

COS 126

General Computer Science
Fall 2012

Robert Sedgewick

What is COS 126? Broad, but technical, introduction to **computer science**.

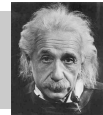
Goals.

- Demystify computer systems.
- Empower you to exploit available technology.
- Build awareness of substantial intellectual underpinnings.

Topics.

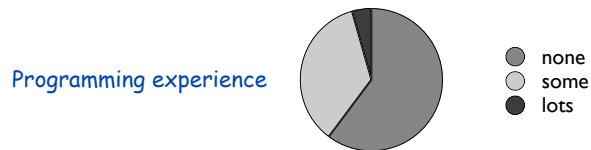
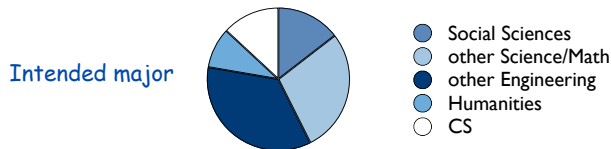
- **Programming** in Java.
- Machine architecture.
- Theory of computation.
- **Applications** to science, engineering, and commercial computing.

“Computers are incredibly fast, accurate, and stupid; humans are incredibly slow, inaccurate, and brilliant; together they are powerful beyond imagination.” – Albert Einstein

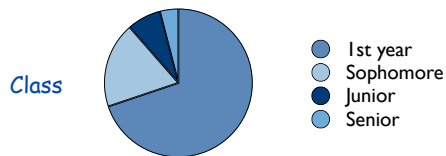


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Who are you?



[data from 2011-12]



Over half of all Princeton students take COS 126

The Basics

■ Lectures. [Sedgewick]

■ RS office hours. ← everyone needs to meet me!

■ Precepts. [Gabai · Moretti · August · Finkelstein · Hadidi · Homilius · Lee · Nadimpalli · Pritchard · Przytycki · Ravi · Wetzel]

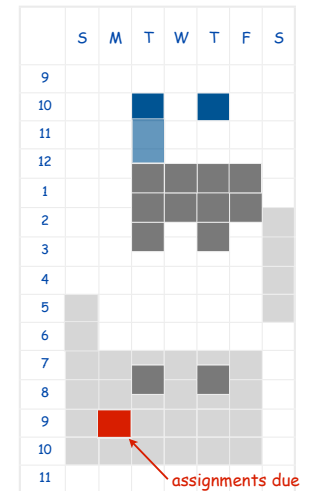
- Tips on assignments / worked examples
- Questions on lecture material.
- Informal and interactive.

■ Friend 016/017 lab. [Ugrad assistants]

- Help with systems/debugging.
- No help with course material.

Piazza.

- Best chance of quick response to a question.
- Post to class or private post to staff.



See www.princeton.edu/~cos126 for full current details and office hours.

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Grades

Course grades. No preset curve or quota.

9 programming assignments. 40%.

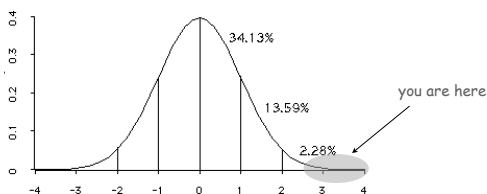
2 written exams (in class, 10/11 and 12/13). 35%.

2 programming exams (evenings, 10/25 and 12/13). 15%.

Final programming project (due Dean's date - 1). 10%.

Extra credit / staff discretion. Adjust borderline cases.

participation helps, frequent absence hurts



Due dates

	Su	Mo	Tu	We	Th	Fr	Sa
						13	14
						20	21
SEP	16	17	18	19	20	21	22
	23	24	25	26	27	28	29
	30						
		1	2	3	4	5	6
	7	8	9	10	11	12	13
OCT	14	15	16	17	18	19	20
	21	22	23	24	25	26	27
	28	29	30	31			
					1	2	3
	4	5	6	7	8	9	10
NOV	11	12	13	14	15	16	17
	18	19	20	21	22	23	24
	25	26	27	28	29	30	
							1
	2	3	4	5	6	7	8
	9	10	11	12	13	14	15
DEC	16	17	18	19	20	21	22
	23	24	25	26	27	28	29
	30	31					
JAN	6	7	8	9	10	11	12
	13	14					

Reading period.

No lectures; precepts T and W.

<http://www.princeton.edu/~cos126>

bookmark this!

Course Website

PRINCETON UNIVERSITY

**Computer Science 126
General Computer Science
Fall 2012**

[Course Information](#) | [People](#) | [Assignments](#) | [Lectures](#) | [Precepts](#) | [Exams](#) | [Booksite](#)

COURSE INFORMATION

Course description. An introduction to computer science in the context of scientific, engineering, and commercial applications. The goal of the course is to teach basic principles and practical issues, while at the same time preparing students to use computers effectively for applications in computer science, physics, biology, chemistry, engineering, and other disciplines. Topics include: programming in Java; hardware and software systems; algorithms and data structures; fundamental principles of computation; and scientific computing, including simulation, optimization, and data analysis.

Instructor. Robert Sedgewick.

Lectures. Lectures meet on Tuesdays and Thursdays at 10am in McCosh 10.

Preceptors. David August · Adam Finkelstein · Donna Gabai (co-lead) · Bobak Hadidi · Max Homilus · Kevin Lee · Christopher Moretti (co-lead) · Shilpa Nadimpalli · David Pritchard · Pawel Przytycki · Sachin Ravi · Josh Wetzel

Precepts. Precepts meet twice a week on Tuesdays and Thursdays or Wednesdays and Fridays. Precepts begin either September 13 or 14.

Undergraduate coordinator. For enrollment problems, see Colleen Kenny-McGinley in CS 210.

Course website. The course website contains a wealth of information, including precept rosters, office hours, lecture slides, programming assignments, and old exams.
<http://www.princeton.edu/~cos126>

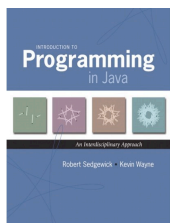
Computing facilities. Undergraduate lab TAs are available most evenings in Friend Center 017 to provide general help with using your operating system and assist with debugging your programs. Lab hours are posted [here](#).

Online forum. If you have general questions about the assignments, lectures, textbook, or other course materials, please post via [Piazza](#). Posts marked private are viewable only by instructors.

Grading. Two written exams (35%), two programming exams (15%), nine programming assignments (40%), final programming project (10%), and staff discretion. We record grades in Blackboard.

Textbook and Booksite

Textbook.



← for use while learning and studying

Booksite.

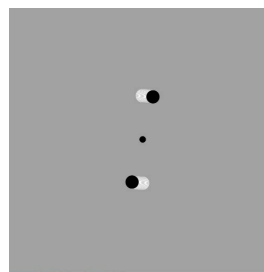
- Summary of content.
- Code, exercises, examples.
- Supplementary material.
- NOT the textbook

for use while online

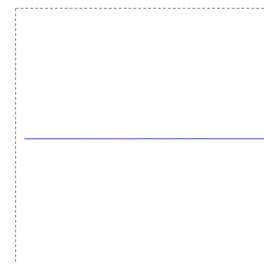
Programming Assignments

Desiderata.

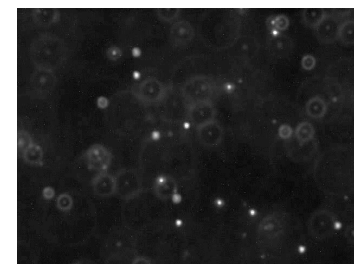
- Address an important scientific or commercial problem.
- Illustrate the importance of a fundamental CS concept.
- You solve problem from scratch!



N-body simulation



pluck a guitar string



estimate Avogadro's number

Desiderata.

- Address an important scientific or commercial problem.
- Illustrate the importance of a fundamental CS concept.
- You solve problem from scratch!

Due. Mondays 9pm via Web submission.

Computing equipment.

- Your laptop. [OS X, Windows, Linux, iPhone, ...]
- OIT desktop. [Friend 016 and 017 labs]

Lecture 2. Intro to Java.

Precept 1. Meets today/tomorrow.

Not registered? Go to any precept now; officially register ASAP.

Change precepts? Use SCORE.

see Colleen Kenny-McGinley in CS 210 if the only precept you can attend is closed

Assignment 0. [www.princeton.edu/~cos126/assignments.php]

- Due Monday 9PM.
- Read Sections 1.1 and 1.2 in textbook.
- Install Java programming environment + a few exercises.
- Lots of help available, don't be bashful.

END OF ADMINISTRATIVE STUFF

0. Prologue: A Simple Machine

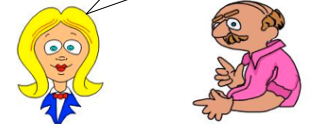


Secure Chat with a One-Time Pad

Alice wants to send a secret message to Bob

- Sometime in the past, they exchange a **one-time pad**.
- Alice uses the pad to encrypt the message.

"use yT25a5i/S if I ever send you an encrypted message"



```
Secure Chat 1.0 [alice]
[alice]: Hey, Bob
[bob]: Hi, Alice!
[alice]: SENDMONEY

Encrypt SENDMONEY with yT25a5i/S
```

```
Secure Chat 1.0 [bob]
[alice]: Hey, Bob
[bob]: Hi, Alice!
[alice]: gX76W3v7K
```

Key point: Without the pad, Eve cannot understand the message.



Encryption Machine

Goal. Design a machine to encrypt and decrypt data.

S E N D M O N E Y

encrypt

g X 7 6 W 3 v 7 K

decrypt

S E N D M O N E Y



Enigma encryption machine.

- "Unbreakable" German code during WWII.
- Broken by Turing bombe.
- One of first uses of computers.
- Helped win Battle of Atlantic by locating U-boats.

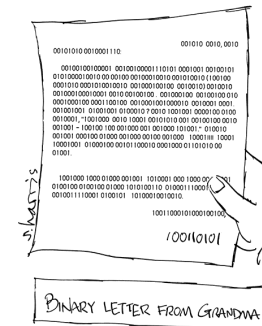
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A Digital World

Data is a sequence of bits. [bit = 0 or 1] ← can use decimal digits, letters, or some other system, but bits are more easily encoded physically ("on-off", "up-down", "hot-cold", ...)

- Text.
- Programs, executables.
- Documents, pictures, sounds, movies, ...

thousands of bits



Copyright 2004, Sidney Harris
http://www.sciencecartoonplus.com

billions of bits

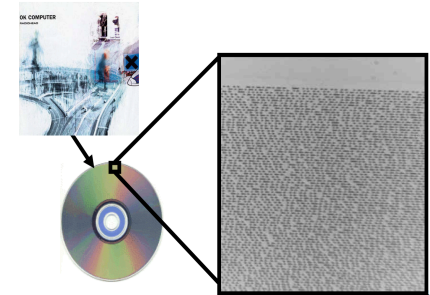


image courtesy of David August

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A Digital World

Data is a sequence of bits. [bit = 0 or 1]

- Text.
- Programs, executables.
- Documents, pictures, sounds, movies, ...

Ex. Base64 encoding of text.

- Simple method for representing A-Z, a-z, 0-9, +, /
- 6 bits to represent each symbol (64 symbols)

000000	A	001000	I	010000	Q	011000	Y	100000	g	101000	o	110000	w	111000	4
000001	B	001001	J	010001	R	011001	Z	100001	h	101001	p	110001	x	111001	5
000010	C	001010	K	010010	S	011010	a	100010	i	101010	q	110010	y	111010	6
000011	D	001011	L	010011	T	011011	b	100011	j	101011	r	110011	z	111011	7
000100	E	001100	M	010100	U	011100	c	100100	k	101100	s	110100	0	111100	8
000101	F	001101	N	010101	V	011101	d	100101	l	101101	t	110101	1	111101	9
000110	G	001110	O	010110	W	011110	e	100110	m	101110	u	110110	2	111110	+
000111	H	001111	P	010111	X	011111	f	100111	n	101111	v	110111	3	111111	/

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Secure Chat with a One-Time Pad

First challenge: Create a one-time pad.

Good choice: A **random** sequence of bits (stay tuned).

Note: any sequence of bits can be encoded as characters

110010	010011	110110	111001	011010	111001	100010	111111	010010	one-time pad
y	T	2	5	a	5	i	/	s	encoded as characters

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One-Time Pad Encryption

Encryption.

- Convert text message to N bits.

Base64 Encoding		
char	dec	binary
A	0	000000
B	1	000001
...
M	12	001100
...

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64

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One-Time Pad Encryption

Encryption.

- Convert text message to N bits.
- Use N random bits as one-time pad.

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	one-time pad
y	T	2	5	a	5	i	/	S	

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One-Time Pad Encryption

Encryption.

- Convert text message to N bits.
- Use N random bits as one-time pad.
- Take bitwise XOR of two bitstrings.

XOR Truth Table		
x	y	$x \wedge y$
0	0	0
0	1	1
1	0	1
1	1	0

sum corresponding pair of bits: 1 if sum is odd, 0 if even

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	one-time pad
100000	010111	111011	111010	010110	110111	101111	111011	001010	XOR

$0 \wedge 1 = 1$

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One-Time Pad Encryption

Encryption.

- Convert text message to N bits.
- Use N random bits as one-time pad.
- Take bitwise XOR of two bitstrings.
- Convert binary back into text.

Base64 Encoding		
char	dec	binary
A	0	000000
B	1	000001
...
w	22	010110
...

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	one-time pad
100000	010111	111011	111010	010110	110111	101111	111011	001010	XOR
g	X	7	6	w	3	v	7	K	encrypted

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Typical Exam Question (TEQ)

Encrypt the message **E A S Y** with the pad **1 2 3 4**.

Secure Chat with a One-Time Pad

Alice wants to send a secret message to Bob

- Sometime in the past, they exchange a **one-time pad**.
- Alice uses the pad to encrypt the message.

"use **yT25a5i/S** if I ever send you an encrypted message"



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Encrypt SENDMONEY with yT25a5i/S
```

```
Secure Chat 1.0 [bob]
[alice]: Hey, Bob
[bob]: Hi, Alice!
[alice]: gX76W3v7K
```

Key point: Without the pad, Eve cannot understand the message. But how can **Bob** understand the message?



Secure Chat with a One-Time Pad

Alice wants to send a secret message to Bob

- Sometime in the past, they exchange a **one-time pad**.
- Alice uses the pad to encrypt the message.
- **Bob uses the same pad to decrypt the message.**

"use **yT25a5i/S** if I ever send you an encrypted message"



```
Secure Chat 1.0 [alice]
[alice]: Hey, Bob
[bob]: Hi, Alice!
[alice]: SENDMONEY

Encrypt SENDMONEY with yT25a5i/S
```

```
Secure Chat 1.0 [bob]
[alice]: Hey, Bob
[bob]: Hi, Alice!
[alice]: gX76W3v7K
        SENDMONEY

Decrypt with yT25a5i/S
```

Key point: Without the pad, Eve cannot understand the message.



One-Time Pad Decryption

Decryption.

- Convert encrypted message to binary.

g X 7 6 W 3 v 7 K encrypted

One-Time Pad Decryption

Decryption.

- Convert encrypted message to binary.

Base64 Encoding		
char	dec	binary
A	0	000000
B	1	000001
...
W	22	010110
...

g	x	7	6	W	3	v	7	K
100000	010111	111011	111010	010110	110111	101111	111011	001010

encrypted
base64

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One-Time Pad Decryption

Decryption.

- Convert encrypted message to binary.
- Use **same** N "random" bits (one-time pad).

g	x	7	6	W	3	v	7	K
100000	010111	111011	111010	010110	110111	101111	111011	001010
110010	010011	110110	111001	011010	111001	100010	111111	010010
y	T	2	5	a	5	i	/	s

encrypted
base64
one-time pad

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One-Time Pad Decryption

Decryption.

- Convert encrypted message to binary.
- Use same N random bits (one-time pad).
- Take bitwise XOR of two bitstrings.

XOR Truth Table		
x	y	$x \wedge y$
0	0	0
0	1	1
1	0	1
1	1	0

g	x	7	6	W	3	v	7	K
100000	010111	111011	111010	010110	110111	101111	111011	001010
110010	010011	110110	111001	011010	111001	100010	111111	010010
010010	000100	001101	000011	001100	001110	001101	000100	011000

encrypted
base64
one-time pad
XOR

$1 \wedge 1 = 0$

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One-Time Pad Decryption

Decryption.

- Convert encrypted message to binary.
- Use same N random bits (one-time pad).
- Take bitwise XOR of two bitstrings.
- Convert back into text.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...
M	12	001100
...

g	x	7	6	W	3	v	7	K
100000	010111	111011	111010	010110	110111	101111	111011	001010
110010	010011	110110	111001	011010	111001	100010	111111	010010
010010	000100	001101	000011	001100	001110	001101	000100	011000
S	E	N	D	M	O	N	E	Y

encrypted
base64
one-time pad
XOR
message



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Why Does It Work?

Crucial property. Decrypted message = original message.

Notation	Meaning
a	original message bit
b	one-time pad bit
^	XOR operator
a ^ b	encrypted message bit
(a ^ b) ^ b	decrypted message bit

Why is crucial property true?

- Use properties of XOR.

$$(a \oplus b) \oplus b = a \oplus (b \oplus b) = a \oplus 0 = a$$

↑ associativity of ^
 ↑ always 0
 ↑ identity

XOR Truth Table

x	y	x ^ y
0	0	0
0	1	1
1	0	1
1	1	0

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One-Time Pad Decryption (with the wrong pad)

Decryption.

- Convert encrypted message to binary.

g	x	7	6	w	3	v	7	k
---	---	---	---	---	---	---	---	---

encrypted

30

One-Time Pad Decryption (with the wrong pad)

Decryption.

- Convert encrypted message to binary.

g	x	7	6	w	3	v	7	k
---	---	---	---	---	---	---	---	---

encrypted

100000	010111	111011	111010	010110	110111	101111	111011	001010
--------	--------	--------	--------	--------	--------	--------	--------	--------

base64

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One-Time Pad Decryption (with the wrong pad)

Decryption.

- Convert encrypted message to binary.
- Use **wrong** N bits (bogus one-time pad).

g	x	7	6	w	3	v	7	k
---	---	---	---	---	---	---	---	---

encrypted

100000	010111	111011	111010	010110	110111	101111	111011	001010
--------	--------	--------	--------	--------	--------	--------	--------	--------

base64

101000	011100	110101	101111	010010	111001	100101	101010	001010
--------	--------	--------	--------	--------	--------	--------	--------	--------

wrong bits

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One-Time Pad Decryption (with the wrong pad)

Decryption.

- Convert encrypted message to binary.
- Use **wrong** N bits (bogus one-time pad).
- Take bitwise XOR of two bitstrings.

g	x	7	6	w	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
101000	011100	110101	101111	010010	111001	100101	101010	001010	wrong bits
001000	001011	001110	010101	000100	001110	001010	010001	000000	XOR

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One-Time Pad Decryption (with the wrong pad)

Decryption.

- Convert encrypted message to binary.
- Use **wrong** N bits (bogus one-time pad).
- Take bitwise XOR of two bitstrings.
- Convert back into text: **Oops.**

g	x	7	6	w	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
101010	110000	000011	100000	011011	000011	101110	011010	101111	wrong bits
001010	100111	111000	011010	001101	110100	000001	100001	100101	XOR
K	n	4	a	N	0	B	h	l	wrong message [usually gibberish]

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Eve's Problem (one-time pads)

Key point: Without the pad, Eve cannot understand the message.



But Eve has a computer. Why not try all possible pads?

One problem: it might take a long time [stay tuned].

Worse problem: she would see all possible messages!

- 54 bits
- 2^{54} possible messages, all different.
- 2^{54} possible **encoded** messages, all different.
- No way for Eve to distinguish real message from any other message.

One-time pad is "provably secure".

AAAAAAAA	gX76W3v7K
AAAAAAAAAB	gX76W3v7L
AAAAAAAAAC	gX76W3v7I
...	
qwDgbDuav	Kn4aN0Bh1
...	
tTtpWk+1E	NEWTATTOO
...	
yT25a5i/S	SENDMONEY
...	
////////+	fo7FpIQE0
/////////	fo7FpIQE1

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Goods and Bads of One-Time Pads

Good.

- Easily computed by hand.
- Very simple encryption/decryption processes.
- Provably unbreakable if bits are truly random. [Shannon, 1940s]

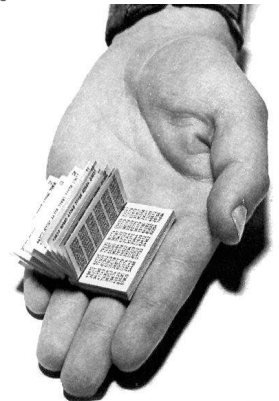
← eavesdropper Eve sees only random bits

Bad.

- Easily breakable if pad is re-used.
- **Pad must be as long as the message.**
- Truly random bits are very hard to come by.
- Pad must be distributed securely.

"one time" means one time only

← impractical for Web commerce



a Russian one-time pad

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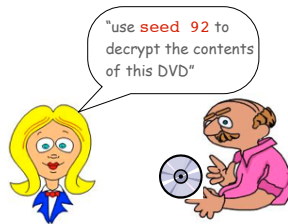
Pseudo-Random Bit Generator

Practical middle-ground.

- Make a "random" bit generator gadget.
- Alice and Bob each get identical small gadgets
[same gadget works for both]
- Alice and Bob also each get identical books of small **seeds**.

← instead of identical large one-time pads

Goal. Small gadget that produces a long sequence of bits.

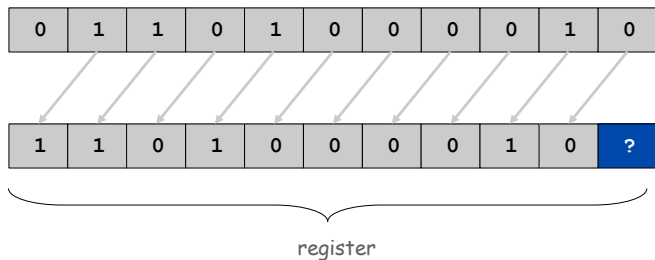


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Shift Register

Shift register terminology.

- Bit: 0 or 1.
- Cell: storage element that holds one bit.
- Register: sequence of cells.
- Seed: initial sequence of bits.
- Shift register: when clock ticks, bits propagate one position to left.



time t

time t + 1

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Pseudo-Random Bit Generator

Small deterministic gadgets that produce long sequences of pseudo-random bits:

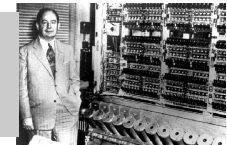
- Enigma
- **Linear feedback shift register.**
- Linear congruential generator.
- Blum-Blum-Shub generator.
- [many others have been invented]

Pseudo-random? Bits are not really random:

- Bob's and Alice's gadgets must produce the same bits from the same seed.
- Bits must have as many properties of random bits as possible (to foil Eve).

↑
Ex 1. approx 1/2 0s and 1/2 1s
Ex 2. approx 1/4 each of 00, 01, 10 11

"Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin."
– John von Neumann (left)
– ENIAC (right)

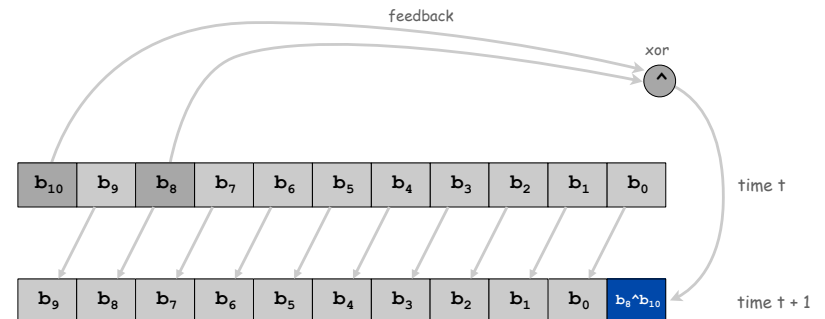


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Linear Feedback Shift Register (LFSR)

{8, 10} linear feedback shift register.

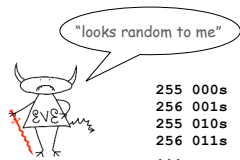
- Shift register with 11 cells.
- Bit b_0 is XOR of previous bits b_8 and b_{10} .
- Pseudo-random bit = b_0 .



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Random Numbers

Q. Are these 2000 numbers random?
If not, what is the pattern?



255 000s
256 001s
255 010s
256 011s
...

```
1100100100111101101110010110101110011000101111101001000010011010010111100110010011111011100
00010101100010000111010100110100001110010011001101111101010000100001000101001010100011000
001011100010010010101011100011010011011001110101110010001001101010111010000010100100010001
0001010101011100000001010000010011000101101010010101001100001111100110000011111000110
0001011100110100111010011100100110110101010101000000000100000000101000001000100001
01010100100000011010000011001000110110101101010001010000100100010101010100001100001
00111100101110011100101110110010010101101100001010110010000101110100100101011000011101
110110010101011100000010011000010111100100100011101010110001100011011101101010100100110
00010011001111011100001010011001000111110101010000100011001010101100001101011001110001
11110110100010101101001101010011100001100110011011111101000000010010000010110100010011
00101011110000100001100101001111000110001101101101101101010101010000010110000110101011
010100010110010111011100101010011000001101100011010110111000101010101000000110010000111
110100110001001111010110001000101010100110000001111000010001100111011110010100001110
001001101010111011000100101110101000111000011000110011101111001010000111010001110
001001101010111011000100101110101000111000101000111000101000111000111000111001100101
1111110010000001101000010100100111001101101110101010001000001010100001000101000101
10001010011101000111010010101001100110011111111000000001100000001110000011001100011111
1011000000101100001001010010100111001111001110011101101111011110001010001101000
1011100101001011000110010101110011010001111001010001110011001101101101010100010001100101
011111100010000010101000111000010110110010011011101110100101001001100110111101101000101
0100101000011000100011101010110010000011101000100100101111011001000101110101001000100011
01101001110100110101111010001000100101010100000001110000010110000111011001101010111
100001000110001010111010
```

A. No. This is output of {8, 10} LFSR with seed 01101000010!

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LFSR Decryption

Decryption.

- Convert encrypted message to binary.
- Initialize identical LFSR with **same** seed
- Generate N bits **with LFSR**.
- Take bitwise XOR of two bitstrings.
- Convert back into text.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...
M	12	001100
...

g	x	7	6	w	3	v	7	k	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	LFSR bits
010010	000100	001101	000011	001100	001110	001101	000100	011000	XOR
S	E	N	D	M	O	N	E	Y	message

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LFSR Encryption

Encryption.

- Convert text message to N bits.
- Initialize LFSR with given seed
- Generate N bits **with LFSR**.
- Take bitwise XOR of two bitstrings.
- Convert binary back into text.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...
w	22	010110
...

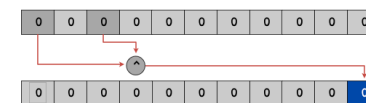
S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	LFSR bits
100000	010111	111011	111010	010110	110111	101111	111011	001010	XOR
g	x	7	6	w	3	v	7	k	encrypted

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Key properties of LFSRs

Property 1: A zero fill (all 0s) produces all 0s.

- Don't use all 0s as a seed!
- Fill of all 0s will not otherwise occur.



Property 2: Bitstream must eventually cycle.

- $2^N - 1$ nonzero fills in an N-bit register.
- Future output completely determined by current fill.

Ex: (1, 2) LFSR

001
010
101
011
111
110
100
001 $2^3 - 1 = 7$

Property 3: Cycle length in an N-bit register is at most $2^N - 1$.

- Could be smaller; cycle length depends on tap positions.
- Need higher math (theory of finite groups) to know tap positions for given N.

Bottom line: 11-bit register generates at most 2047 bits before cycling, so use a longer register (say, N = 61).

challenge for the bored: what tap positions?

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Eve's Problem (LFSR encryption/decryption)

Key point: Without the (short) seed

Eve cannot understand the (long) message.



But Eve has a computer. Why not try all possible seeds?

- Seeds are short, messages are long.
- All seeds give a tiny fraction of all messages.
- Extremely likely that all but real seed will produce gibberish.

assume Eve has a machine
(knows LFSR length and taps)

Bad news (for Eve): There are still too many possibilities!

- Ex: 61-bit register implies 2^{61} possibilities.
- If Eve could check 1 million seeds per second, it would take her **730 centuries** to try them all!

Exponential growth dwarfs technological improvements [stay tuned].

- 1000 bits: 2^{1000} possibilities.
- Age of the universe in microseconds: 2^{70}



Goods and Bads of LFSRs

Good.

- Easily computed with simple machine.
- Very simple encryption/decryption processes.
- Bits have many of the same properties as random bits.
- Scalable: 20 cells for 1 million bits; 30 cells for 1 billion bits.
[but need theory of finite groups to know where to put taps]



a commercially available LFSR

Bad.

- Still need secure, independent way to distribute LFSR seed.
- The bits are not truly random.
[bits in our 11-bit LFSR cycle after $2^{11} - 1 = 2047$ steps]
- Experts have cracked LFSR encryption.
[need more complicated machines]

Other LFSR Applications

What else can we do with a LFSR?

- DVD encryption with CSS.
- DVD decryption with DeCSS!
- Subroutine in military cryptosystems.

```

/*  efdtt.c  Author: Charles M. Hannum <root@ihack.net>  */
/*  Usage is: cat title-key scrambled.vob | efdtt >clear.vob  */

#define m(i) (x[i]^s[i+84])<<

    unsigned char x[5]          ,y,s[2048];main(
n){for( read(0,x,5          );read(0,s ,n=2048
); write(1 ,s,n          )if(s
[y=s [13]%8+20] /16%4 ==1 )if(s
i=m( 1)17 ^256 +m(0) 8,k =m(2)
0,j= m(4) 17^ m(3) 9*k* 2-k%8
^8,a =0,c =26;for (s[y] -=16;
--c;j *=2)a= a*2^i% 1,i=i /2^j&1
<<24;for(j= 127; ++j<n;c=c>
y)
c
+*=i^i/8^i>>4^i>>12,
i=i>>8^y<<17,a^=a>>14,y=a^a*8^a<<6,a=a
>>8^y<<9,k=s[j],k =7Wo-'G_\216"[k
&7]+2^"cr3sfw6v;*k->>n." [k]>>4]*2^k+257/
8,s[j]=k^(k&k*2&34)*6^c++y
;)}
    
```

<http://www.cs.cmu.edu/~dst/DeCSS/gallery>

Typical Exam Question (TEQ) on LFSRs 1

Give first 10 steps of {3, 4} LFSR with initial fill 00001.

Goal. Decrypt/encrypt 300 characters (1800 bits).

Challenge. Is it a good idea to use an 11-bit LFSR?

A. Yes, no problem.

B. No, the bits it produces are not truly random.

C. No, need a longer LFSR.

D. No, experts have cracked LFSRs

Important properties.

- Built from simple components.
- Scales to handle huge problems.
- Requires a deep understanding to use effectively.

Basic Component	LFSR	Computer
control	start, stop, load	same
clock	regular pulse	2.8 GHz pulse
memory	11 bits	1 GB
input	seed	sequence of bits
computation	shift, XOR	logic, arithmetic, ...
output	pseudo-random bits	Sequence of bits

Critical difference. General purpose machine can be programmed to simulate ANY abstract machine.

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A Profound Idea

Programming. Can write a Java program to simulate the operations of **any** abstract machine.

- Basis for theoretical understanding of computation. [stay tuned]
- Basis for bootstrapping real machines into existence. [stay tuned]

Stay tuned. See Assignment 5.

```
public class LFSR
{
    private int[] seed;
    private int tap;
    private int N;

    public LFSR(String seed, int tap) { ... }

    public int step() { ... }

    public static void main(String[] args)
    {
        LFSR lfsr = new LFSR("01101000010", 8);
        for (int i = 0; i < 2000; i++)
            StdOut.println(lfsr.step());
    }
}
```

```
% java LFSR
1100100100111101101110010110101
1100110001011111101001000010011
0100101111001100100111...
```

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A Profound Question

Q. What is a random number?

LFSR does not produce random numbers.

- It is a very simple deterministic machine.
- Not obvious how to distinguish the bits it produces from random.
- Experts have figured out how to do so.

Q. Are random processes found in nature?

- Motion of cosmic rays or subatomic particles?
- Mutations in DNA?

Q. Is the natural world a (not-so-simple) deterministic machine?

"God does not play dice."
– Albert Einstein



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