Ray Tracing

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Princeton University
COS 526, Fall 2010

Overview

- Rendering equation
  - Rendering is integration
- Solution methods
  - Direct illumination
  - Radiosity
  - Ray tracing
  - Path tracing

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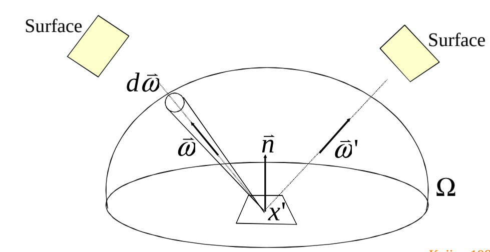
Rendering Equation

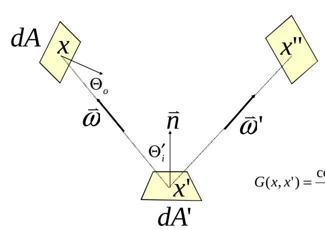
$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}')L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n})d\vec{\omega}$$



Kajiya 1986

Rendering Equation (2)

$$L(x'\to x'') = L_e(x'\to x'') + \int_{S} f_r(x\to x'\to x'')L(x\to x')V(x,x')G(x,x')dA$$



$$G(x,x') = \frac{\cos\Theta_i' \cos\Theta_o}{\|x-x'\|^2}$$

Kajiya 1986

Rendering Equation

- Rendering = integration
  - Antialiasing
  - Soft shadows
  - Indirect illumination
  - Caustics

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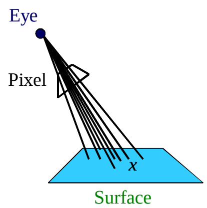

$$L_P = \int_S L(x \rightarrow e)\, dA$$

Rendering Equation

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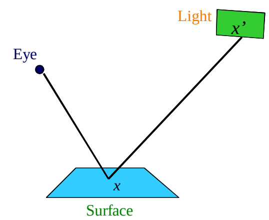

$$L(x,\vec{w}) = L_e(x,x \rightarrow e) + \int_S f_r(x,x' \rightarrow x, x \rightarrow e) L(x' \rightarrow x) V(x,x') G(x,x')\, dA$$

Rendering Equation

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Herf

$$L(x,\vec{w}) = L_e(x,x \rightarrow e) + \int_S f_r(x,x' \rightarrow x, x \rightarrow e) L(x' \rightarrow x) V(x,x') G(x,x')\, dA$$

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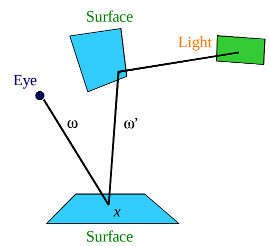

$$L_o(x,\vec{w}) = L_e(x,\vec{w}) + \int_\Omega f_r(x,\vec{w}',\vec{w}) L_i(x,\vec{w}')(\vec{w}' \bullet \vec{n})\, d\vec{w}$$

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Debevec

$$L_o(x,\vec{w}) = L_e(x,\vec{w}) + \int_\Omega f_r(x,\vec{w}',\vec{w}) L_i(x,\vec{w}')(\vec{w}' \bullet \vec{n})\, d\vec{w}$$

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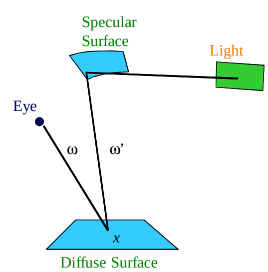

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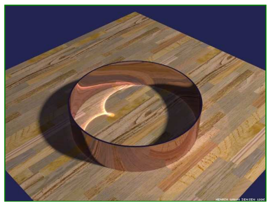
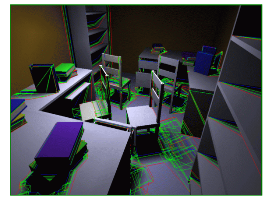


Jensen

$$L_o(x,\vec{w}) = L_e(x,\vec{w}) + \int_{\Omega} f_r(x,\vec{w}',\vec{w}) L_i(x,\vec{w}')(\vec{w}' \bullet \vec{n}) d\vec{w}$$

Challenge

- Rendering integrals are difficult to evaluate
  - Multiple dimensions
  - Discontinuities
    - » Partial occluders
    - » Highlights
    - » Caustics



Drettakis

$$L(x,\vec{w}) = L_e(x,x \rightarrow e) + \int_{S} f_r(x,x' \rightarrow x, x \rightarrow e) L(x' \rightarrow x) V(x,x') G(x,x') dA$$

Challenge

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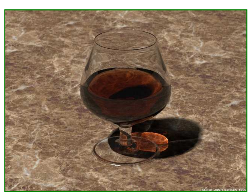

Jensen

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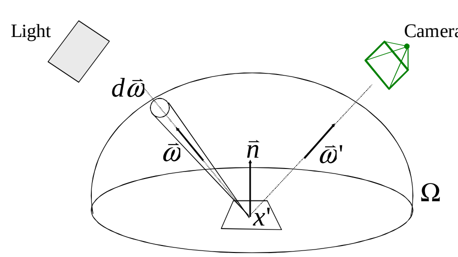
Direct Illumination

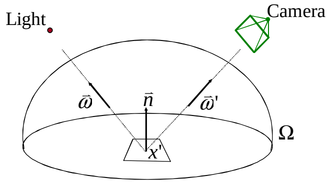
$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega_L} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n}) d\vec{\omega}$$



OpenGL

$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}')L_i(x',\vec{\omega})(\vec{\omega}\bullet\vec{n})d\vec{\omega}$$

Assume direct illumination from point lights and ignore visibility



$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \sum_{i=1}^{nlights} f_r(x',\vec{\omega},\vec{\omega}')L_i(x',\vec{\omega})(\vec{\omega}\bullet\vec{n})$$

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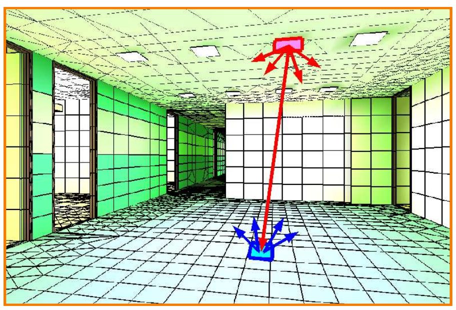
Radiosity



Radiosity

$$L(x'\to x'') = L_e(x'\to x'') + \int_S f_r(x\to x'\to x'')L(x\to x')V(x,x')G(x,x')dA$$

Assume everything is Lambertian

$$\rho(x') = f_r(x\to x'\to x'')\pi$$

$$L(x') = L_e(x') + \frac{\rho(x')}{\pi}\int_S L(x)V(x,x')G(x,x')dA$$

Convert to Radiosities

$$B = \int_{\Omega} L_o \cos\theta d\omega \qquad L = \frac{B}{\pi}$$

$$B(x') = B_e(x') + \frac{\rho(x')}{\pi}\int_S B(x)V(x,x')G(x,x')dA$$

Radiosity

$$B(x') = B_e(x') + \frac{\rho(x')}{\pi}\int_S B(x)V(x,x')G(x,x')dA$$

Discretize the surfaces into "elements"

$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

Rearrange terms and write as matrix

$$\begin{bmatrix} 1-\rho_1 F_{1,1} & . & . & . & -\rho_1 F_{1,n} \\ -\rho_2 F_{2,1} & 1-\rho_2 F_{2,2} & . & . & -\rho_2 F_{2,n} \\ . & . & . & . & . \\ . & . & . & . & . \\ -\rho_{n-1} F_{n-1,1} & . & . & . & -\rho_{n-1} F_{n-1,n} \\ -\rho_n F_{n,1} & . & . & . & 1-\rho_n F_{n,n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ . \\ . \\ . \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ . \\ . \\ . \\ E_n \end{bmatrix}$$

Radiosity

Radiosity?

Paul Debevec

Radiosity?

Herf

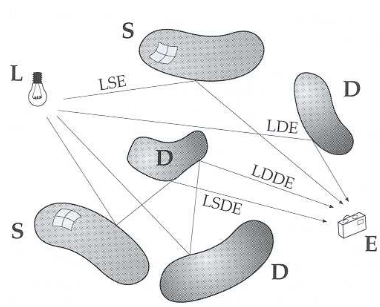
Radiosity?

Jensen

Radiosity?

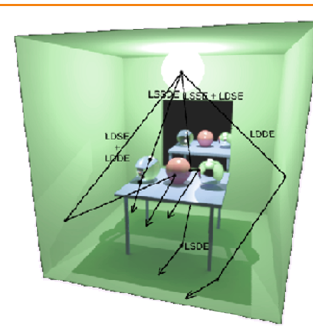
Henrik Wann Jensen

Path Types



Rendering Methods – Path Types

- Radiosity
  - LD*E
- Ray tracing
  - LDS*E
- Path tracing
  - L(D|S)*E
- OpenGL
  - LDE



John Hart

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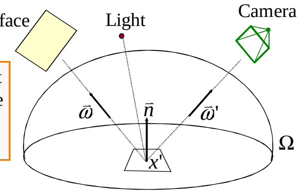
Ray Tracing

$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n}) d\vec{\omega}$$



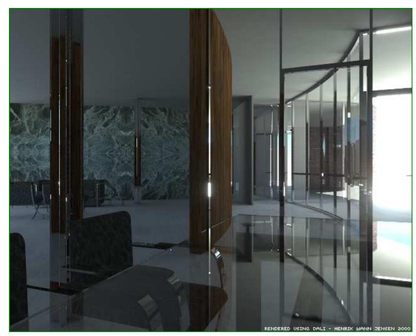
Assume only significant indirect illumination due to perfect specular reflection and refraction

$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \sum_{i=1}^{nlights} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n}) + specular$$

Ray Tracing?

Paul Debevec

Ray Tracing?

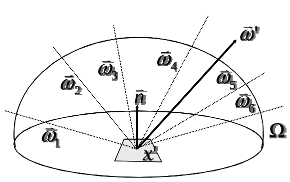


Jensen

Distribution Ray Tracing

$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n}) d\vec{\omega}$$

Estimate integral for each reflection by random sampling



Also:
- Depth of field
- Motion blur
- etc.

Distribution Ray Tracing?

Henrik Wann Jensen

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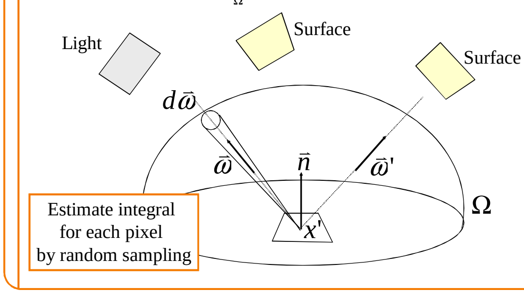
Path Tracing

$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}')L_i(x',\vec{\omega})(\vec{\omega}\bullet\vec{n})d\vec{\omega}$$



Path Tracing

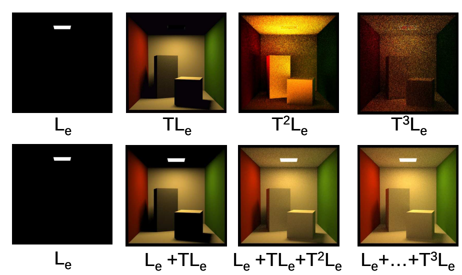
$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}')L_i(x',\vec{\omega})(\vec{\omega}\bullet\vec{n})d\vec{\omega}$$

Perform Neumann series expansion

$L = L_e + TL$ where $T(x,\vec{\omega}') = \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}')g(x,\vec{\omega})(\vec{\omega}\bullet\vec{n})d\vec{\omega}$

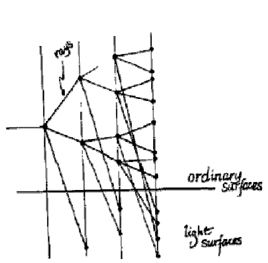
$$L = L_e + TL_e + T^2L_e + T^3L_e + \ldots$$

Path Tracing

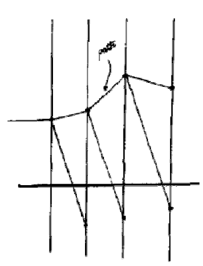


Dutré

Ray Tracing vs. Path Tracing



Ray tracing

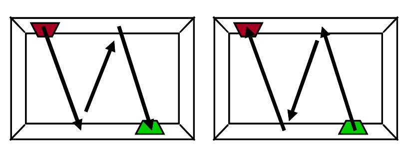


Path tracing

Kajiya

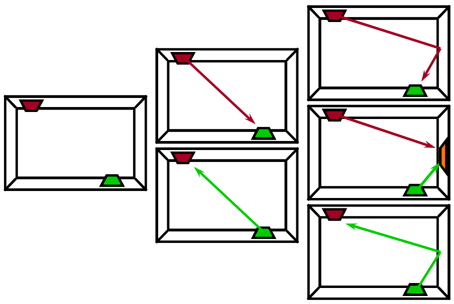
Bidirectional Path Tracing

- Role of source and receiver can be switched, flux does not change



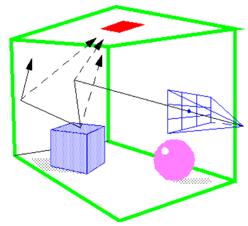
Dutré

Bidirectional Path Tracing



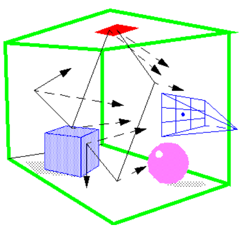
Dutré

Tracing From Eye



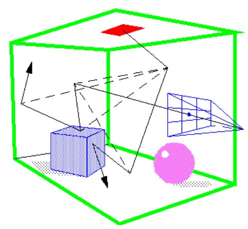
Dutré

Tracing from Lights



Dutré

Bidirectional Path Tracing



Dutré

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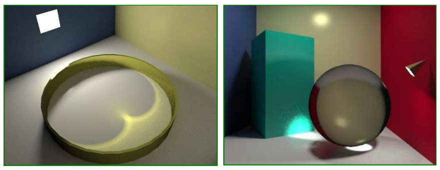

(RenderPark 98)

Dutré

Bidirectional Ray Tracing?

Henrik Wann Jensen

Summary

- Rendering is integration
  - Rendering equation
- Different solution methods for different path types
  - OpenGL - LDE
  - Radiosity - LD*E
  - Ray tracing – LDS*E
  - Path tracing – L(SD)*E