Linear Programming   by example

Diet Problem

		T tofuburger	H hamburger	P pizza
calories	c	0.1	.6	.7
vitamins	v	.9	.9	.5
fat	f	0.1	1.0	1.5

$$
\begin{cases}
\text{total calories} = 0.1\,x_T + 0.6\,x_H + 0.7\,x_P \geq \text{Cal Req.} \\
\text{total vitamins} = 0.9\,x_T + 0.9\,x_H + 0.5\,x_P \geq \text{Vit Req.} \\
\text{total fat} \;\;= 0.1\,x_T + 1.0\,x_H + 1.5\,x_P \leq \text{Fat Limit}
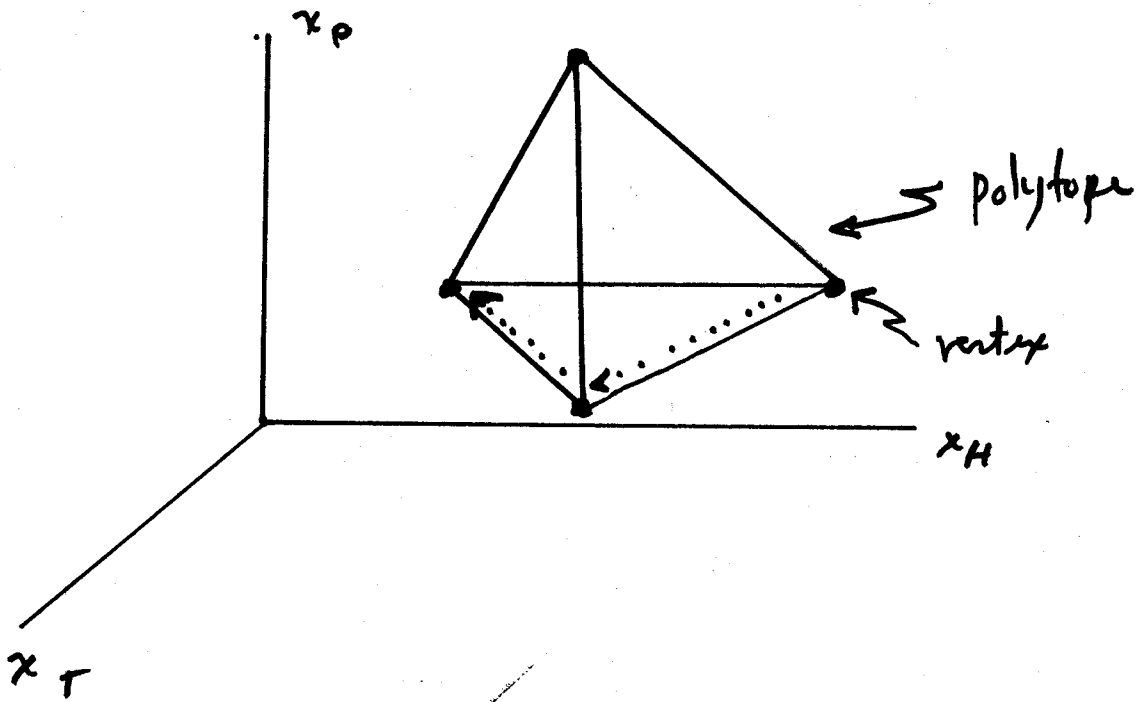\end{cases}
$$

$$x_T, x_H, x_P \geq 0$$

$$\min \; \text{cost} = \pi_T \cdot x_T + \pi_H \, x_H + \pi_P \, x_P$$

Can always be put in __standard form__:

$$
\begin{aligned}
&\min \; \underline{c}' \underline{x} \\
&\underline{\underline{A}} \, \underline{x} = \underline{b} \\
&\underline{x} \geq 0
\end{aligned}
$$

x_P

x_H

x_T

polytope

vertex

<u>Simplex algorithm</u>   practical, worst-case bad,
                              widely used

<u>Interior-point algorithms</u>   sometimes practical,
                                      <u>polynomial!</u>
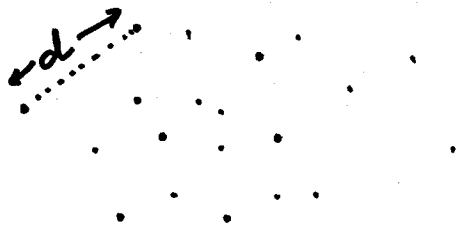

"worst-case bad"   exponential time, measured in terms
                     of input length

basic step, move from vertex to vertex, is
called a <u>pivot</u>. More later, when we
discuss solving linear equations.

See, e.g., [PS 98]

Intractable Problems

usual example: traveling Salesman Problem
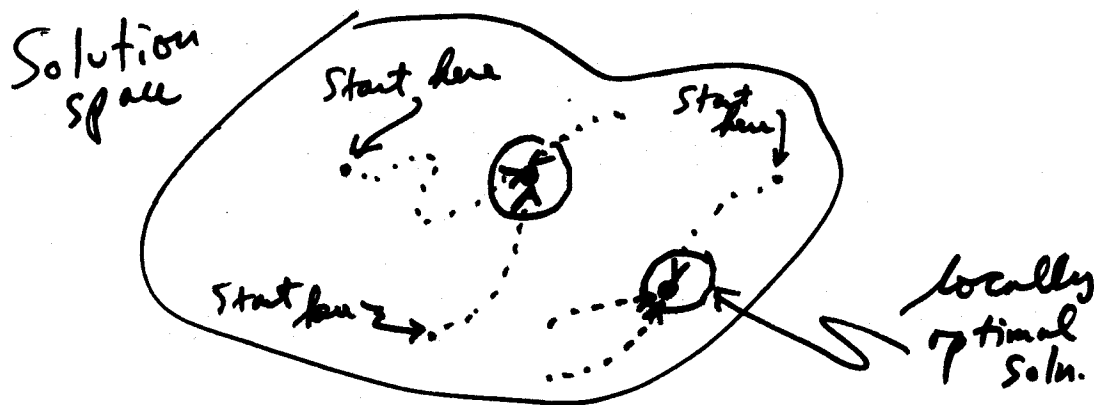
Given graph & distances

find closed tour of minimum total length

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belongs to class <u>NP-complete</u>
If one can be solved efficiently, all can.
Believed to be essentially hard.

One common computational approach: <u>Local Search</u>

Solution space
Start here
Start here
Start here 2
locally optimal soln.

<u>basins of attraction</u>

often good quality, practical, no guarantee
(usually)