4.4 Symbol Tables

"I no longer teach the meaning of life. I now focus on search engine optimization, which, in my opinion, is the meaning of life."

Symbol Table

Symbol table. Key-value pair abstraction.
- Insert a key with specified value.
- Given a key, search for the corresponding value.

Ex. [DNS lookup]
- Insert URL with specified IP address.
- Given URL, find corresponding IP address.

URL	IP address
www.cs.princeton.edu	128.112.136.11
www.princeton.edu	128.112.128.15
www.yale.edu	130.132.143.21
www.harvard.edu	128.103.060.55
www.simpsons.com	209.052.165.60

key value

Symbol Table Applications

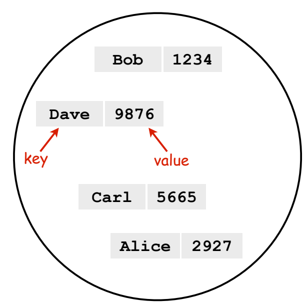
Application	Purpose	Key	Value
phone book	look up phone number	name	phone number
bank	process transaction	account number	transaction details
file share	find song to download	name of song	computer ID
file system	find file on disk	filename	location on disk
dictionary	look up word	word	definition
web search	find relevant documents	keyword	list of documents
book index	find relevant pages	keyword	list of pages
web cache	download	filename	file contents
genomics	find markers	DNA string	known positions
DNS	find IP address given URL	URL	IP address
reverse DNS	find URL given IP address	IP address	URL
compiler	find properties of variable	variable name	value and type
routing table	route Internet packets	destination	best route

Symbol Table API

```
public class  ST<Key extends Comparable<Key>, Value>

              ST()                     create a symbol table
         void put(Key key, Value v)    put key-value pair into the table
        Value get(Key key)             return value paired with key, null if key not in table
      boolean contains(Key key)        is there a value paired with key?
```

symbol table is a set of key-value pairs

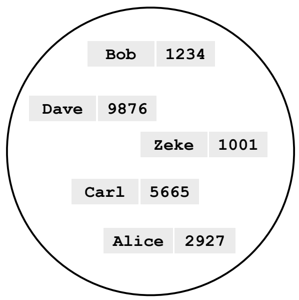


Symbol Table API

public class ST<Key extends Comparable<Key>, Value>

	ST()	*create a symbol table*
void	put(Key key, Value v)	*put key-value pair into the table*
Value	get(Key key)	*return value paired with key, null if key not in table*
boolean	contains(Key key)	*is there a value paired with key?*

put("Zeke", 1001);
adds key-value pair

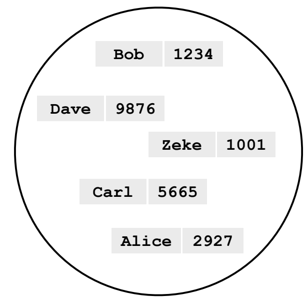


Symbol Table API

public class ST<Key extends Comparable<Key>, Value>

	ST()	*create a symbol table*
void	put(Key key, Value v)	*put key-value pair into the table*
Value	get(Key key)	*return value paired with key, null if key not in table*
boolean	contains(Key key)	*is there a value paired with key?*

get("Alice");
returns 2927



Symbol Table API

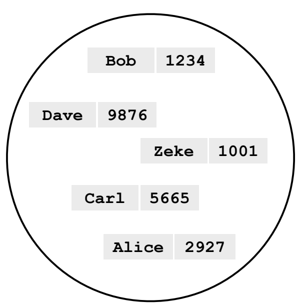
public class *ST<Key extends Comparable<Key>, Value>

	*ST()	*create a symbol table*
void	put(Key key, Value v)	*put key-value pair into the table*
Value	get(Key key)	*return value paired with key, null if key not in table*
boolean	contains(Key key)	*is there a value paired with key?*

Note: Implementations should also implement the Iterable<Key> *interface to enable clients to access keys in sorted order with foreach loops.*

contains("Alice");
returns true

contains("Fred");
returns false



Symbol Table API

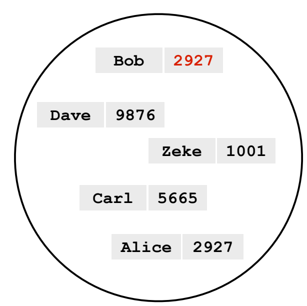
public class *ST<Key extends Comparable<Key>, Value>

	*ST()	*create a symbol table*
void	put(Key key, Value v)	*put key-value pair into the table*
Value	get(Key key)	*return value paired with key, null if key not in table*
boolean	contains(Key key)	*is there a value paired with key?*

Note: Implementations should also implement the Iterable<Key> *interface to enable clients to access keys in sorted order with foreach loops.*

put("Bob", 2927);
changes Bob's value

"Associative array" notation
st["Bob"] = 2927;
is legal in some languages
(not Java)



Symbol Table Client Example 1: Index

Indexing
- Key: string
- Value: `Queue` of integers
- Read a key from standard input.
- If key is in symbol table, add its position to queue
  If key is not in symbol table, create a queue first

```
% more tiny.txt
it was the best of times it was the worst of times
it was the age of wisdom it was the age of foolishness
it was the epoch of belief it was the epoch of incredulity
it was the season of light it was the season of darkness
it was the spring of hope it was the winter of despair
```

```
public class Index
{
   public static void main(String[] args)          // key type, value type
   {
      ST<String, Queue> st = new ST<String, Queue>();
      int i = 0;
      while (!StdIn.isEmpty())
      {
         String key = StdIn.readString();
         if (!st.contains(key)) st.put(key, new Queue());
         st.get(key).enqueue(i++);
      }
                                     // enhanced for loop (stay tuned)
      for (String s : st)
         StdOut.println(s + " " + st.get(s));
   }
}
```

```
%  java Index < tiny.txt
age 15 21
belief 29
best 3
darkness 47
despair 59
epoch 27 33
foolishness 23
hope 53
incredulity 35
it 0 6 12 18 24 30 36 42 48 54
light 41
of 4 10 16 22 28 34 40 46 52 58
season 39 45
spring 51
the 2 8 14 20 26 32 38 44 50 56
times 5 11
was 1 7 13 19 25 31 37 43 49 55
winter 57
wisdom 17
worst 9
```


Symbol Table Client Example 2: Frequency Counter

Frequency counter. [e.g., web traffic analysis, linguistic analysis]
- Key: string
- Value: `Integer` counter
- Read a key from standard input.
- If key is in symbol table, increment counter by 1;
  If key is not in symbol table, insert it with counter = 1.

```
public class Freq
{
   public static void main(String[] args)          // key type, value type
   {
      ST<String, Integer> st = new ST<String, Integer>();
                                                    // calculate frequencies
      while (!StdIn.isEmpty())
      {
         String key = StdIn.readString();
         if (st.contains(key)) st.put(key, st.get(key) + 1);
         else                  st.put(key, 1);
      }
                                     // enhanced for loop (stay tuned)
      for (String s : st)                           // print results
         StdOut.println(st.get(s) + " " + s);
   }
}
```

```
$ java Freq < tiny.txt
2 age
1 belief
1 best
1 darkness
1 despair
2 epoch
1 foolishness
1 hope
1 incredulity
10 it
1 light
10 of
2 season
1 spring
10 the
2 times
10 was
1 winter
1 wisdom
1 worst
```


Sample datasets

Linguistic analysis. Compute word frequencies in a piece of text.

File	Description	Words	Distinct
mobydick.txt	Melville's Moby Dick	210,028	16,834
leipzig100k.txt	100K random sentences	2,121,054	144,256
leipzig200k.txt	200K random sentences	4,238,435	215,515
leipzig1m.txt	1M random sentences	21,191,455	534,580

Reference: Wortschatz corpus, Univesität Leipzig
http://corpora.informatik.uni-leipzig.de

Zipf's Law

Linguistic analysis. Compute word frequencies in a piece of text.

```
% java Freq < mobydick.txt
4583 a
2 aback
2 abaft
3 abandon
7 abandoned
1 abandonedly
2 abandonment
2 abased
1 abasement
2 abashed
1 abate
…
```

```
% java Freq < mobydick.txt | sort -rn
13967 the
6415 of
6247 and
4583 a
4508 to
4037 in
2911 that
2481 his
2370 it
1940 i
1793 but
…
```

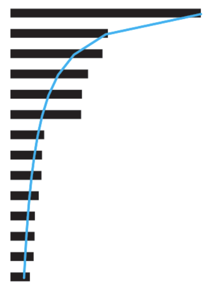
e.g., most frequent word occurs about twice as often as second most frequent one

Zipf's law. Frequency of i^{th} most common word is inversely proportional to i.

Zipf's Law

Linguistic analysis. Compute word frequencies in a piece of text.

```
% java Freq < leipzig1m.txt | sort -rn
1160105 the
593492 of
560945 to
472819 a
435866 and
430484 in
205531 for
192296 The
188971 that
172225 is
148915 said
147024 on
141178 was
118429 by
…
```

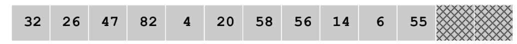


Zipf's law. Frequency of i^{th} most common word is inversely proportional to i.

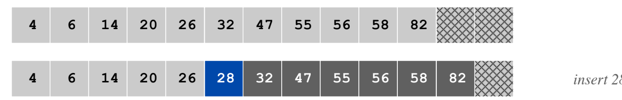
Challenge: Develop symbol-table implementation for such experiments.

Symbol Table: Elementary Implementations

Unordered array.
- Put: add key to the end (if not already there).
- Get: scan through all keys to find desired value.



Ordered array.
- Put: find insertion point, and shift all larger keys right.
- Get: binary search to find desired key.



Symbol Table: Implementations Cost Summary

Unordered array. Hopelessly slow for large inputs.

Ordered array. Acceptable if many more searches than inserts;
too slow if large number of inserts.

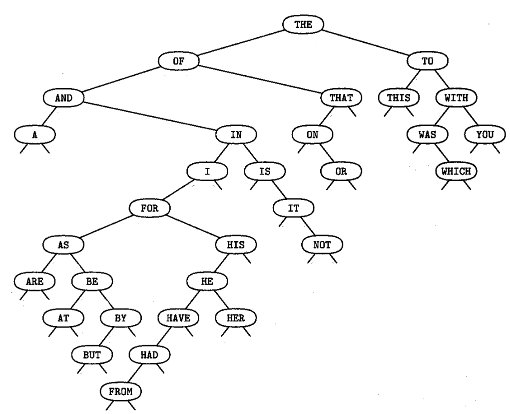
	Running Time		Frequency Count			
implementation	get	put	Moby	100K	200K	1M
unordered array	N	N	170 sec	4.1 hr	-	-
ordered array	$\log N$	N	5.8 sec	5.8 min	15 min	2.1 hr

too slow: ~N² to build entire table

doubling test: quadratic

Challenge. Make all ops logarithmic.
Note: Linked lists are not much help (have to traverse list)

Binary Search Trees



Reference: Knuth, The Art of Computer Programming

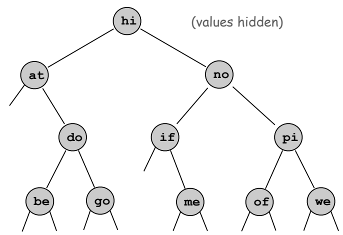
Binary Search Trees

Def. A binary search tree is a binary tree, with keys in symmetric order.

Binary tree is either:

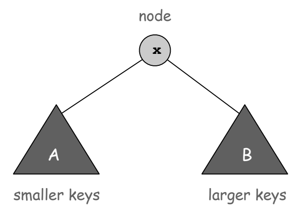
- Empty.
- A key-value pair and two binary trees.

we suppress values from figures

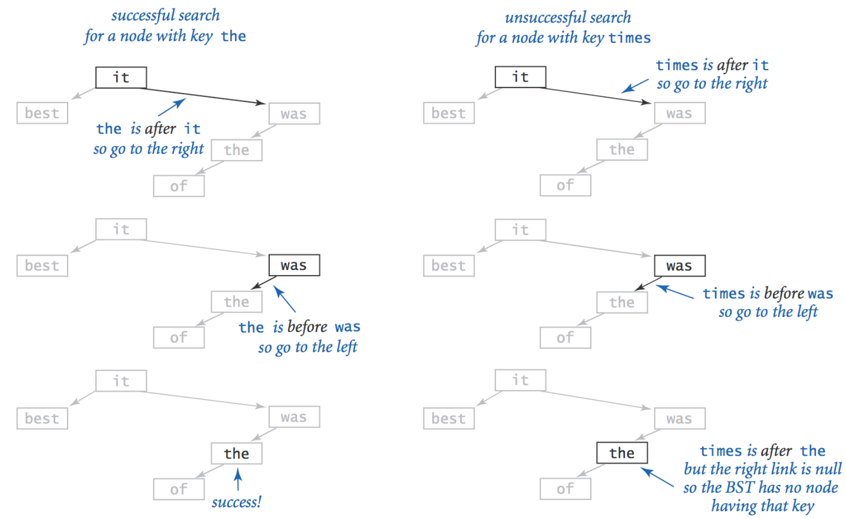


Symmetric order.

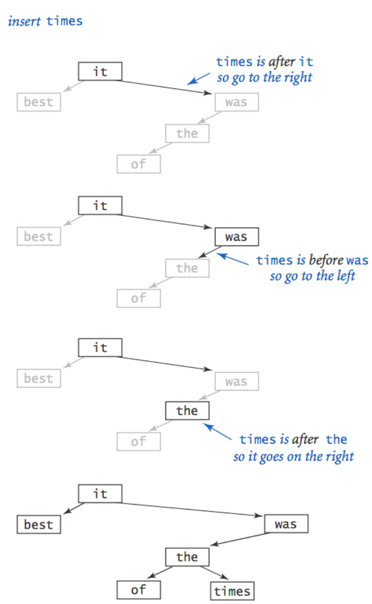
- Keys in left subtree are smaller than parent.
- Keys in right subtree are larger than parent.



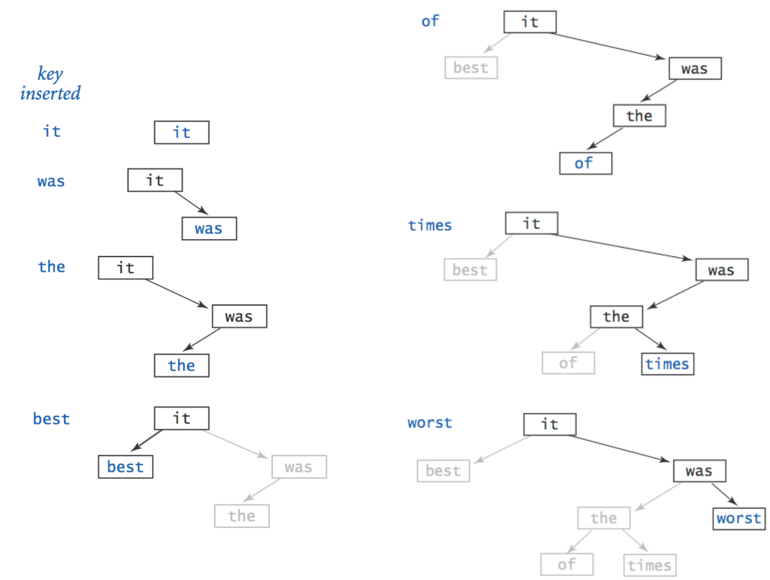
BST Search



BST Insert



BST Construction

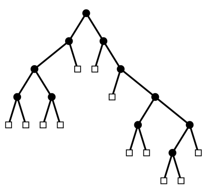


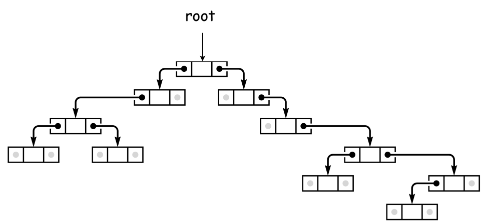
Binary Search Tree: Java Implementation

To implement: use two links per **`Node`**.

A **`Node`** is comprised of:
- A key.
- A value.
- A reference to the left subtree.
- A reference to the right subtree.

```
private class Node
{
   private Key key;
   private Value val;
   private Node left;
   private Node right;
}
```





BST: Skeleton

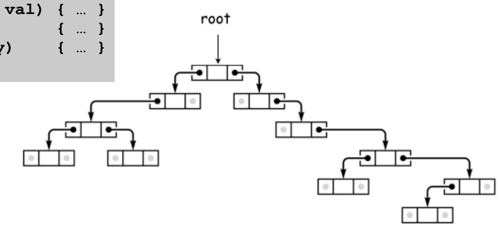
BST. (with generic keys and values).

```
public class BST<Key, Value>
{
   private Node root;   // root of the BST

   private class Node
   {
      private Key key;
      private Value val;
      private Node left, right;

      private Node(Key key, Value val)
      {
         this.key = key;
         this.val = val;
      }
   }

   public void put(Key key, Value val)  { … }
   public Value get(Key key)            { … }
   public boolean contains(Key key)     { … }
}
```



BST: Get

Get. Return **`val`** corresponding to given **`key`**, or **`null`** if no such key.

```
public Value get(Key key)
{
   return get(root, key);
}

private Value get(Node x, Key key)
{
   if (x == null) return null;
   int cmp = key.compareTo(x.key);
   if      (cmp < 0)  return get(x.left,  key);
   else if (cmp > 0)  return get(x.right, key);
   else if (cmp == 0) return x.val;
}

public boolean contains(Key key)
{
   return (get(key) != null);
}
```


BST: Put

Put. Associate **`val`** with **`key`**.
- Search, then insert.
- Concise (but tricky) recursive code.

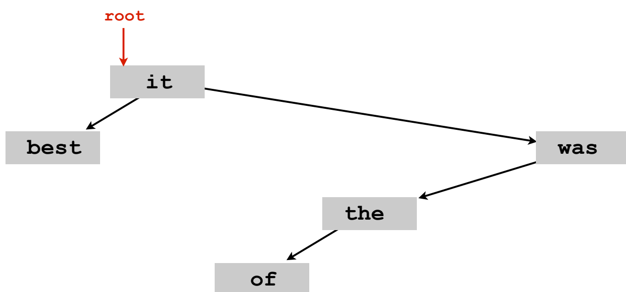
```
public void put(Key key, Value val)
{
   root = put(root, key, val);
}

private Node put(Node x, Key key, Value val)
{
   if (x == null) return new Node(key, val);
   int cmp = key.compareTo(x.key);
   if      (cmp < 0)  x.left  = put(x.left,  key, val);
   else if (cmp > 0)  x.right = put(x.right, key, val);
   else               x.val = val;   // overwrite old value with new
   return x;
}
```

Inserting a new node in a BST

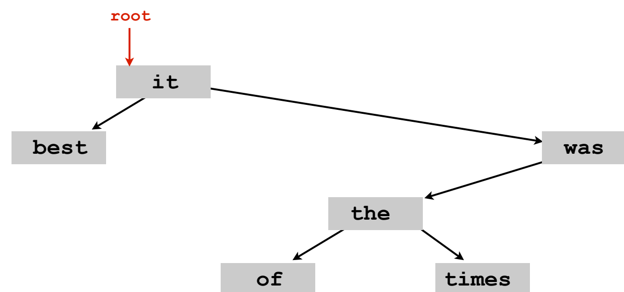
```
public void put(Key key, Value val)
{
   root = put(root, key, val);
}
```

key
times



Inserting a new node in a BST

```
public void put(Key key, Value val)
{
   root = put(root, key, val);
}
```



BST Implementation: Practice

Bottom line. Difference between a practical solution and no solution.

	Running Time		Frequency Count			
implementation	get	put	Moby	100K	200K	1M
unordered array	N	N	170 sec	4.1 hr	-	-
ordered array	$\log N$	N	5.8 sec	5.8 min	15 min	2.1 hr
BST	?	?	.95 sec	7.1 sec	14 sec	69 sec

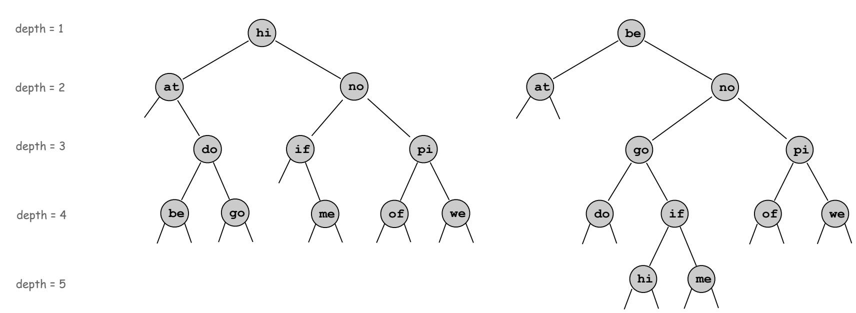
doubling test: linear

BST: Analysis

Running time per put/get.

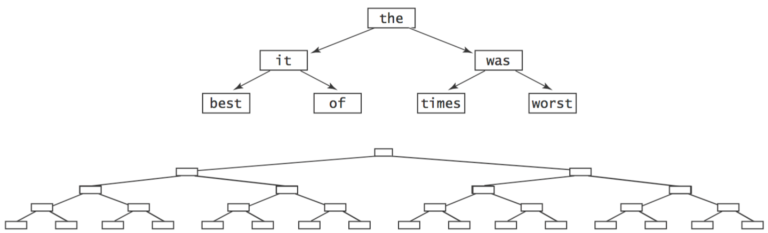
- There are many BSTs that correspond to same set of keys.
- Cost is proportional to depth of node.

number of nodes on path from root to node



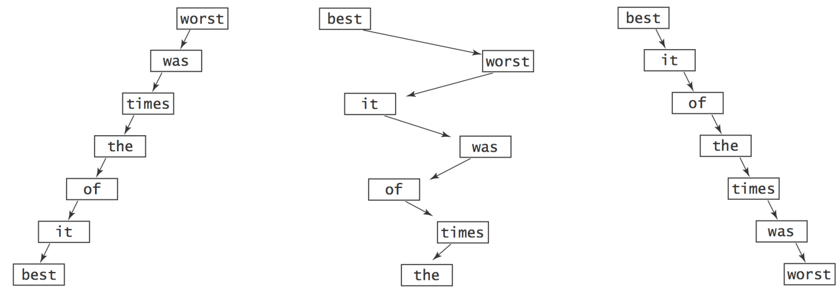
BST: Analysis

Best case. If tree is perfectly balanced, depth is at most $\lg N$.



BST: Analysis

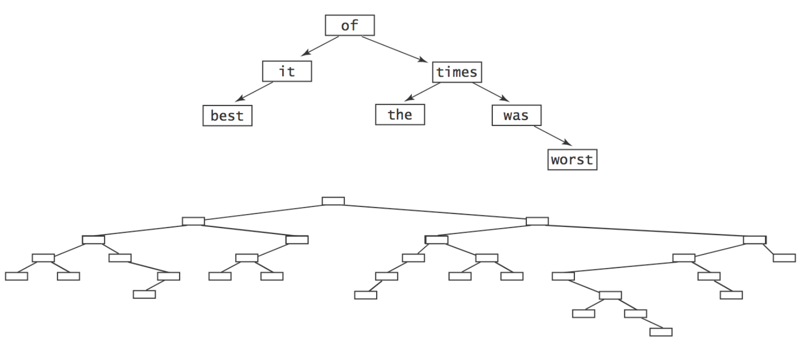
Worst case. If tree is unbalanced, depth is N.



BST: Analysis

Average case. If keys are inserted in random order, average depth is $2 \ln N$.

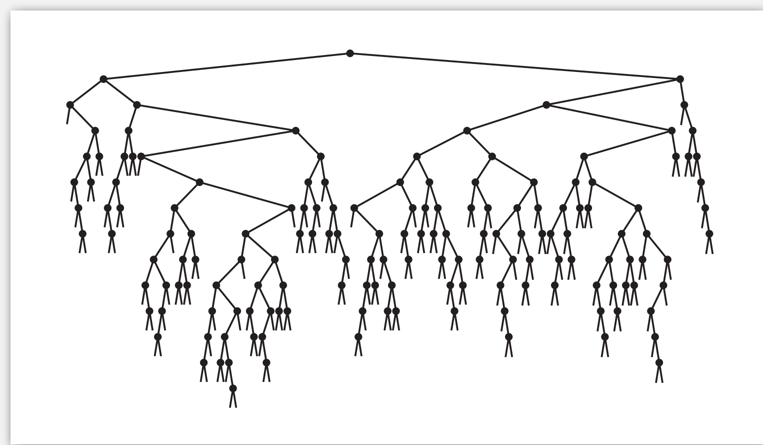
requires proof (see COS 226)



Typical BSTs constructed from randomly ordered keys

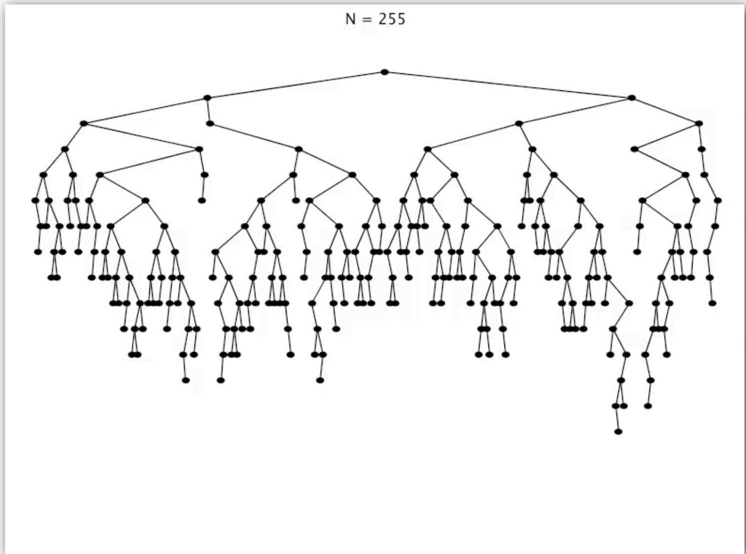
BST insertion: random order

Observation. If keys inserted in random order, tree stays relatively flat.



BST insertion: random order visualization

Ex. Insert keys in random order.



Symbol Table: Implementations Cost Summary

BST. Logarithmic time ops if keys inserted in random order.

	Running Time		Frequency Count			
implementation	get	put	Moby	100K	200K	1M
unordered array	N	N	170 sec	4.1 hr	-	-
ordered array	$\log N$	N	5.8 sec	5.8 min	15 min	2.1 hr
BST	$\log N$ †	$\log N$ †	.95 sec	7.1 sec	14 sec	69 sec

† assumes keys inserted in random order

Q. Can we guarantee logarithmic performance?

Red-Black Tree

Red-black tree. A clever BST variant that guarantees depth ≤ $2 \lg N$.

see COS 226

Java red-black tree library implementation

```
import java.util.TreeMap;
import java.util.Iterator;

public class ST<Key>, Value> implements Iterable<Key>
{

   private TreeMap<Key, Value> st = new TreeMap<Key, Val>();

   public void put(Key key, Value val)
   {
      if (val == null) st.remove(key);
      else             st.put(key, val);
   }
   public Value get(Key key)        { return st.get(key);              }
   public Value remove(Key key)     { return st.remove(key);           }
   public boolean contains(Key key) { return st.containsKey(key);      }
   public Iterator<Key> iterator()  { return st.keySet().iterator(); }
}
```


Red-Black Tree

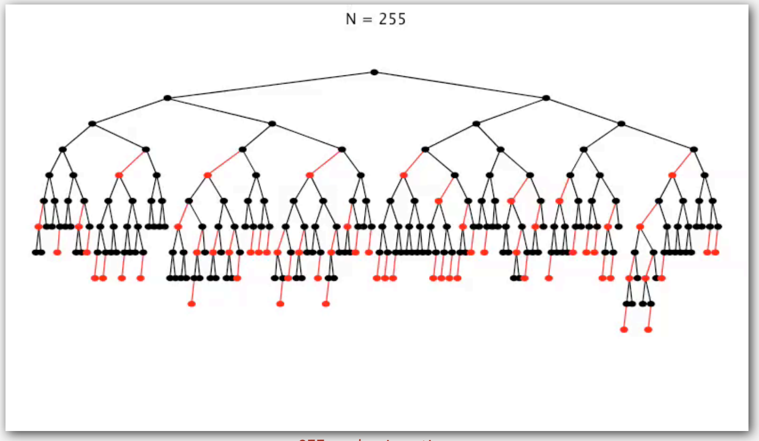
Red-black tree. A clever BST variant that guarantees depth ≤ $2 \lg N$.

see COS 226

	Running Time		Frequency Count			
implementation	get	put	Moby	100K	200K	1M
unordered array	N	N	170 sec	4.1 hr	-	-
ordered array	$\log N$	N	5.8 sec	5.8 min	15 min	2.1 hr
BST	$\log N$ †	$\log N$ †	.95 sec	7.1 sec	14 sec	69 sec
red-black	$\log N$	$\log N$	.95 sec	7.0 sec	14 sec	74 sec

† assumes keys inserted in random order

Insertion in a LLRB tree: visualization

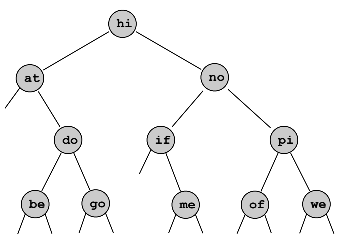


255 random insertions

Iteration

Inorder Traversal

Inorder traversal.
- Recursively visit left subtree.
- Visit node.
- Recursively visit right subtree.



inorder: at be do go hi if me no of pi we

```
public inorder()
{  inorder(root);  }

private void inorder(Node x)
{
   if (x == null) return;
   inorder(x.left);
   StdOut.println(x.key);
   inorder(x.right);
}
```


Enhanced For Loop

Enhanced for loop. Enable client to iterate over items in a collection.

```
ST<String, Integer> st = new ST<String, Integer>();
…

for (String s : st)
   StdOut.println(st.get(s) + " " + s);
```

Enhanced For Loop with BST

BST. Add following code to support enhanced for loop (uses a stack).

see COS 226 for details

```
import java.util.Iterator;
import java.util.NoSuchElementException;

public class BST<Key extends Comparable<Key>, Value> implements Iterable<Key>
{
   private Node root;

   private class Node { … }

   public void put(Key key, Value val) { … }
   public Value get(Key key)           { … }
   public boolean contains(Key key)    { … }

   public Iterator<Key> iterator() { return new Inorder(); }
   private class Inorder implements Iterator<Key>
   {
       Inorder() { pushLeft(root); }
       public boolean hasNext() { return !stack.isEmpty();                  }
       public Key next()
       {
          if (!hasNext()) throw new NoSuchElementException();
          Node x = stack.pop();
          pushLeft(x.right);
          return x.key;
       }
       public void pushLeft(Node x)
       {
          while (x != null) {
             stack.push(x);
             x = x.left;
          }
       }
   }
}
```


Symbol Table: Summary

Symbol table. Quintessential database lookup data type.

Choices. Ordered array, unordered array, BST, red-black, hash, ….
- Different performance characteristics.
- Fast search and insert is available.
- Java libraries: `TreeMap`, `HashMap`.

Remark. Better symbol table implementation improves all clients.