Snakes, Strings, Balloons
and Other Active Contour Models
Goal

• Start with image and initial closed curve
• Evolve curve to lie along “important” features
  – Edges
  – Corners
  – Detected features
  – User input
Applications

• Region selection in Photoshop
• Segmentation of medical images
• Tracking
Corpus Callosum

[Davatzikos and Prince]
Corpus Callosum

[Davatzikos and Prince]
User-Visible Options

- **Initialization**: user-specified, automatic
- **Curve properties**: continuity, smoothness
- **Image features**: intensity, edges, corners, …
- **Other forces**: hard constraints, springs, attractors, repulsors, …
- **Scale**: local, multiresolution, global
Behind-the-Scenes Options

• **Framework:** energy minimization, forces acting on curve

• **Curve representation:** ideal curve, sampled, spline, implicit function

• **Evolution method:** calculus of variations, numerical differential equations, local search
Snakes: Active Contour Models

- Introduced by Kass, Witkin, and Terzopoulos
- Framework: energy minimization
  - Bending and stretching curve = more energy
  - Good features = less energy
  - Curve evolves to minimize energy
- Also “Deformable Contours”
Snakes Energy Equation

- Parametric representation of curve

\[ \mathbf{v}(s) = (x(s), y(s)) \]

- Energy functional consists of three terms

\[ \varepsilon = \int \left[ \varepsilon_{\text{int}}(\mathbf{v}(s)) + \varepsilon_{\text{img}}(\mathbf{v}(s)) + \varepsilon_{\text{con}}(\mathbf{v}(s)) \right] ds \]
Internal Energy

\[ \varepsilon_{\text{int}}(v(s)) = \left( \alpha(s) \| v_{s} (s) \|^2 + \beta(s) \| v_{ss} (s) \|^2 \right)/2 \]

- First term is “membrane” term – minimum energy when curve minimizes length (“soap bubble”)
- Second term is “thin plate” term – minimum energy when curve is smooth
Internal Energy

\[ \varepsilon_{\text{int}}(\mathbf{v}(s)) = \left( \alpha(s)\|\mathbf{v}_s(s)\|^2 + \beta(s)\|\mathbf{v}_{ss}(s)\|^2 \right)/2 \]

- Control \( \alpha \) and \( \beta \) to vary between extremes
- Set \( \beta \) to 0 at a point to allow corner
- Set \( \beta \) to 0 everywhere to let curve follow sharp creases – “strings”
Image Energy

- Variety of terms give different effects
- For example,

\[ \varepsilon_{img} = w \cdot |I(x, y) - I_{desired}| \]

minimizes energy at intensity \( I_{desired} \)
Edge Attraction

- Gradient-based:

\[ \varepsilon_{img} = -w \cdot \| \nabla I(x, y) \|^2 \]

- Laplacian-based:

\[ \varepsilon_{img} = w \cdot \| \nabla^2 I(x, y) \|^2 \]

- In both cases, can smooth with Gaussian
Corner Attraction

• Can use corner detector we saw last week
• Alternatively, let $\theta = \tan^{-1} \frac{l_y}{l_x}$ and let $\mathbf{n}_\perp$ be a unit vector perpendicular to the gradient. Then

$$\varepsilon_{img} = w \cdot \left| \frac{\partial \theta}{\partial \mathbf{n}_\perp} \right|$$
Constraint Forces

- Spring

\[ \mathcal{E}_{con} = k \cdot \left\| \mathbf{v} - \mathbf{x} \right\|^2 \]

- Repulsion

\[ \mathcal{E}_{con} = \frac{k}{\left\| \mathbf{v} - \mathbf{x} \right\|^2} \]
Evolving Curve

• Computing forces on \( v \) that locally minimize energy gives differential equation for \( v \)
  – Euler-Lagrange formula
    \[
    \frac{d^2}{ds^2} \left( \frac{\partial \varepsilon}{\partial \dot{v}} \right) + \frac{d}{ds} \left( \frac{\partial \varepsilon}{\partial \ddot{v}} \right) + \frac{\partial \varepsilon}{\partial v} = 0
    \]

• Discretize \( v \): samples \((x_i, y_i)\)
  – Approximate derivatives with finite differences

• Iterative numerical solver
Other Curve Evolution Options

- Exact solution: calculus of variations
- Write equations directly in terms of forces, not energy
- Implicit equation solver
- Search neighborhood of each \((x_i, y_i)\) for pixel that minimizes energy
  - Shah & Williams paper
Variants on Snakes

• Balloons [Cohen 91]
  – Add inflation force

\[ F_{infl} = k \ n(s) \]

  – Helps avoid getting stuck on small features
Balloons

Snakes

Balloons
Balloons
Other Energy or Force Terms

- Results of previously-run local algorithms
  – e.g., Canny edge detector output convolved with Gaussian
- Automatically-evolved control points
- Others...
Brain Cortex Segmentation

Add energy term for constant-color regions of a single color

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Find features and add constraints

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Scale

- In the simplest snakes algorithm, image features only attract locally

- Greater region of attraction: smooth image
  - Curve might not follow high-frequency detail

- Multiresolution processing
  - Start with smoothed image to attract curve
  - Finish with unsmoothed image to get details

- Heuristic for global minimum vs. local minima
Diffusion-Based Methods

• Another way to attract curve to localized features: vector flow or diffusion methods

• Example:
  – Find edges using Canny
  – For each point in entire image, compute distance to nearest edge
  – Push curve along gradient of distance field
Gradient Vector Fields

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Simple Snake

With Gradient Vector Field

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