COS522: Computational Complexity Fall 2011

Princeton University

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Lecture 4 notes Sept 27.

1. Recall meta theorem about space bounded computation: NSPACE(S(n)) is essentially PATH problem on directed graphs of size exp(S(n)). Furthermore, reduction can be carried out in space S(n) as well. Recall coNL=NL.
2. PSPACE. Contains P, NP, coNP.
3. Essence of PSPACE: 2-person games of perfect information. Specifically, does the first player has a winning strategy?
4. TQBF, a complete problem for PSPACE. Show it is in PSPACE. (Meta theorem: simple computations on trees of depth S can be done in O(S) space. )
5. Proof of PSPACE-completeness (sketch). Recall Cook’s theorem.  
   Inspires us to do something similar. Problem: Size of tableau is exponential!
6. Recall: P, NP, coNP. Complete problem for it.
7. Today’s topic: Polynomial hierarchy
8. Motivation: exact clique, Min –EQ-DNF = phi, k where there is a DNF formula of size at most k that is equiv. to phi. (Note: k must be in unary.)
9. Sigma\_2, sigma\_i using a certificate-type definition. PH is the union of Sigma\_i for all finite i.
10. Why study PH: (a) seems to capture interesting computational problems. (b) Arises unexpectedly in all kinds of complexity investigations. Will see an example next time: Fortnow’s theorem.