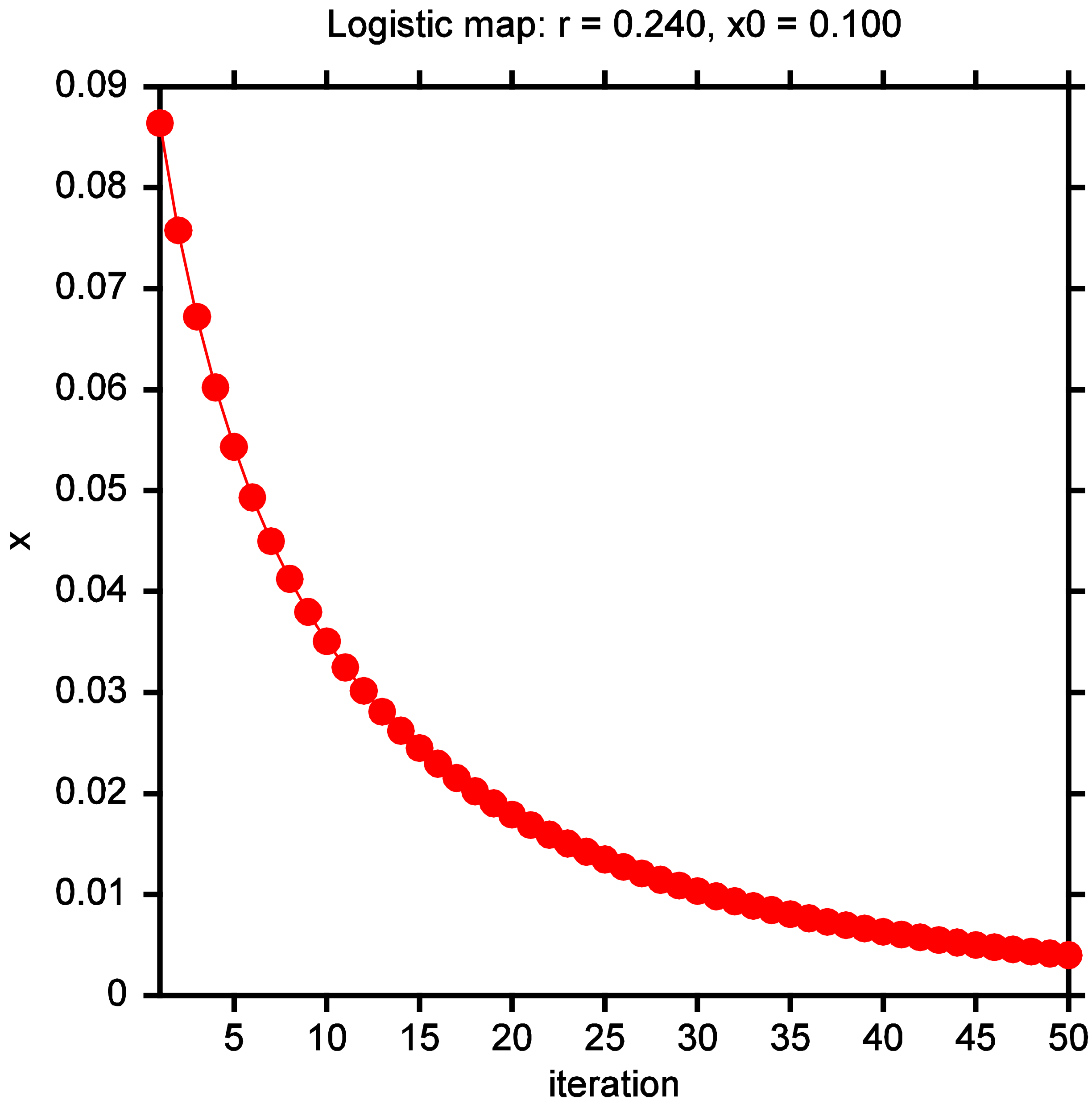
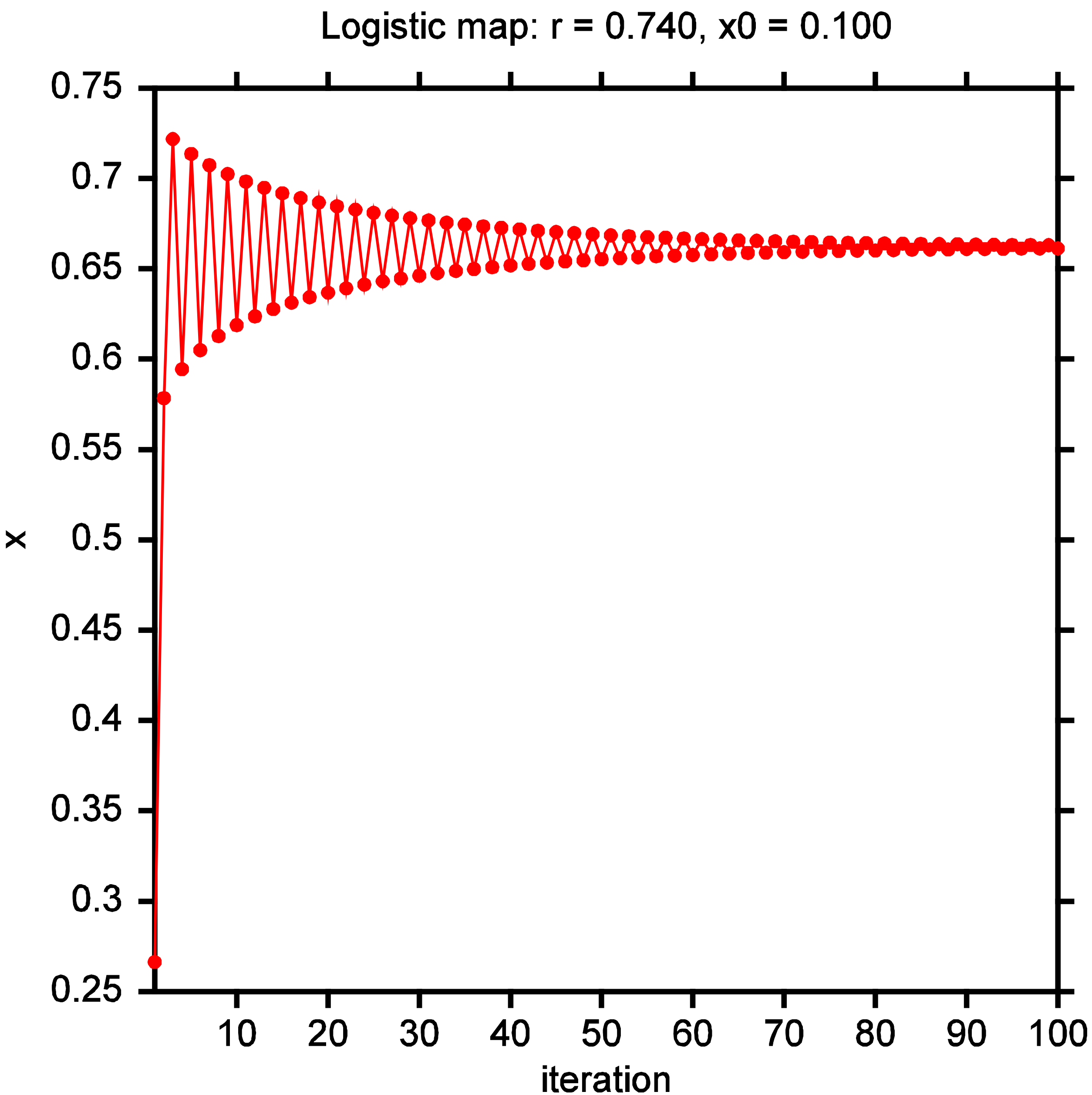
Chaos

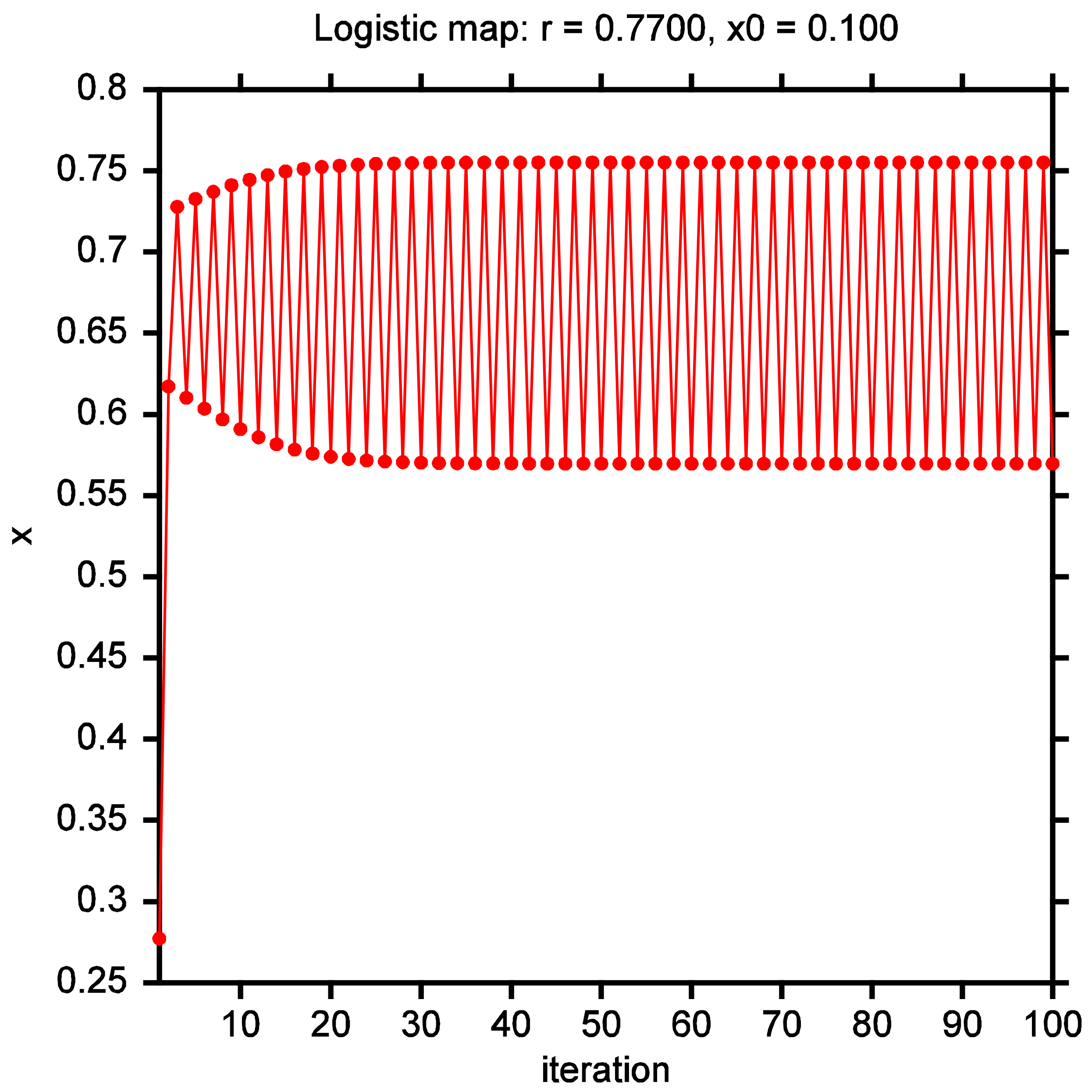
Let's look at difference equation: Logistic Map

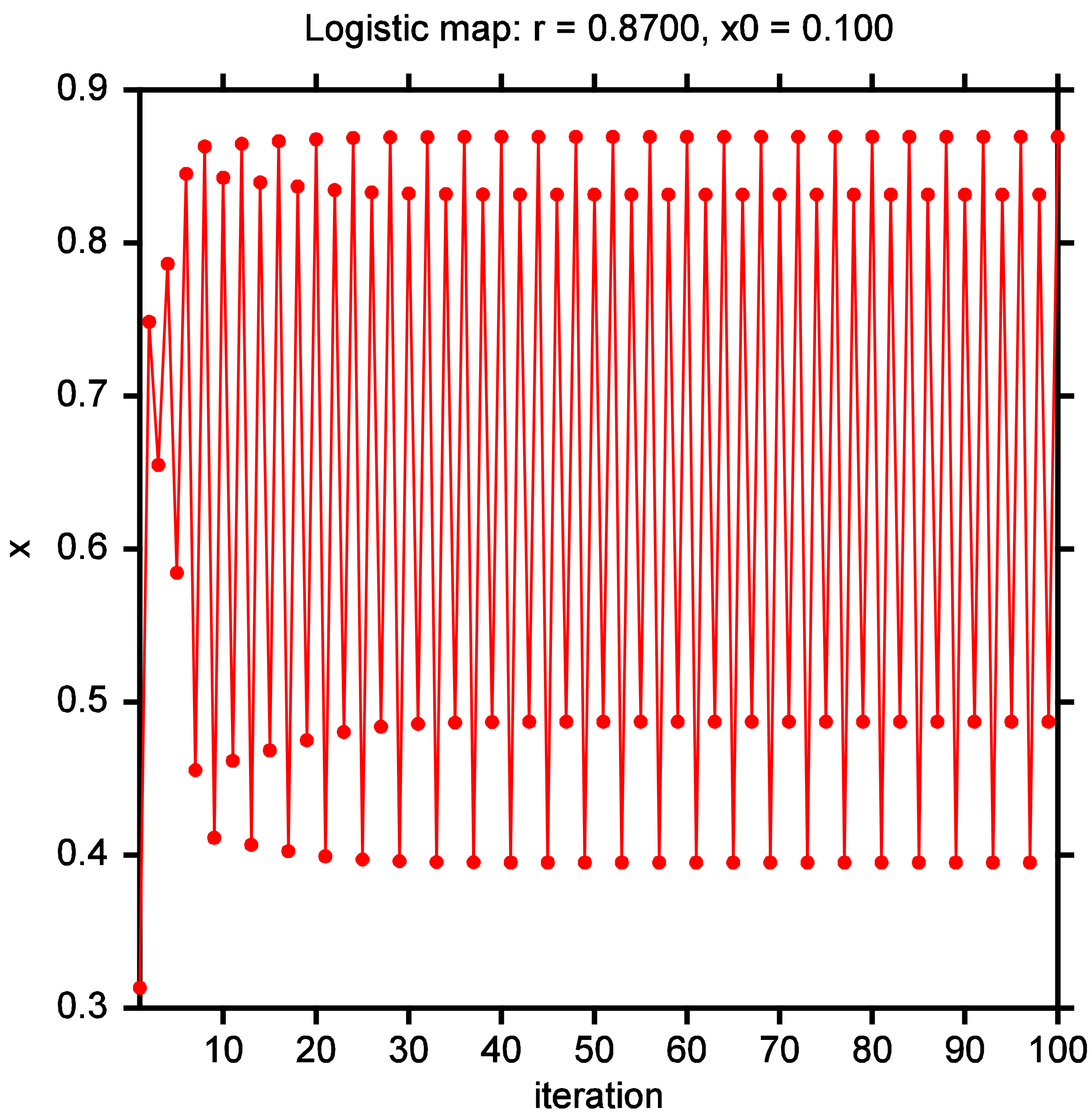
$$x_{n+1} = 4rx_n(1 - x_n)$$

Maps [0,1] → [0,1]

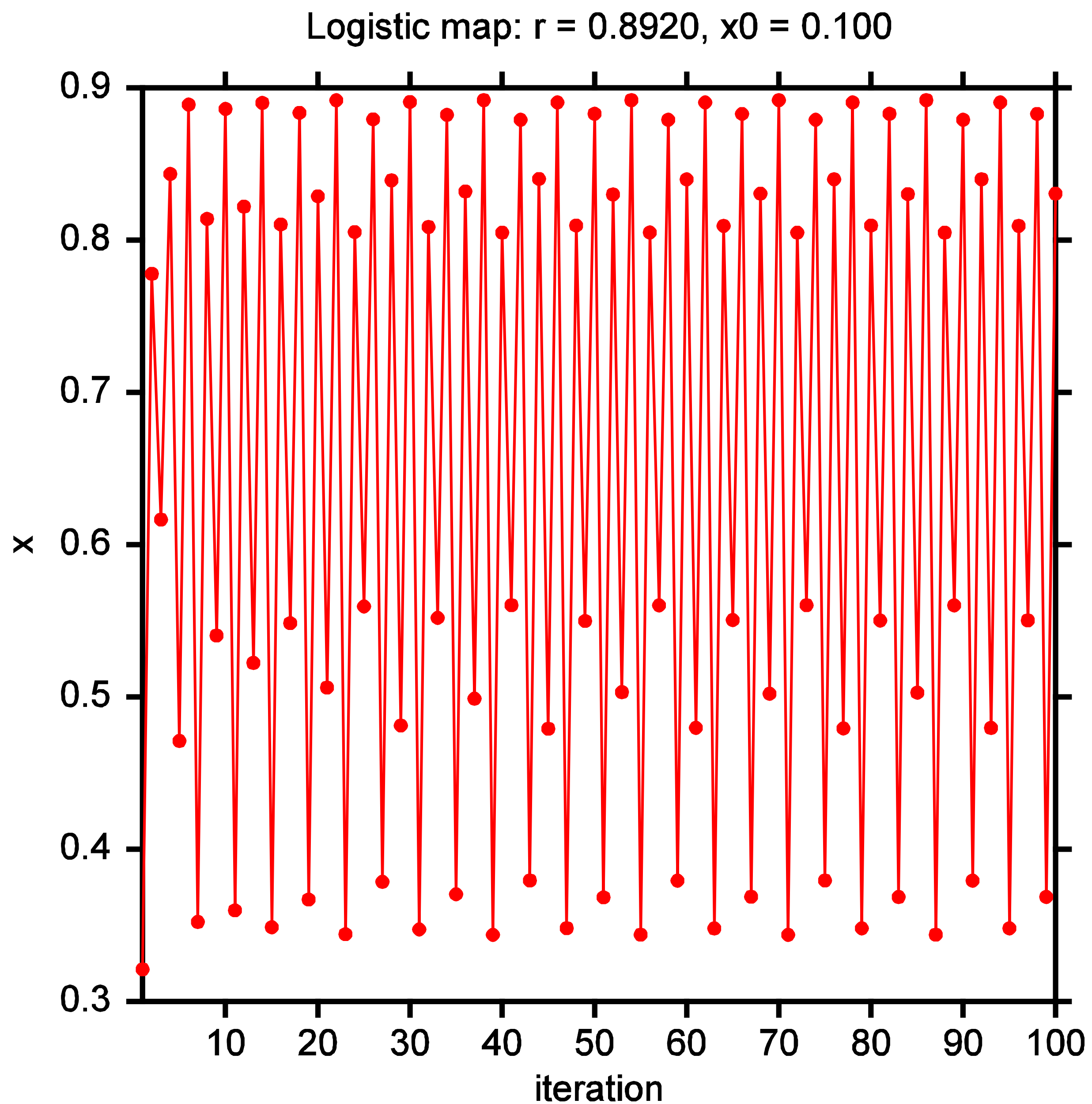

Logistic map: r = 0.240, x0 = 0.100
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
x
5
10
15
20
25
30
35
40
45
50
iteration


Logistic map: r = 0.740, x0 = 0.100
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
x
10
20
30
40
50
60
70
80
90
100
iteration

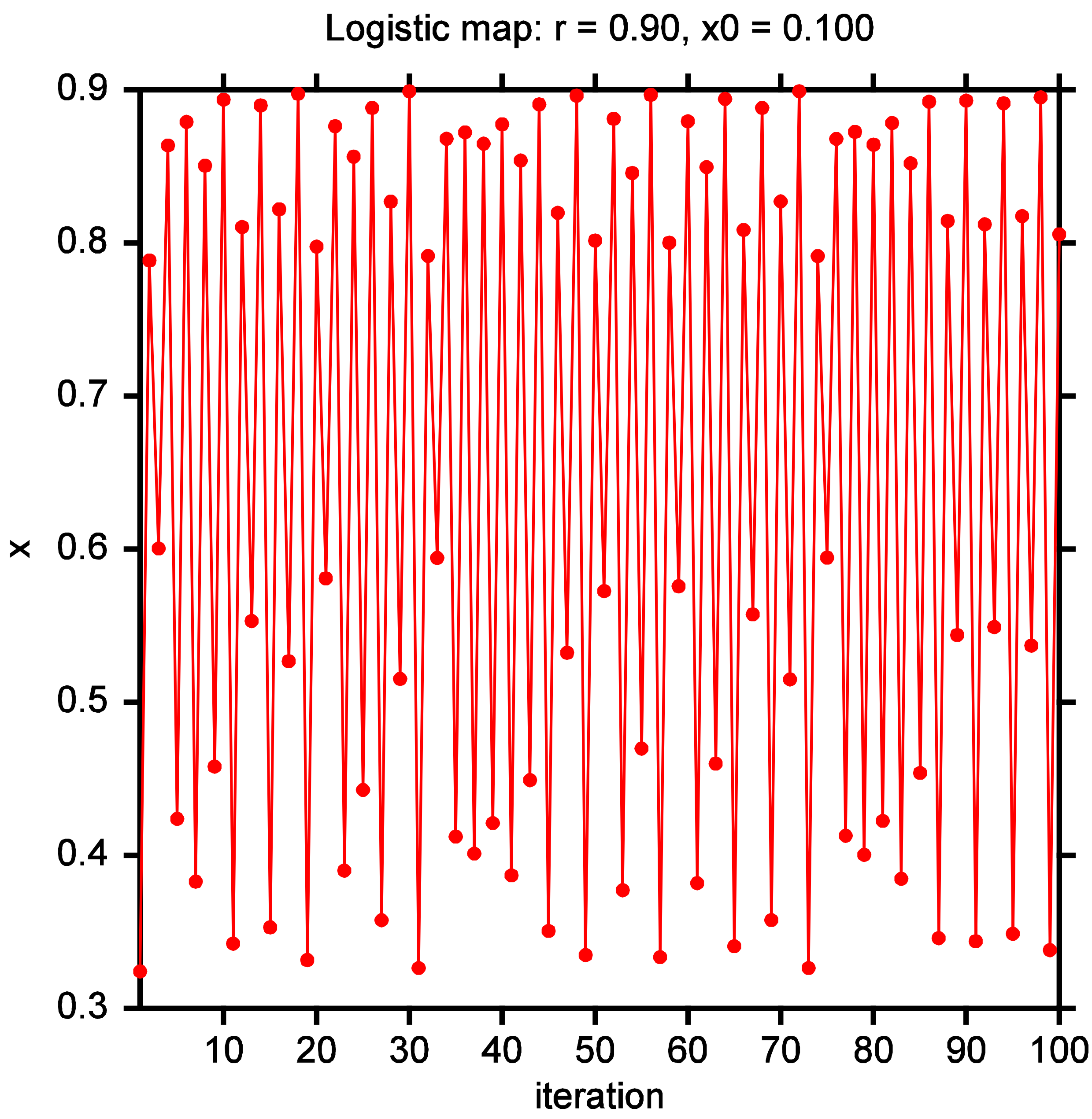

Logistic map: r = 0.7700, x0 = 0.100
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
x
10
20
30
40
50
60
70
80
90
100
iteration



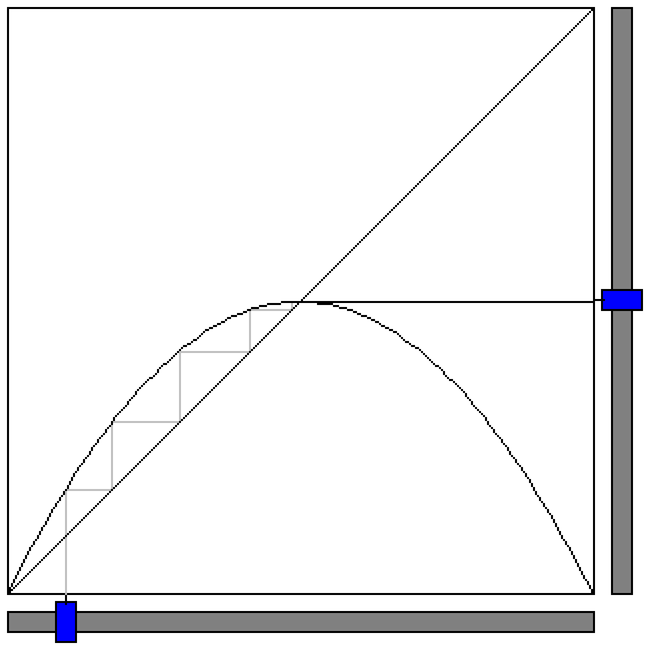
Logistic map: r = 0.8700, x0 = 0.100
0.9
0.8
0.7
0.6
0.5
0.4
0.3
x
10
20
30
40
50
60
70
80
90
100
iteration



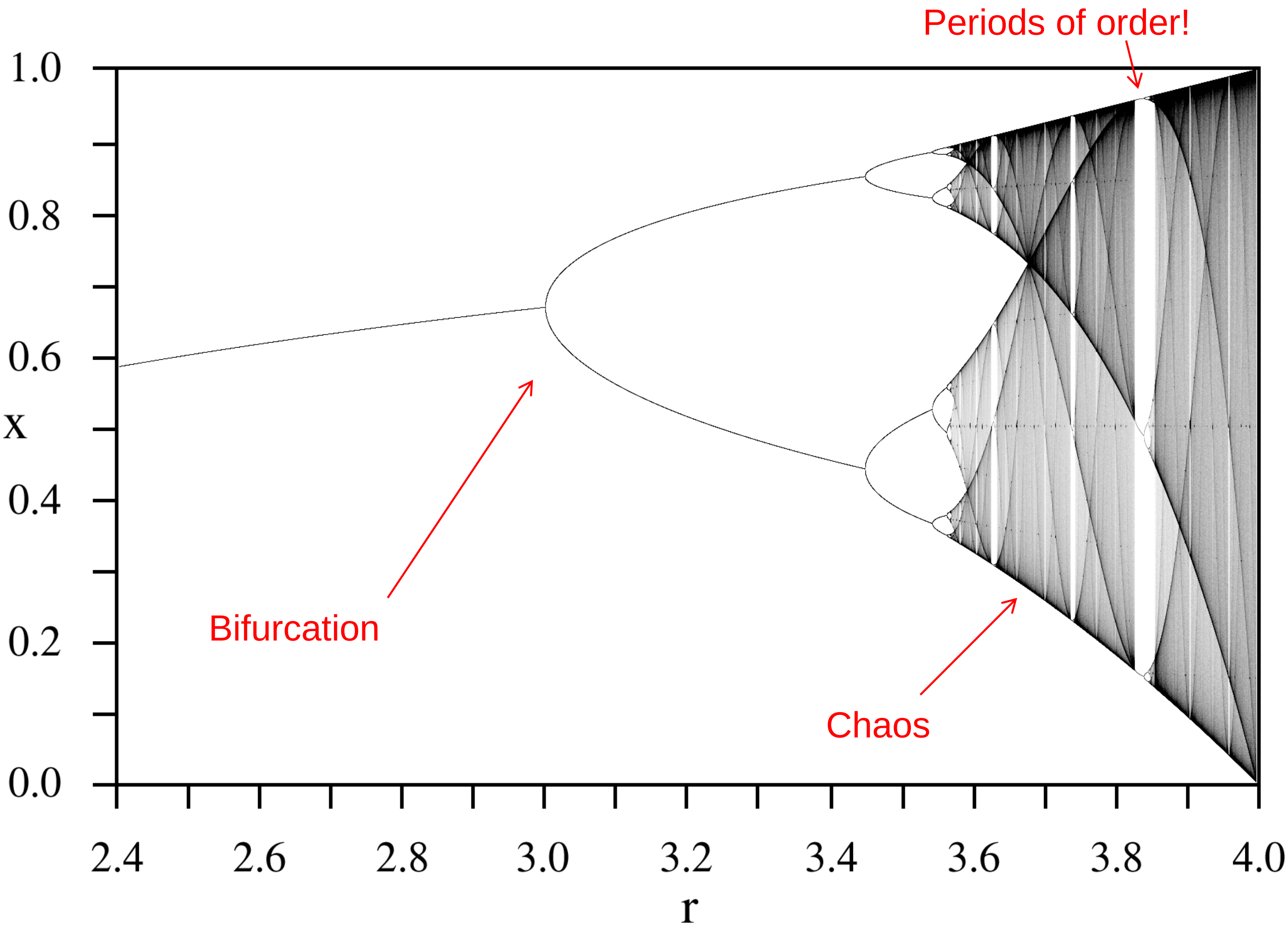
Logistic map: r = 0.8920, x0 = 0.100
0.9
0.8
0.7
0.6
0.5
0.4
0.3
x
10
20
30
40
50
60
70
80
90
100
iteration



Logistic map: r = 0.90, x0 = 0.100
x
iteration
0.3
0.4
0.5
0.6
0.7
0.8
0.9
10
20
30
40
50
60
70
80
90
100

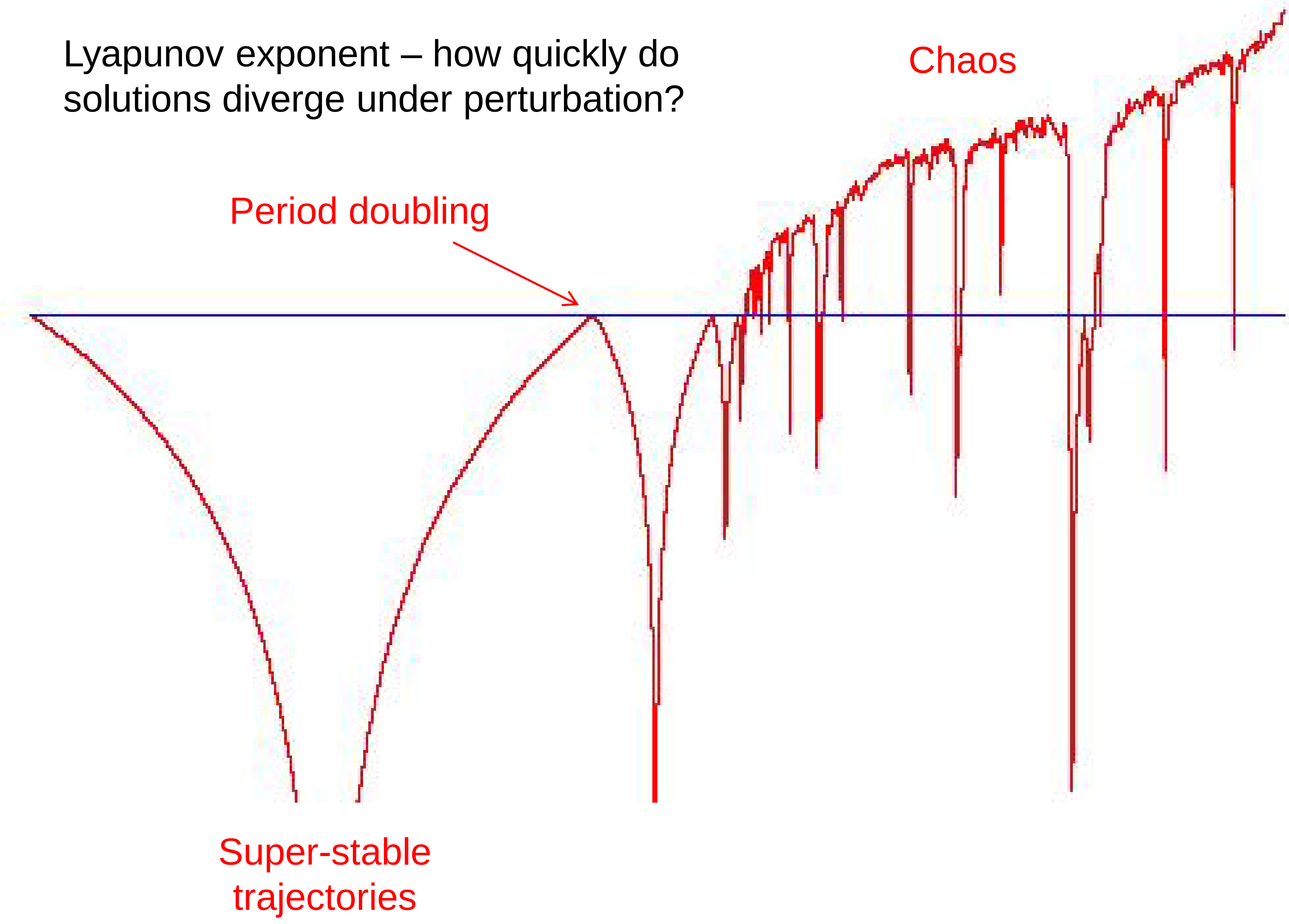


Iterated Logistic Map Demo

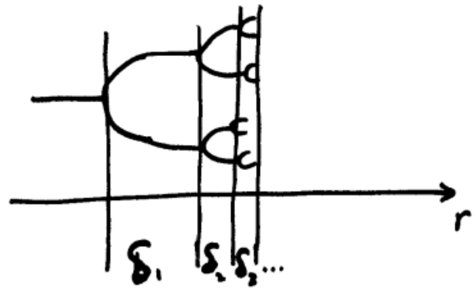


Bifurcation diagram

Lyapunov exponent – how quickly do solutions diverge under perturbation?



Pitchfork bifurcations & Approach to Chaos



get smaller & smaller

define $\delta = \lim \frac{\delta_k}{\delta_{k+1}}$

$\delta = 4.669\,201\,609\,102\,991\ldots$

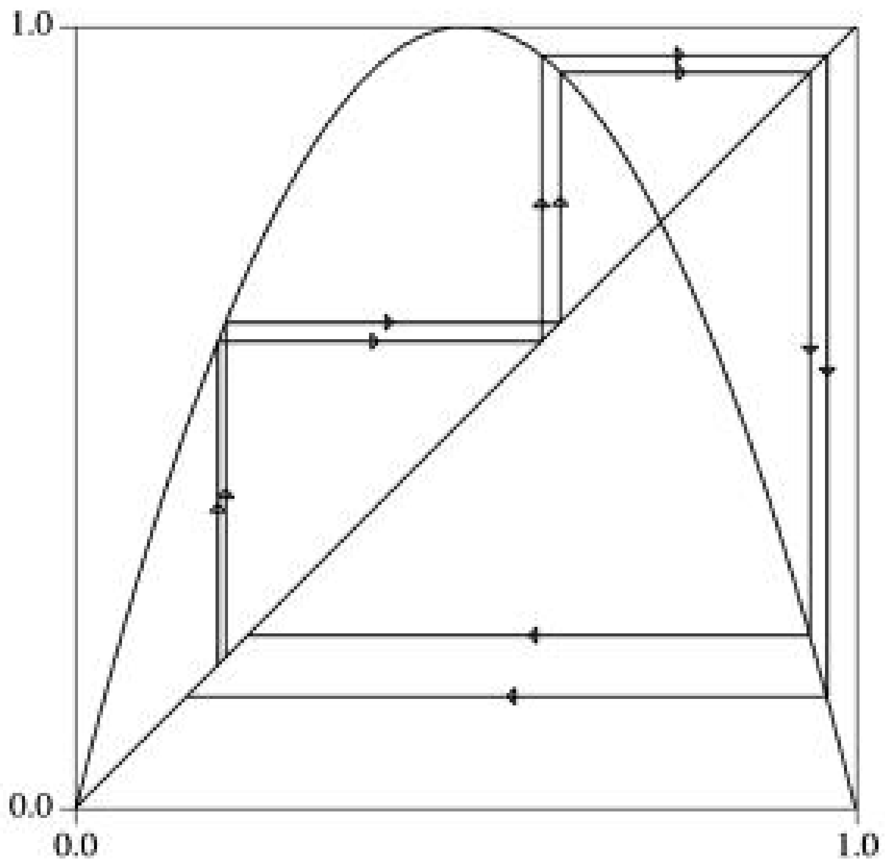
Feigenbaum constant

Feigenbaum showed that δ is <u>independent</u> of the shape of the map, as long as map has $f'(x)=0$ & $f''(x)<0$ at max.

this "route to chaos" is <u>universal</u>!

- Electrical circuits (ODEs)
- water flow (PDEs)

Iterated logistic map



Economic applications: see Medio 92, Puu 03

Corn-Hog cycle:

[Corn-Hog cycle (William King, Drexel)]