Snakes, Strings, Balloons and Other Active Contour Models

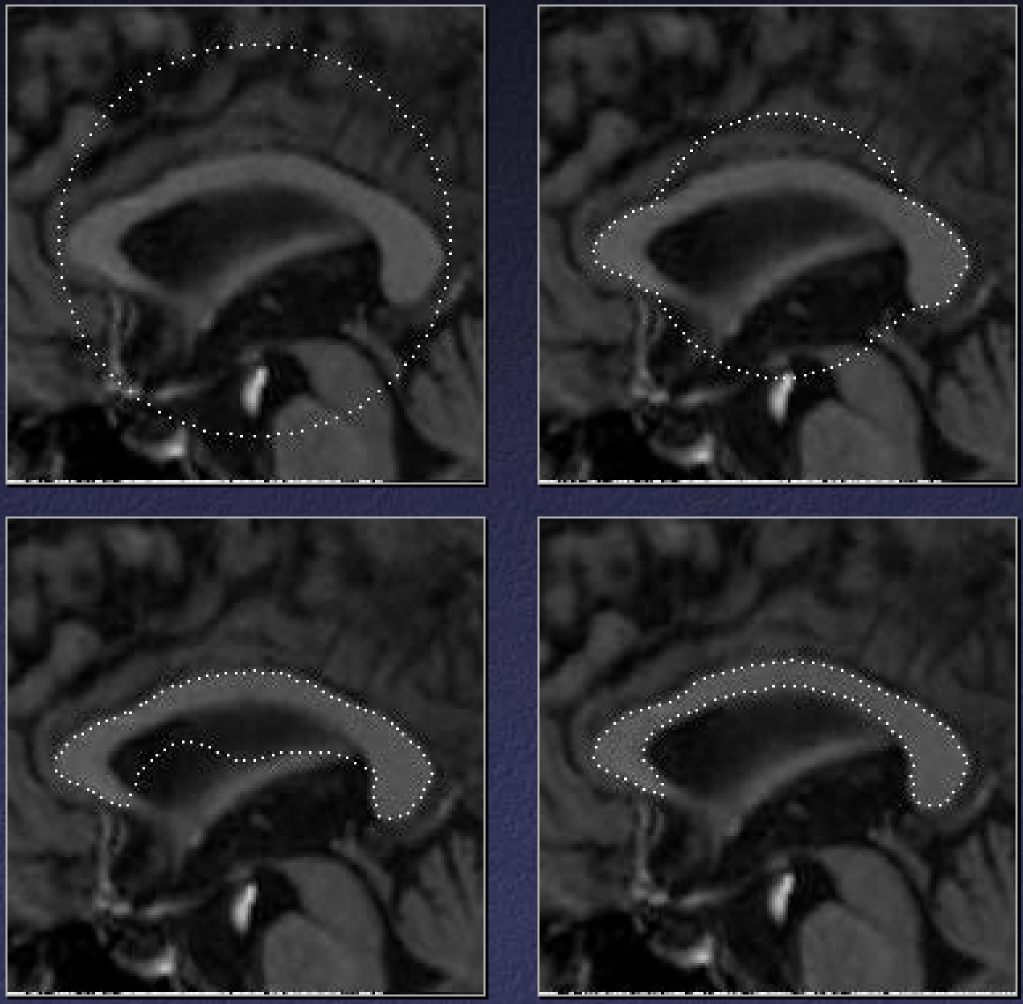
Goal

- Start with image and initial closed curve
- Evolve curve to lie along “important” features
  - Edges
  - Corners
  - Detected features
  - User input

Applications

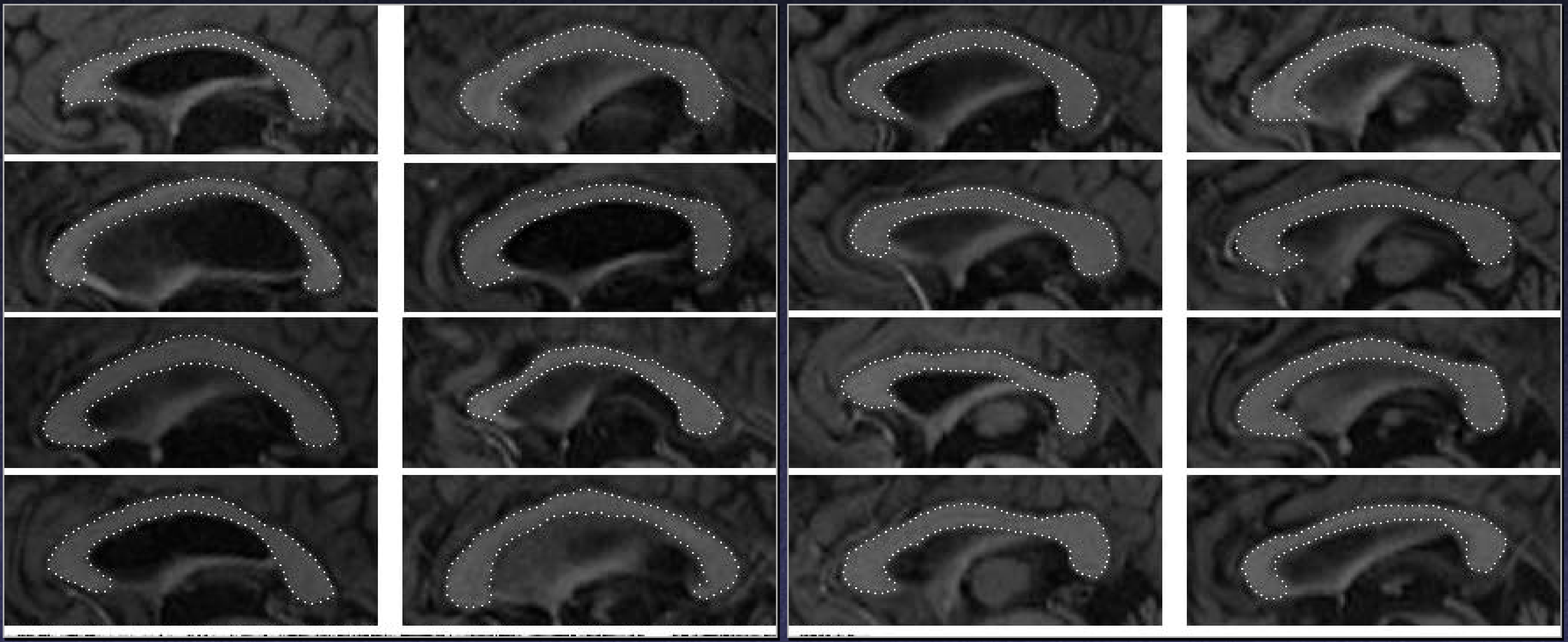
- Region selection in Photoshop
- Segmentation of medical images
- Tracking

Corpus Callosum



[Davatzikos and Prince]

Corpus Callosum



[Davatzikos and Prince]

User-Visible Options

- Initialization: user-specified, automatic
- Curve properties: continuity, smoothness
- Image features: intensity, edges, corners, …
- Other forces: hard constraints, springs, attractors, repulsors, …
- Scale: local, multiresolution, global

Behind-the-Scenes Options

- Framework: energy minimization, forces acting on curve
- Curve representation: ideal curve, sampled, spline, implicit function
- Evolution method: calculus of variations, numerical differential equations, local search

Snakes: Active Contour Models

- Introduced by Kass, Witkin, and Terzopoulos
- Framework: energy minimization
  - Bending and stretching curve = more energy
  - Good features = less energy
  - Curve evolves to minimize energy
- Also “Deformable Contours”

Snakes Energy Equation

- Parametric representation of curve

$$\mathbf{v}(s) = \big(x(s), y(s)\big)$$

- Energy functional consists of three terms

$$\varepsilon = \int \Big[\varepsilon_{\text{int}}\big(\mathbf{v}(s)\big) + \varepsilon_{img}\big(\mathbf{v}(s)\big) + \varepsilon_{con}\big(\mathbf{v}(s)\big)\Big] ds$$

Internal Energy

$$\varepsilon_{\text{int}}\left(\mathbf{v}(s)\right)=\left(\alpha(s)\left\|\mathbf{v}_s(s)\right\|^2+\beta(s)\left\|\mathbf{v}_{ss}(s)\right\|^2\right)\Big/2$$

- First term is "membrane" term – minimum energy when curve minimizes length ("soap bubble")
- Second term is "thin plate" term – minimum energy when curve is smooth

Internal Energy

$$\varepsilon_{\text{int}}(\mathbf{v}(s)) = \left(\alpha(s)\left\|\mathbf{v}_s(s)\right\|^2 + \beta(s)\left\|\mathbf{v}_{ss}(s)\right\|^2\right)/2$$

- Control α and β to vary between extremes
- Set β to 0 at a point to allow corner
- Set β to 0 everywhere to let curve follow sharp creases – “strings”

Image Energy

- Variety of terms give different effects
- For example,

$$\varepsilon_{img} = w \cdot \left| I(x, y) - I_{desired} \right|$$

minimizes energy at intensity $I_{desired}$

Edge Attraction

- Gradient-based:

$$\varepsilon_{img} = -w \cdot \left\| \nabla I(x, y) \right\|^2$$

- Laplacian-based:

$$\varepsilon_{img} = w \cdot \left| \nabla^2 I(x, y) \right|^2$$

- In both cases, can smooth with Gaussian

Corner Attraction

- Can use corner detector we saw last week
- Alternatively, let $\theta = \tan^{-1} I_y / I_x$
  and let $\mathbf{n}_\perp$ be a unit vector perpendicular to the gradient. Then

$$\varepsilon_{img} = w \cdot \left| \frac{\partial \theta}{\partial \mathbf{n}_\perp} \right|$$

Constraint Forces

- Spring

$$\varepsilon_{con} = k \cdot \left\| \mathbf{v} - \mathbf{x} \right\|^2$$

- Repulsion

$$\varepsilon_{con} = \frac{k}{\left\| \mathbf{v} - \mathbf{x} \right\|^2}$$

Evolving Curve

- Computing forces on v that locally minimize energy gives differential equation for v
  - Euler-Lagrange formula

$$\frac{d^2}{ds^2}\left(\frac{\partial \varepsilon}{\partial \ddot{v}}\right)+\frac{d}{ds}\left(\frac{\partial \varepsilon}{\partial \dot{v}}\right)+\frac{\partial \varepsilon}{\partial v}=0$$

- Discretize *v*: samples (x_i, y_i)
  - Approximate derivatives with finite differences
- Iterative numerical solver

Other Curve Evolution Options

- Exact solution: calculus of variations
- Write equations directly in terms of forces, not energy
- Implicit equation solver
- Search neighborhood of each (x_i, y_i) for pixel that minimizes energy
  - Shah & Williams paper

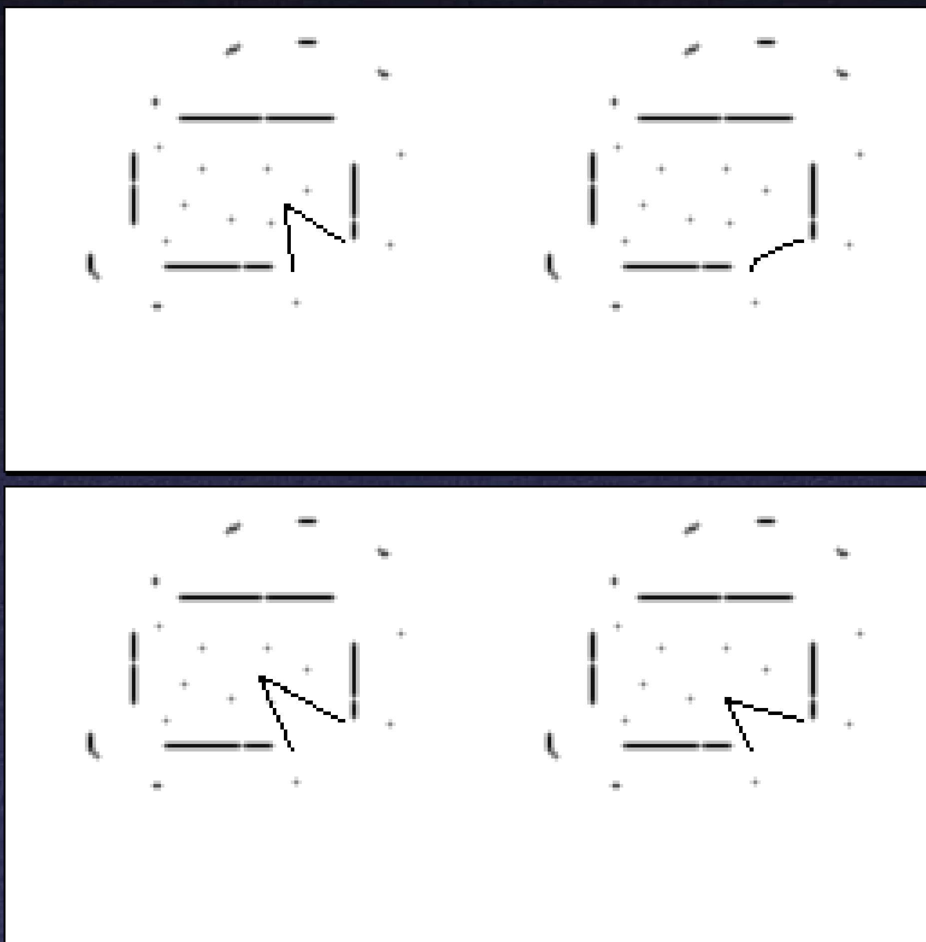
Variants on Snakes

- Balloons [Cohen 91]
  - Add inflation force

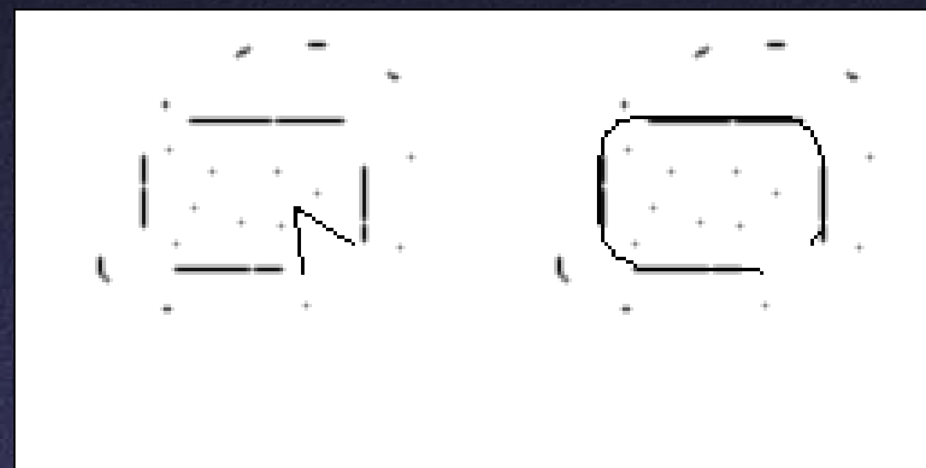
$$F_{infl} = k\,\mathbf{n}(s)$$

  - Helps avoid getting stuck on small features

Balloons



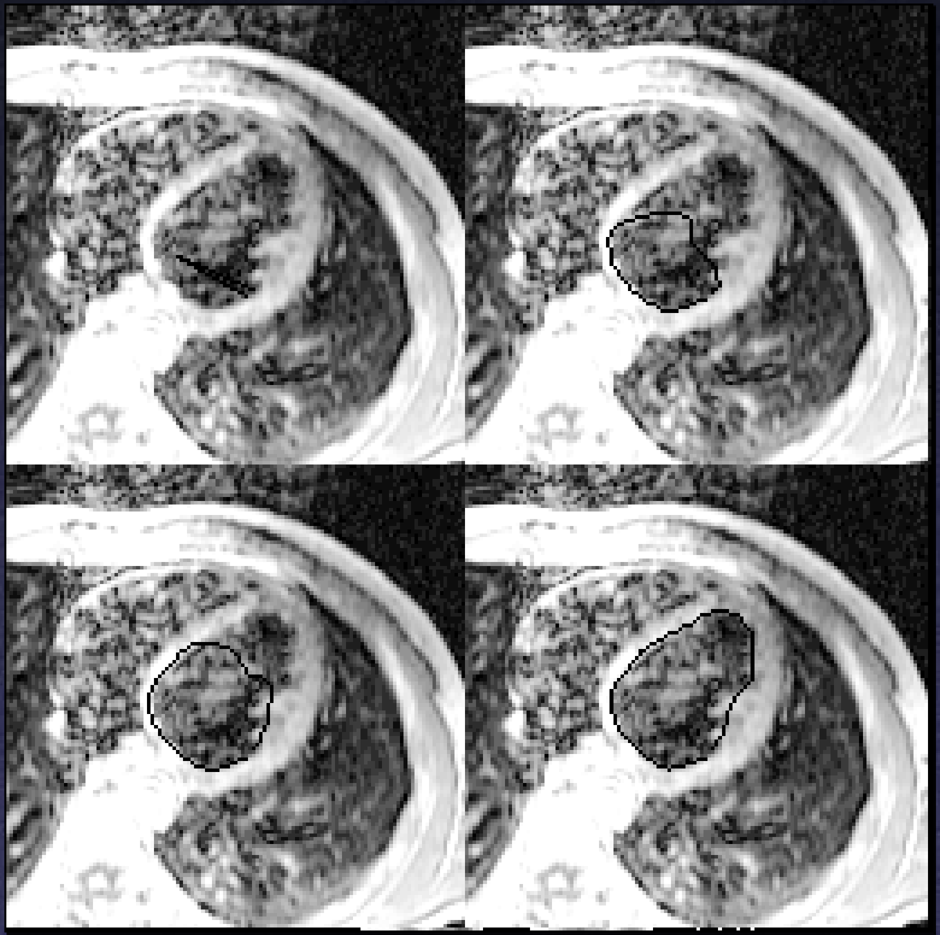
Snakes



Balloons

[Cohen 91]

Balloons

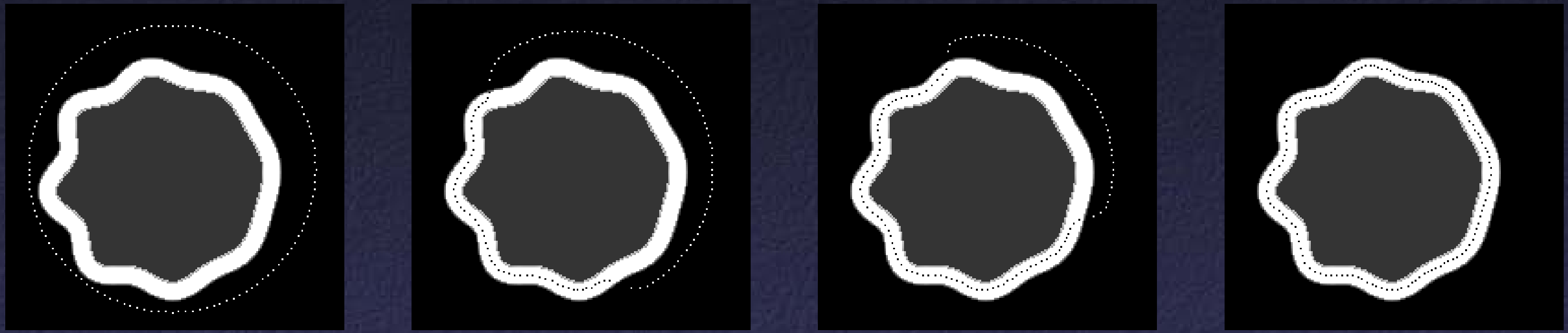


[Cohen 91]

Other Energy or Force Terms

- Results of previously-run local algorithms
  - e.g., Canny edge detector output convolved with Gaussian
- Automatically-evolved control points
- Others…

Brain Cortex Segmentation



Add energy term for constant-color regions of a single color

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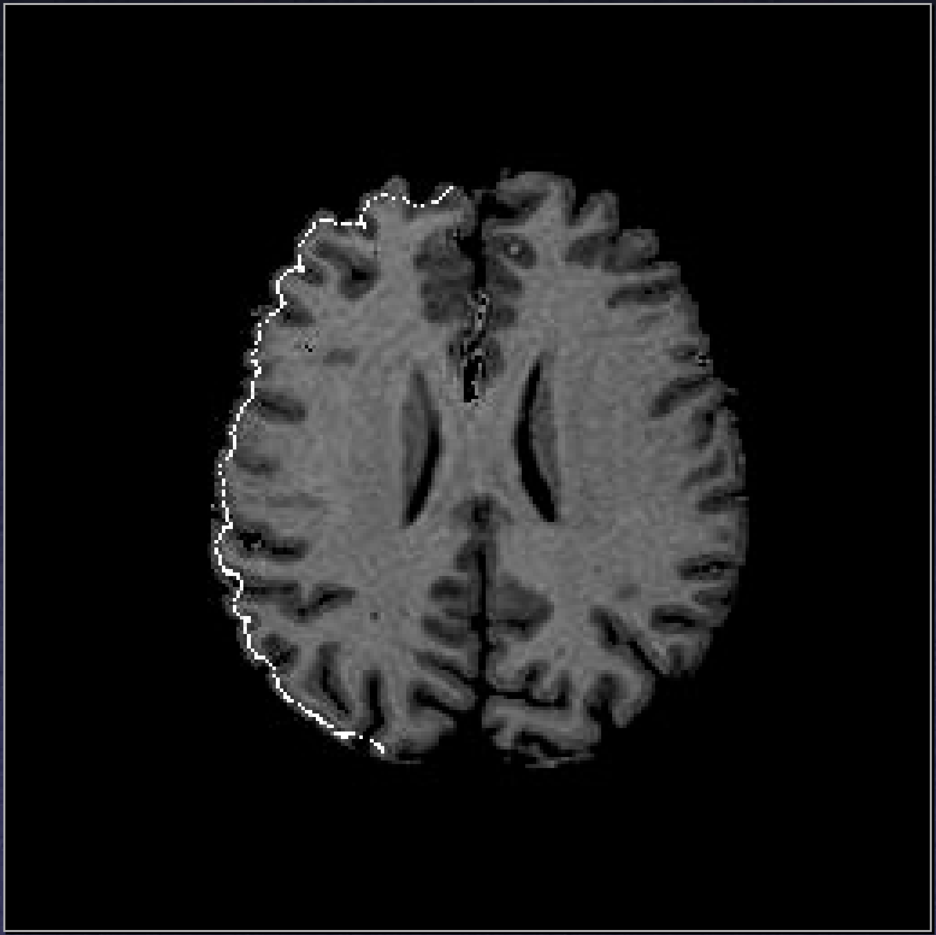
Brain Cortex Segmentation



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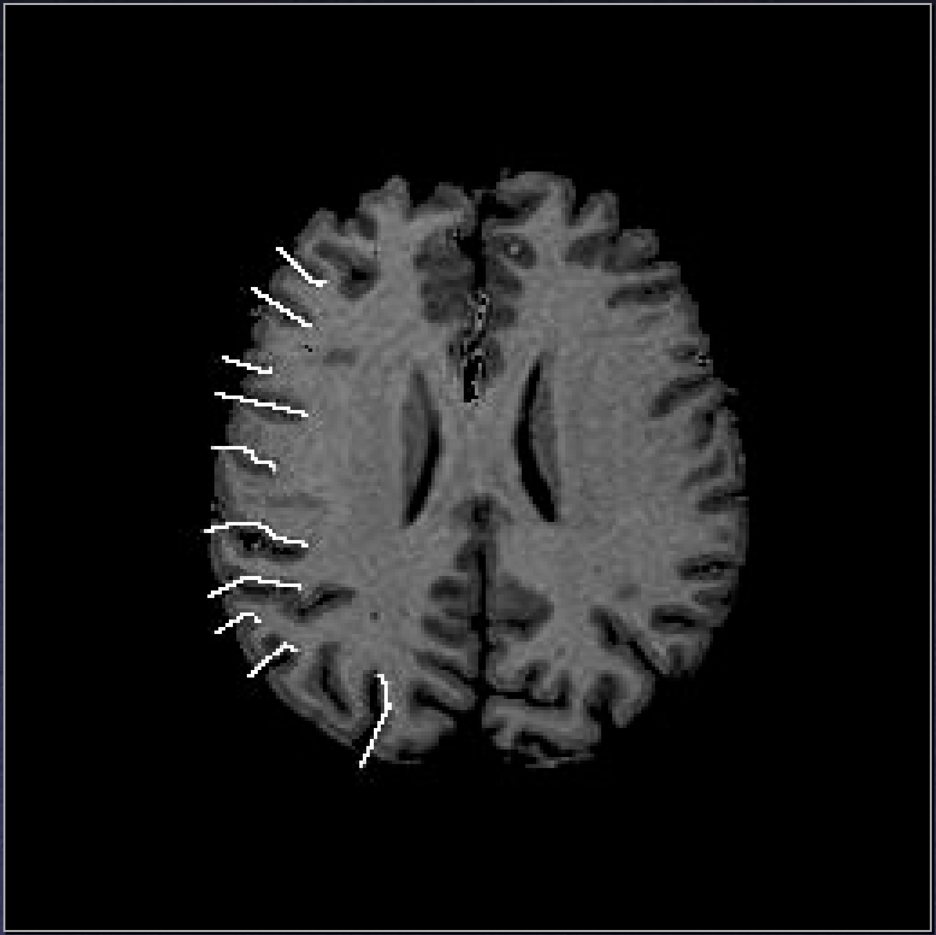
Brain Cortex Segmentation



Find features and add constraints

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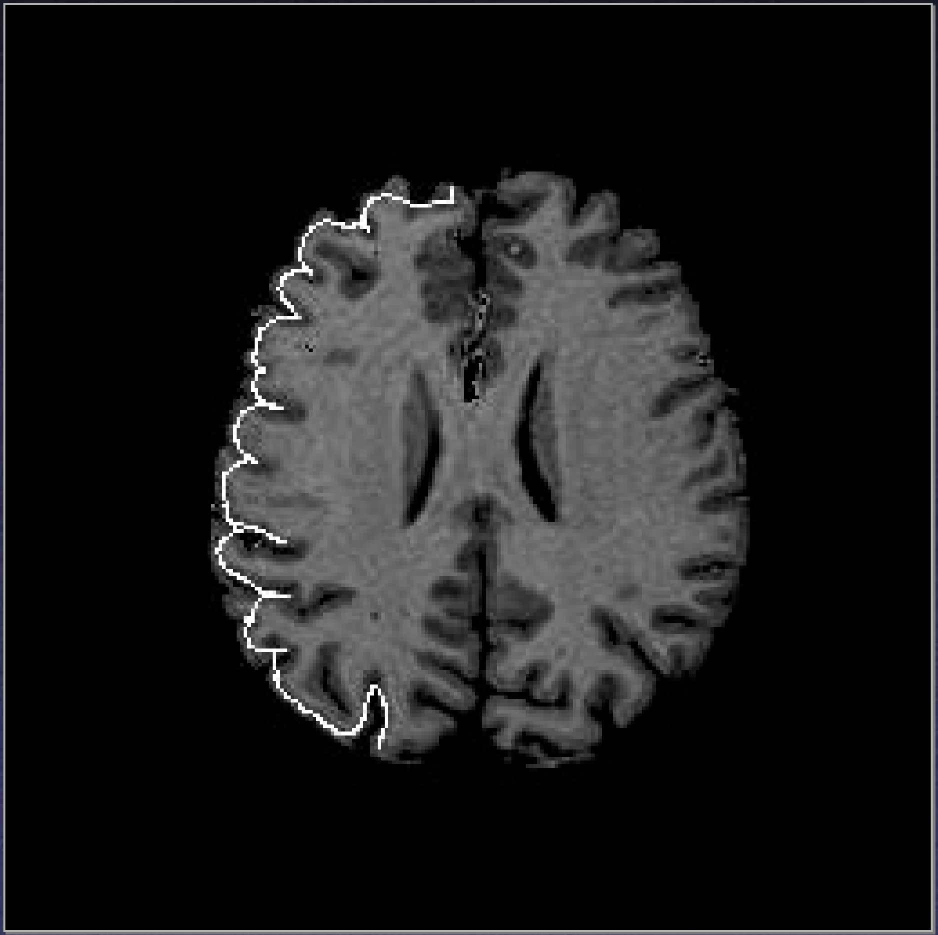
Brain Cortex Segmentation



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Scale

- In the simplest snakes algorithm, image features only attract locally
- Greater region of attraction: smooth image
  - Curve might not follow high-frequency detail
- Multiresolution processing
  - Start with smoothed image to attract curve
  - Finish with unsmoothed image to get details
- Heuristic for global minimum vs. local minima

Diffusion-Based Methods

- Another way to attract curve to localized features: vector flow or diffusion methods
- Example:
  - Find edges using Canny
  - For each point in entire image, compute distance to nearest edge
  - Push curve along gradient of distance field

Gradient Vector Fields

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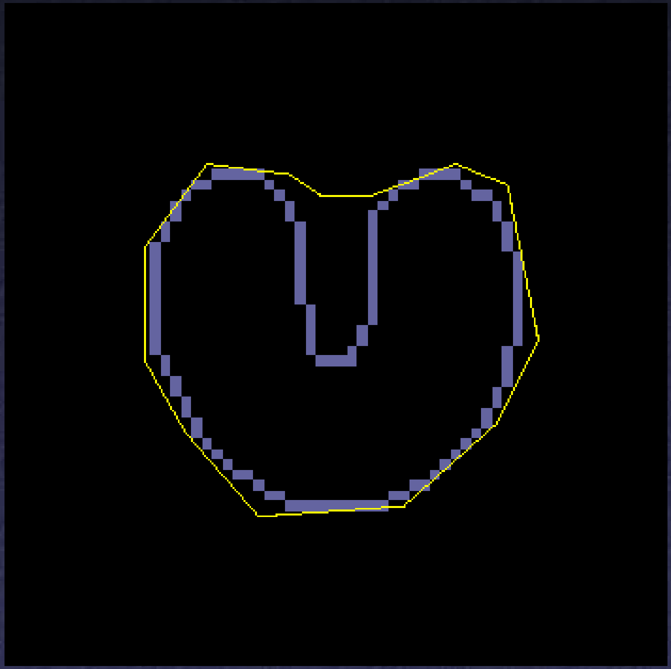
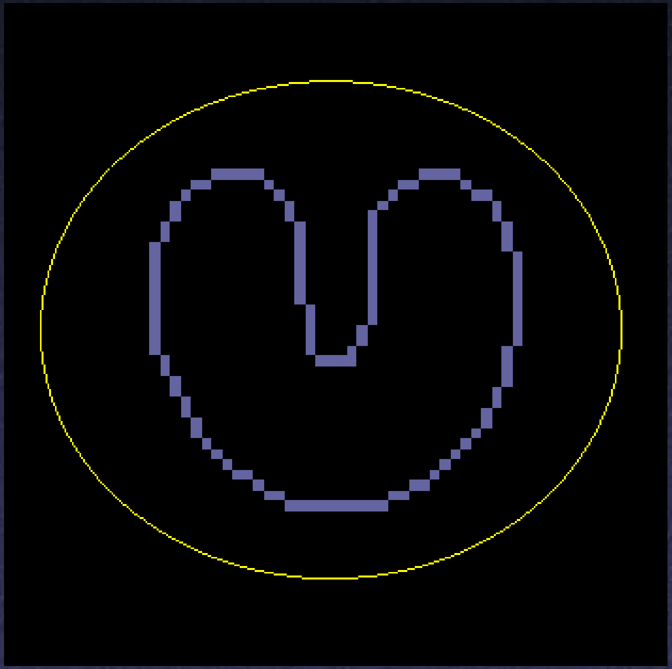
Gradient Vector Fields



Simple Snake



With Gradient Vector Field

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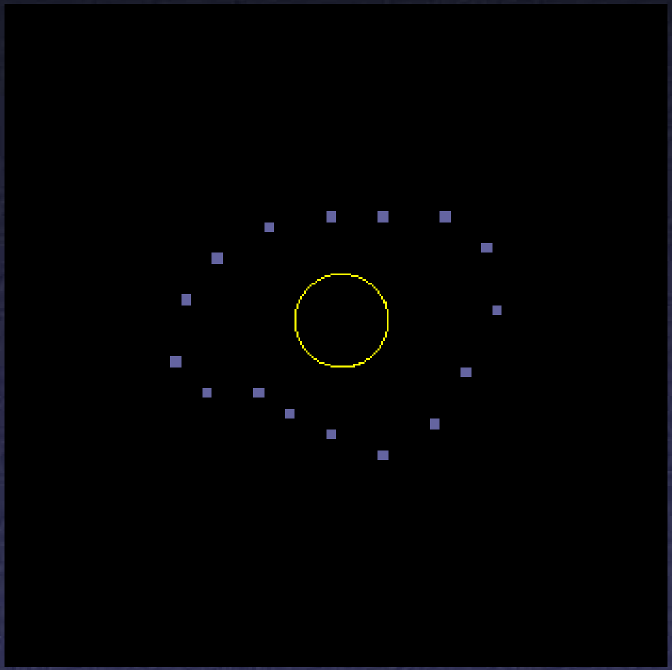
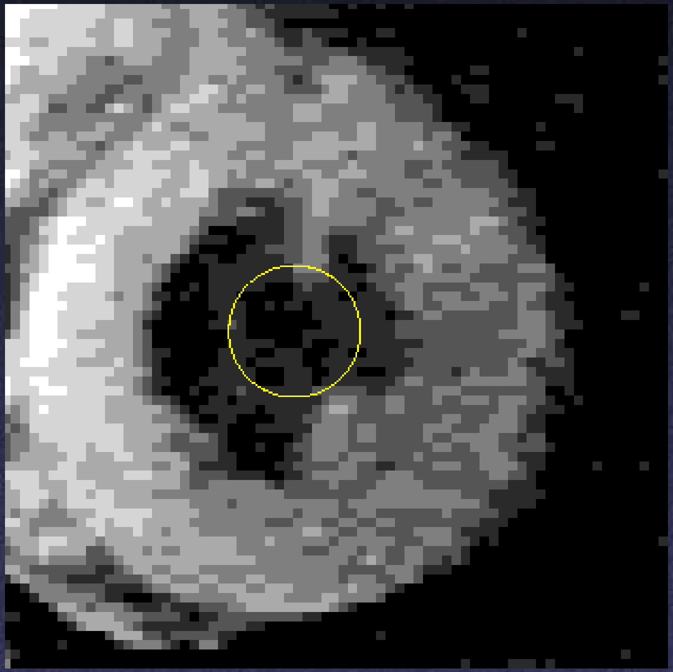
Gradient Vector Fields





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