Electrical Networks   Ley, Lutz, Rehberg

These four branch currents are independent of each other, for if we

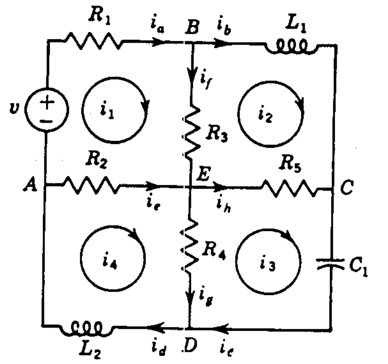

FIG. 1-15. A four-mesh problem.

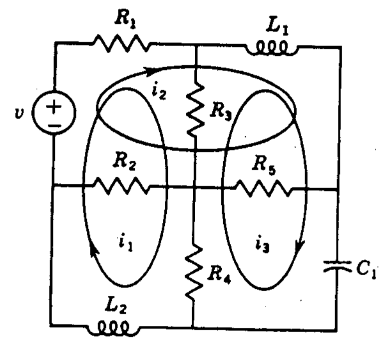

FIG. 1-16. An insufficient number of meshes for the circuit of Fig. 1-15.

identify the branch current i_a with the mesh current i_1, i_b with i_2, i_c with i_3, and i_d with i_4, it is clear from the figure that each of these mesh currents can be given a value that will be independent of the others. Kirchhoff's current law will thus be satisfied at every node since any mesh current entering a node also leaves it. The network equations can therefore be written in terms of the four mesh currents i_1, i_2, i_3, and i_4.

$$
\begin{aligned}
(R_1 + R_2 + R_3)i_1 - R_3 i_2 \qquad\qquad\qquad\qquad - R_2 i_4 &= v \\
-R_3 i_1 + (R_3 + R_5 + L_1 D) i_2 - R_5 i_3 \qquad\qquad &= 0 \\
-R_5 i_2 + \left(R_4 + R_5 + \frac{1}{C_1 D}\right) i_3 - R_4 i_4 &= 0 \\
-R_2 i_1 \qquad\qquad - R_4 i_3 + (R_2 + R_4 + L_2 D) i_4 &= 0
\end{aligned}
\tag{1-46}
$$

Obviously, the student would not attempt to solve the network of Fig. 1-15 in terms of only the mesh currents i_1, i_2, and i_3, for not all the branches would be included in the resulting equations (e.g., the branch containing L_2). However, if the mesh currents were drawn as shown in Fig. 1-16, every branch would be included and the student might be

Kirchhoff's Laws lead to Simultaneous Linear Eqns.

CHEMICAL

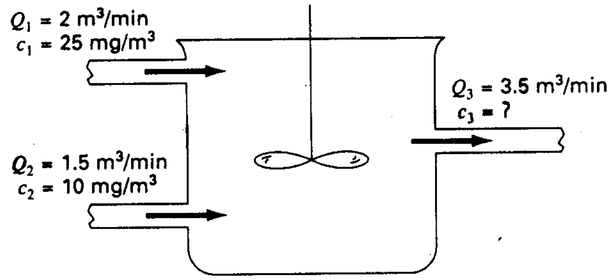

FIGURE 12.2
A steady-state, completely mixed reactor with two inflow pipes and one outflow pipe. The flows Q are in cubic meters per minute, and the concentrations c are in milligrams per cubic meter.

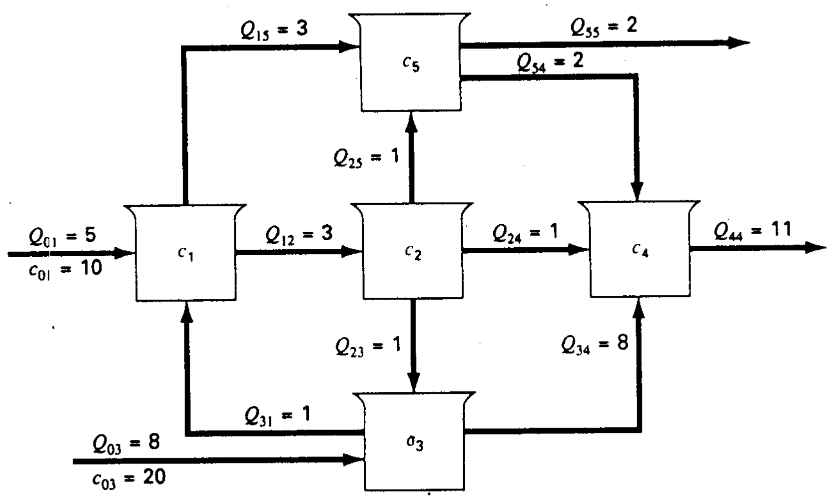

FIGURE 12.3
reactors linked by pipes.

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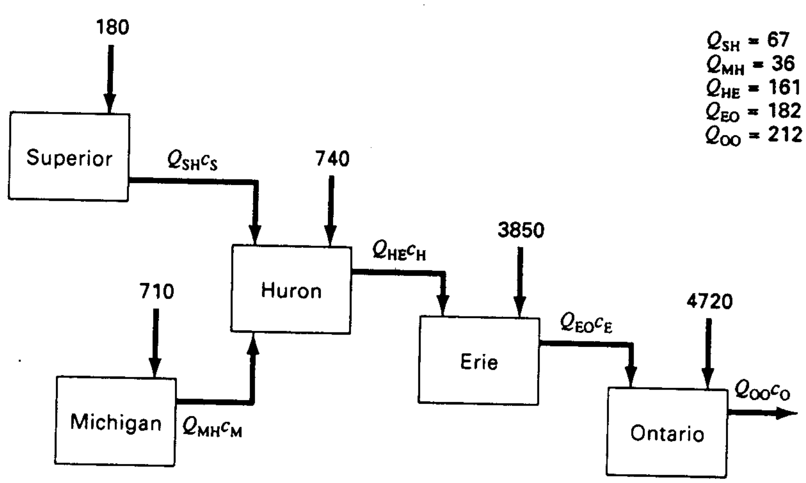


Figure P12.7
A chloride balance for the Great Lakes. Numbered arrows are direct inputs.

MECHANICAL

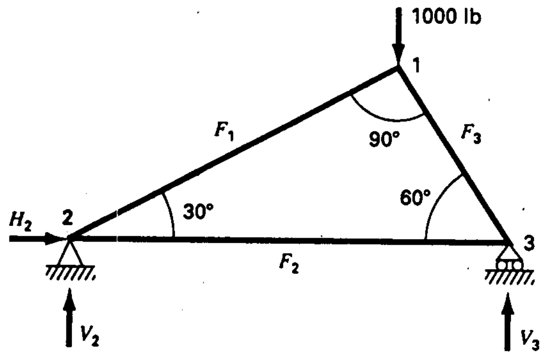


FIGURE 12.4
Forces on a statically determinate truss.

Therefore, for node 1,

$$\Sigma F_H = 0 = -F_1 \cos 30° + F_3 \cos 60° + F_{1,h}$$
$$\Sigma F_V = 0 = -F_1 \sin 30° - F_3 \sin 60° + F_{1,v}$$

for node 2,

$$\Sigma F_H = 0 = F_2 + F_1 \cos 30° + F_{2,h} + H_2$$
$$\Sigma F_V = 0 = F_1 \sin 30° + F_{2,v} + V_2$$

for node 3,

$$\Sigma F_H = 0 = -F_2 - F_3 \cos 60° + F_{3,h}$$
$$\Sigma F_V = 0 = F_3 \sin 60° + F_{3,v} + V_3$$

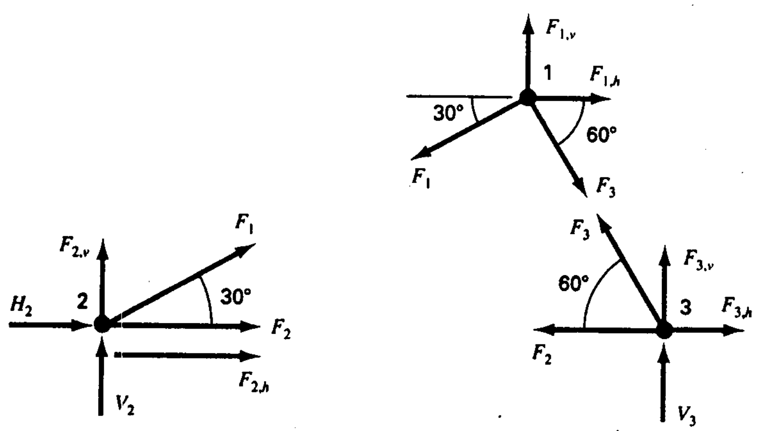
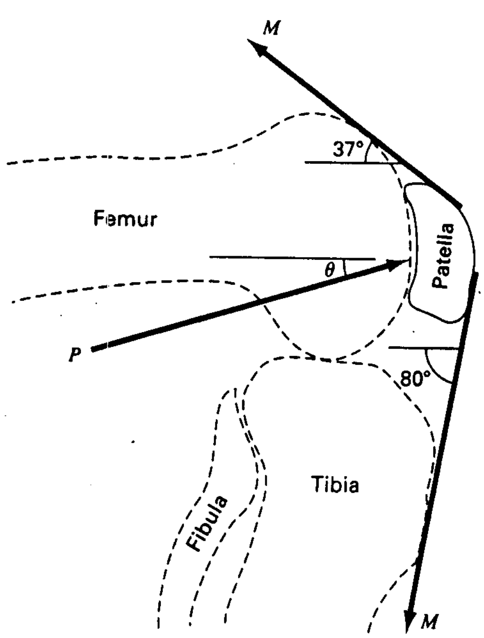


FIGURE 12.5
Free-body force diagrams for the nodes of a statically determinate truss.



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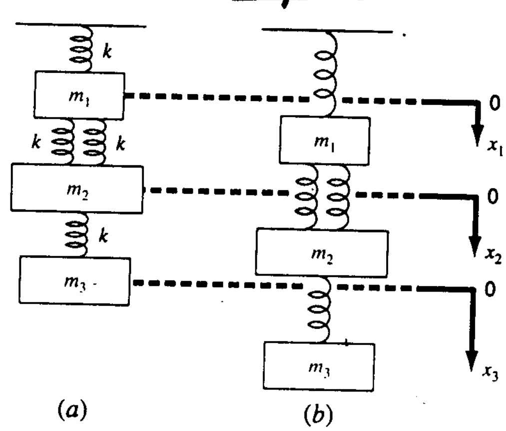

Numerical Recipes in C, Press et al.

2.11 Is Matrix Inversion an N^3 Process?

We close this chapter with a little entertainment, a bit of algorithmic prestidigitation which probes more deeply into the subject of matrix inversion. We start with a seemingly simple question:

How many individual multiplications does it take to perform the matrix multiplication of two 2×2 matrices,

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \tag{2.11.1}$$

Eight, right? Here they are written explicitly:

$$\begin{aligned} c_{11} &= a_{11} \times b_{11} + a_{12} \times b_{21} \\ c_{12} &= a_{11} \times b_{12} + a_{12} \times b_{22} \\ c_{21} &= a_{21} \times b_{11} + a_{22} \times b_{21} \\ c_{22} &= a_{21} \times b_{12} + a_{22} \times b_{22} \end{aligned} \tag{2.11.2}$$

Do you think that one can write formulas for the c's that involve only *seven* multiplications? (Try it yourself, before reading on.)

Such a set of formulas was, in fact, discovered by Strassen [1]. The formulas are:

$$\begin{aligned} Q_1 &\equiv (a_{11} + a_{22}) \times (b_{11} + b_{22}) \\ Q_2 &\equiv (a_{21} + a_{22}) \times b_{11} \\ Q_3 &\equiv a_{11} \times (b_{12} - b_{22}) \\ Q_4 &\equiv a_{22} \times (-b_{11} + b_{21}) \\ Q_5 &\equiv (a_{11} + a_{12}) \times b_{22} \\ Q_6 &\equiv (-a_{11} + a_{21}) \times (b_{11} + b_{12}) \\ Q_7 &\equiv (a_{12} - a_{22}) \times (b_{21} + b_{22}) \end{aligned} \tag{2.11.3}$$

in terms of which

$$\begin{aligned} c_{11} &= Q_1 + Q_4 - Q_5 + Q_7 \\ c_{21} &= Q_2 + Q_4 \\ c_{12} &= Q_3 + Q_5 \\ c_{22} &= Q_1 + Q_3 - Q_2 + Q_6 \end{aligned} \tag{2.11.4}$$

Applied recursively to submatrices,

$O(N^{\log_2 7}) = O(N^{2.807})$