COS522: Computational Complexity Fall 2011

Princeton University

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Lecture 5 notes Sept 29. Continuing discussion of PH.

1. P=NP implies PH=P.
2. Alternating TMs.
3. Unlimited # of alternations: AP =PSPACE.
4. Show that SAT cannot be solved simultaneously in time n^{1.2} and space n^{0.2}. (Fortnow’s theorem. Sharpest version due to Williams.)
5. Step 1: TISP(n^{12}, n^2) is contained in \sigma\_2-TIME(n^8). Proof: Guess n^6 in-between configurations of the tape, and for every i in [n^6] verify using \forall guessing that configuration C\_i leads to C\_{i+1} in n^6 steps.
6. Step 2: If NTIME(n) is in DTIME(n^{1.2}) then \Sigma\_2(n^8) is in NTIME(N^{9.6}). Proof: Simple padding argument.
7. Putting them together: if the theorem statement were false then NTIME(n^{10}) would be in TISP(n^{12}, n^2), which by the prev. steps is in NTIME(n^{9.6}) which violates NTIME hierarchy thm.
8. Mention oracle definition of PH. \Sigma\_i is exactly the languages decided by NP machines with \Sigma\_{i-1} oracles.
9. NP^{SAT} = \Sigma\_2. Nontrivial side: NP^{SAT} is in \Sigma\_2.
10. Circuits. Defn. Deciding languages with a circuit: One circuit for each input size.
11. Undecidable languages can have small circuits.
12. If a language is decidable in T(n) time it has circuits of size T(n)^2. (Can be improved to T(n) log T(n).)
13. Equivalence to Straight line programs. Note: different program for each input size.
14. If NP has poly size circuits, PH = \Sigma\_2. Guess the SAT circuit? How to check it is the right circuit?
15. Main idea: search reduces to decision problem for SAT. There is also a poly size circuit that generates the true assignment for every satisfiable formula. Guess that.