Please be succinct, and give a proof sketch rather than gory details.

1. What is the expected number of triangles in $G(n, p)$ when you plant a random clique of size $k$ in it? Try to prove concentration of this quantity.

2. Let the Frobenius norm of a real symmetric $n \times n$ matrix $M$, denote $|M|_F^2$, be defined as the sum of squares of its entries. Let its spectral norm, denoted $|M|$, be the magnitude of the largest eigenvalue. Show that if $M$ has rank $k$ then $|M|_F^2 \leq k|M|^2$.

3. Suppose $M$ is an $n \times n$ symmetric matrix and $u_1, u_2, \ldots, u_k$ are the eigenvectors corresponding to its $k$ largest eigenvalues (in absolute value). Let $\hat{M}$ be the matrix obtained by replacing each column of $M$ by its projection in the $k$-dimensional subspace spanned by $u_1, u_2, \ldots, u_k$. Note that $\hat{M}$ has rank at most $k$. Show that for every matrix $X$ of rank $k$, $|M - \hat{M}| \leq |M - X|$.

4. The trace of a matrix, denoted $Tr(M)$, is the sum of its diagonal entries. Show that for every real symmetric matrix, $\lim_{k \to \infty} Tr(M^{2k})^{1/2k}$ exists and is equal to $|M|$. (Here the limit is taken over integral $k$.)