

Ex 1: 6 5 4 3 2 1 - - - - - -

Ex 2: 1 - 2 - 3 - 4 - 5 - 6 -

Ex 3: 4 1 - 3 2 - - - 6 5 - -

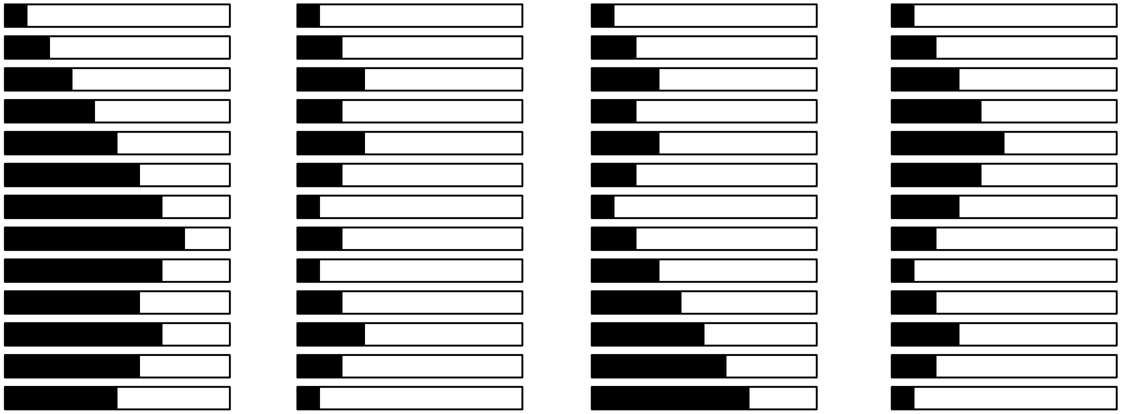
Array Stack: Performance

Running time. Push and pop take constant time. ✓

Memory. Proportional to client-supplied capacity, not number of items. ✗

Problem.

- Original API does not call for capacity (never good to change API)
- Client might have multiple stacks
- Client might not know what capacity to use (depends on its client)



Challenge. Stack implementation where space use is not fixed ahead of time.

Example: potential stack client

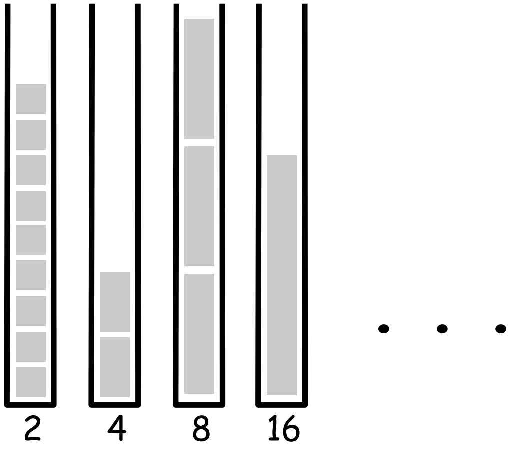
Possible implementation of Java memory management system (sketch)

Maintain N stacks

- stack i: blocks of contiguous 2^i byte chunks of memory
- `new`: pop from stack t, where 2^t is smallest block that will hold new object
- stack t empty? pop from t+1, split in half, push 2 blocks on stack t
- garbage collector: periodically finds unused memory blocks ← How? See COS 226. and pushes onto appropriate stack.

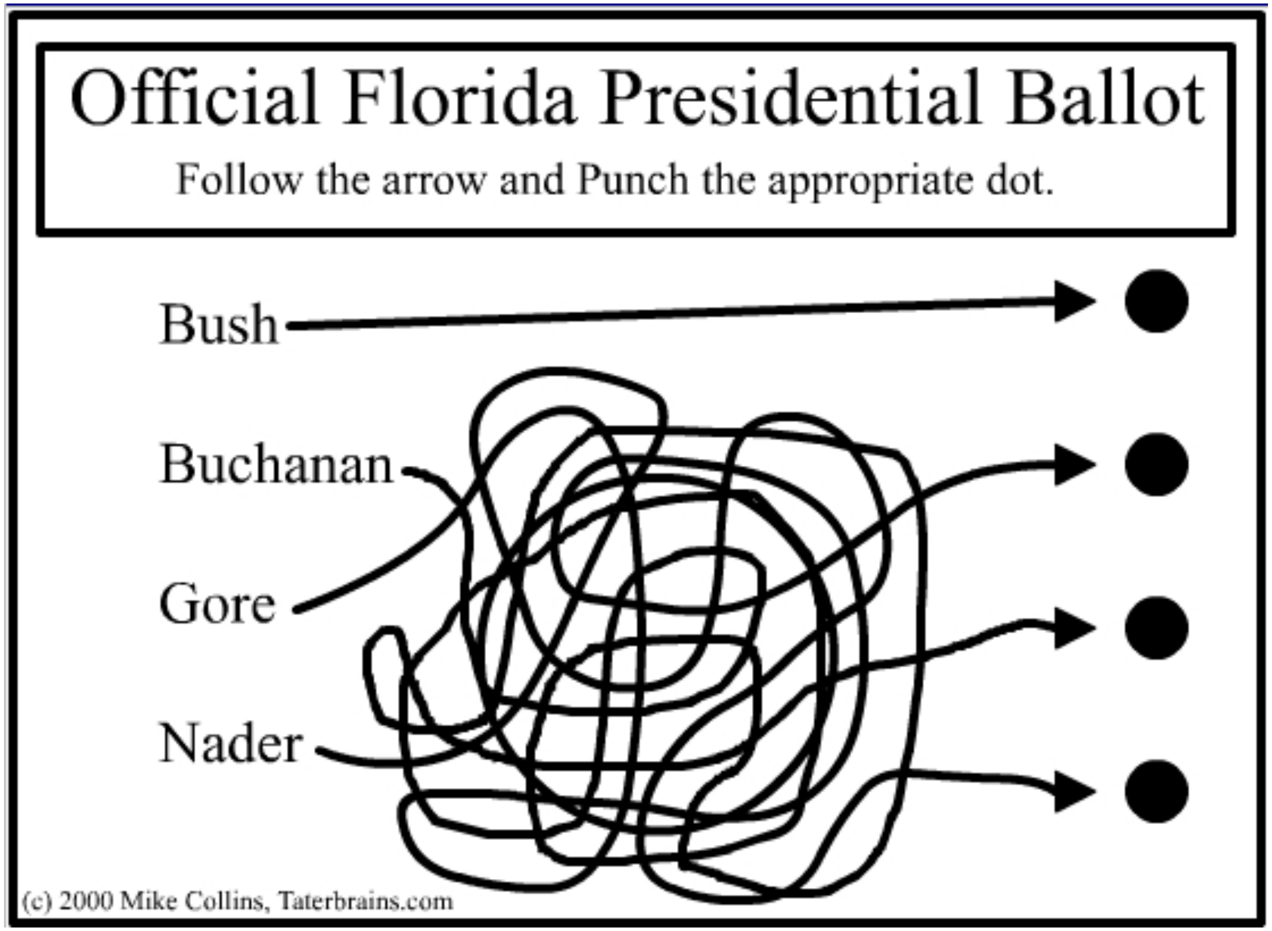
Properties

- many stacks
- stack size unpredictable



Stack implementation without capacity restriction (as in API) is a requirement

Linked Lists


Official Florida Presidential Ballot
Follow the arrow and Punch the appropriate dot.
Bush
Buchanan
Gore
Nader

(c) 2000 Mike Collins, Taterbrains.com

Sequential vs. Linked Data Structures

Sequential data structure. Put object one next to another.

- TOY: consecutive memory cells.
- Java: array of objects.

Linked data structure. Include in each object a link to the another one.

- TOY: link is memory address of next object.
- Java: link is reference to next object.

Key distinctions.

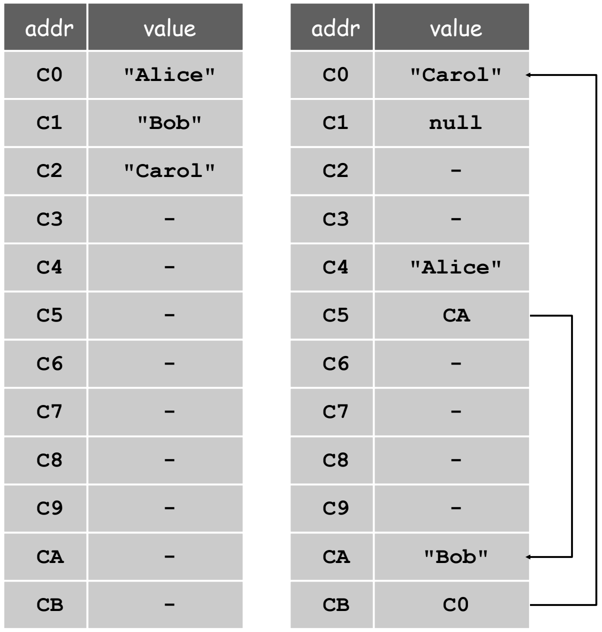
get ith element

- Array: arbitrary access, fixed size.
- Linked list: sequential access, variable size.

get next element

Linked structures.

- Not intuitive, overlooked by naive programmers
- Flexible, widely used method for organizing data



addr	value
C0	"Alice"
C1	"Bob"
C2	"Carol"
C3	-
C4	-
C5	-
C6	-
C7	-
C8	-
C9	-
CA	-
CB	-

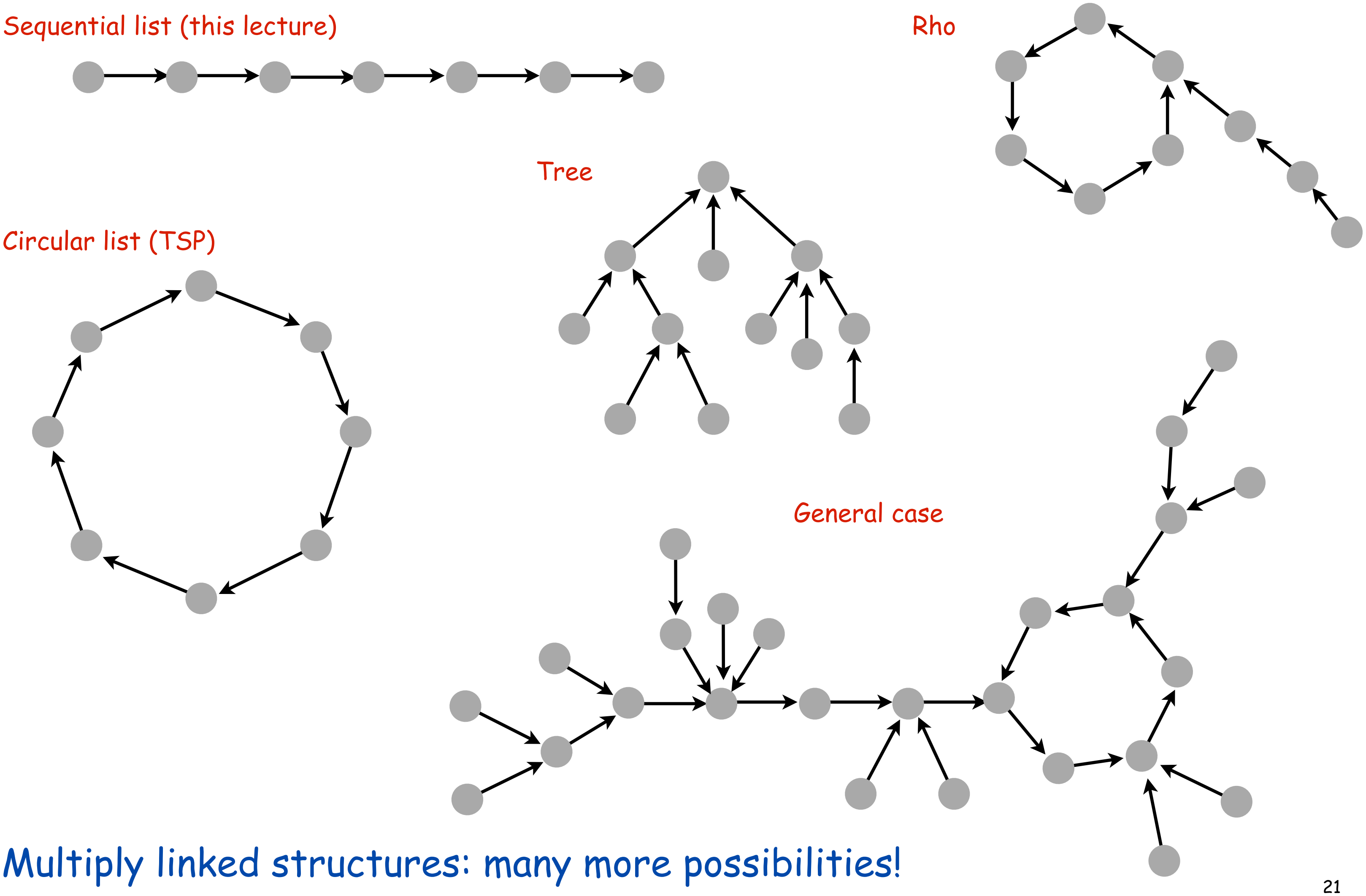
array

addr	value
C0	"Carol"
C1	null
C2	-
C3	-
C4	"Alice"
C5	CA
C6	-
C7	-
C8	-
C9	-
CA	"Bob"
CB	C0

linked list

Singly-linked data structures

From the point of view of a particular object, all of these structures look the same: ●⟶



Sequential list (this lecture)

Rho

Tree

Circular list (TSP)

General case

Multiply linked structures: many more possibilities!

Linked Lists

Linked list.

- Simplest linked structure.
- A recursive data structure.
- An item plus a pointer to another linked list (or empty list).
- Unwind recursion: linked list is a sequence of items.

Node data type.

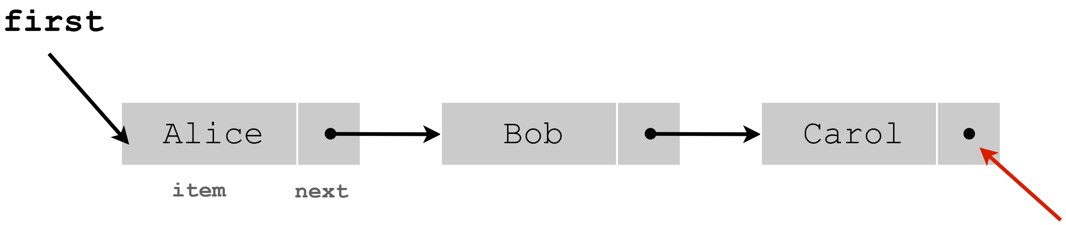
- A reference to a `String`.
- A reference to another `Node`.

```
public class Node
{
   private String item;
   private Node next;
}
```

Confusing point:

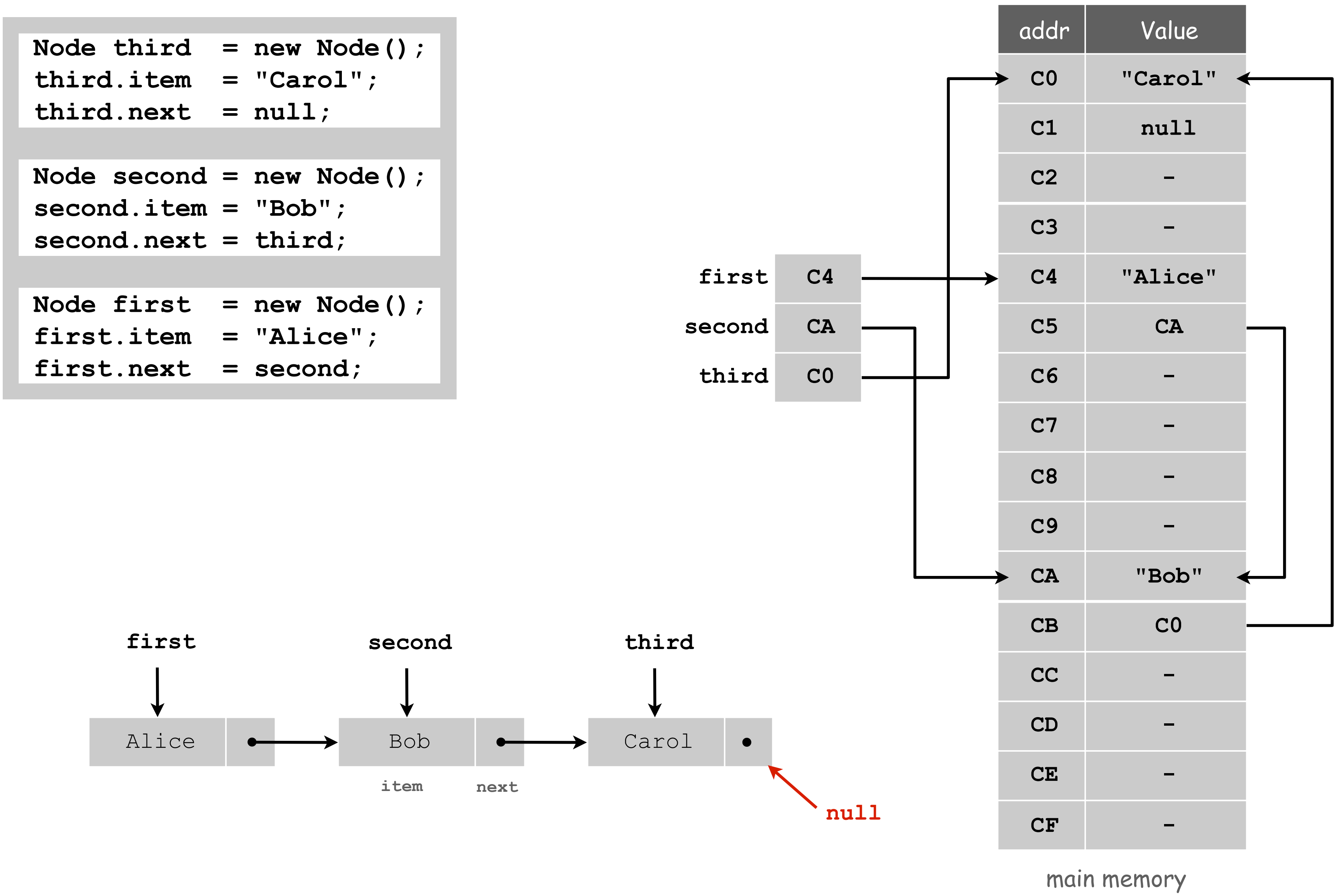
Purpose of data structure is to represent data in a data type
but, we also use data types to implement data structures

Example: The data type Node acts behind the scenes to implement the linked list data structure. It is not visible to the client.



special pointer value `null` terminates list

Building a Linked List



```
Node third  = new Node();
third.item  = "Carol";
third.next  = null;

Node second = new Node();
second.item = "Bob";
second.next = third;

Node first  = new Node();
first.item  = "Alice";
first.next  = second;
```

first	C4
second	CA
third	C0

addr	Value
C0	"Carol"
C1	null
C2	-
C3	-
C4	"Alice"
C5	CA
C6	-
C7	-
C8	-
C9	-
CA	"Bob"
CB	C0
CC	-
CD	-
CE	-
CF	-

main memory

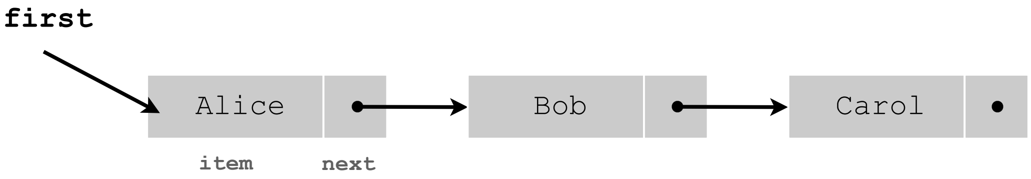
Traversing a List

Iteration. Idiom for traversing a null-terminated linked list.

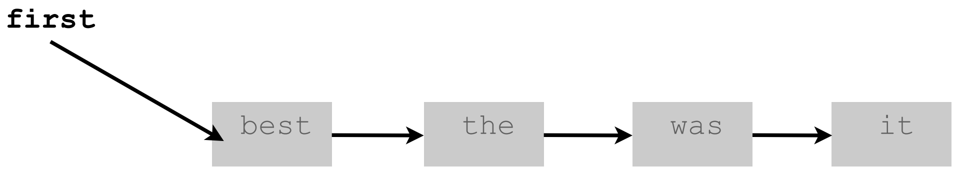
```
Node x = first;
while (x != null)
{
   StdOut.println(x.item);
   x = x.next;
}
```

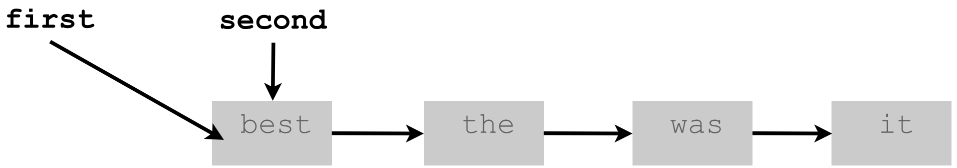
shorthand version

```
for (Node x = first; x != null; x = x.next)
   StdOut.println(x.item);
```

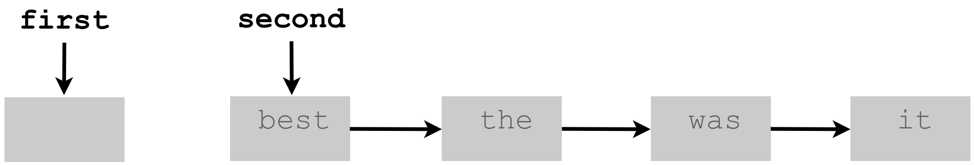


Stack Push: Linked List Implementation

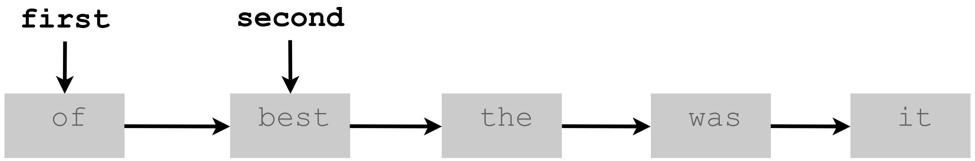




```
second = first;
```

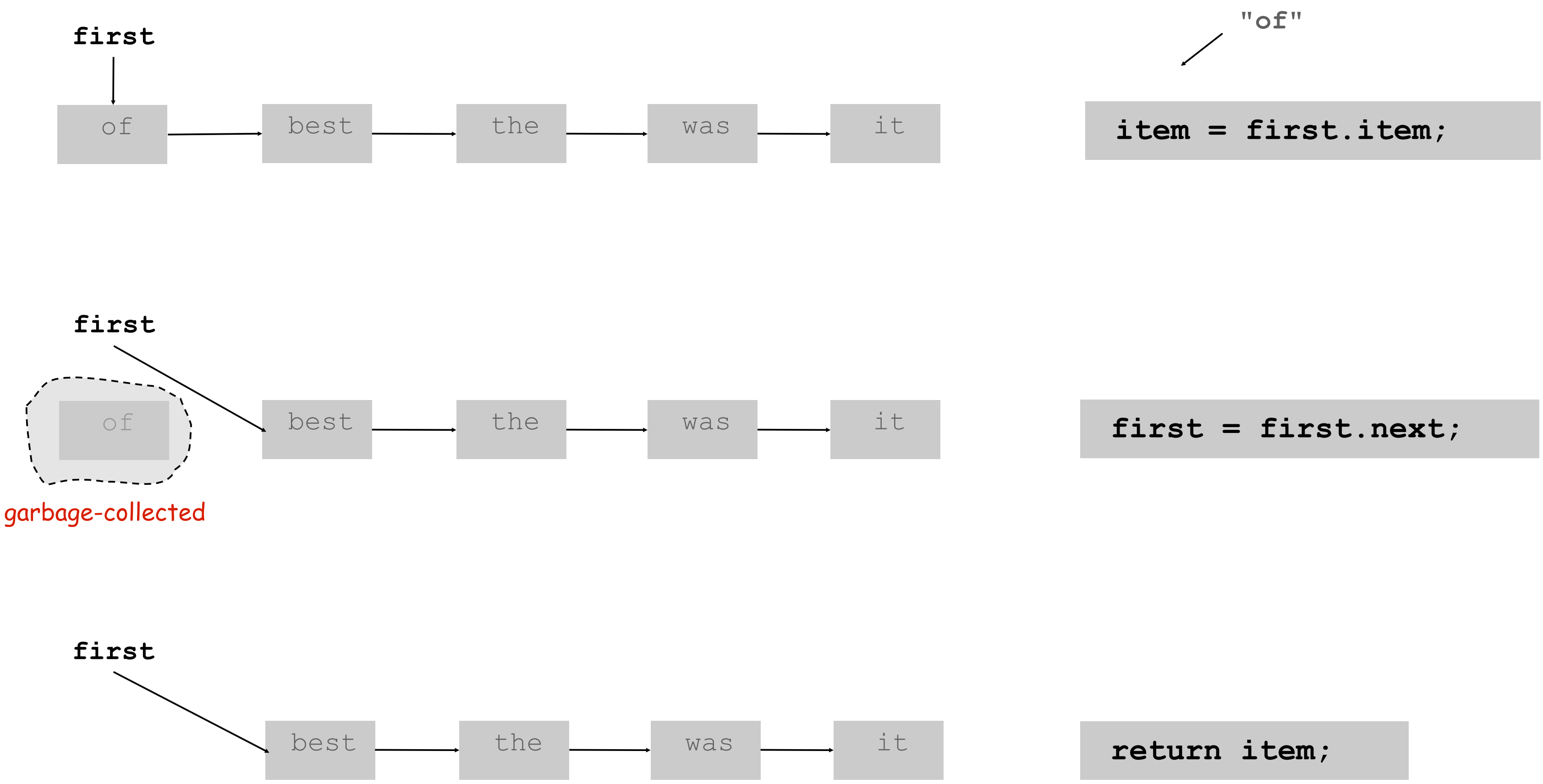


```
first = new Node();
```



```
first.item = item;
first.next = second;
```

Stack Pop: Linked List Implementation



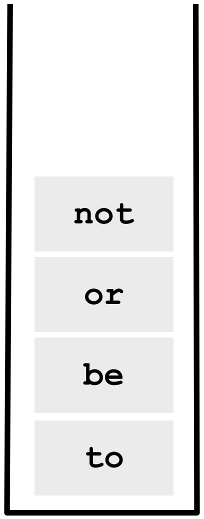
```
item = first.item;
```

```
first = first.next;
```

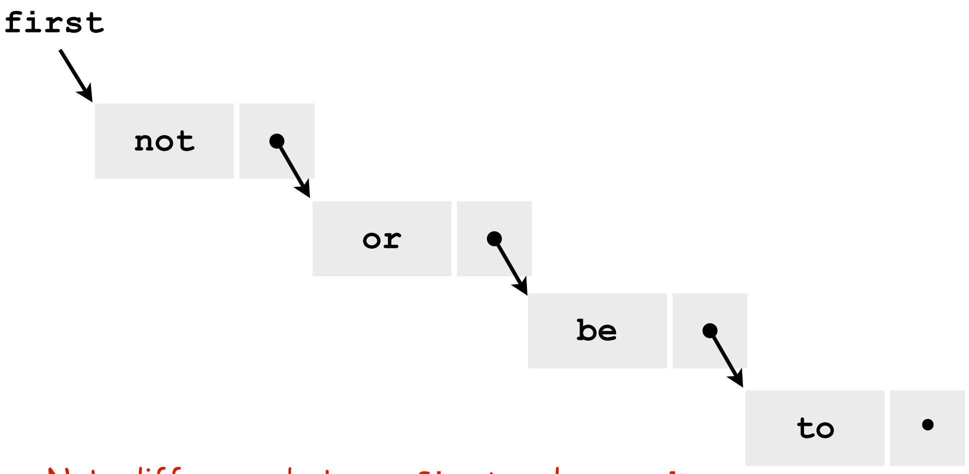
```
return item;
```

Stack: Linked List Implementation

```
public class LinkedStackOfStrings
{
   private Node first = null;

   private class Node
   {
      private String item;
      private Node next;
   }

   public boolean isEmpty()
   {  return first == null;   }

   public void push(String item)
   {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```



stack contents after 4th push() operation in test client



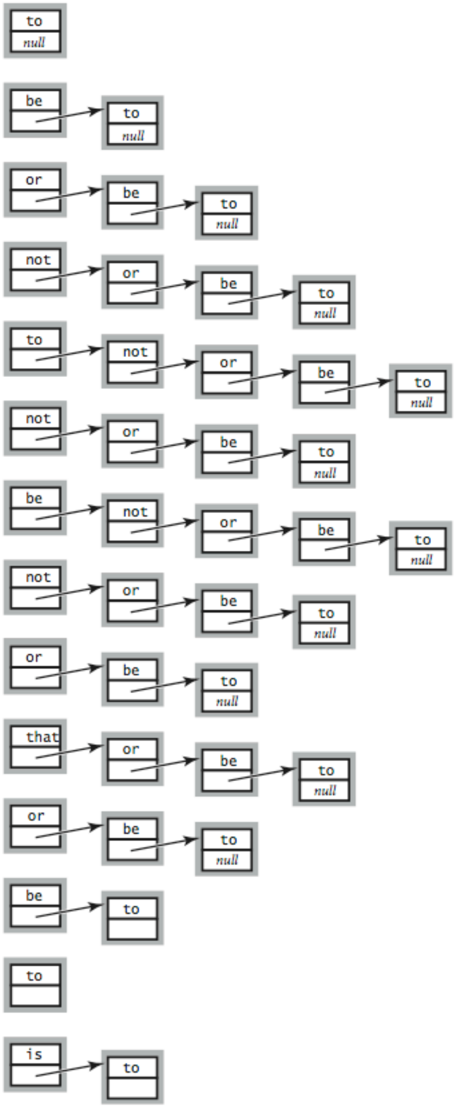
Note difference between **`first`** and **`second`**:

`first`: an instance variable that retains state

`second`: a local variable that goes out of scope

Linked List Stack: Trace

	StdIn	StdOut
push	to	
	be	
	or	
	not	
	to	
pop	-	to
	be	
	-	be
	-	not
	that	
	-	that
	-	or
	-	be
	is	



Linked-List Stack: Performance

Running time. Push and pop take constant time. ✓

Memory. Always proportional to number of items in stack. ✓

Stack Data Structures: Tradeoffs

Two data structures to implement the Stack data type.

Array.

- Every push/pop operation take constant time.
- But does not implement API… (must fix max capacity ahead of time).

Linked list.

- Every push/pop operation takes constant time.
- But… uses extra space and time to deal with references.

Client can evaluate performance tradeoffs to choose among APIs
(implicitly choosing among underlying data structures)

TEQ on List Processing 1

What does the following code do?

```
...
Node list = null;
while (!StdIn.isEmpty())
{
   Node old = list;
   list = new Node();
   list.item = StdIn.readString();
   list.next = old;
}
for (Node t = list; t != null; t = t.next)
   StdOut.println(t.item);
...
```

TEQ on List Processing 2

What does the following code do?

```
...
Node list = new Node();
list.item = StdIn.readString();
Node last = list;
while (!StdIn.isEmpty())
{
   last.next = new Node();
   last = last.next;;
   last.item = StdIn.readString();
}
...
```

Parameterized Data Types

Parameterized Data Types

We implemented: `StackOfStrings`.

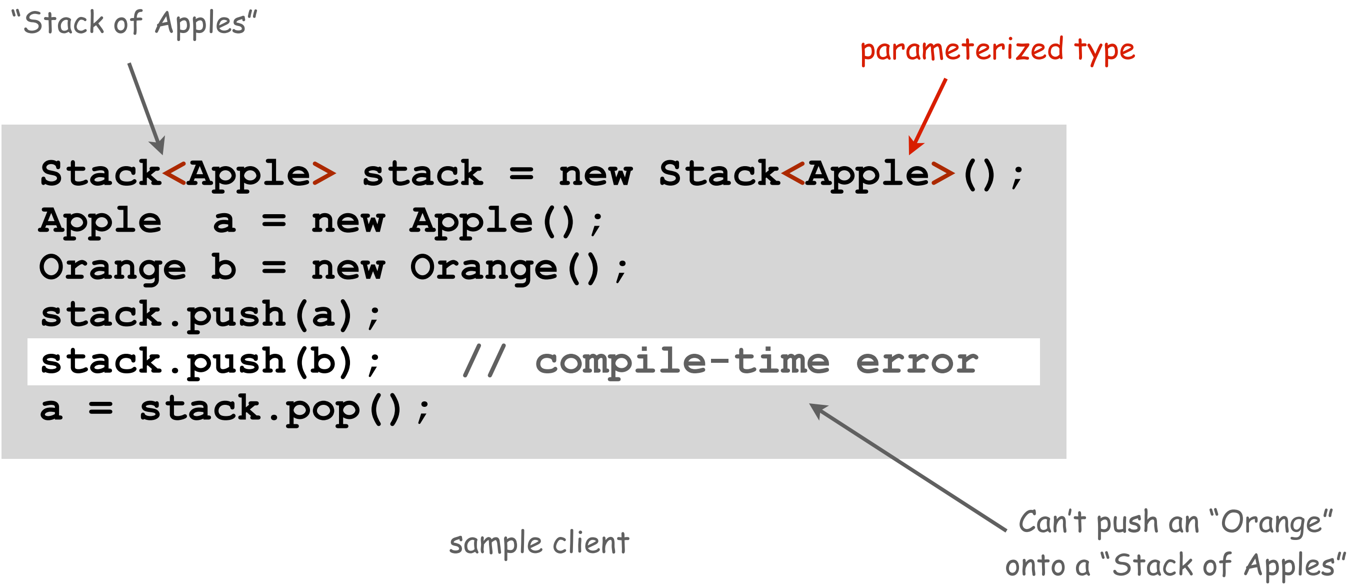
We also want: `StackOfMemoryBlocks`, `StackOfURLs`, `StackOfInts`, …

Strawman. Implement a separate stack class for each type.

- Rewriting code is tedious and error-prone.
- Maintaining cut-and-pasted code is tedious and error-prone.

Generics

Generics. Parameterize stack by a single type.



“Stack of Apples”

parameterized type

```
Stack<Apple> stack = new Stack<Apple>();
Apple  a = new Apple();
Orange b = new Orange();
stack.push(a);
stack.push(b);    // compile-time error
a = stack.pop();
```

Can’t push an “Orange” onto a “Stack of Apples”

sample client

Generic Stack: Linked List Implementation

String stack (for reference)

```
public class LinkedStackOfStrings
{
   private Node first = null;

   private class Node
   {
      private String item;
      private Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

```
public class Stack<Item>
{
   private Node first = null;

   private class Node
   {
      private Item item;
      private Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(Item item)
   {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
   }

   public Item pop()
   {
      Item item = first.item;
      first = first.next;
      return item;
   }
}
```

parameterized
type name
chosen by
programmer

Autoboxing

Generic stack implementation.

- Cannot use primitives with parameterized data types
- Can only substitute a reference type name for a parameterized name.

Wrapper type.

- Each primitive type has a wrapper reference type.
- Ex: `Integer` is wrapper type for `int`.
- Wrapper type has larger set of operations than primitive type.
- Values of wrapper type are objects.

Autoboxing. Automatic cast from primitive type to wrapper type.

Autounboxing. Automatic cast from wrapper type to primitive type.

```
Stack<Integer> stack = new Stack<Integer>();
stack.push(17);           // Autobox     (int -> Integer)
int a = stack.pop();      // Auto-unbox (Integer -> int)
```

Stack Applications

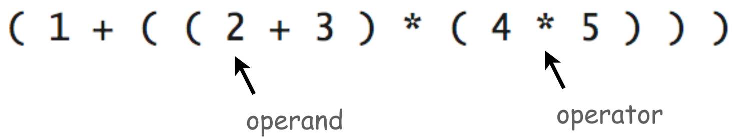
Real world applications.

- Parsing in a compiler.
- Java virtual machine.
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
- Implementing function calls in a compiler.

Stack Client 4: Arithmetic Expression Evaluation

value stack
operator stack

Goal. Evaluate infix expressions.



(1 + ((2 + 3) * (4 * 5)))

Two stack algorithm. [E. W. Dijkstra]

- Value: push onto the value stack.
- Operator: push onto the operator stack.
- Left parens: ignore.
- Right parens: pop operator and two values; push the result of applying that operator to those values onto the operand stack.

Arithmetic Expression Evaluation

```
public class Evaluate
{
   public static void main(String[] args)
   {
      Stack<String> ops  = new Stack<String>();
      Stack<Double> vals = new Stack<Double>();
      while (!StdIn.isEmpty())
      {
         String s = StdIn.readString();
         if      (s.equals("("))               ;
         else if (s.equals("+"))    ops.push(s);
         else if (s.equals("*"))    ops.push(s);
         else if (s.equals(")"))
         {
            String op = ops.pop();
            if      (op.equals("+"))
               vals.push(vals.pop() + vals.pop());
            else if (op.equals("*"))
               vals.push(vals.pop() * vals.pop());
         }
         else vals.push(Double.parseDouble(s));
      }
      StdOut.println(vals.pop());
   }
}
```

```
% java Evaluate
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
101.0
```

Correctness

Why correct? When algorithm encounters an operator surrounded by two values within parentheses, it leaves the result on the value stack.

```
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
```

So it's as if the original input were:

```
( 1 + ( 5 * ( 4 * 5 ) ) )
```

Repeating the argument:

```
( 1 + ( 5 * 20 ) )
( 1 + 100 )
101
```

Extensions. More ops, precedence order, associativity, whitespace.

```
1 + (2 - 3 - 4) * 5 * sqrt(6*6 + 7*7)
```

Postfix

Observation 1. Remarkably, the 2-stack algorithm computes the same value if the operator occurs after the two values.

```
( 1 ( ( 2 3 + ) ( 4 5 * ) * ) + )
```

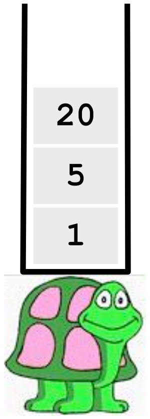
Observation 2. Now all of the parentheses are redundant!

```
1 2 3 + 4 5 * * +
```

Jan Lukasiewicz

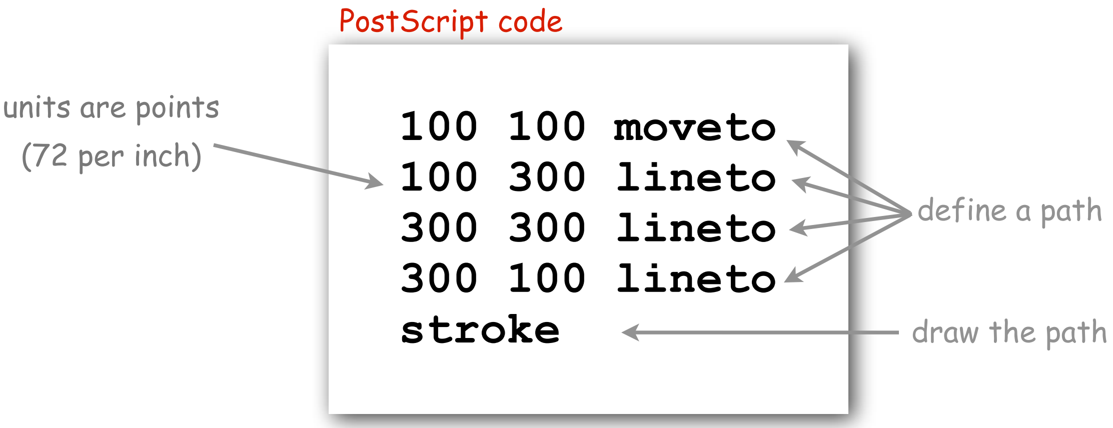
Bottom line. Postfix or "reverse Polish" notation.

Real-World Stack Application: PostScript



PostScript (Warnock-Geschke, 1980s). A turtle with a stack.

- postfix program code
- add commands to drive virtual graphics machine
- add loops, conditionals, functions, types



PostScript code

```
100 100 moveto
100 300 lineto
300 300 lineto
300 100 lineto
stroke
```

units are points
(72 per inch)

define a path

draw the path

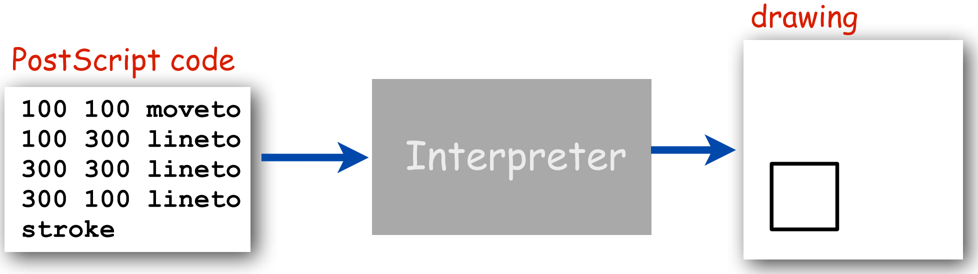
Simple virtual machine, but not a toy.

- Easy to specify published page.
- Easy to implement on various specific printers
- Revolutionized world of publishing.
- Virtually all printed material is PostScript.

Context/Definitions/Summary

Interpreter.

- Takes a program as input
- Does what that program would do.
- Simulates a virtual machine.



PostScript code

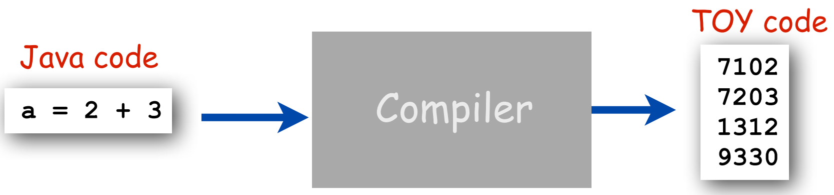
```
100 100 moveto
100 300 lineto
300 300 lineto
300 100 lineto
stroke
```

Interpreter

drawing

Compiler.

- Takes a program as input
- Produces a program as output.
- Produces code for a (real) machine.



Java code

```
a = 2 + 3
```

Compiler

TOY code

```
7102
7203
1312
9330
```

TOY is our proxy for a real machine

Virtual machines you have used

- LFSR
- Stack
- TOY
- PostScript
- Java Virtual Machine (another stack machine)

Data Type and Virtual Machine are the same thing!

- Set of values = machine state.
- Operations on values = machine operations.

Data Structure.

- Represent data and relationships among data in a data type.
- array, linked list, compound, multiple links per node