Radiosity
COS 526, Fall 2010

Overview
- Radiosity equation
- Solution methods
  - Computing form factors
  - Selecting basis functions for radiosities
  - Solving linear system of equations
  - Meshing surfaces into elements
  - Rendering images

Radiosity Equation
Assume everything is Lambertian
\[ \mathbf{L}(x \rightarrow z \rightarrow x') = \mathbf{L}(x')/z \]
Convert to Radiosities
\[ \mathbf{R} = \sum_x \mathbf{L} \cdot \mathbf{S} \]

Rendering Equation

Radiosity Approximation
Discretize surfaces into elements
\[ r(x) = \frac{1}{4} \sum_{x'} I(x') \frac{S(x \rightarrow x')}{S(x') \cdot S(x)} \]
where \( r(x) \) is the radiosity at point \( x \).
System of Equations

\[ \begin{align*}
    x - x_0 &= \sum_{j \neq i} \frac{e_j}{|e_j|} \cdot \sum_{k \neq i} \eta_{jk} \\
    e_i &= \sum_{j \neq i} \frac{e_j}{|e_j|} \cdot \sum_{k \neq i} \eta_{jk}
\end{align*} \]

Intuition

Overview

- Radiosity equation
- Solution methods
  - Computing form factors
  - Selecting basis functions for radiosities
  - Solving linear system of equations
  - Meshing surfaces into elements
  - Rendering images

Form Factor

- Fraction of energy leaving element \( i \) that arrives at element \( j \)

\[ F_{ij} = \frac{1}{4} \sum_{k \neq j} \frac{e_i \cdot e_k}{|e_i| |e_k|} d_i d_k \]

Form Factor Intuition

Projection to hemisphere

\[ F_{\text{proj}} = \frac{1}{4} \sum_{k \neq j} \frac{e_i \cdot e_k}{|e_i| |e_k|} \]

Projection to disk

\[ \text{Divide by area of disk} \]

Computing Form Factors

Cohen & Wallace
Computing Form Factors

- Derive equation for projected area
  - Possible only for simple cases

Analytic Form Factors

- Compute form factor with image-space precision
  - Render scene from centroid of $A_i$
  - Use z-buffer to determine visibility of other surfaces
  - Count “pixels” to determine projected areas

Hemicube

- Approximating $A_i$ with point leads to errors
  - Regular sampling leads to aliasing artifacts

Monte Carlo Sampling

- Compute form factor by random sampling
  - Select random points on elements
  - Intersect line segment to evaluate $V_{ij}$
  - Evaluate $F_{ij}$ by Monte Carlo integration

$$F_{ij} = \frac{1}{A_i} \int \frac{V_i \cos \theta_i \cos \theta}{\Delta^2} dA_i dA_j$$
Overview

- Radiosity equation
- Solution methods
  - Computing form factors
  - Selecting basis functions for radiosities
  - Solving linear system of equations
  - Meshing surfaces into elements
  - Rendering images

Selecting a Basis Function

- Store radiosity function on surface mesh
  - Piecewise-constant, piecewise-linear, wavelets, etc.

Overview

- Radiosity equation
- Solution methods
  - Computing form factors
  - Selecting basis functions for radiosities
  - Solving linear system of equations
  - Meshing surfaces into elements
  - Rendering images

Solving the System of Equations

- Challenges:
  - Size of matrix
  - Cost of computing form factors
  - Computational complexity

Solving the System of Equations

- Challenges:
  - Size of matrix
  - Cost of computing form factors
  - Computational complexity

Gauss-Seidel Iteration

1. for all $i$
2. $B_i = E_i$
3. while not converged
4. for each $i$ in turn
5. $B_i = E_i + \rho \sum_{j \neq i} B_j E_{ij}$
6. display the image using $B_i$ as the intensity of patch $i$. 
Gauss-Seidel Iteration

- Iteratively relax rows of linear system
- Effectiveness depends on sparsity of matrix

Progressive Radiosity

1. for all \( i \)
2. \( B_i = E_i \)
3. \( \Delta B_i = E_i \)
4. while not converged
5. pick \( i \) such that \( \Delta B_i + A_i \) is largest
6. for every patch \( j \)
7. \( \Delta \text{rad} = \Delta B_i \ast \mu_j \rho_{jj} \)
8. \( \Delta B_j = \Delta B_j + \Delta \text{rad} \)
9. \( B_j = B_j + \Delta \text{rad} \)
10. \( \Delta B_i = 0 \)
11. display the image using \( B_i \) as the intensity of patch \( i \).

Progressive Radiosity

- Iteratively shoot “unshot” radiosity from elements
- Select shooters in order of unshot radiosity

Progressive Radiosity

- Adaptive refinement

Graph showing comparison of Gauss-Seidel and Progressive Radiosity

Progressive Radiosity

The above images show increasing levels of global diffuse illumination. From left to right: 0 bounces, 1 bounce, 3 bounces.
Progressive Radiosity

Overview

- Radiosity equation
- Solution methods
  - Computing form factors
  - Selecting basis functions for radiosities
  - Solving linear system of equations
  - Meshing surfaces into elements
  - Rendering images

Surface Meshing Goals

- Store radiosity across surface
  - Represents function well
  - Few elements
  - Few visible artifacts

Artifacts of Bad Surface Meshing

Figure 6.3: Error image. Figure 6.4: Error image.

Error Image
Adaptive Meshing

- Refine mesh in areas of high residual

Adaptive Meshing

Uniform mesh
Adaptive mesh

Error Comparison

Uniform
Adaptive

Adaptive Meshing

Adaptive Meshing
Discontinuity Meshing

• Capture discontinuities in radiosity across a surface with explicit mesh boundaries

Hierarchical Radiosity

• Estimate errors, refine elements if too large

Hierarchical Radiosity

• Select resolution of interaction based on error tolerance
Hierarchical Radiosity

Overview

- Radiosity equation
- Solution methods
  - Computing form factors
  - Selecting basis functions for radiosity
  - Solving linear system of equations
  - Meshing surfaces into elements
  - Rendering images

Displaying Radiosity

- Usually, simple interpolation (Gouraud shading)
- Can also try to preserve discontinuities...

Extensions

- Non-diffuse environments
  - Directional radiosity functions
  - Extended form factors
  - Multipass methods
- Participating media
  - Path integrals in form factors
- Dynamic scenes
  - Incremental updates
- Parallel solvers
  - Decomposition
  - Scheduling