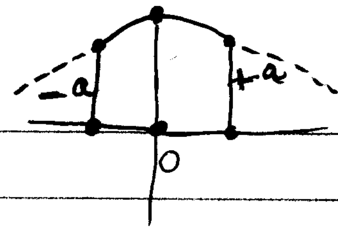
Simpson's Rule



Interpolate quadratic (Lagrange form)

$$f(x) = (f_-)\frac{x(x-a)}{2a^2} + (f_0)\frac{(x+a)(x-a)}{-a^2} + (f_+)\frac{x(x+a)}{2a^2}$$

$$= \frac{1}{2a^2}\left[f_- \boxed{x(x-a)} - 2\boxed{f_0(x-a^2)} + f_+\boxed{x(x+a)} \right]$$

$$\int_{-a}^{+a} x(x-a)\,dx = \int_{-a}^{+a} (x^2-ax)\,dx = \left[\frac{x^3}{3} - a\frac{x^2}{2}\right]\Big|_{-a}^{a}$$

$$= \frac{a^3}{3} - a\cancel{\frac{a^2}{2}} + \frac{a^3}{3} + \cancel{\frac{a^3}{2}} = \frac{2a^3}{3}$$

$$\int_{-a}^{a} (x^2-a^2)\,dx = \left[\frac{x^3}{3} - a^2x\right]_{-a}^{a} = \frac{a^3}{3} - a^3 + \frac{a^3}{3} - a^3$$

$$= \frac{2}{3}a^3 - 2a^3 = \left[\frac{2-6}{3}\right]a^3 = -\frac{4}{3}a^3$$

$$\int_{-a}^{a} x(x+a)\,dx = \int_{-a}^{a}(x^2+ax)\,dx = \left[\frac{x^3}{3} + \frac{ax^2}{2}\right]_{-a}^{a}$$

$$= \frac{a^3}{3} + \cancel{\frac{a^3}{2}} + \frac{a^3}{3} - \cancel{\frac{a^3}{2}} = \frac{2a^3}{3}$$

$$\boxed{\int_{-a}^{a} f(x)\,dx} = \frac{a^3}{2a^2}\left[\frac{2}{3}f_- + \frac{8}{3}f_0 + \frac{2}{3}f_+\right]$$

$$= \cancel{\frac{2}{3}}\,\frac{a}{\cancel{2}}\left[f_- + 4f_0 + f_+\right]$$

$$= \frac{a}{3}\left[f_- + 4f_0 + f_+\right]$$