Radiometry & Shape from Shading

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Shape acquisition recap

- stereo
  - observe objects from multiple positions
  - establish correspondences
  - triangulation gives depth ("Z") maps, meshes
- structure from motion

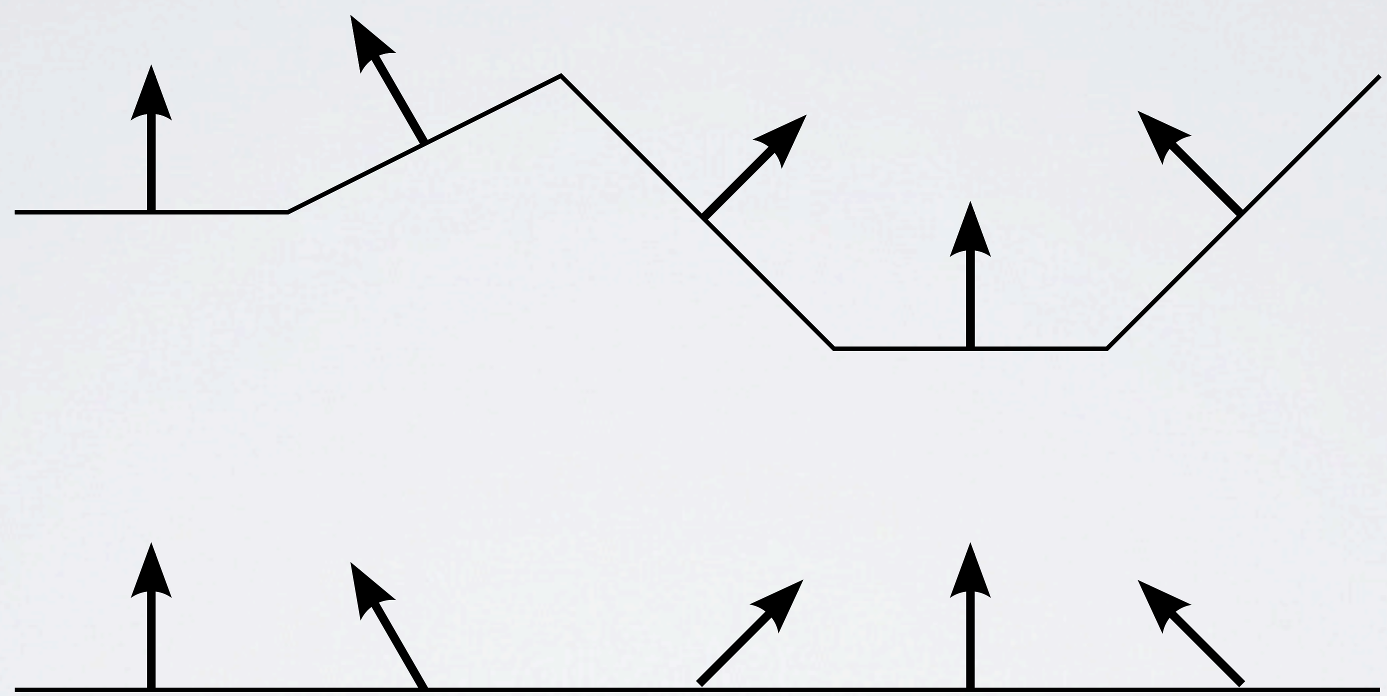
Shape acquisition recap

- passive stereo has problems finding correspondences
- active stereo has resolution limits
  - multi-megapixel cameras: cheap + available
  - scan line techniques take lots of time for hi-res models
  - multi-megapixel projectors: expensive

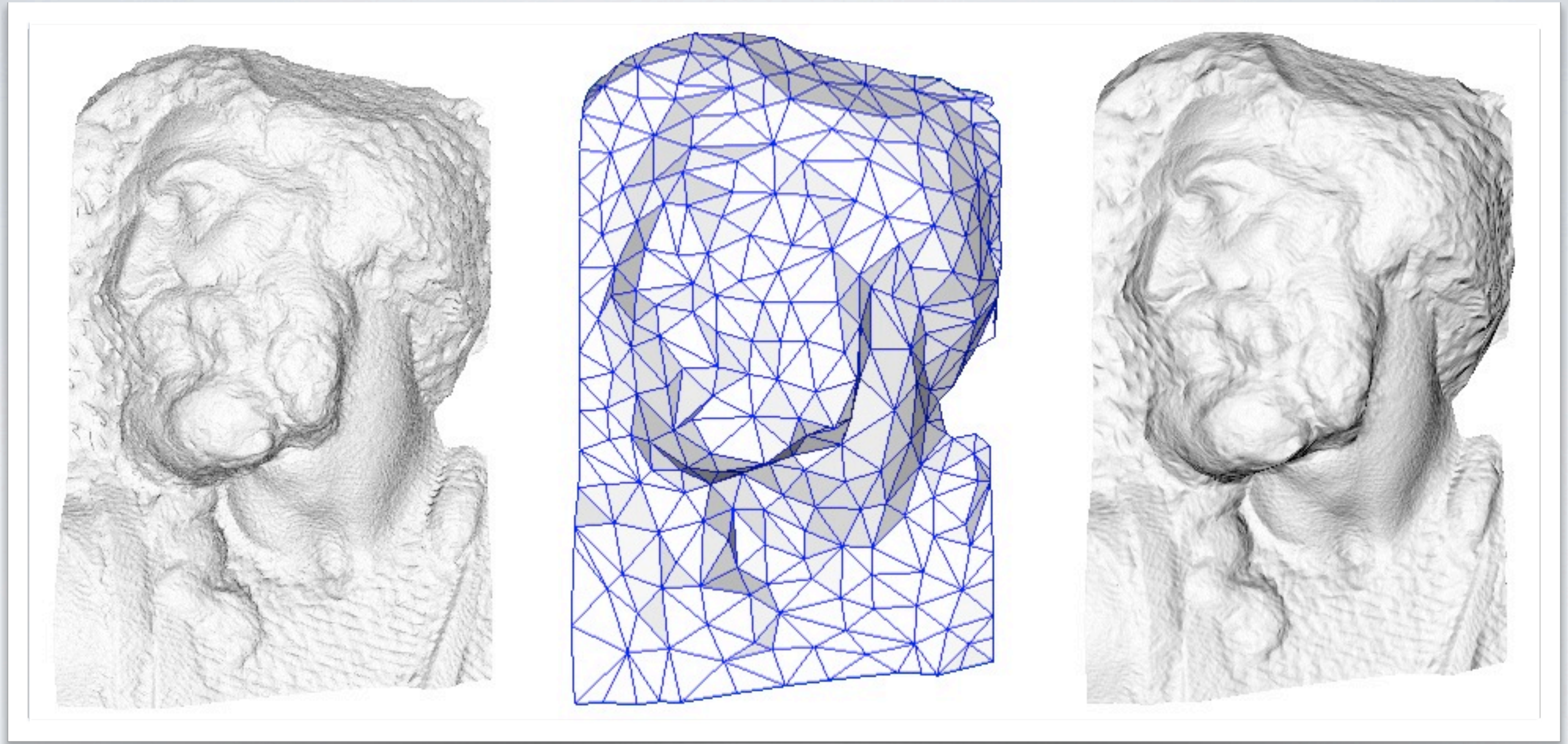
Shape from appearance

- material appearance gives cues for shape
- can be precise locally (per-pixel)
- challenging to get full geometry

Normal maps

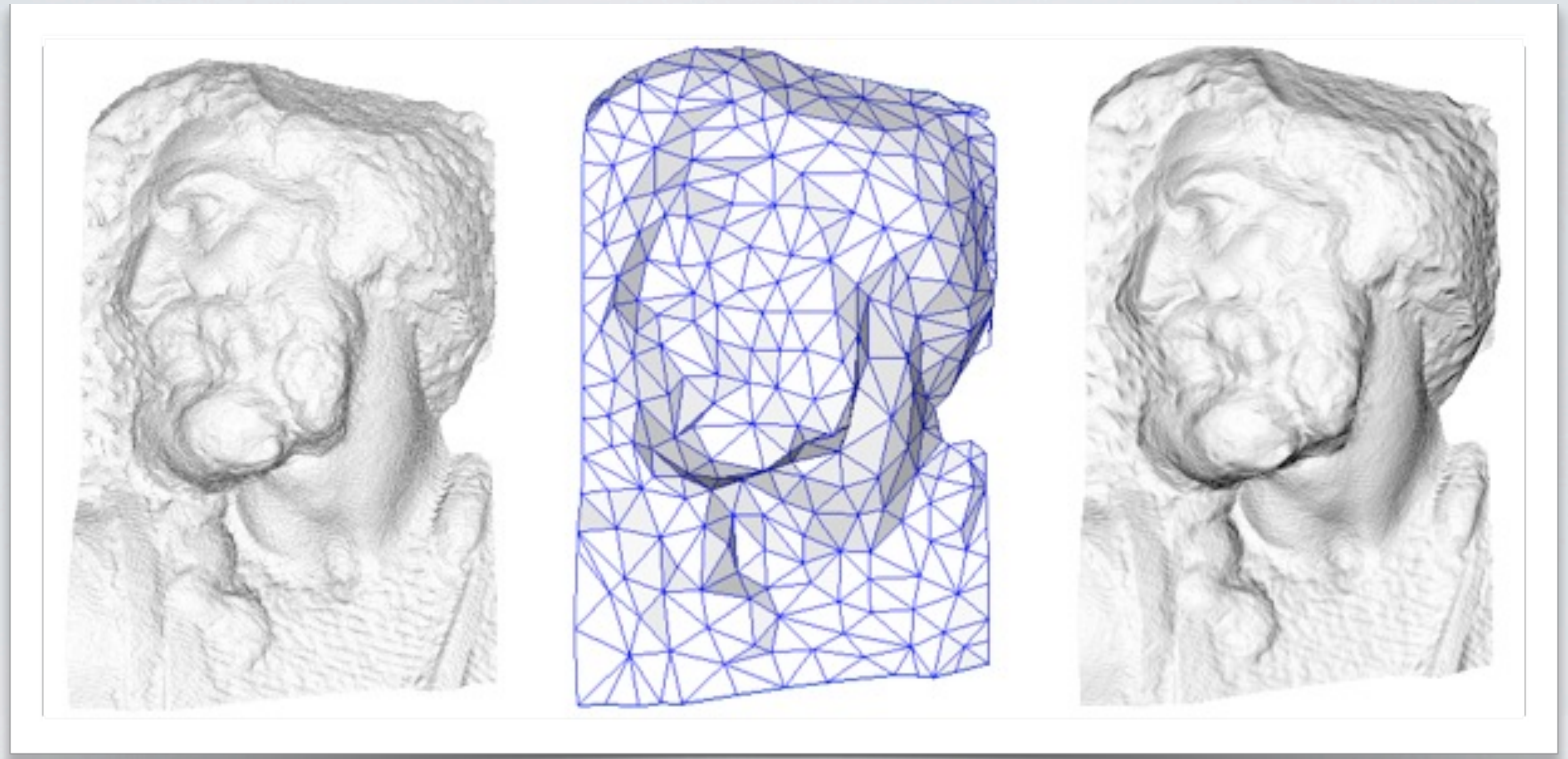


Normal maps



http://en.wikipedia.org/wiki/File:Normal_map_example.png

Normal maps



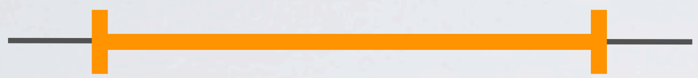
http://en.wikipedia.org/wiki/File:Normal_map_example.png

- normal maps can be measured in every surface point
  - with active methods
  - by looking at local appearance

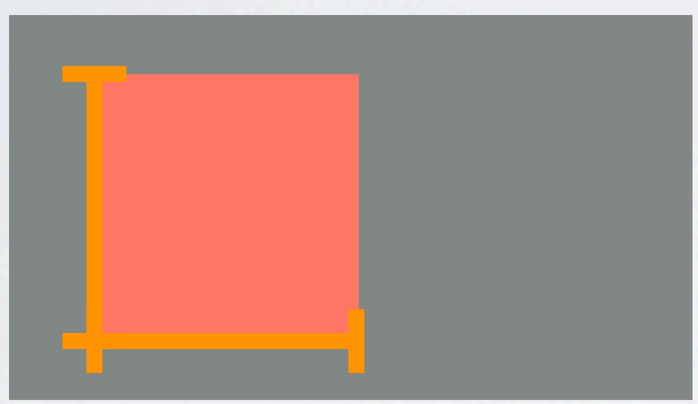
Today's Menu

- Radiometry
- Lambert's Cosine law
- Photometric Stereo
- Shape from Shading

Solid Angle

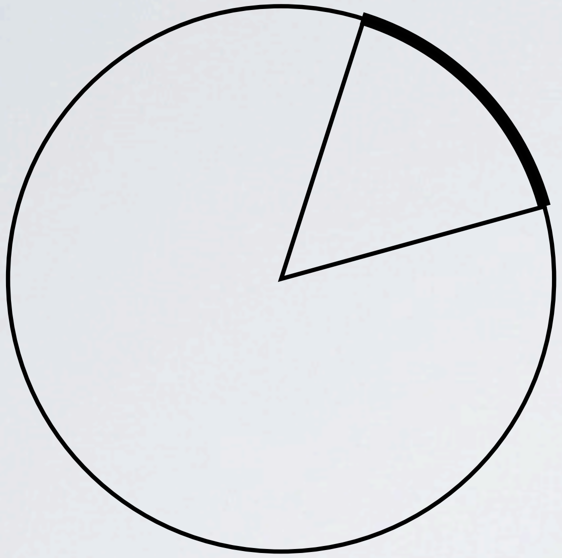


length, unit $1\,\mathrm{meter} = 1\,\mathrm{m}$

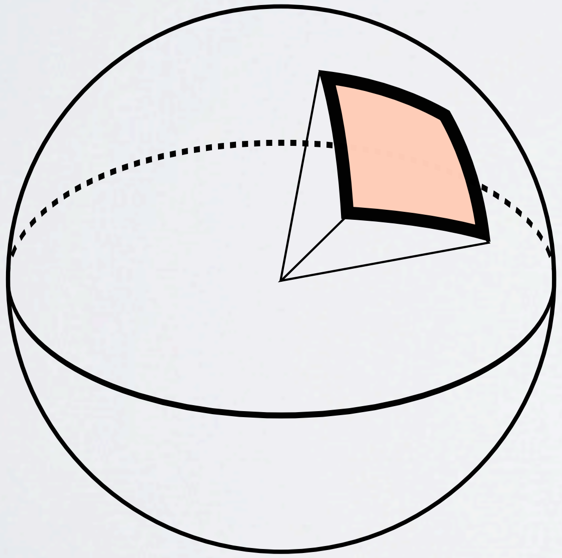


area, unit $1\,\mathrm{m}^2$

Solid Angle



angle = arc length of projection on unit circle, unit $1\,\mathrm{radian} = 1\,\mathrm{rad}$



solid angle = area of projection on unit sphere, unit $1\,\mathrm{steradian} = 1\,\mathrm{sr}$

Radiometry

Name Energy

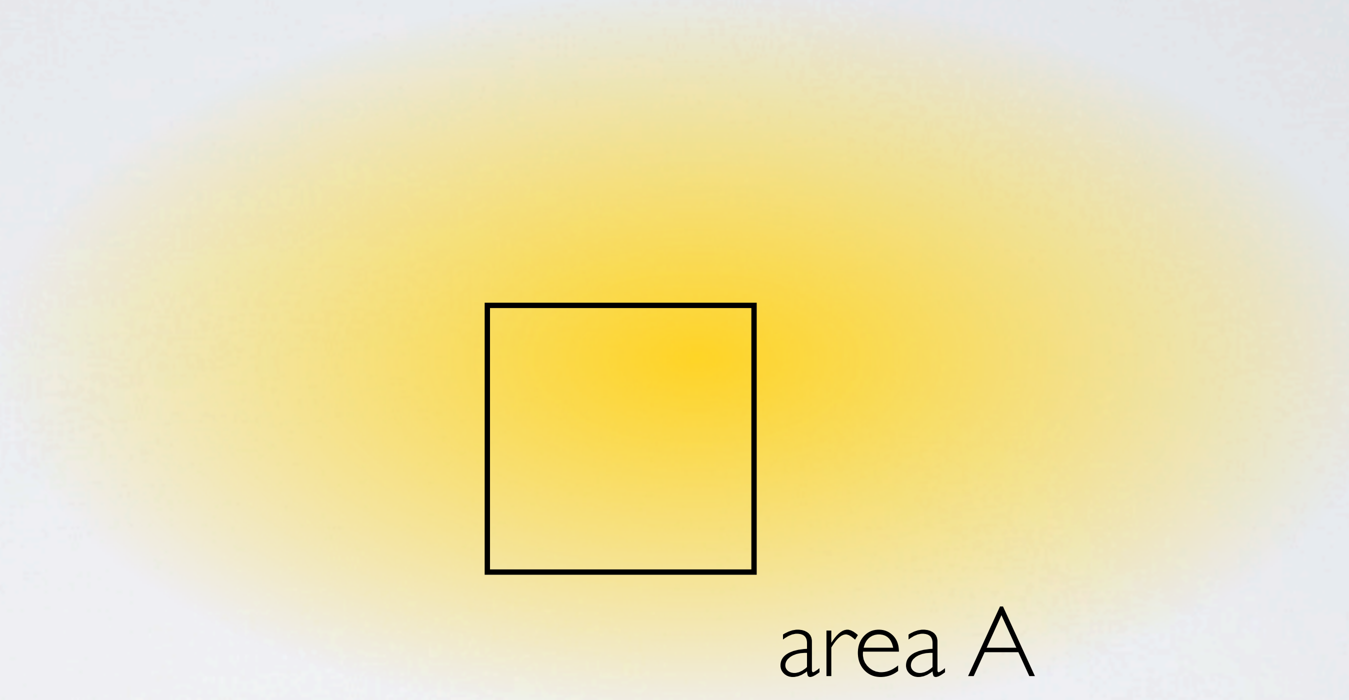
Symbol Q

Unit $[Q] = 1\,\mathrm{Joule} = 1\,\mathrm{J} = 1\,\mathrm{kg}\,\mathrm{m}^2\,\mathrm{s}^{-2}$

Radiometry

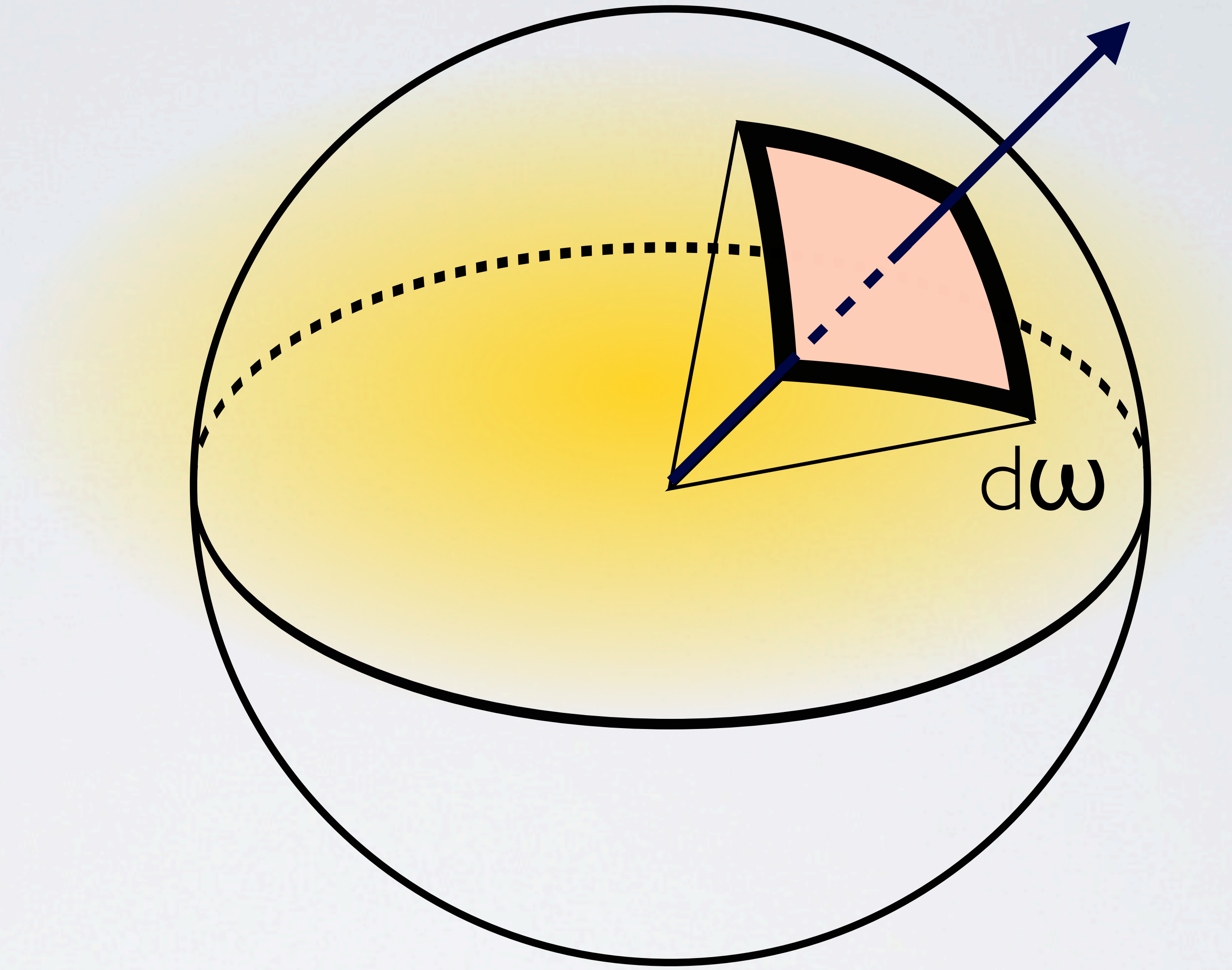
Name	radiant power, flux
Symbol	$P = \frac{dQ}{dt}, \quad \Phi$
Unit	$[P] = 1\,\mathrm{J\,s^{-1}} = 1\,\mathrm{Watt} = 1\,\mathrm{W}$

Radiometry



Name	radiant power, flux
Symbol	$P = \frac{dQ}{dt}, \quad \Phi$
Unit	$[P] = 1\,\mathrm{J\,s^{-1}} = 1\,\mathrm{Watt} = 1\,\mathrm{W}$

Radiometry

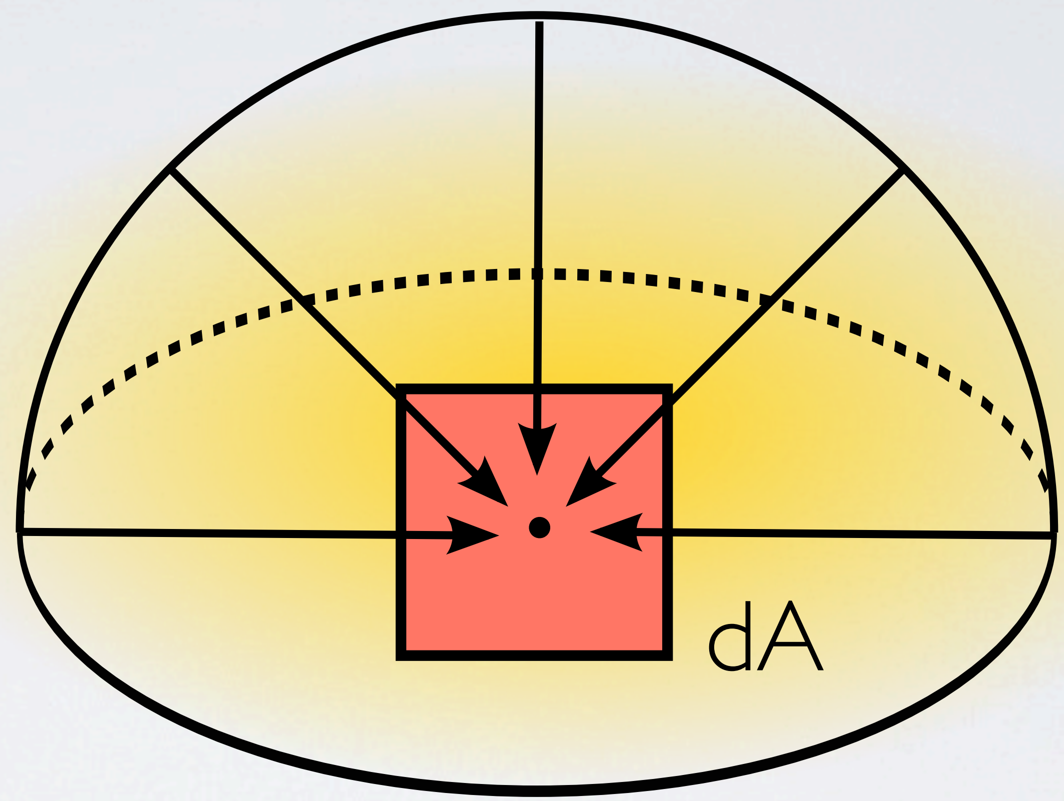


Name radiant intensity (in direction, of a point light source)

Symbol $I = \frac{dP}{d\boldsymbol{\omega}}$

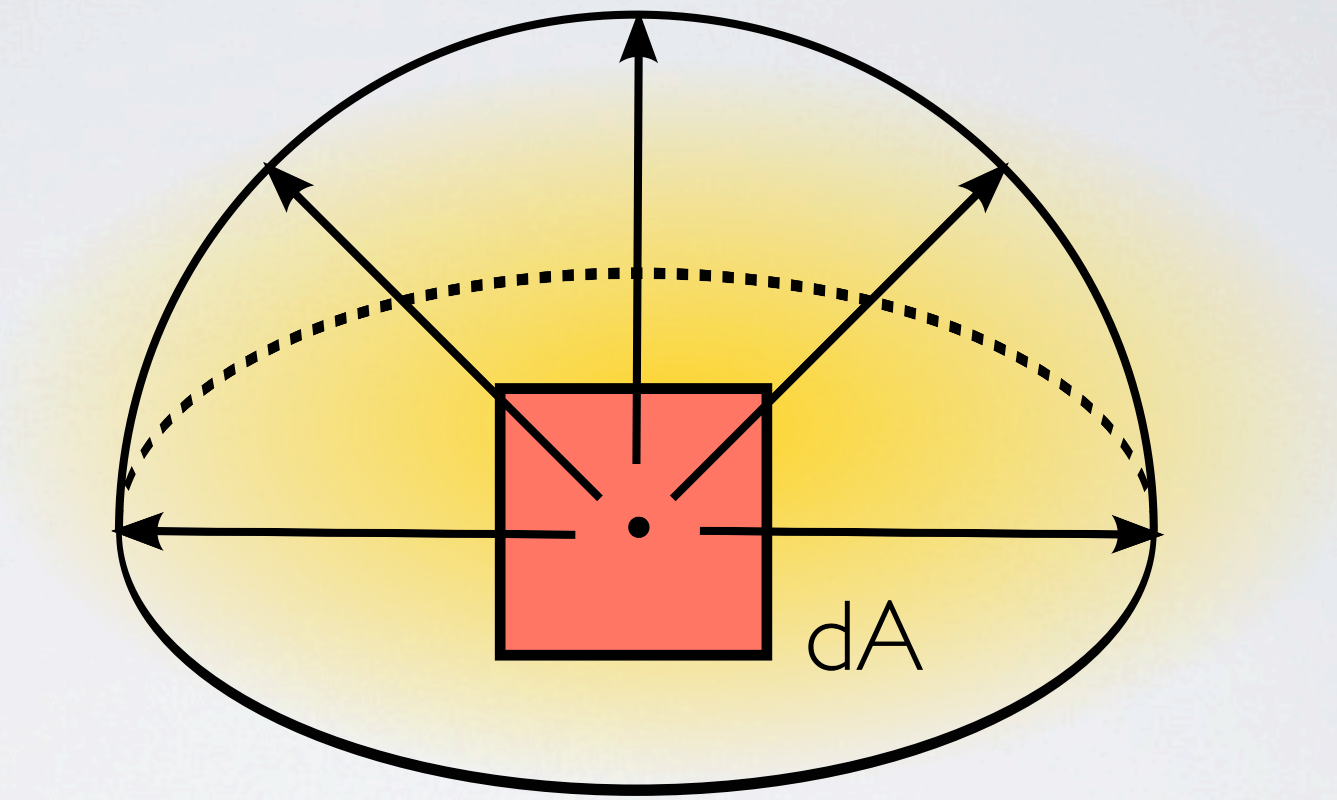
Unit $[I] = 1\,\mathrm{W\,sr}^{-1}$

Radiometry



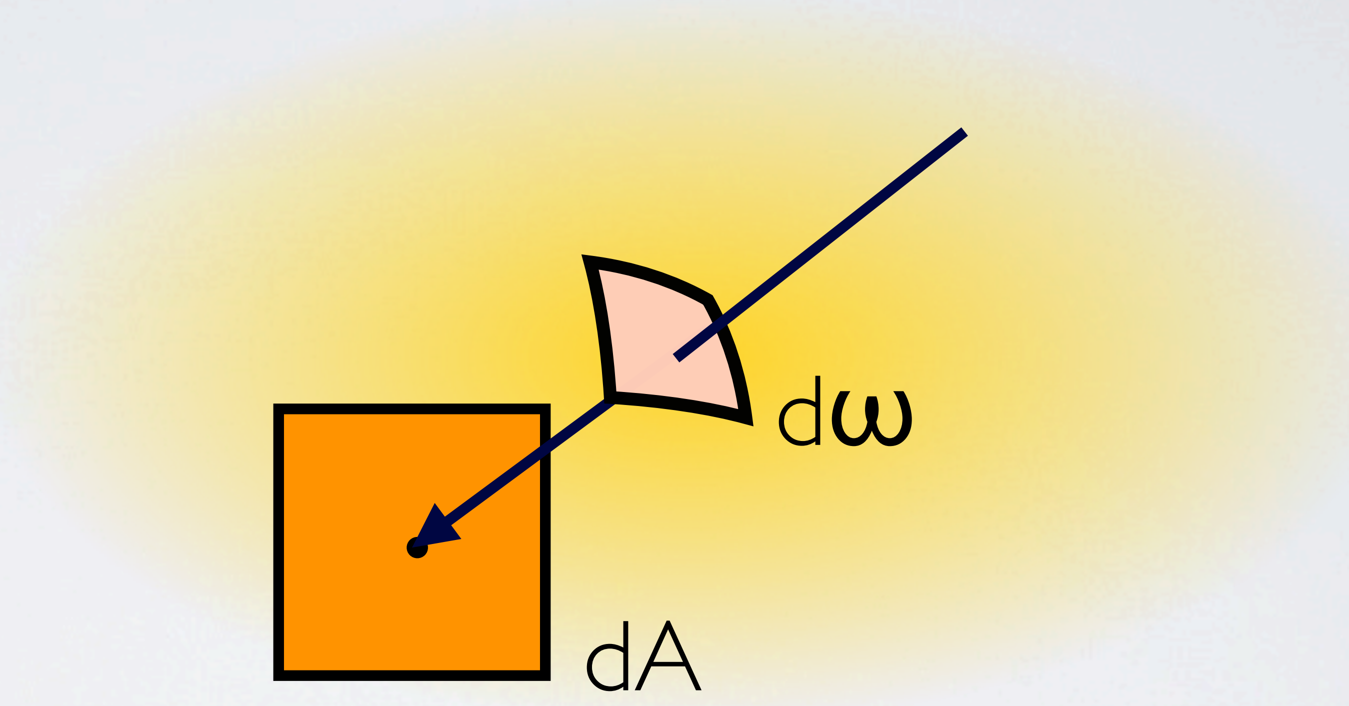
Name	irradiance (in a point on the surface)
Symbol	$E = \frac{dP}{dA}$
Unit	$[E] = 1\,\mathrm{W\,m^{-2}}$

Radiometry



Name	radiant exitance, radiosity (from a point)
Symbol	$B = \frac{dP}{dA}$
Unit	$[B] = 1\,\mathrm{W\,m^{-2}}$

Radiometry

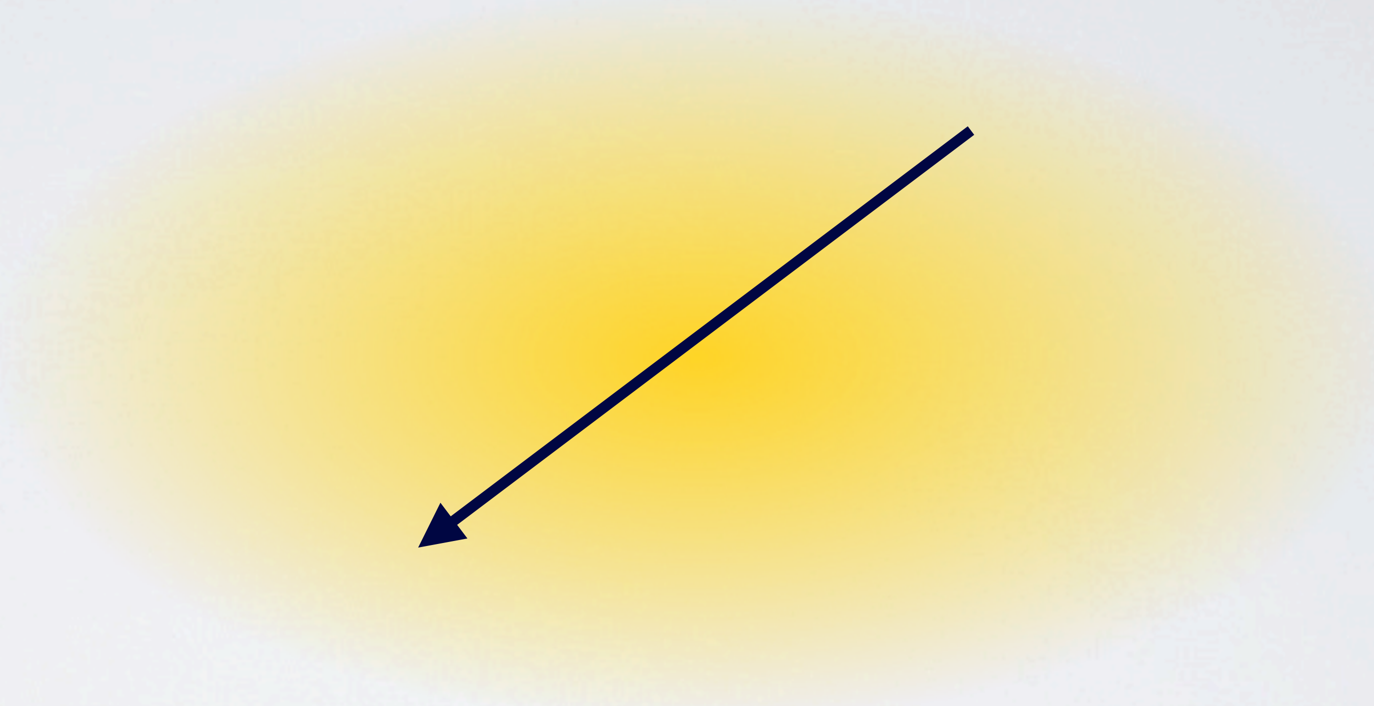


Name radiance

Symbol $L = \frac{d^2 P}{d\omega \, dA}$

Unit $[L] = 1\ W \mathrm{sr}^{-}1\mathrm{m}^{-2}$

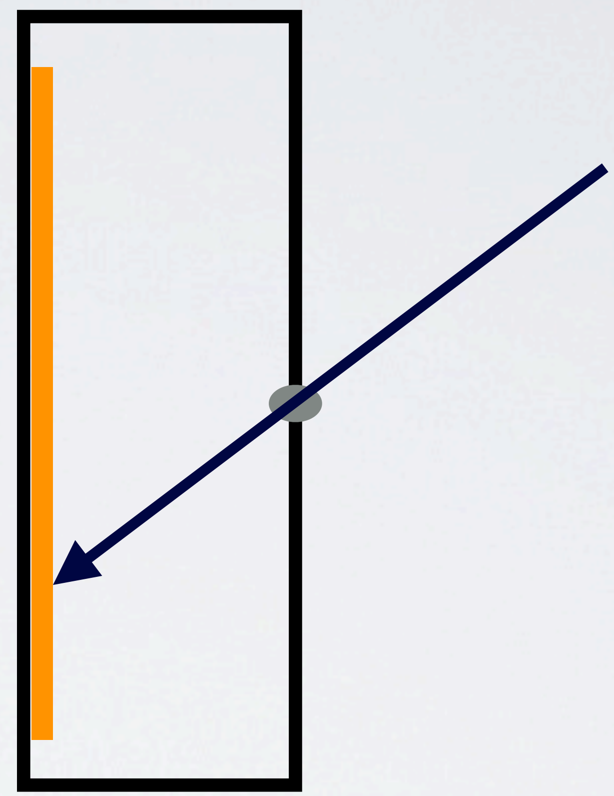
Radiometry



In vacuum,

radiance is constant along rays!

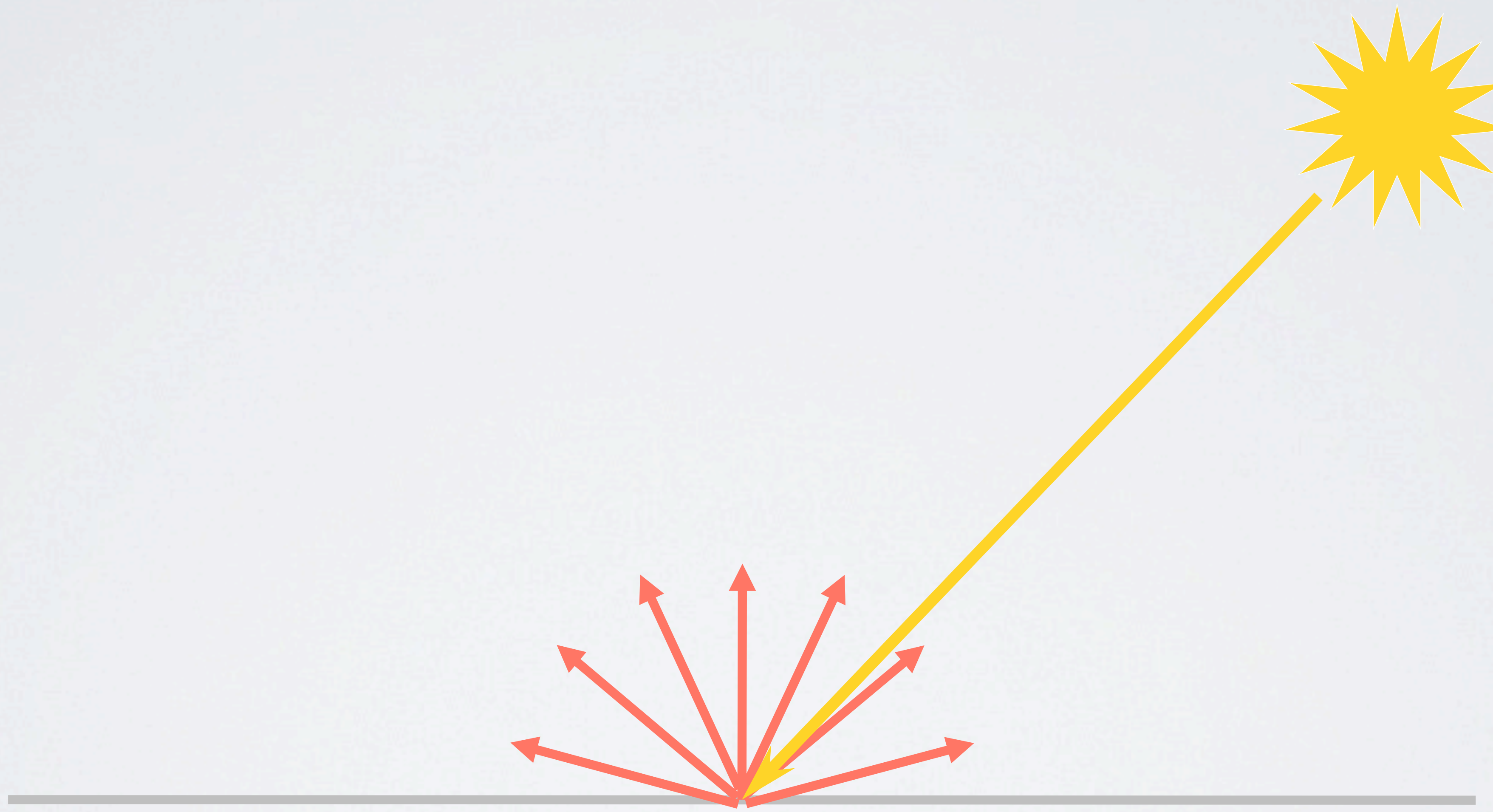
Radiometry



Pinhole cameras record radiance.

Lambertian Surfaces

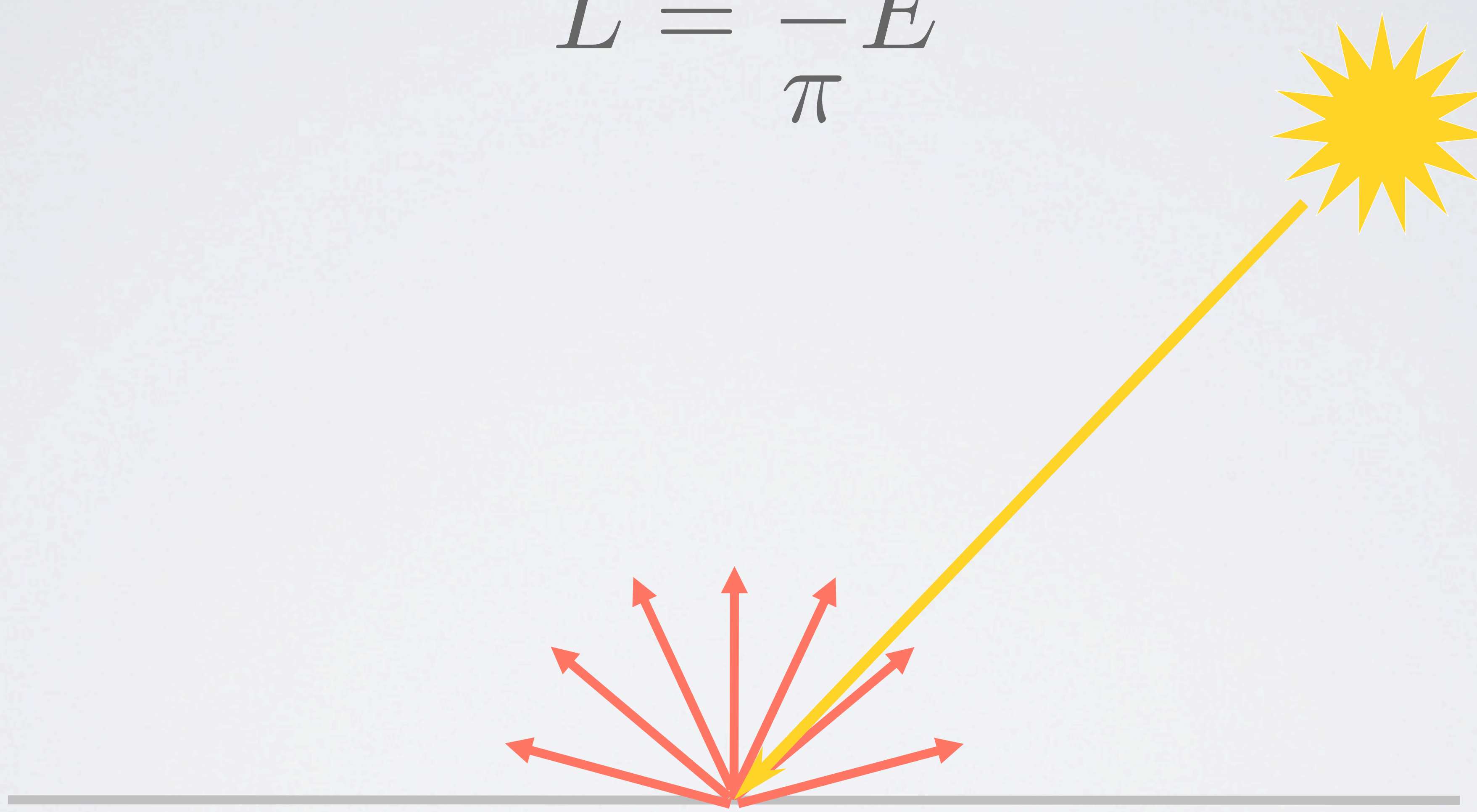
appear equally bright from all directions



Lambertian Surfaces

appear equally bright from all directions

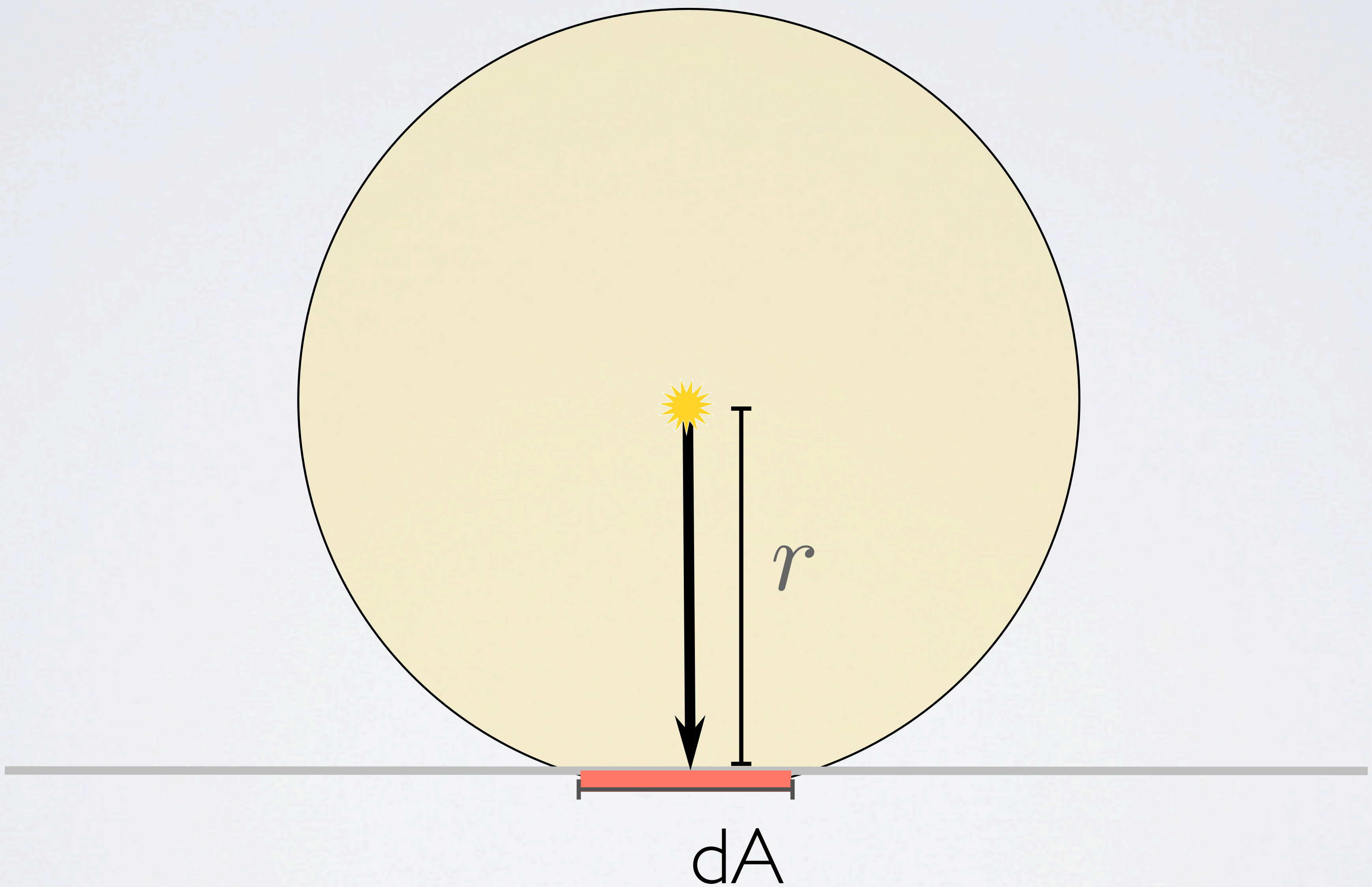
$$L = \frac{\rho}{\pi} E$$



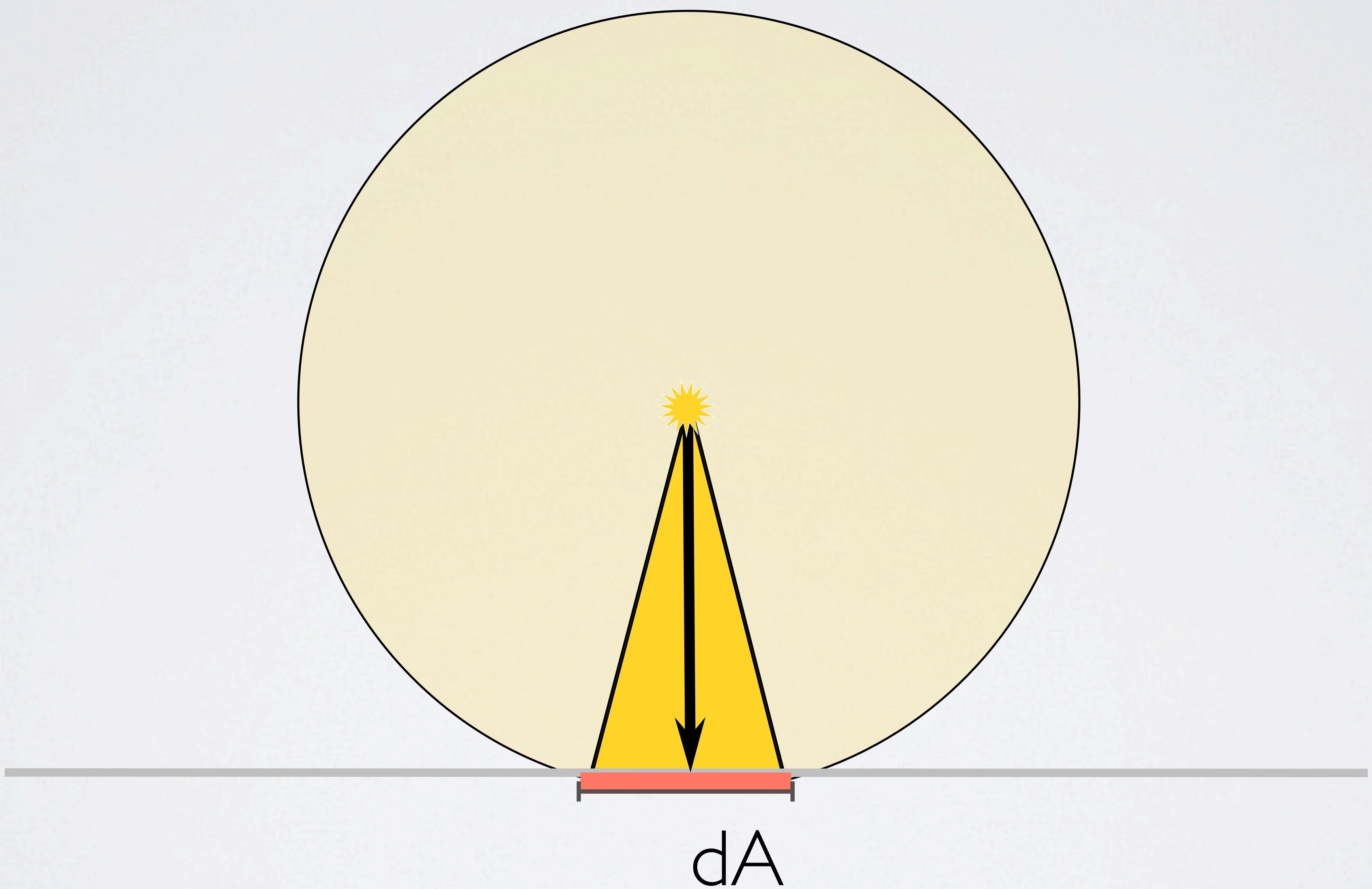
Albedo **ρ** ranges from 0 (perfect black) to 1 (perfect white).

How is E computed?

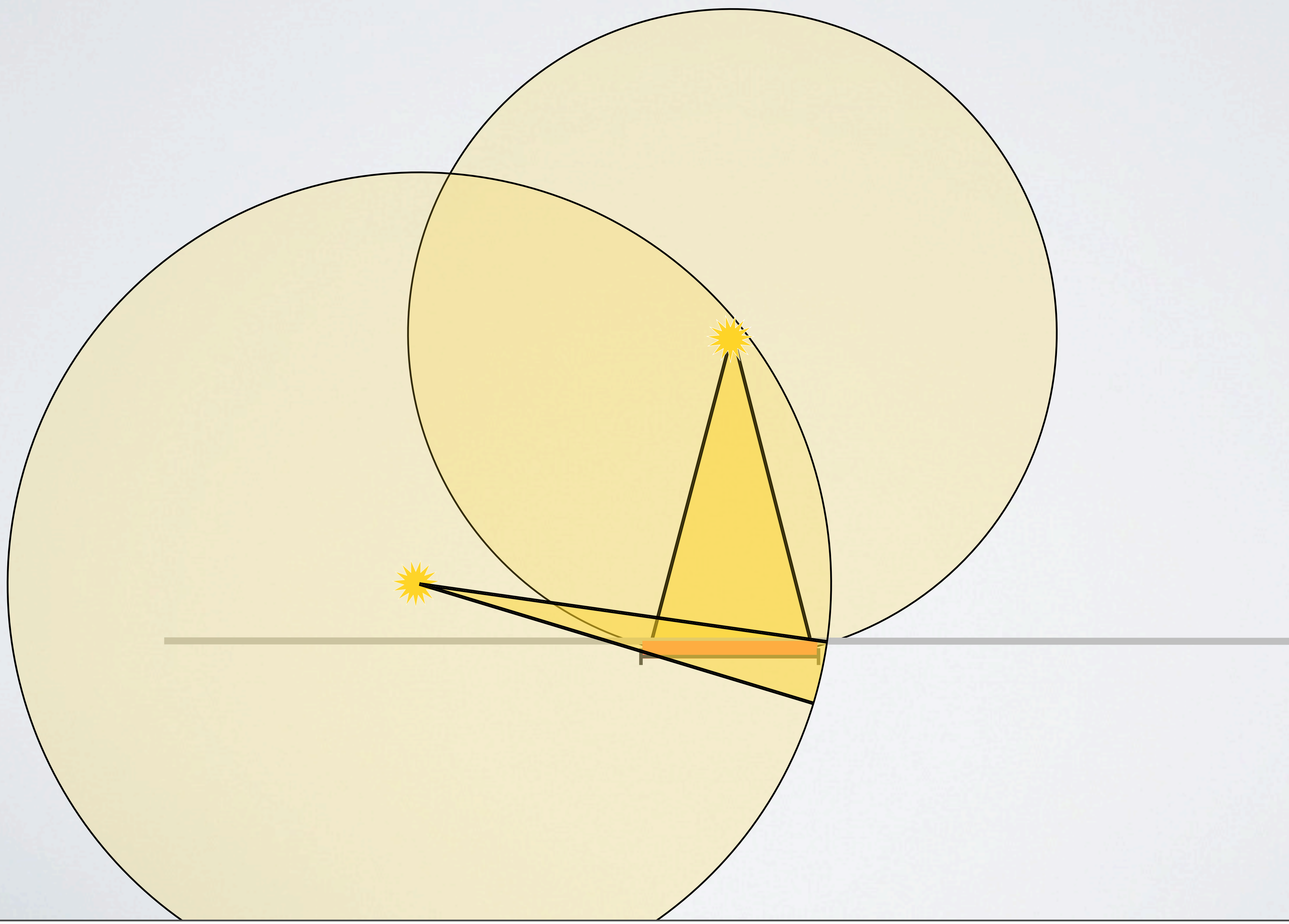
$$E \sim \frac{I}{r^2}$$



How is E computed?

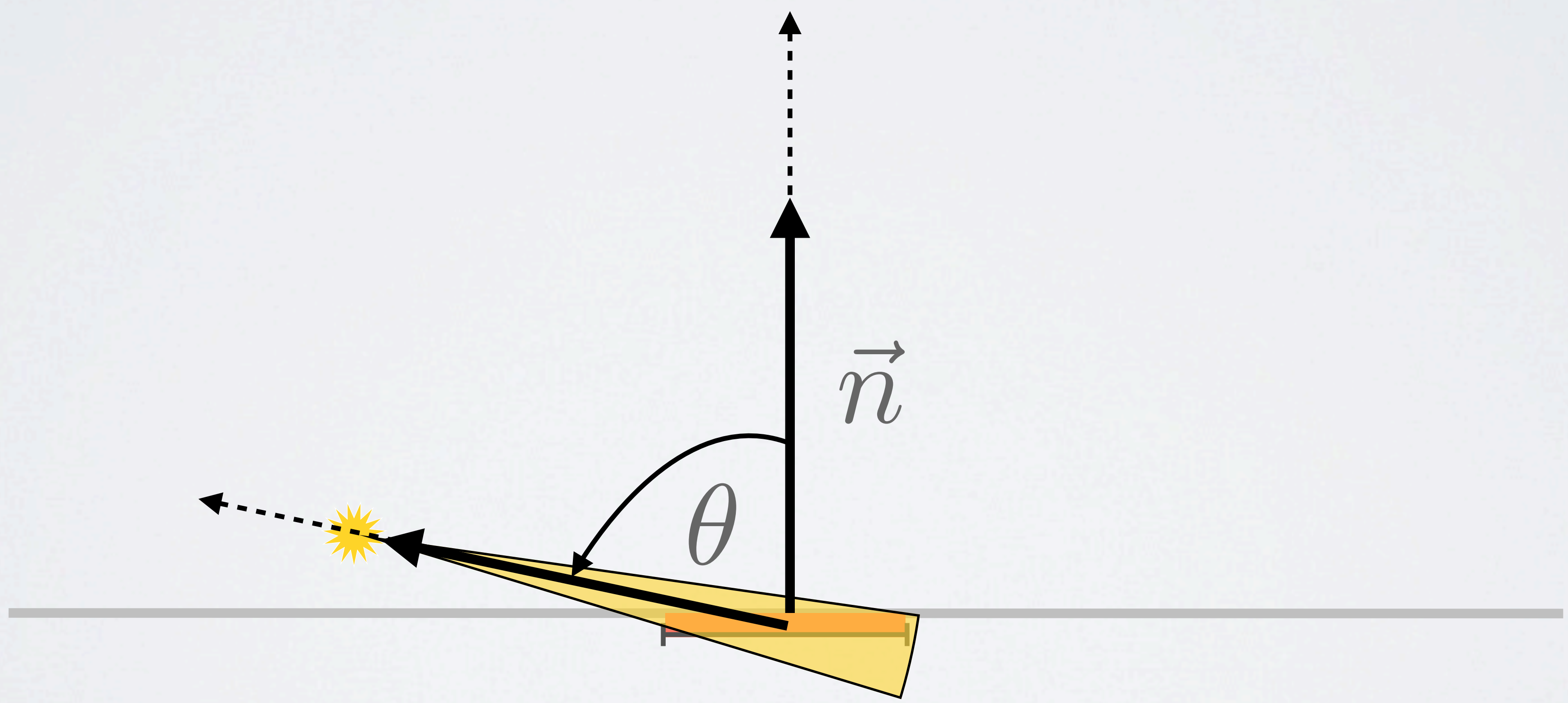


How is E computed?



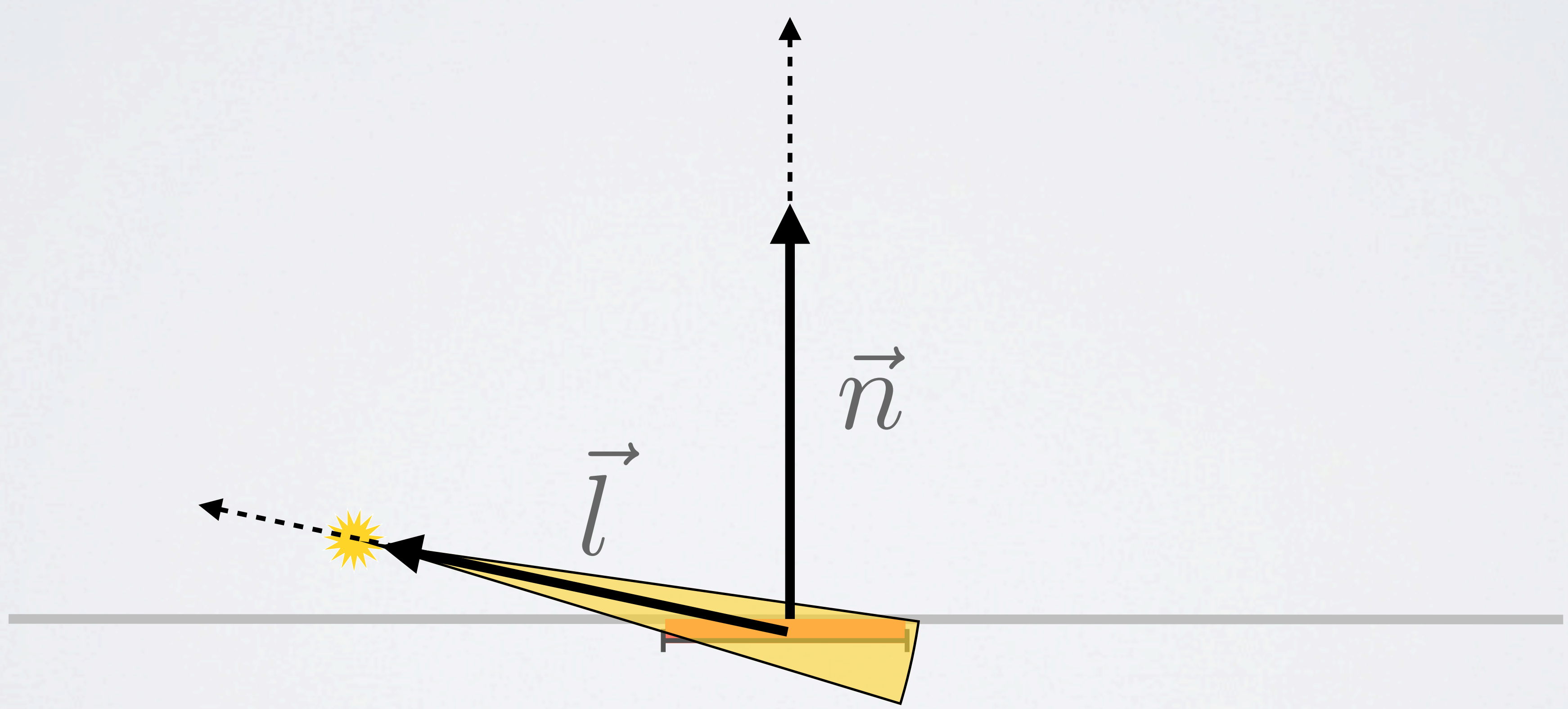
Lambert's Cosine Law

$$E \sim \cos\theta \cdot I$$



Lambert's Cosine Law

$$E \sim \left\langle \vec{l}, \vec{n} \right\rangle \cdot I$$

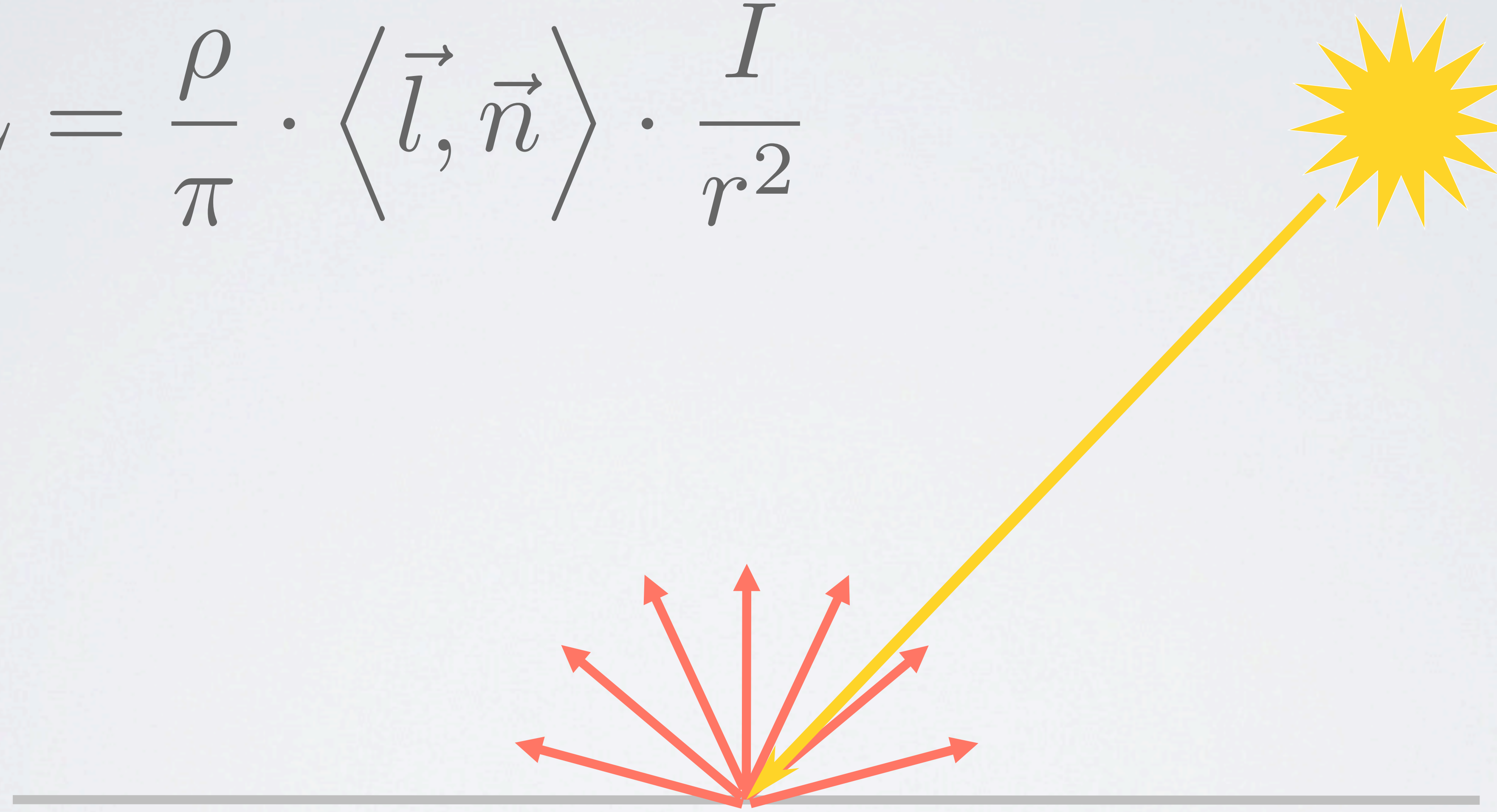


$$\left\| \vec{l} \right\| = \|\vec{n}\| = 1$$

Lambertian Surfaces

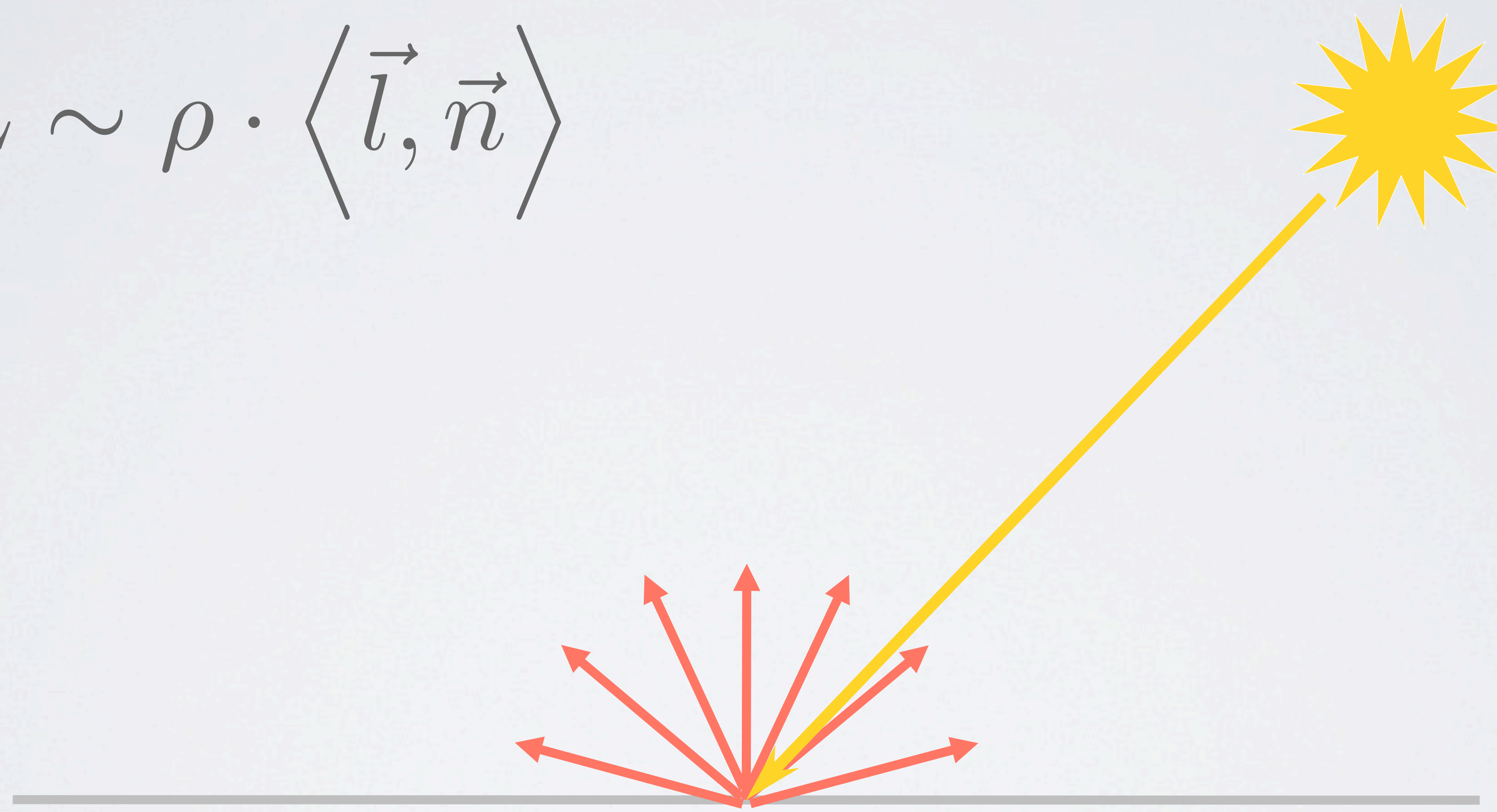
$$L = \frac{\rho}{\pi} E$$

$$L = \frac{\rho}{\pi} \cdot \left\langle \vec{l}, \vec{n} \right\rangle \cdot \frac{I}{r^2}$$



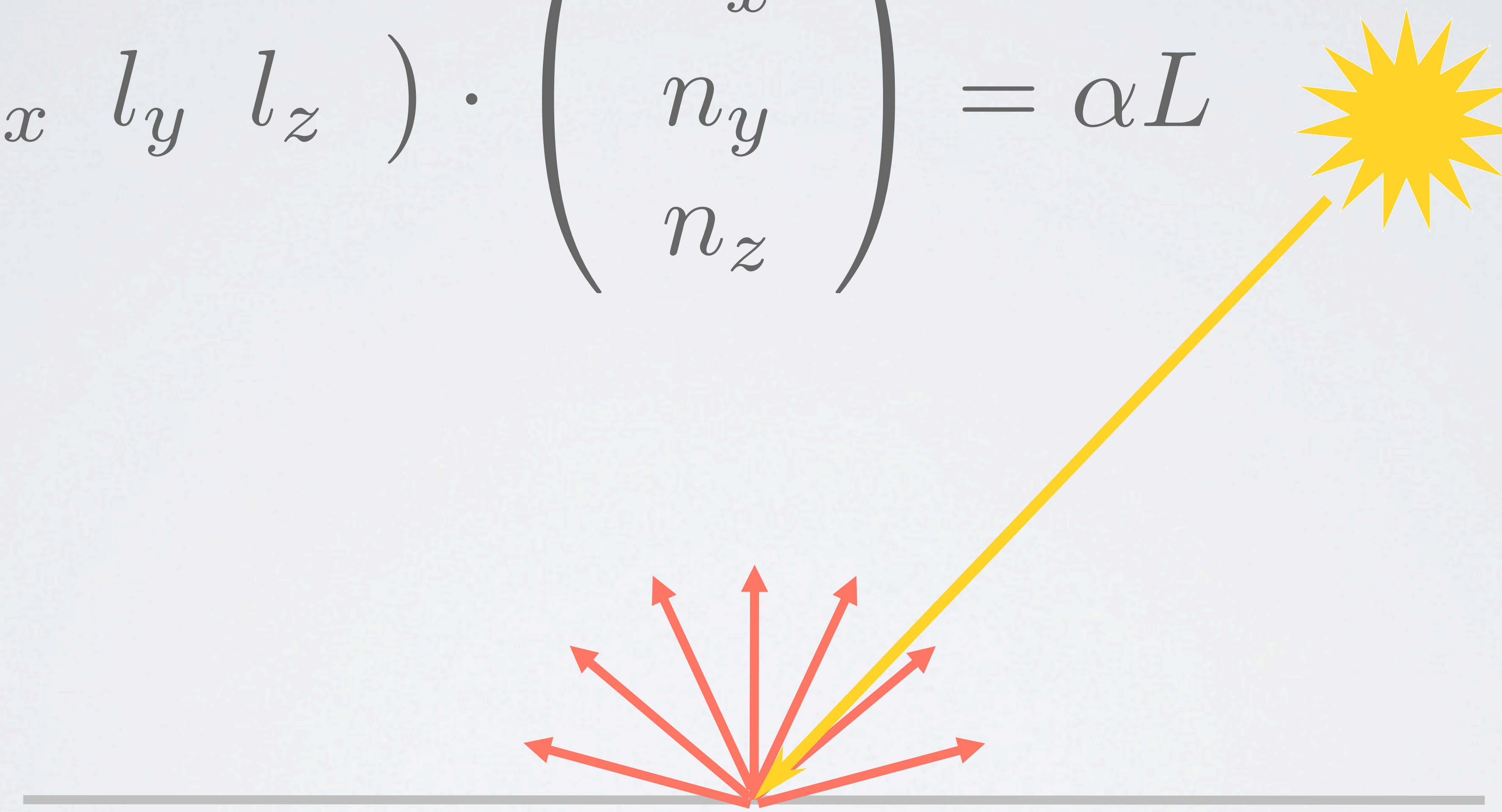
Lambertian Surfaces

$$L \sim \rho \cdot \left\langle \vec{l}, \vec{n} \right\rangle$$



Photometric Stereo

$$\rho \cdot \begin{pmatrix} l_x & l_y & l_z \end{pmatrix} \cdot \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} = \alpha L$$

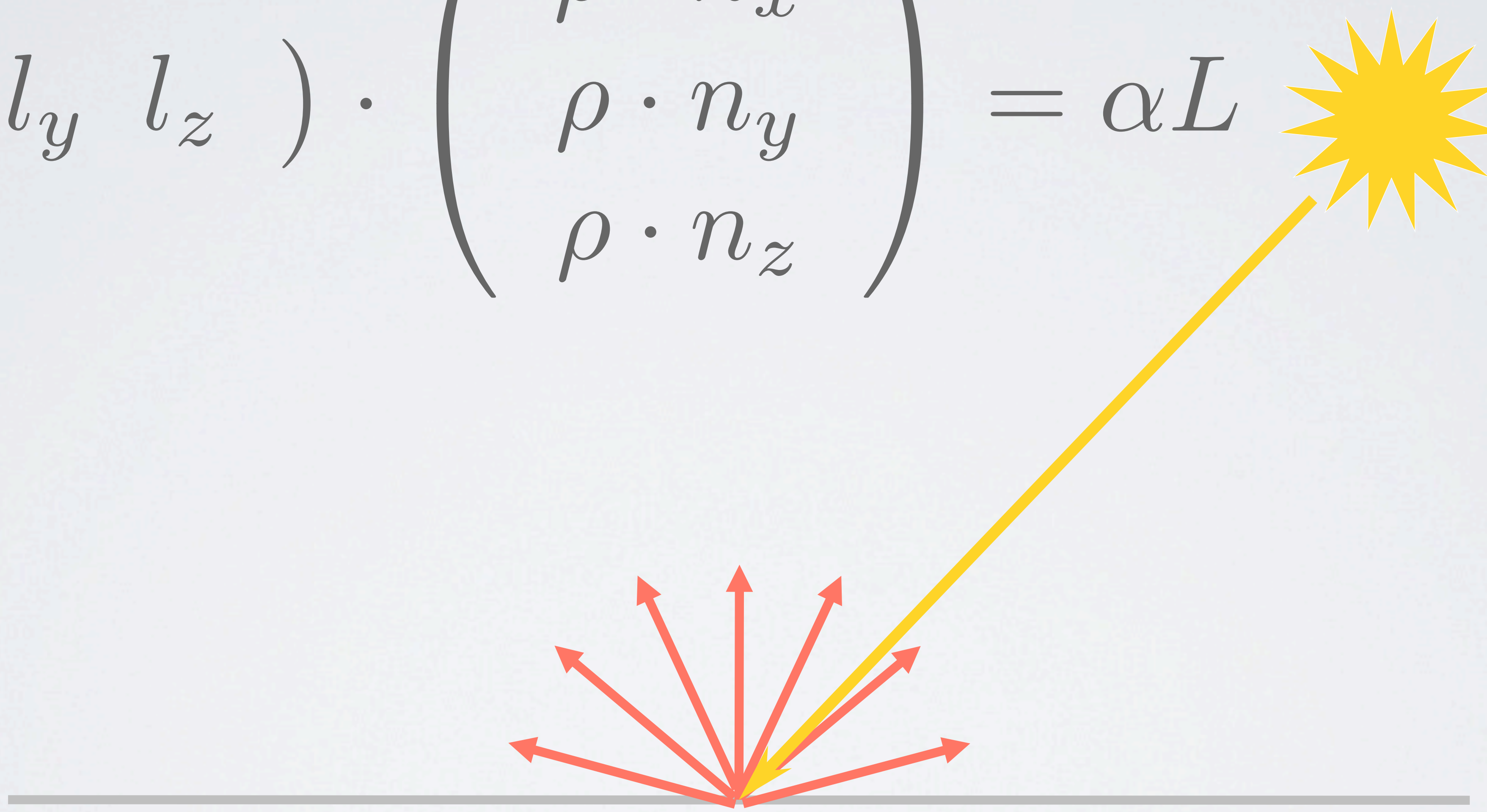


α subsumes camera properties, light source brightness, r^2

ρ and $\vec{n}$ are unknown

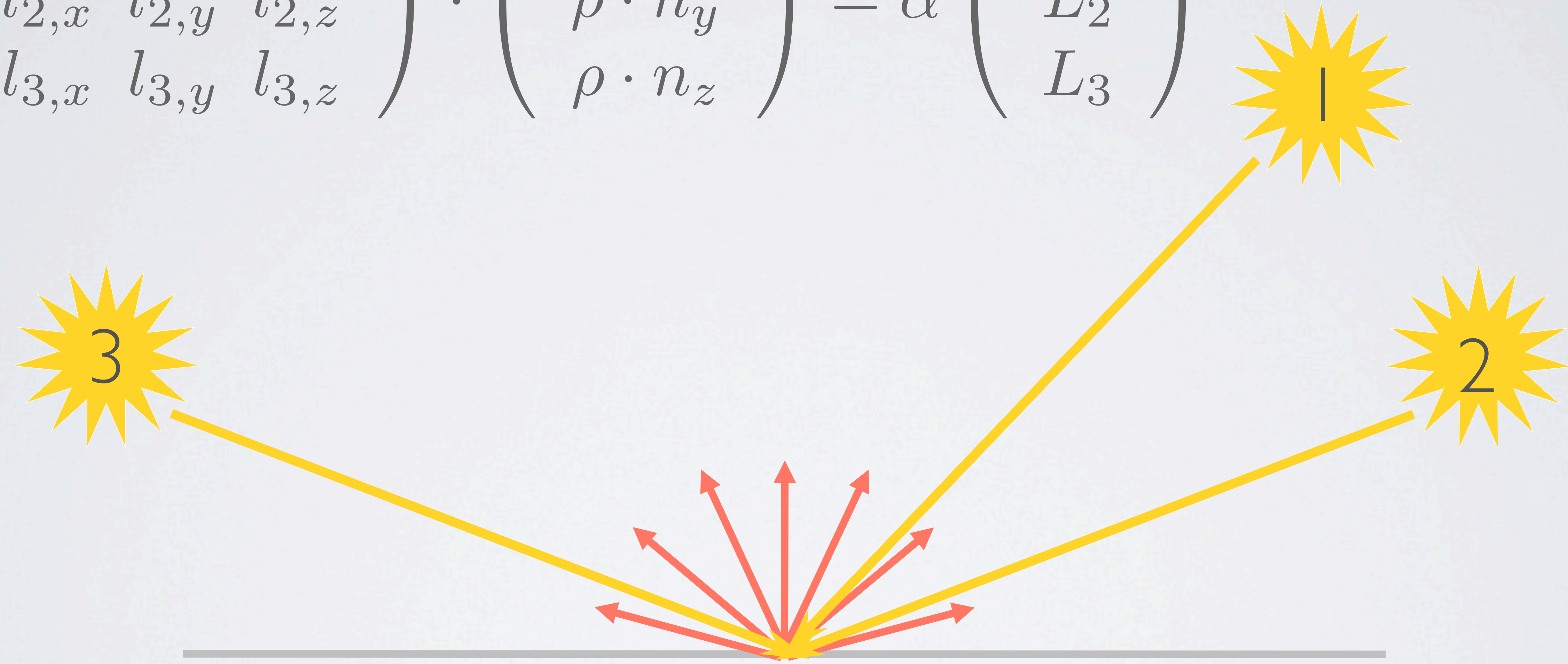
Photometric Stereo

$$\begin{pmatrix} l_x & l_y & l_z \end{pmatrix} \cdot \begin{pmatrix} \rho \cdot n_x \\ \rho \cdot n_y \\ \rho \cdot n_z \end{pmatrix} = \alpha L$$



Photometric Stereo

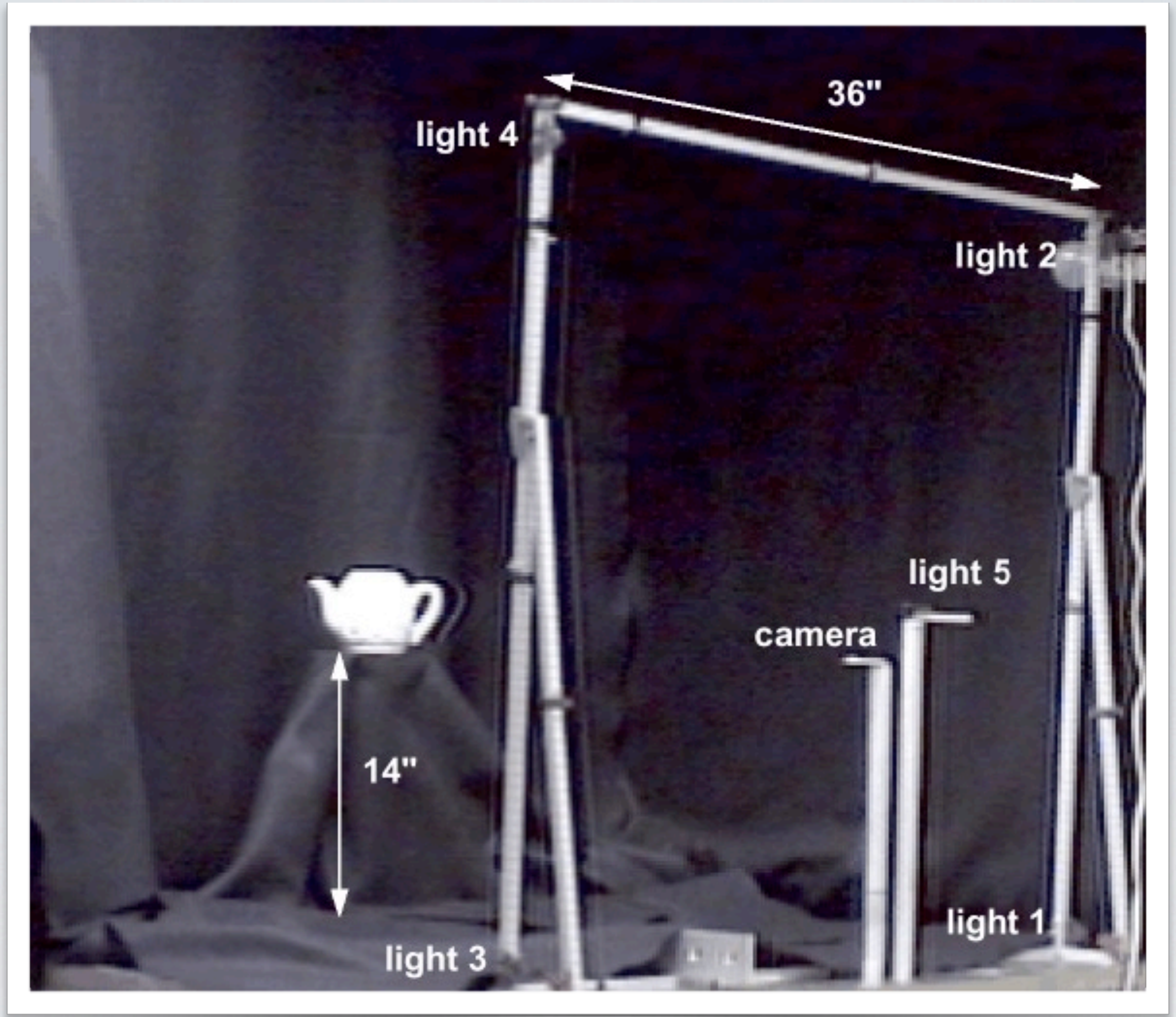
$$\begin{pmatrix} l_{1,x} & l_{1,y} & l_{1,z} \\ l_{2,x} & l_{2,y} & l_{2,z} \\ l_{3,x} & l_{3,y} & l_{3,z} \end{pmatrix} \cdot \begin{pmatrix} \rho \cdot n_x \\ \rho \cdot n_y \\ \rho \cdot n_z \end{pmatrix} = \alpha \begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix}$$



Photometric Stereo

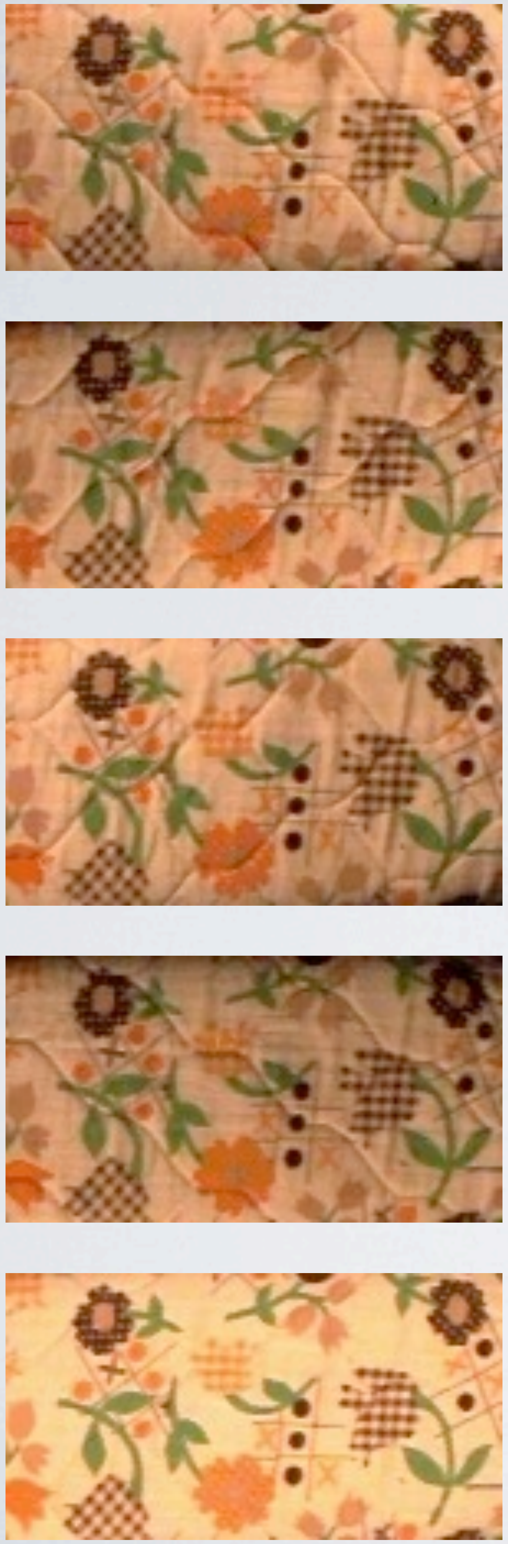
- In practice, use more light sources
  - discard darkest observations: shadowed
  - discard brightest observations: not Lambertian
  - use least-squares solution for over-determined system

Photometric Stereo



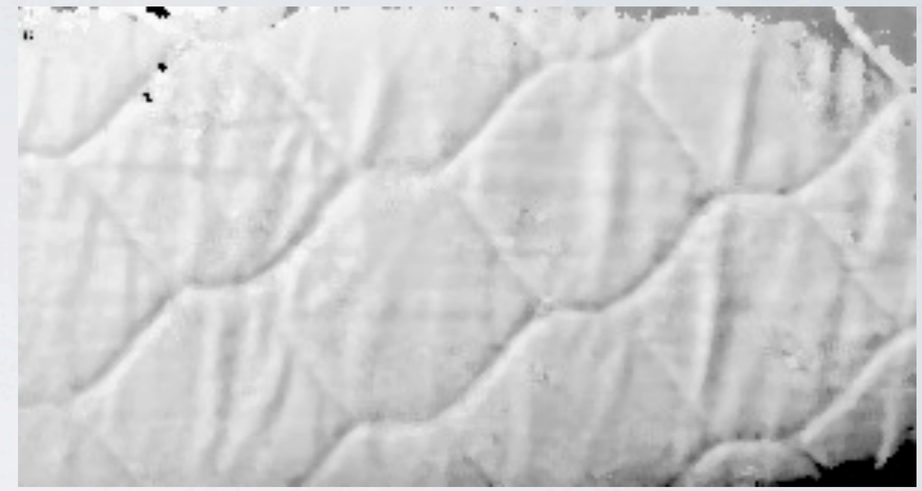
[Rushmeier et al., 1997]

Photometric Stereo

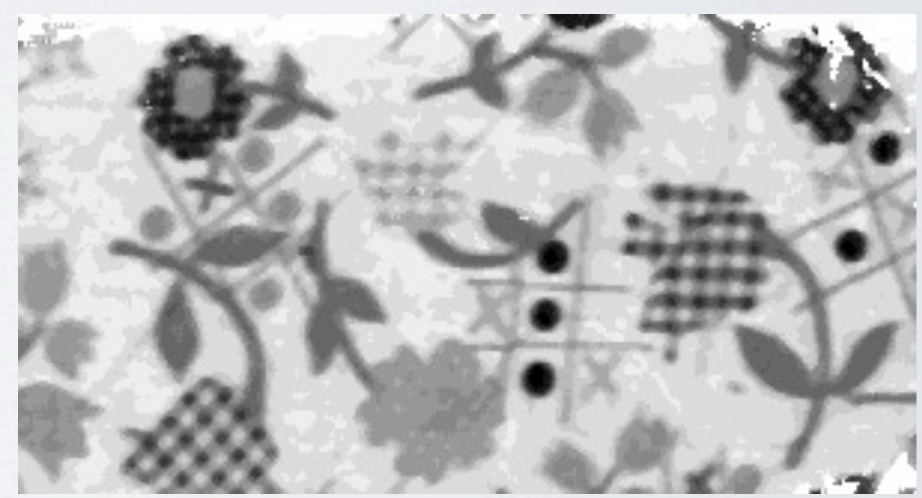


Input images

Estimated normals (lit)



Estimated albedo



[Rushmeier et al., 1997]

Photometric Stereo

Practical considerations:

- illumination: flashes vs. steady light sources
    - subtract dark frame
- calibrate: light source positions
    - either build or measure (mirror balls)
- calibrate: light source intensities
    - move same light source: very precise
    - observe on known reflectance target: more flexible
- some pre-computation possible

Photometric Stereo

- good:
  - fast (real-time)
  - high resolution (one normal vector per pixel!)
- bad:
  - normals only
  - active light only

Shape from Shading

- like photometric stereo, but with **one** image and **one** light source only!
- useful for astronomy etc.
- underconstrained problem. Assume:
  - uniform, known albedo
  - known illumination
  - smooth variation in surface normal

Shape from Shading

- Further simplifications:
  - Camera far from object: (x, y) in image = (x, y) in world
- $z(x, y)$ denotes the depth in pixel

Shape from Shading

Definitions:

Let $p := \frac{\partial}{\partial x} z, \quad q := \frac{\partial}{\partial y} z$

Then, $\begin{pmatrix} 1 \\ 0 \\ p \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ q \end{pmatrix}$ are tangents, and the normal is

$$\vec{n} = \frac{1}{\sqrt{1 + p^2 + q^2}} \begin{pmatrix} 1 \\ 0 \\ p \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ q \end{pmatrix}$$

Shape from Shading

Definitions:

Let $p := \frac{\partial}{\partial x} z, \quad q := \frac{\partial}{\partial y} z$

Then, $\begin{pmatrix} 1 \\ 0 \\ p \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ q \end{pmatrix}$ are tangents, and the normal is

$$\vec{n} = \frac{1}{\sqrt{1 + p^2 + q^2}} \begin{pmatrix} -p \\ -q \\ 1 \end{pmatrix}$$

Shape from Shading

Definitions:

Let

$$R(p,q) := \rho \cdot \left\langle \vec{l}, \frac{1}{\sqrt{1+p^2+q^2}} \begin{pmatrix} -p \\ -q \\ 1 \end{pmatrix} \right\rangle \cdot \frac{1}{\alpha}$$

be the radiance for a hypothesis of p and q, and

$$L$$

be the radiance in the captured picture.

Shape from Shading

Minimize the functional:

$$\int_A (L - R(p,q))^2 + \lambda(|\nabla p|^2 + |\nabla q|^2)\, dx\, dy$$

Data term | Smoothness Term

for a control parameter λ

Shape from Shading

$$\int_A (L - R(p,q))^2 + \lambda(|\nabla p|^2 + |\nabla q|^2)\, dx\, dy$$

Solve by iteration:

- initialize with an estimate for p and q
- after step k , set

$$p_{k+1} = \overline{p}_k + \frac{1}{4\lambda}\,(L - R(p_k, q_k))\,\frac{\partial R(p_k, q_k)}{\partial p}$$

$$q_{k+1} = \overline{q}_k + \frac{1}{4\lambda}\,(L - R(p_k, q_k))\,\frac{\partial R(p_k, q_k)}{\partial q}$$

for local averages $\overline{p}$ and $\overline{q}$

- iterate

see [Trucco and Verri, 1998] for Details

Shape from Shading

- Problems:
  - end of computation not necessarily a global optimum
  - boundary conditions need to be chosen
    (enforce after each iteration)
- for a consistent surface,

$$\frac{\partial^2 z}{\partial x\,\partial y} = \frac{\partial^2 z}{\partial y\,\partial x} \quad \Rightarrow \quad \frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}$$

but this is not necessarily the case

Shape from Shading

- Enforce constraint in functional: add

$$\left(\frac{\partial p}{\partial y} - \frac{\partial q}{\partial x}\right)^2$$

  to the energy functional (makes things difficult)

- Maintain constraint throughout optimization
  - after each iteration, project p and q to closest integrable function pair (in Fourier space)

Shape form shading

When consistent, p and q can be integrated to yield z:

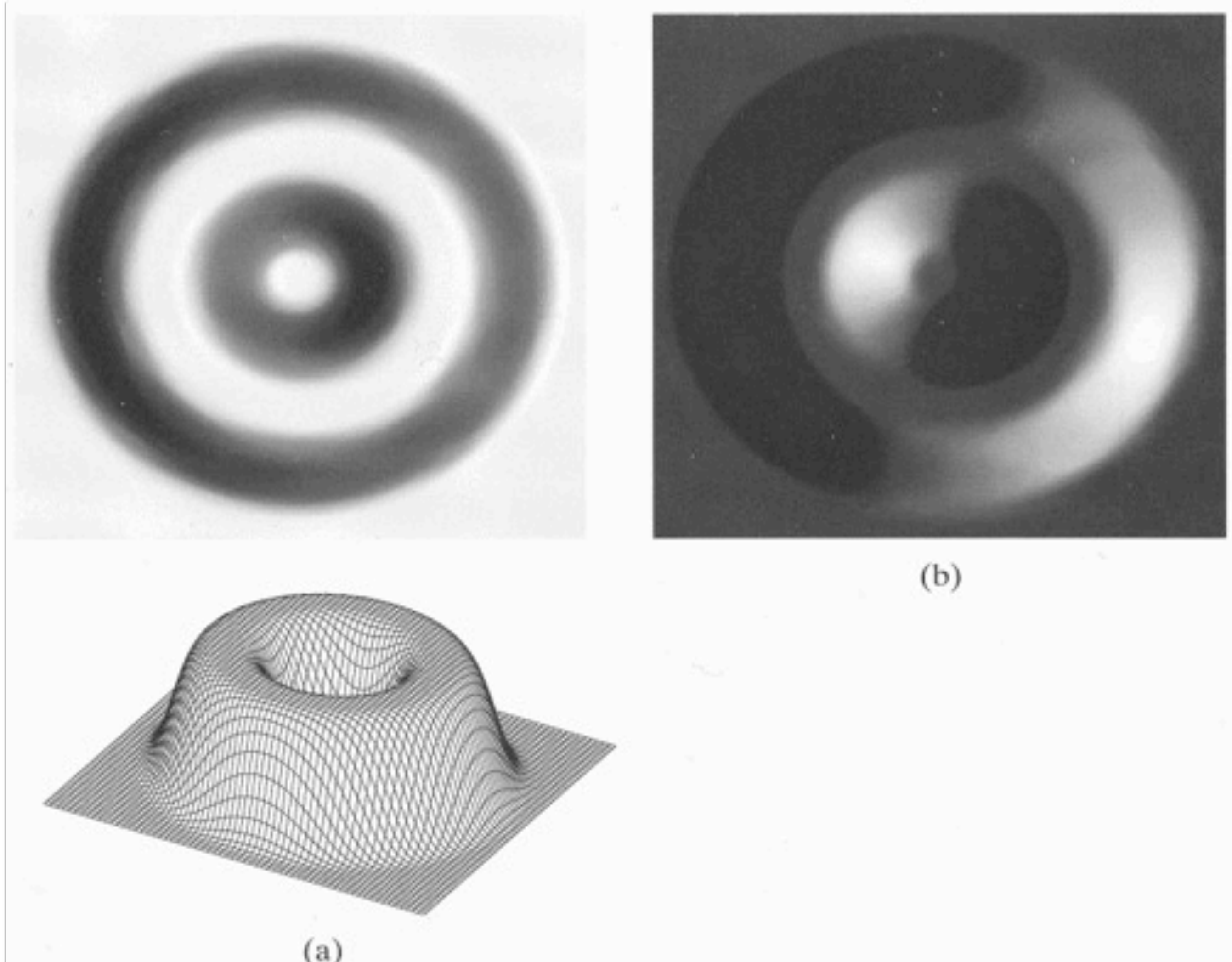


Figure 9.2 Two images of the same Lambertian surface seen from above but illuminated from different directions and 3-D rendering of the surface. Practically all the points in the top left image receive direct illumination ($\mathbf{i} = [0.20, 0, 0.98]^\top$); some regions of the top right image are in the dark due to self-shadowing effects ($\mathbf{i} = [0.94, 0.31, 0.16]^\top$).

[Trucco and Verri, 1998]

Shape form shading

When consistent, p and q can be integrated to yield z:

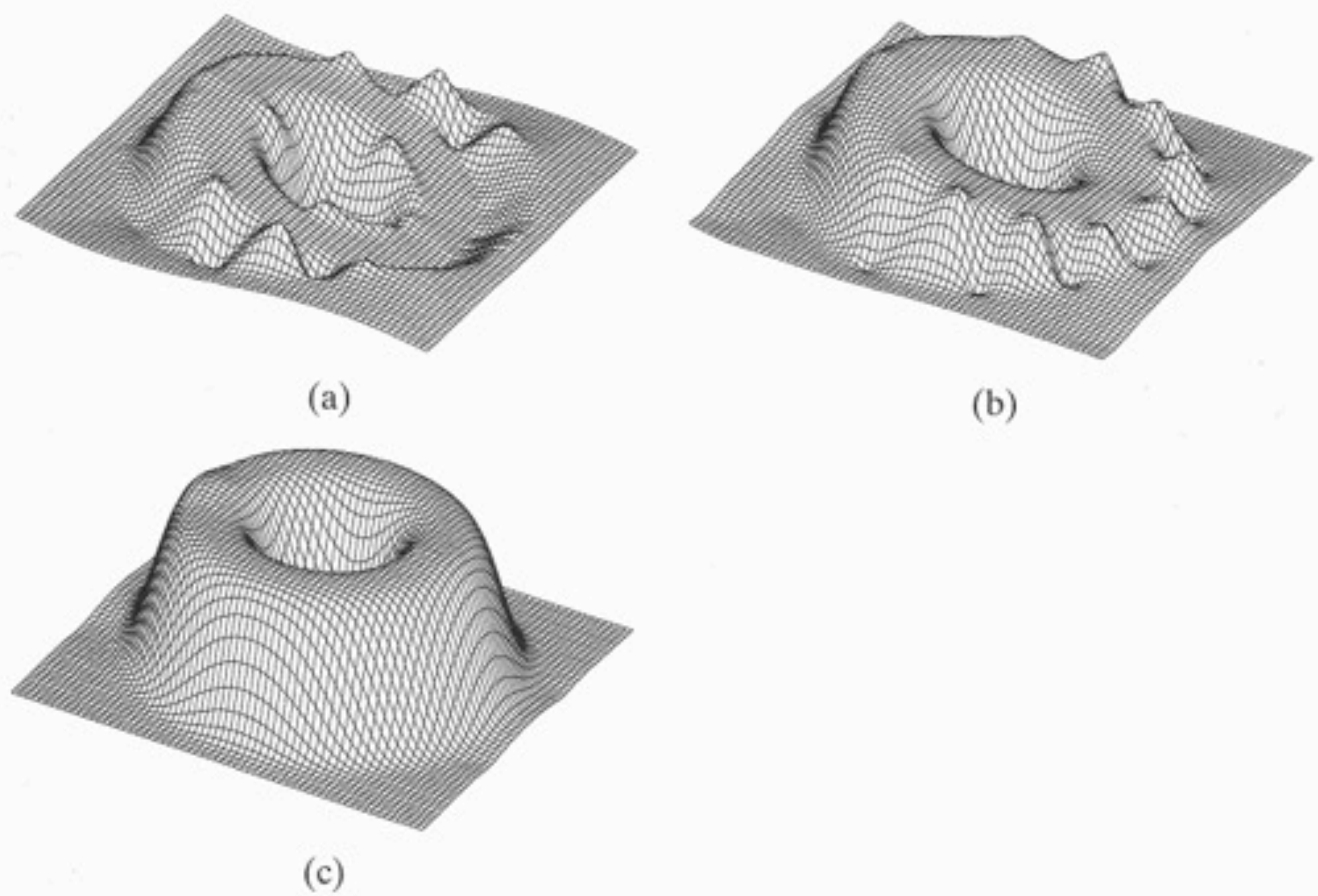


Figure 9.4 Reconstructions of the surface in Figure 9.2 after 100 (a), 1000 (b) and 2000 (c) iterations. The initial surface was a plane of constant height. The asymmetry of the first two reconstruction is due to the illuminant direction.

[Trucco and Verri, 1998]

Limitations

- material properties
- occlusions (shadows)
- how to know albedo and light intensity?
  - further assumptions needed

Conclusions

- appearance can be used to estimate shape
  - assumptions required, Lambertian common
- photometric stereo is precise, but active light required
- shape from shading can be useful where stereo is impossible

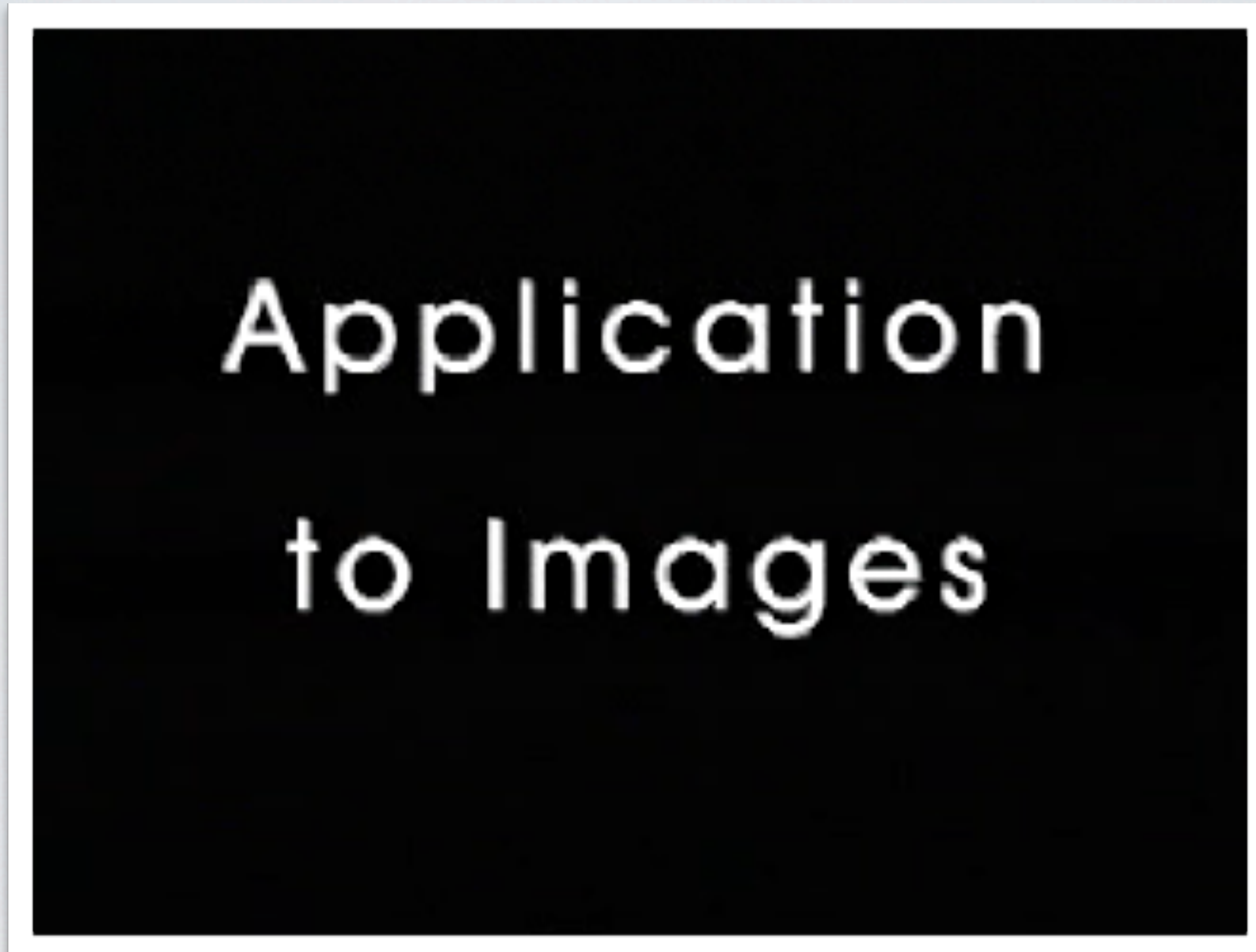
Knowing more ...

- with more assumptions, you can know more about the shape

- a statistical model + a single image can give you
  - shape
  - texture + colors
  - additional semantics ...

[Blanz and Vetter, 1999]

Knowing more ...



Video source: http://mi.informatik.uni-siegen.de/movies/siggraph99.mpg

[Blanz and Vetter, 1999]

Questions ?