Radiometry and Light Transport

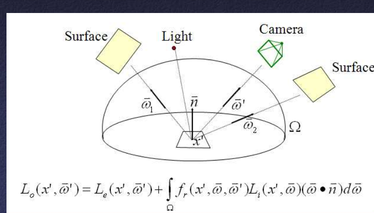
COS 526, Fall 2010

Overview

- Rendering equation
- Radiometry
- Local light transport
- Definition of BRDF
- BRDF properties and common BRDFs

Rendering Equation

- Compute radiance in outgoing direction by integrating reflections over all incoming directions



Rendering Equation

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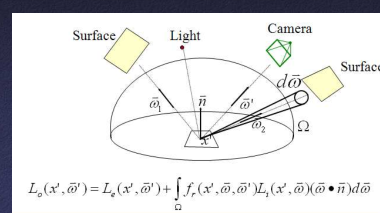

Rendering Equation

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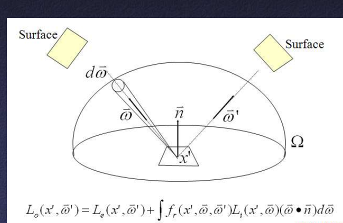
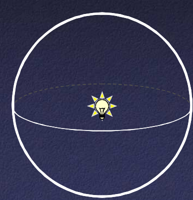


Radiometric Units

- Light is a form of energy
  - Measured in Joules (J)
- Power: energy per unit time
  - Measured in Joules/sec = Watts (W)
  - Also called Radiant Flux (Φ)

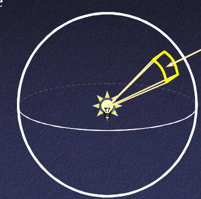
Isotropic Point Source

- Radiant flux leaves point source in all directions
- Flux distributed evenly over sphere



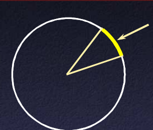
Point Light Source in a Direction

- How to define radiant flux for one direction?
  - Solid angle



Digression – Solid Angle

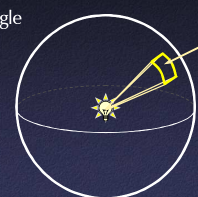
- Angle in radians



- Solid angle in steradians

Point Light Source in a Direction

- How to define radiant flux for one direction?
  - Solid angle



- *Irradiance* (E) = radiant flux per unit solid angle
  - Measured in Watts per steradian (W/sr)

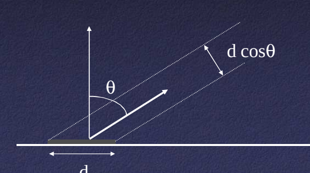
Light Falling on a Surface from a Direction

- Power per unit area – *Irradiance* (E)
  - Measured in W/m^2
- Move surface away from light
  - Inverse square law: $E \sim 1/r^2$

- Tilt surface away from light
  - Cosine law: $E \sim \mathbf{n} \cdot \mathbf{l}$

Why the Cosine Term?

- Foreshortening is by cosine of angle.
- Radiance gives energy by *effective* surface area.



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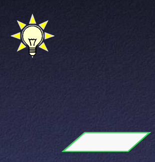
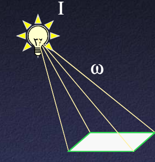


$$E = \frac{\Phi}{A}$$

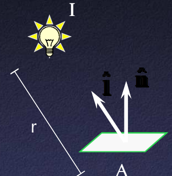
Light Falling on a Surface from a Direction



$$E = \frac{\Phi}{A}$$

$$\Phi = I\omega$$

Light Falling on a Surface from a Direction



$$E = \frac{\Phi}{A}$$

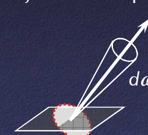
$$\Phi = I\omega$$

$$\omega = \frac{A(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})}{r^2}$$

$$\Rightarrow E = \frac{I(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})}{r^2}$$

Light Emitted from a Surface in a Direction

- Power per unit area per unit solid angle – *Radiance* (L)
  - Measured in $W/m^2/sr$
  - *Projected area* – perpendicular to given direction



$$L = \frac{d\Phi}{dA_p\, d\omega}$$

$$L = \frac{d\Phi}{dA \cos\theta \;\, d\vec{\omega}}$$

Irradiance from Radiance

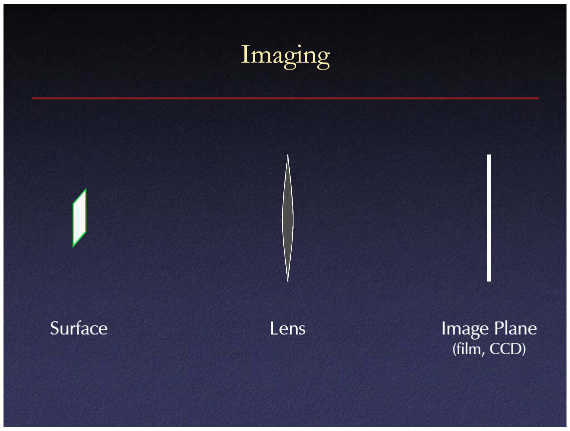
$$E = \int_{\Omega} L\cos\theta \, d\omega$$

- $\cos\theta \;\, d\omega$ is projection of a differential area

Radiance as a unit of measure

- Radiance doesn't change with distance
  - Therefore it's the quantity we want to measure in a ray tracer.
- Radiance proportional to what a sensor (camera, eye) measures.
  - Therefore it's what we want to output.

Imaging



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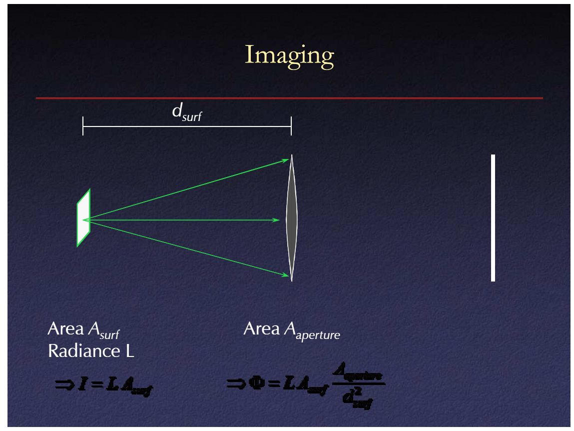

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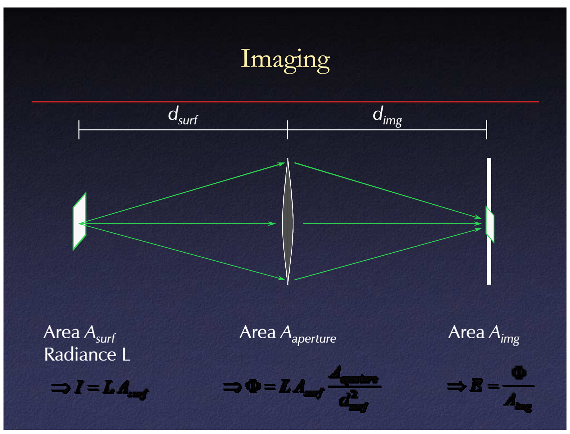


Imaging

$I = L\,A_{surf}$ $\quad \Phi = L\,A_{surf}\frac{A_{aperture}}{d_{surf}^2}$ $\quad E = \frac{\Phi}{A_{img}}$

$$E = L\frac{A_{aperture}A_{surf}}{d_{surf}^2 A_{img}}$$

$$\frac{A_{surf}}{A_{img}} = \left(\frac{d_{surf}}{d_{img}}\right)^2$$

$$E = L\frac{A_{aperture}}{d_{img}^2}$$

← Depends only on camera

- Punch line: cameras “see” radiance

Surface Reflectance – BRDF

- Reflected radiance is proportional to incoming flux and to irradiance (incident power per unit area).

$$dL_r(\vec{\omega}_r) \propto dE(\vec{\omega}_i)$$

Surface Reflectance – BRDF

- Bidirectional Reflectance Distribution Function

$$f_r(\omega_i \to \omega_o) = \frac{L_o(\omega_o)}{E_i(\omega_i)}$$

- 4-dimensional function: also written as

$$f_r(\theta_i,\varphi_i,\theta_o,\varphi_o) = \frac{L_o(\theta_o,\varphi_o)}{E_i(\theta_i,\varphi_i)}$$

(the symbol ρ is also used sometimes)

Surface Reflectance – BRDF

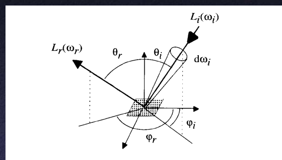


Figure 2.9: *Bidirectional reflection distribution function.*

$$f_r(\vec{\omega}_i \rightarrow \vec{\omega}_r) \equiv \frac{L_r(\vec{\omega}_r)}{L_i(\vec{\omega}_i)\cos\theta_i d\omega_i}$$

Properties of the BRDF

- Energy conservation:

$$\int_\Omega f_r(\theta_i,\varphi_i,\theta_o,\varphi_o)\cos\theta_o d\omega_o \leq 1$$

- Helmholtz reciprocity:

$$f_r(\omega_i \rightarrow \omega_o) = f_r(\omega_o \rightarrow \omega_i)$$

(not always obeyed by "BRDFs" used in graphics)

Isotropy

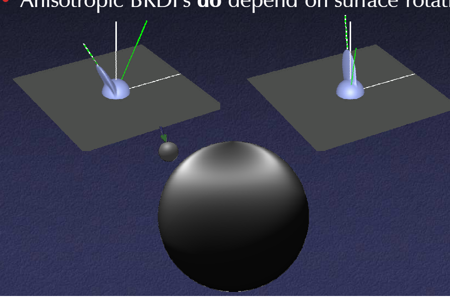
- A BRDF is isotropic if it stays the same when surface is rotated around normal

- Isotropic BRDFs are 3-dimesional functions:

$$f_r(\theta_i,\theta_o,\varphi_i-\varphi_o)$$

Anisotropy

- Anisotropic BRDFs **do** depend on surface rotation



BRDF Representations

- Physically-based vs. phenomenological models
- Measured data
- Desired characteristics:
  - Fast to evaluate
  - Maintain reciprocity, energy conservation
  - For global illumination: easy to importance sample

Diffuse

- The simplest BRDF is "ideal diffuse" or *Lambertian*: just a constant

$$f_r(\omega_i \rightarrow \omega_o) = k_d$$

- Note: does *not* include $\cos(\theta_i)$
  - Remember definition of irradiance

Diffuse BRDF

- Assume BRDF reflects a fraction ρ of light

$$\int_{\Omega} f_{r,Lambertian}(\omega_i \rightarrow \omega_o)\cos\theta_o \, d\omega_o = \rho$$

$$\int_{\substack{\theta\in[0,\pi/2]\\ \varphi\in[0,2\pi]}} k_d \cos\theta_o \sin\theta_o \, d\theta_o d\varphi_o = \rho$$

$$2\pi k_d \int_{\theta\in[0,\pi/2]} \sin\theta_o \cos\theta_o \, d\theta_o = \rho$$

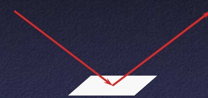
$$\pi k_d = \rho$$

$$\therefore f_{r,Lambertian} = \frac{\rho}{\pi}$$

- The quantity ρ is called the albedo

Ideal Mirror

- All light incident from one direction is reflected into another



- BRDF is zero everywhere except where

$$\theta_o = \theta_i$$

$$\varphi_o = \varphi_i + \pi$$

Ideal Mirror

- To conserve energy,

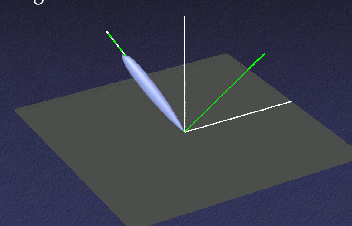
$$\int_{\Omega} f_{r,Mirror}(\omega_i \rightarrow \omega_o)\cos\theta_o \, d\omega_o = \rho$$

- So, BRDF is a delta function at direction of ideal mirror reflection

$$f_{r,Mirror} = \frac{\delta(\theta_i - \theta_o)\delta(\varphi_i + \pi - \varphi_o)}{\cos(\theta_i)}$$

Glossy Reflection

- Non-ideal specular reflection
- Most light reflected *near* ideal mirror direction



Phong BRDF

- Phenomenological model for glossy reflection

$$f_{r,Phong} = k_s(\hat{l}\cdot\hat{r})^n$$

l is a vector to the light source
r is the direction of mirror reflection

- Exponent n determines width of specular lobe
- Constant k_s determines size of lobe

Torrance-Sparrow BRDF

- Physically-based BRDF model
  - Originally used in the physics community
  - Adapted by Cook & Torrance and Blinn for graphics

$$f_{r,T\text{-}S} = \frac{DGF}{\pi\cos\theta_i\cos\theta_o}$$

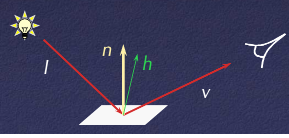
- Assume surface consists of tiny “microfacets” with mirror reflection off each

Torrance-Sparrow BRDF

- *D* term is distribution of microfacets (i.e., how many are pointing in each direction)
- Beckmann distribution

$$D = \frac{e^{-[(\tan\beta)/m]^2}}{4m^2\cos^4\beta}$$

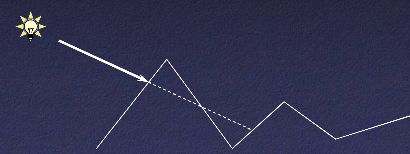
β is angle between n and h
h is halfway between l and v
m is "roughness" parameter



Torrance-Sparrow BRDF

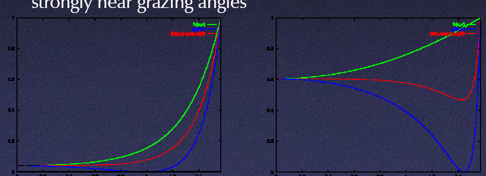
- *G* term accounts for self-shadowing

$$G = \min\left\{1, \frac{2(n\cdot h)(n\cdot v)}{(v\cdot h)}, \frac{2(n\cdot h)(n\cdot l)}{(v\cdot h)}\right\}$$



Torrance-Sparrow BRDF

- *F* term is Fresnel term – reflection from an ideal smooth surface (solution of Maxwell's equations)
- Consequence: most surfaces reflect (much) more strongly near grazing angles

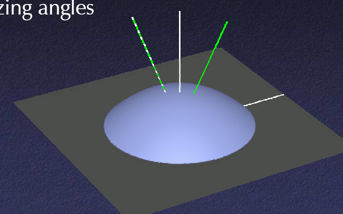


Dielectric

Metal
(note behavior at Brewster's angle)

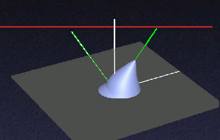
Other BRDF Features

- BRDFs for dusty surfaces scatter light towards grazing angles



Other BRDF Features

- Retroreflection: strong reflection back towards the light source



- Can arise from bumpy diffuse surfaces

- … or from corner reflectors

Beyond BRDFs

- So far, have assumed 4D BRDF
- Function of wavelength: 5D
- Fluorescence (absorb at one wavelength, emit at another): 6D
- Phosphorescence (absorb now, emit later): 7D
- Temporal dependence: 8D
- Spatial dependence: 10D
- Subsurface scattering: 12D
- Polarization
- Wave optics effects (diffraction, interference)
- …

"Cross product" of two plenoptic functions

Radiometric and Photometric Units

Radiant energy Joule (J)	Luminous energy Talbot
Radiant flux or power (F) Watt (W) = J / sec	Luminous power Lumen (lm) = talbot / sec = cd · sr
Radiant intensity (I) W / sr	Luminous intensity Candela (cd)
Irradiance (E) W / m^2	Illuminance Lux = lm / m^2
Radiance (L) W / m^2 / sr	Luminance Nit = lm / m^2 / sr
Radiosity (B) W / m^2	Luminosity Lux = lm / m^2