Due: Monday October 31, in person/mailbox.

1. It turns out that there is an equivalent definition to the definition of graph entropy given in class. Let I_G be the set of all the independent sets of a graph $G(V, E)$. Let $H'(G)$ be the minimum value attained by
$$Q(Y) = \sum_{x \in V} \frac{1}{|V|} \log \frac{1}{\Pr[x \in Y]},$$
where the minimum is taken over all random variables Y taking values in I_G. It is the case that $H'(G) = H(G)$. However, you are only required to prove that $H'(G) \leq H(G)$. *Hint:* Use the fact that $I(X;Y) = \mathbf{E}_y D[X|_y \| X]$.

   Bonus: Show the other inequality. Namely, that $H(G) \leq H'(G)$.

2. If n is a power of 2, show that there is a monotone boolean formula of size only $2n \log n - 1$ computing Th_2^n, and using only $\log n$ AND gates.

3. Recall that a family of graphs on a vertex set V is G-intersecting if the intersection on each pair of graphs in the family contains a copy of G. Let G be a graph whose chromatic number is k (i.e. k is the minimum number of colors needed to color the vertices of G so that each edge gets two colors). Show that any family of G-intersecting graphs on n vertices can consist of at most a $1/2^{k-1}$ fraction of all graphs.