Universal Behavior & Computation

Ken Steiglitz, CS Dept.

- Cellular Automata as dynamic model
- Universal behavior
- Turing Equivalence ($\stackrel{?}{=}$ computation)
  - Life
  - Lattice gasses
  - billiard-balls
- Filter Automata
- Embedded carry-ripple adder
- Embedded computation with optical solitons
- Energy-switching gate with Manakov solitons

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The Great Way is gateless,
Approached in a thousand ways.
Once past this checkpoint
You stride through the universe.

-- Mumon

Cellular Automata: simplest dynamic model

- discrete space

- discrete time
- discrete state values (usually 0/1)
- local dependence

$$a_i^{t+1} = f(a_i^t\text{'s in neighborhood})$$

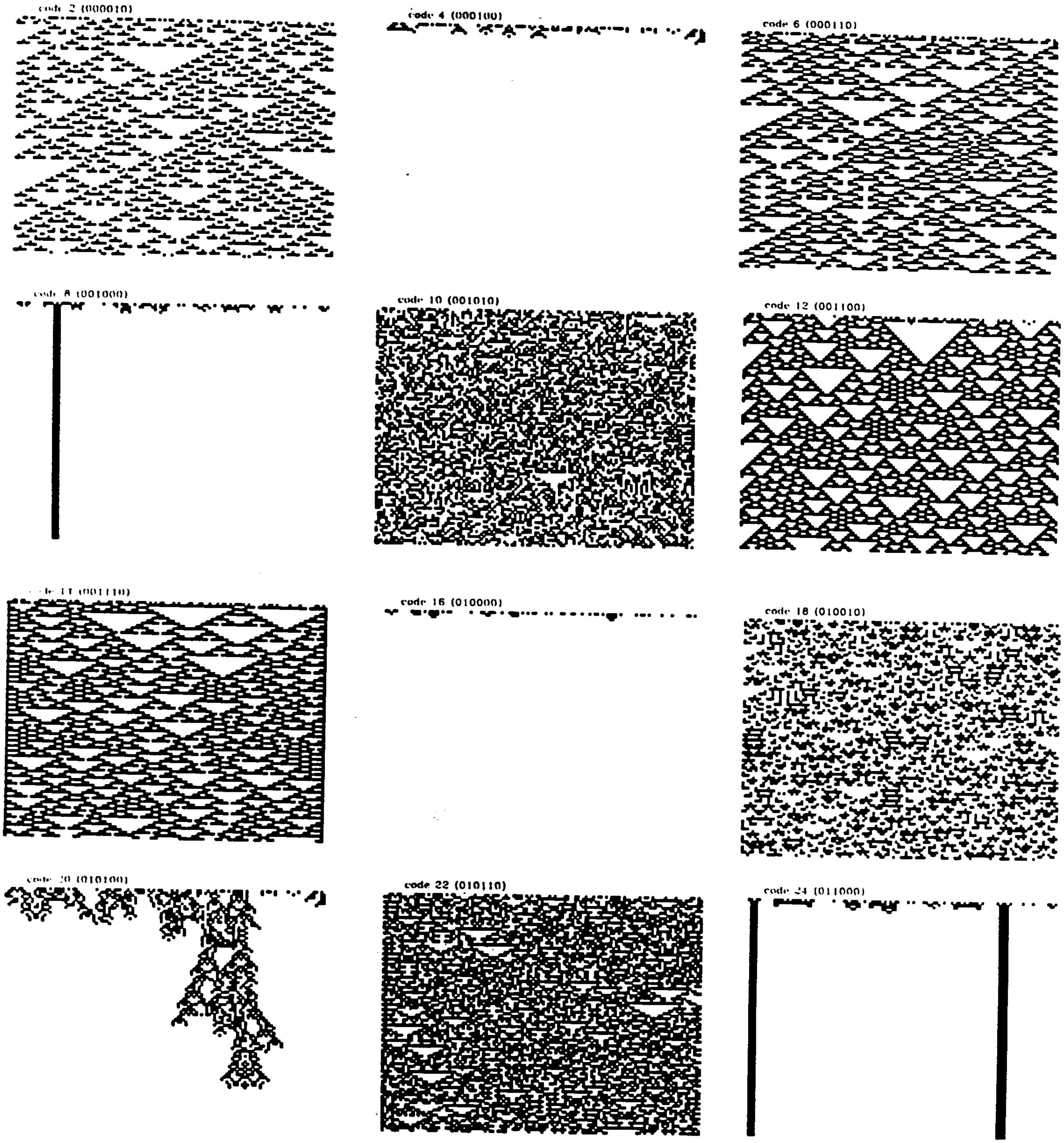
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Fig. 1a.

CA2

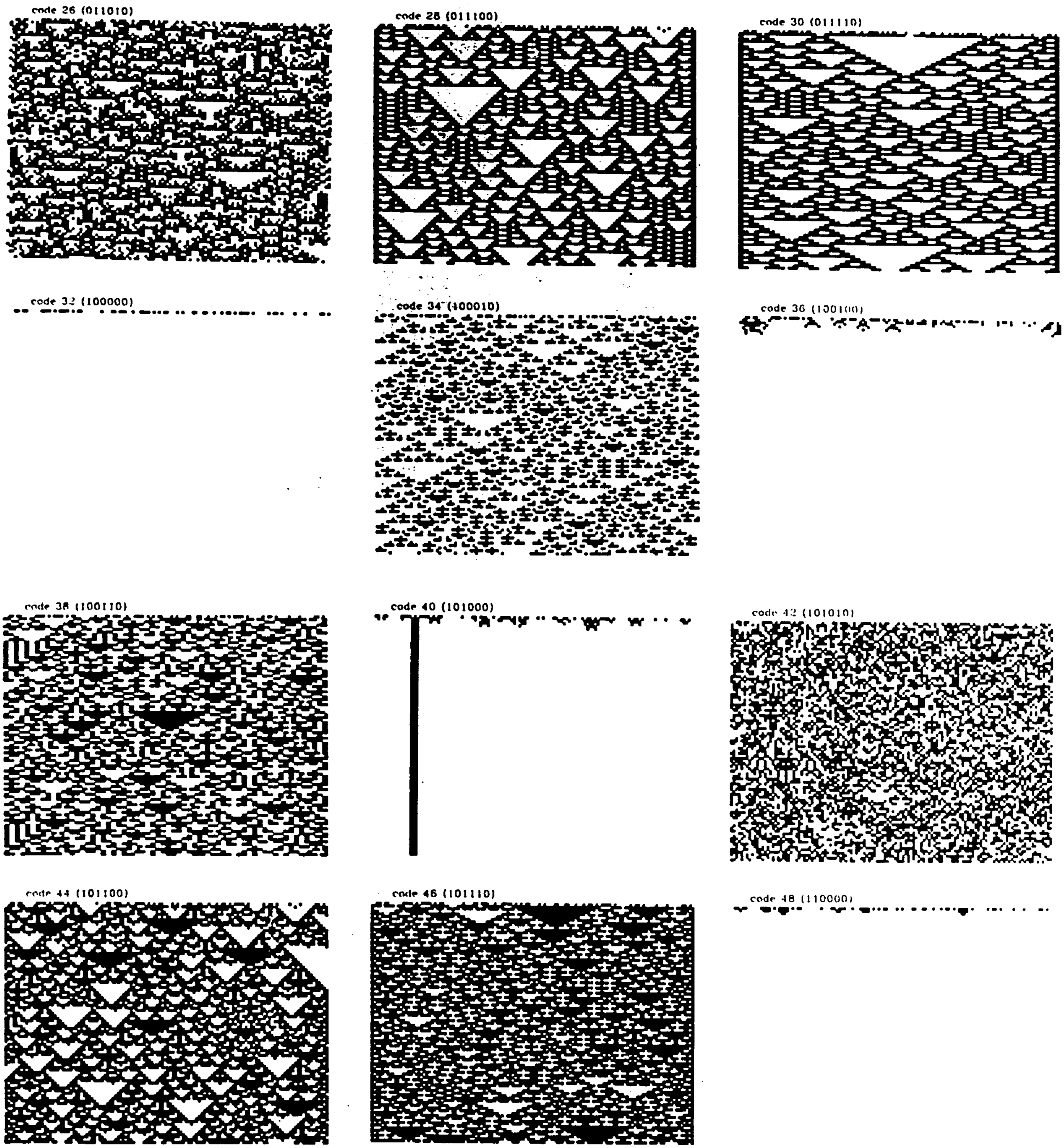
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Fig. 1b.

CA3

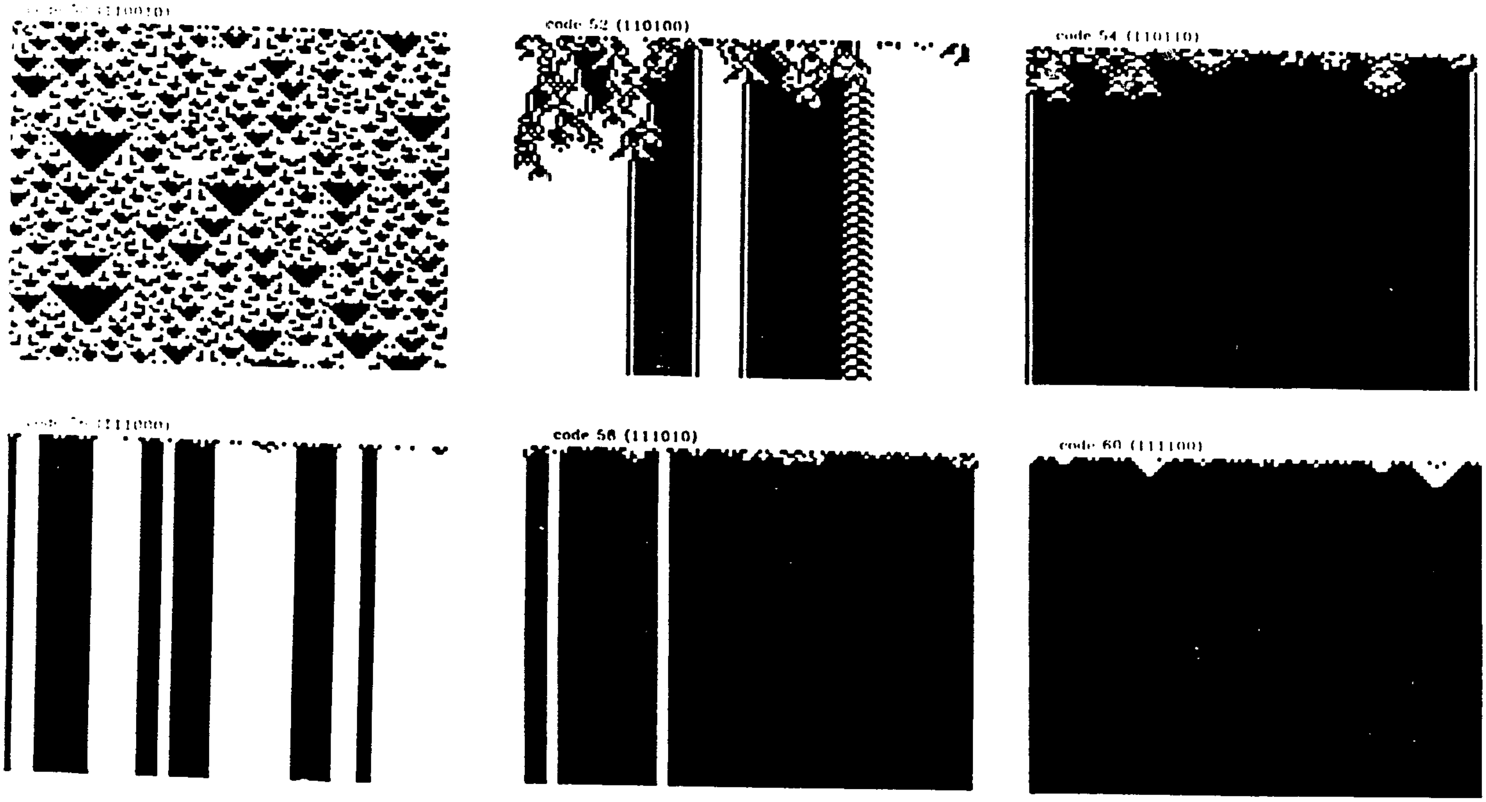

Fig. 1c.

ig. 1a–c. Evolution of all possible legal one-dimensional totalistic cellular automata with $k = 2$ and $r = 2$. k gives the number of ossible values for each site, and r gives the range of the cellular automaton rules. A range $r = 2$ allows the nearest and next-nearest eighbours of a site to affect its value on the next time step. Time evolution for totalistic cellular automata is defined by eqns. (2.2) nd (2.7). The initial state is taken disordered, each site having values 0 and 1 with independent equal probabilities. Configurations btained at successive time steps in the cellular automaton evolution are shown on successive horizontal lines. Black squares represent tes with value 1; white squares sites with value 0. All the cellular automaton rules illustrated are seen to exhibit one of four qualitative

CA 4

Four classes of behavior: (Wolfram)

I trivial (all white, all black)
fixed point

II periodic fixed point

III chaotic

IV complex localized structures,
sometimes long-lived

SPACE-TIME ON A SEASHELL

214 American Scientist, Volume 83

Brian Hayes MAY-JUNE 1995

References

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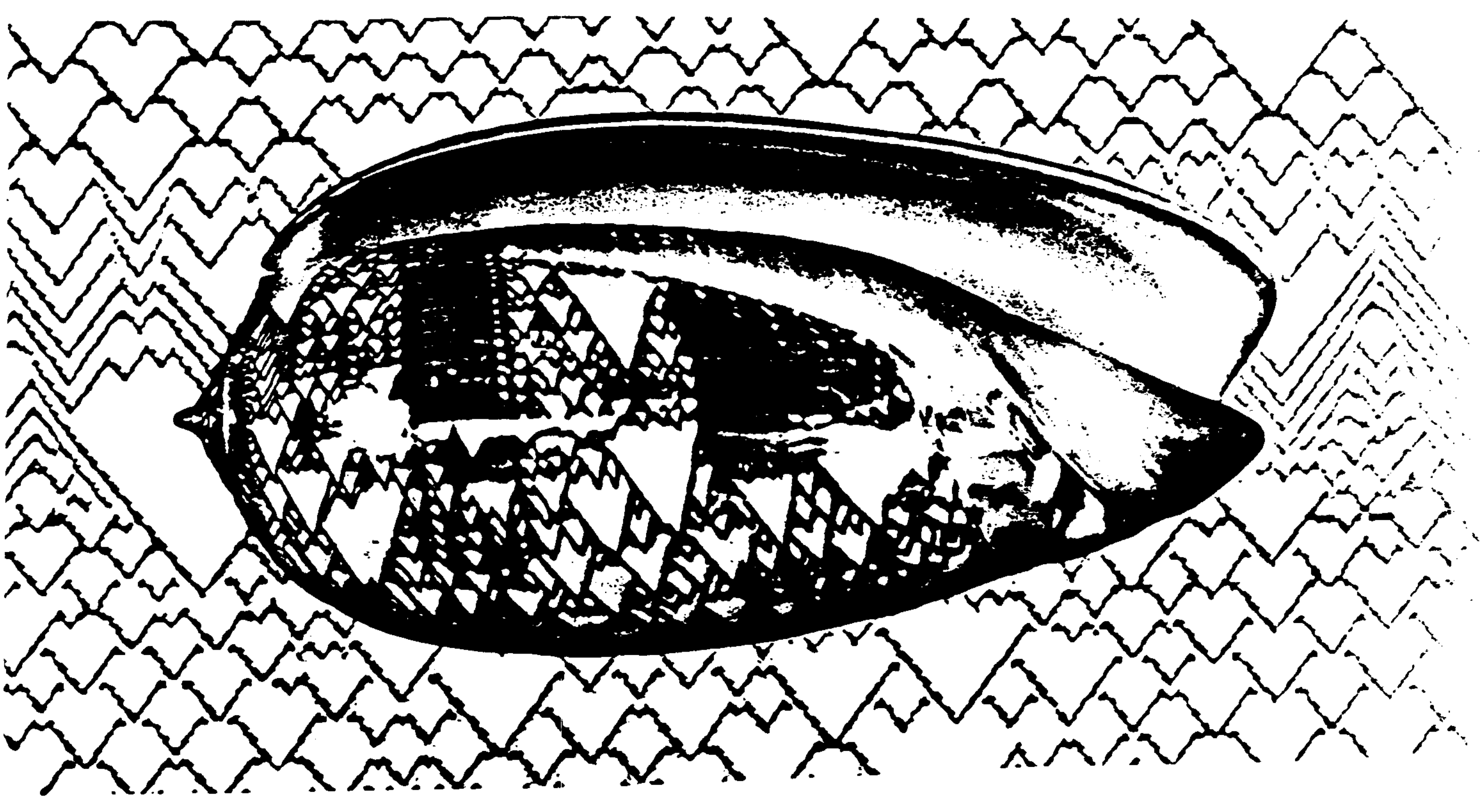

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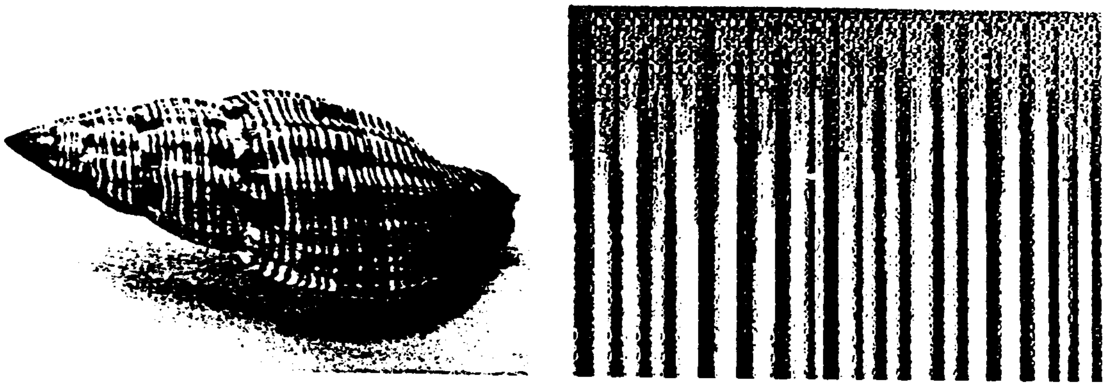

Figure 1. "Vertical" stripes, perpendicular to the growing edge of a mollusk shell, result from a stable spatial pattern of alternating active and dormant pigment-secreting cells. In the simulation *(right)* the pattern emerges spontaneously. The shell *(left)* is a specimen of *Lyra taiwanica*. (All photographs courtesy of Hans Meinhardt.)

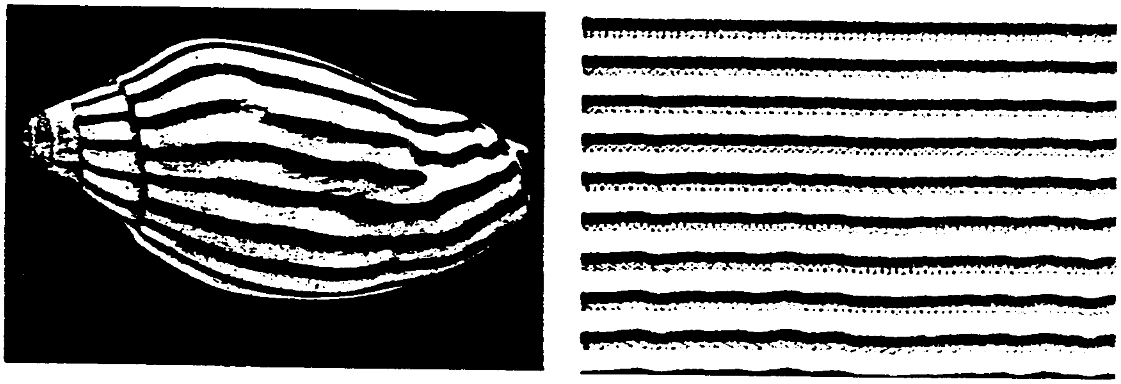

Figure 2. "Horizontal" stripes, parallel to the growing edge, reflect a temporal oscillation in pigment-cell activity. in which all the cells turn on and off almost simultaneously. The shell is *Amoria dampieria*.

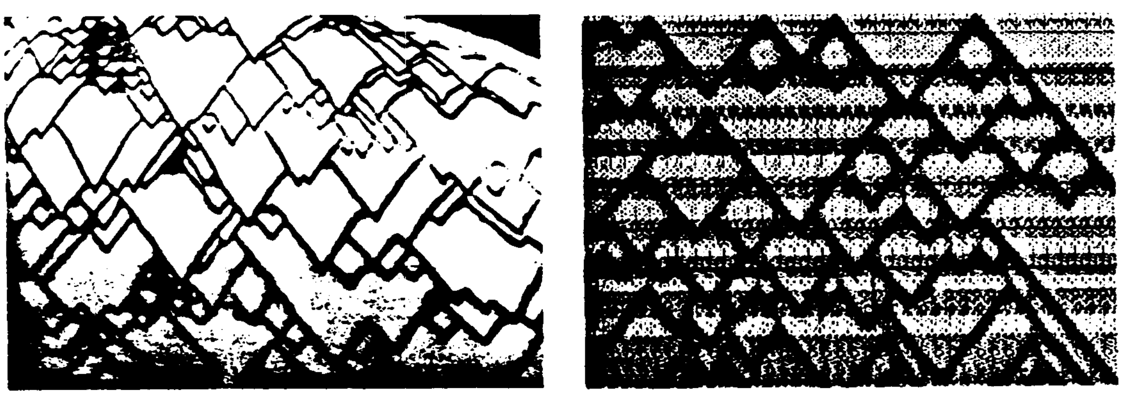

Figure 4. Oblique stripes on *Oliva porphyria (left)* record the passage of traveling waves in the line of pigment cells. The stripes are created and annihilated in pairs, like particles of matter and antimatter. To account for the frequent synchronization of creation events, the model *(right)* includes a global hormone-like signal *(green)*. A larger sample of the output of this simulation, without the green hormone, is shown on the cover of this issue.

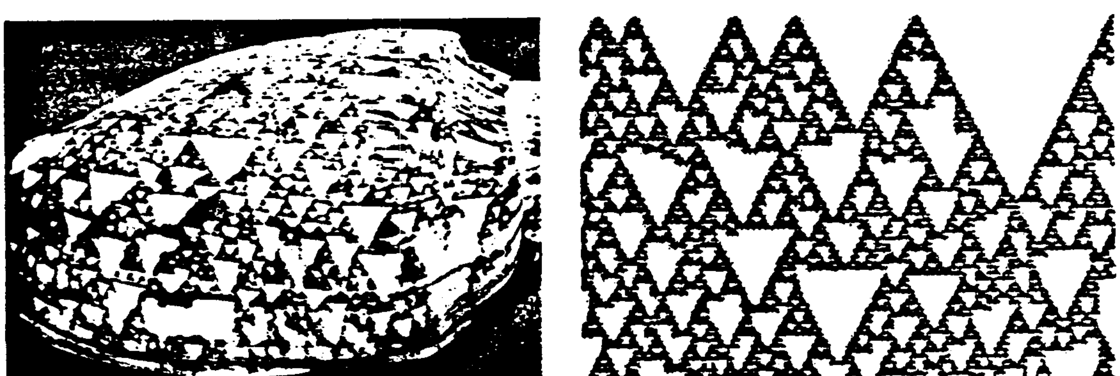

Figure 5. Triangular pattern on *Cymbiola innexa Reeve (left)* also suggests the presence of some synchronizing mechanism, since many secreting cells must shut off at once to form the upper edge of a white triangle. In the simulation *(right)* another global signal is introduced for this purpose.

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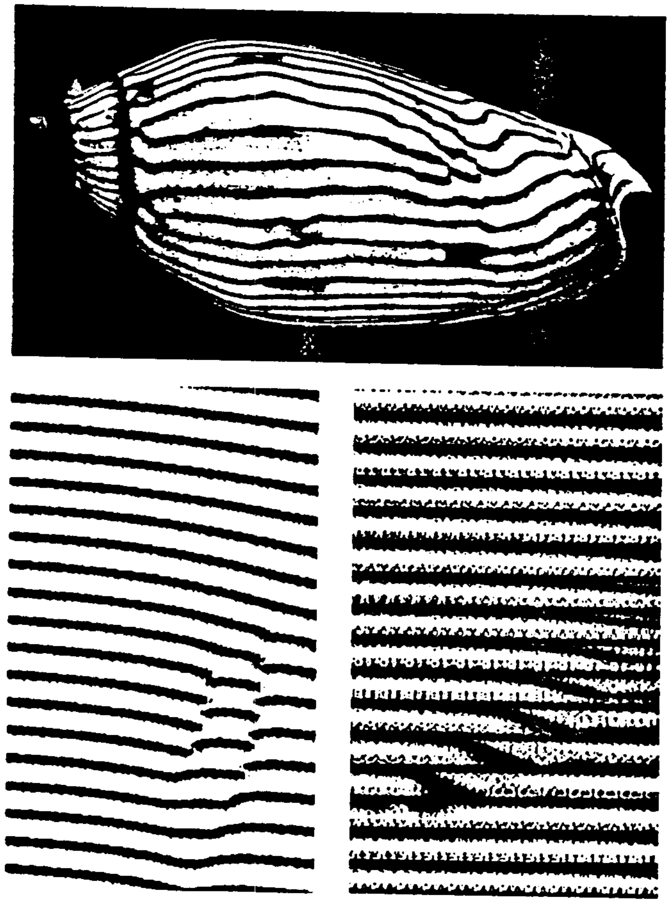


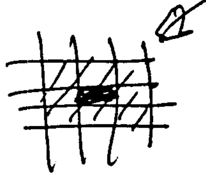
Figure 3. Dislocation in the striped pattern of *Amoria turneri (top)* is a challenge to the simulation scheme. A single oscillator whose frequency varies across the width of the shell *(left)*, produces several discontinuous stripes, whereas the natural pattern is quickly resynchronized. Adding a central hormonelike oscillator (shown here in green, although its actual products are presumed to be invisible) yields a more realistic output *(right)*.

(A7

CONWAY'S GAME OF LIFE

E.R. BERLEKAMP, J.H. CONWAY, R.K. GUY, *Winning ways for your Mathematical Plays*, vol. 2, Academic Press (1982).

2-D BINARY



neighborhood

BIRTH. A cell that's *dead* at time t becomes *live* at $t+1$ only if *exactly three* of its eight neighbors were live at t.

DEATH by overcrowding. A cell that's live at t and has four or more of its eight neighbors live at t will be dead by time $t+1$.

DEATH by exposure. A live cell that has only one live neighbor, or none at all, at time t, will also be dead at $t+1$.

These are the only causes of death, so we can take a more positive viewpoint and describe instead the rule for

SURVIVAL. A cell that was live at time t will remain live at $t+1$ if and only if it had just 2 or 3 live neighbors at time t.

Just 3 for BIRTH
2 or 3 for SURVIVAL

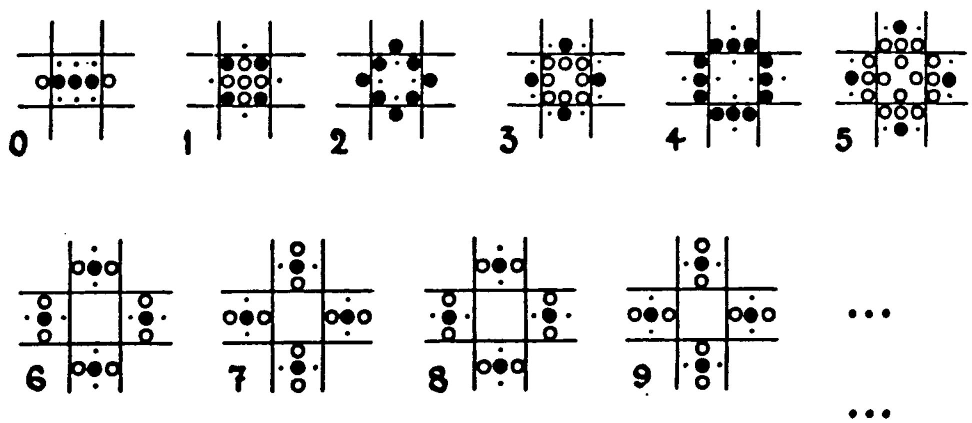


Figure 1. A Line of Five Becomes Traffic Lights.

Time 1–2: The corners will survive, having 3 neighbors each, but everything else will die of overcrowding. There will be 4 births, one by the middle of each side.

2–3: We see a ring in which each live cell has 2 neighbors so everything survives; there are 4 births inside.

3–4: Massive overcrowding kills off all except the 4 outer cells, but neighbors of these are born to form:

4–5: another survival ring with 8 happy events about to take place.

5–6: More overcrowding again leaves just 4 survivors. This time the neighboring births form:

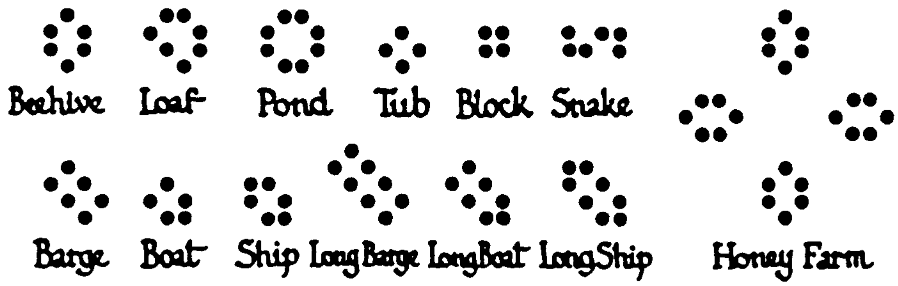
6–7: four separated lines of 3, called **Blinkers**, which will never interact again.

7–8–9–10–... At each generation the tips of the Blinkers die of exposure but the births on each side reform the line in a perpendicular direction.

CA8

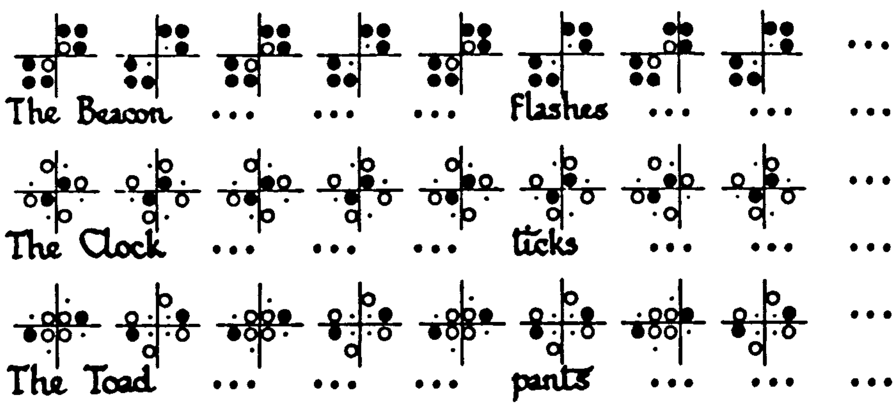
STILL LIFE

It's easy to find other stable configurations. The commonest such Still Life can be seen in Fig. 3 along with their traditional names. The simple cases are usually loops in which each iive cell has two or three neighbors according to local curvature, but the exact shape of the loop is important for effective birth control.



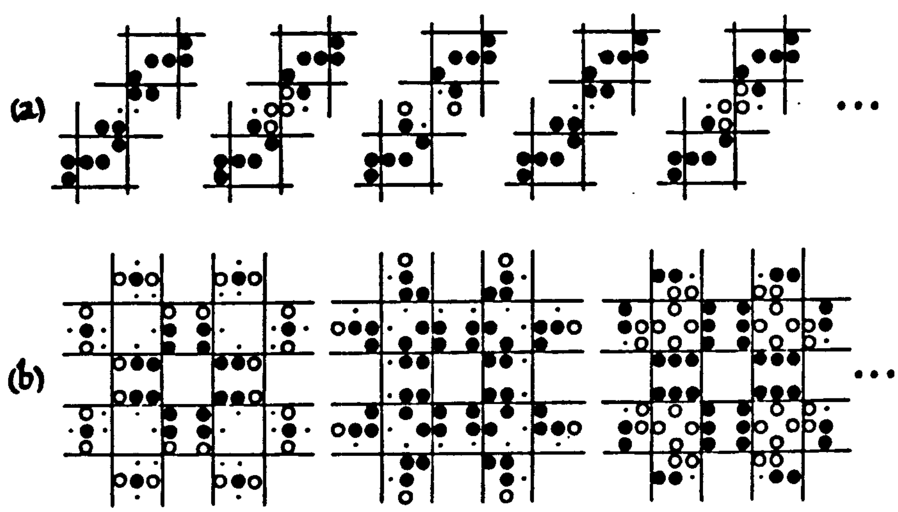
Period 1
(STILL)

Figure 3. Some of the Commoner Forms of Still Life.



PERIOD 2

Figure 4. Three Life Cycles with Period Two.



PERIOD 3

Figure 5. Two Life Cycles with Period Three. (a) Two Eaters Gnash at Each Other. (b) The Cambridge Pulsar CP 48-56-72.

CA9

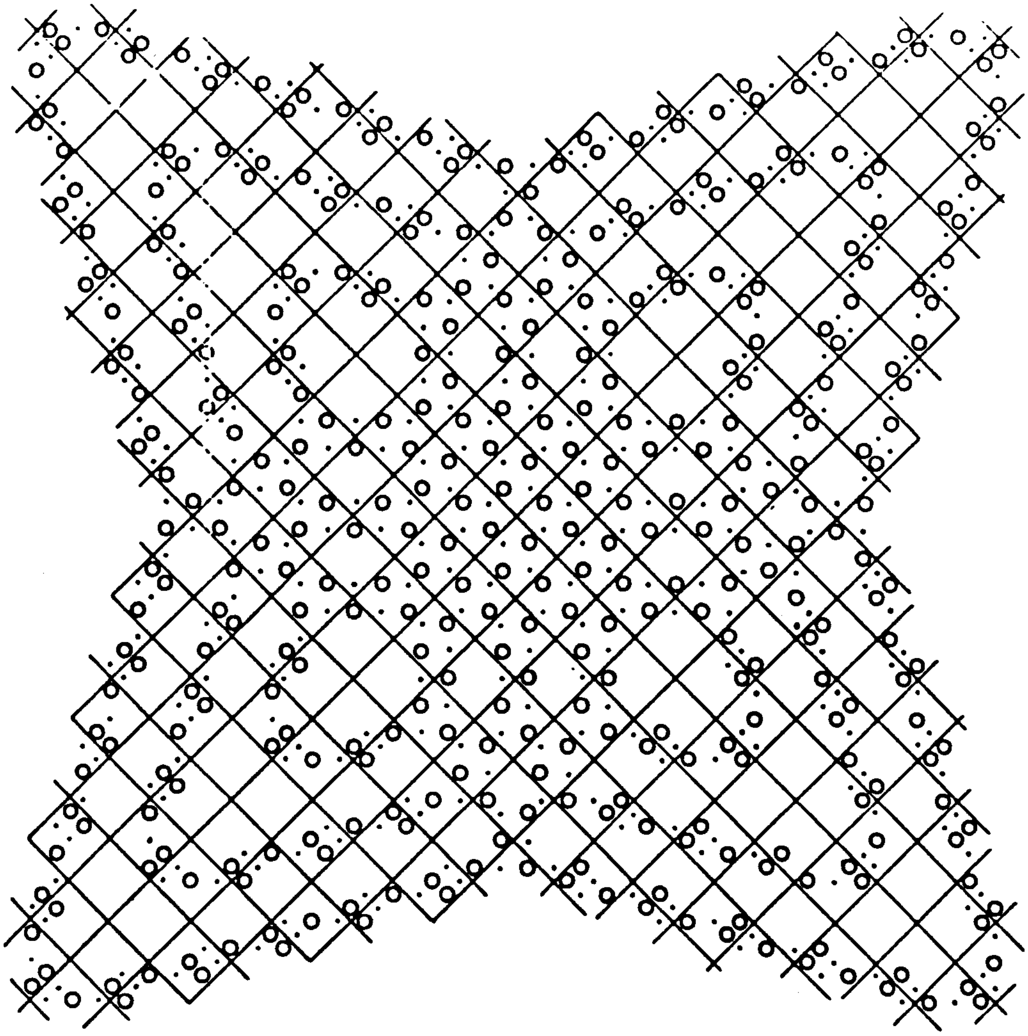


Figure 6. A Flip-Flop by the Gosper Group.

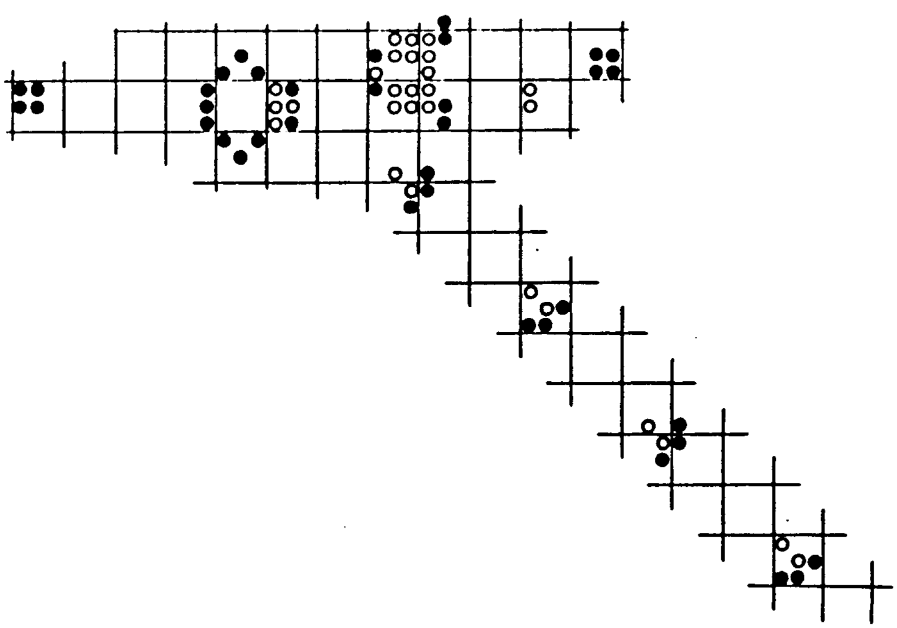


Figure 14. Gosper's Glider Gun.

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MAKING A LIFE COMPUTER

Many computers have been programmed to play the game of Life. We shall now return the compliment by showing how to define Life patterns that can imitate computers. Many remarkable consequences will follow from this idea.

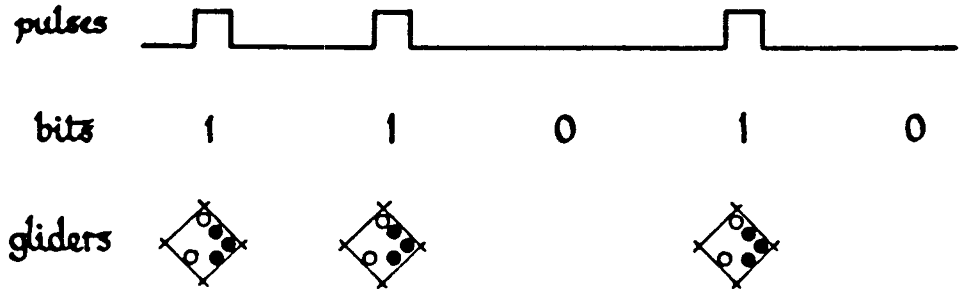


Figure 17. Gliding Pulses.

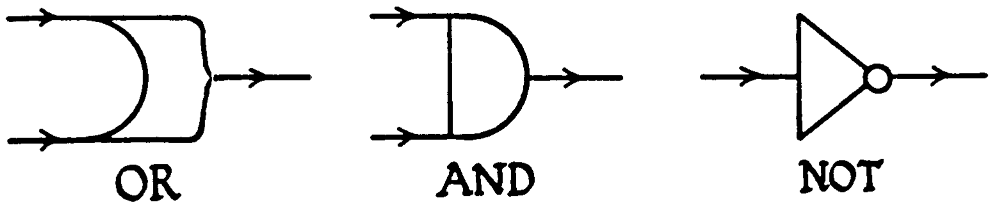


Figure 18. The Three Logical Gates.

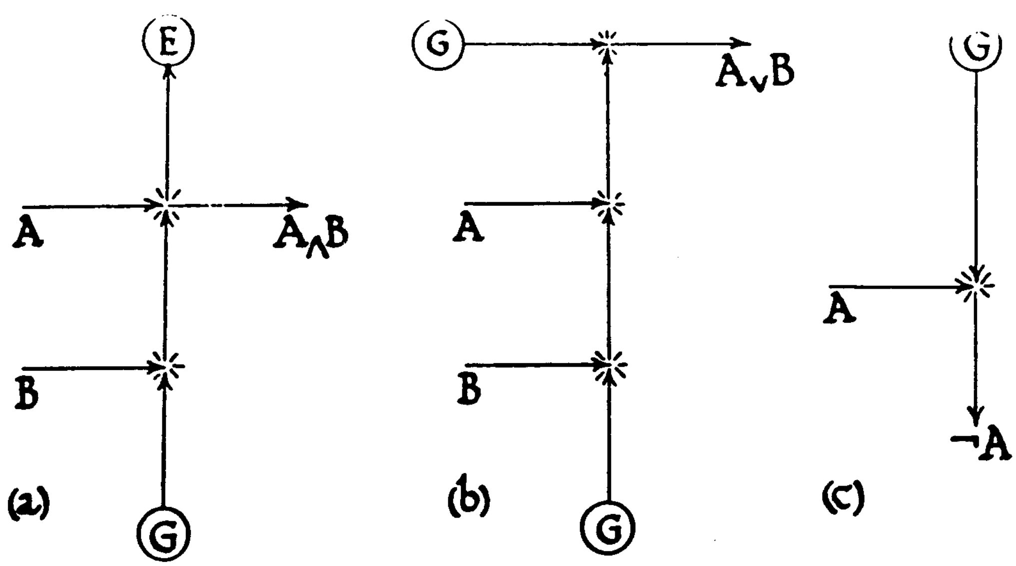


Figure 29. (a) An AND Gate. (b) An OR Gate. (c) A NOT Gate.

HOW TO MAKE A NOT GATE

We can use a vanishing reaction, together with a Glider Gun, to create a NOT gate (Fig. 20). The input stream enters at the left of the figure, and the Glider Gun is positioned and timed so that every space in the input stream allows just one glider to escape from the gun, while a glider in the stream necessarily crashes with one from the gun in a vanishing reaction (indicated by ✱). Figure 20 shows the periodic stream

1 1 0 1 1 0 1 1 0 ...

being complemented to

0 0 1 0 0 1 0 0 1 ....

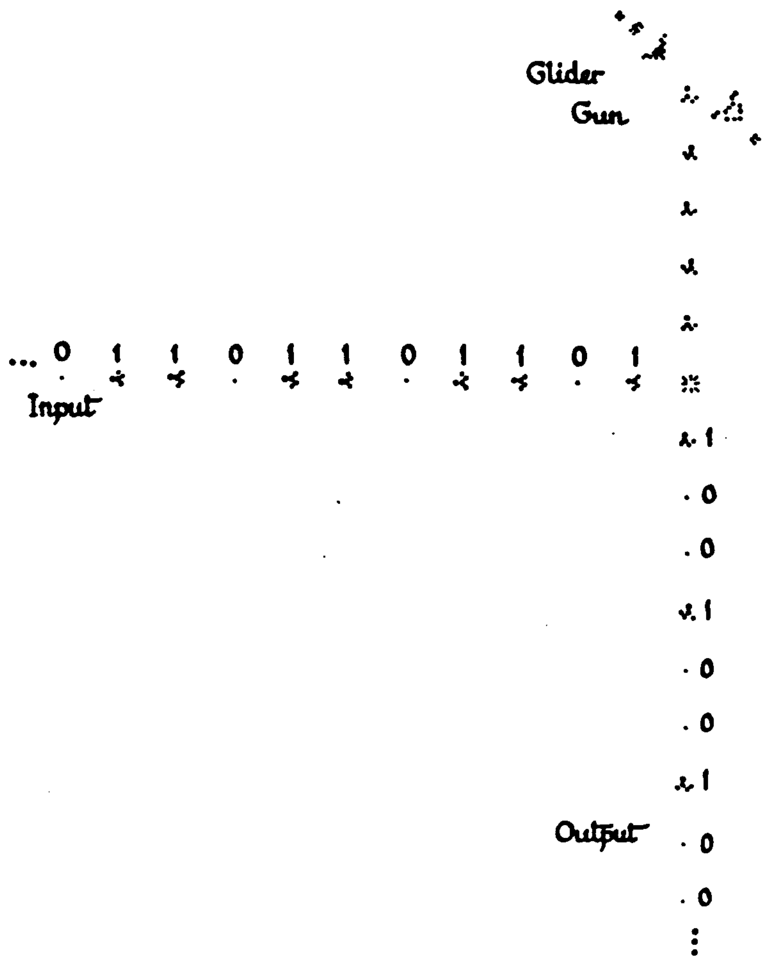


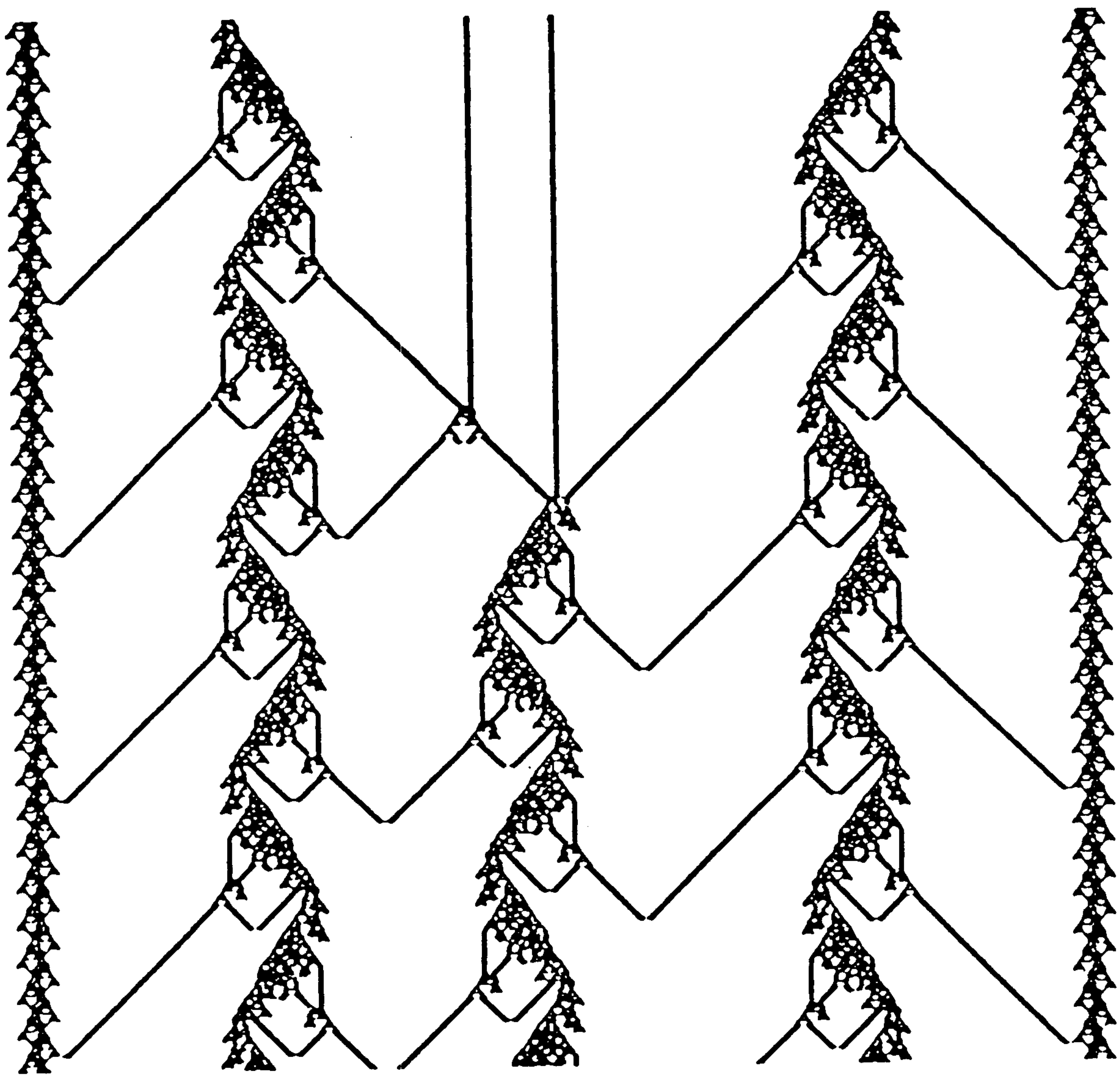
Figure 20. A Glider Gun and a Vanishing Reaction Make a NOT Gate.

LIFE IS UNIVERSAL!

James Park's glider gun in a 1-d CA

space (totalistic) →

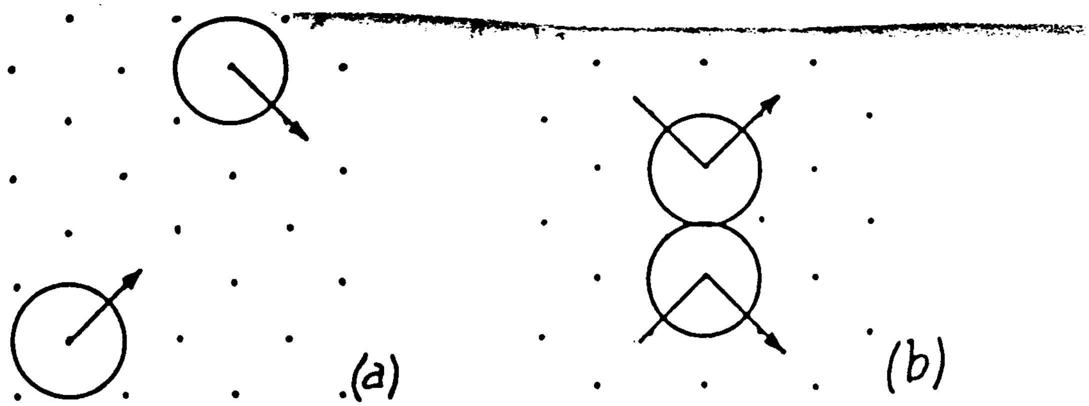
time ↓



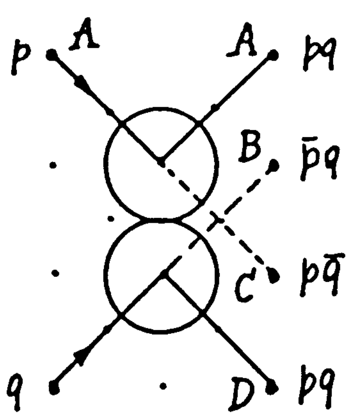
nformation processing structures in a one-dimensional cellular automaton. The pattern 'as produced by evolution according to the k=2, r=3 totalistic cellular automaton rule with code 8. This class 4 rule supports complex localized structures whose evolution and interactions can e used to implement certain logical functions. One suspects that the cellular automaton is capale of universal computation. (Picture by James Park.)

2-d is easier!

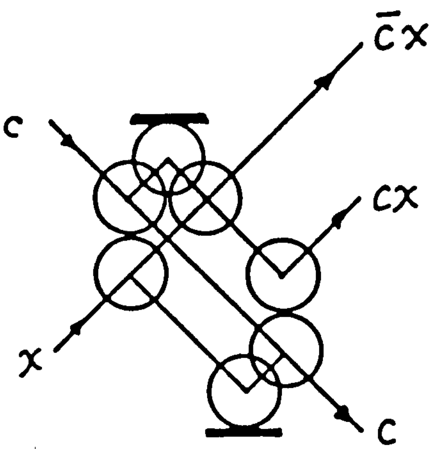
Billiard-Balls (Fredkin-Toffoli)



(a) Balls of radius $1/\sqrt{2}$ traveling on a unit grid. (b) Right-angle elastic collision between two balls.



Billiard ball model realization of the interaction gate.



A simple realization of the switch gate.

FRisch, Hasslacher, Pomeau

LATTICE GASSES

HEX (TRIANGULAR) LATTICE

PARTICLES

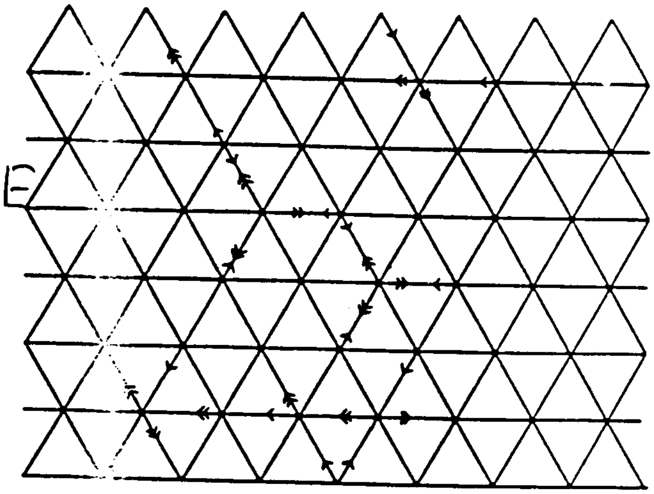


FIG. [illegible] Triangular lattice with hexagonal symmetry and hexagonal lattice-gas rules. Particles at time t and $t+1$ are marked by single and double arrows, respectively.

before   after

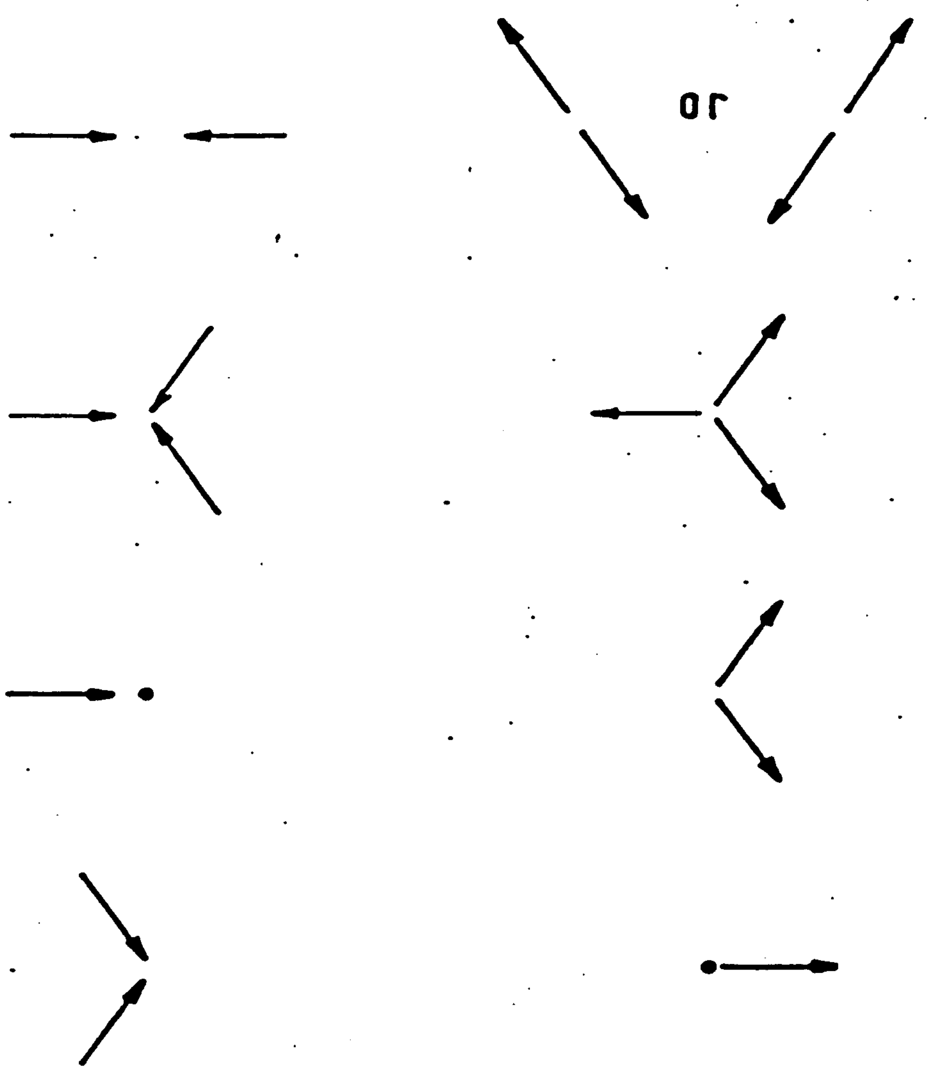


Figure 1. Relaxation to "thermodynamic equilibrium" in the hexagonal lattice cellular automaton (CA) described in the text. Discrete particles are initially in a simple array in the centre of a 32×32 site square box. The upper sequence shows the randomization of this pattern with time; the lower sequence shows the cells visited in the discrete phase space (one particle track is drawn thicker). The graph illustrates the resulting increase of coarse-grained entropy $\sum p_i \log_2 p_i$ calculated from particle densities in 32×32 regions of a 256×256 box.

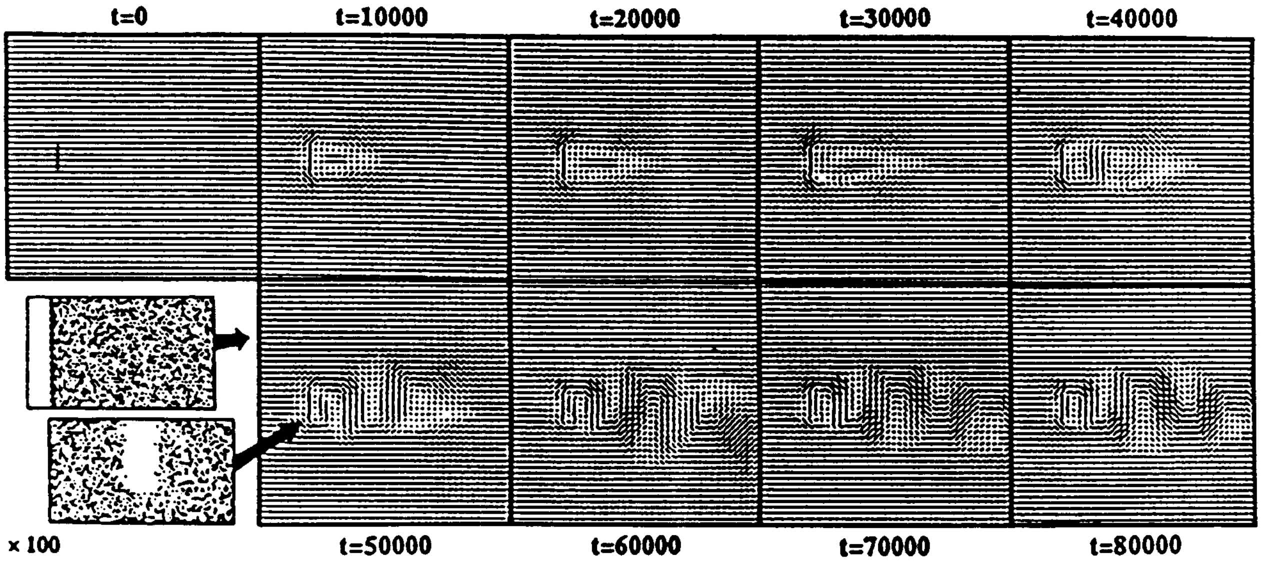


Figure 2. Time evolution of hydrodynamic flow around a plate in the CA of figure 1 on a 4096×4096 site lattice. Hydrodynamic velocities are obtained as indicated by averaging over 96×96 site regions. There is an average density of 0.3 particles per link (giving a total of 3×10^8 particles). An overall velocity U=0.1 is maintained by introducing an excess of particles (here in a regular pattern) on the left hand boundary.

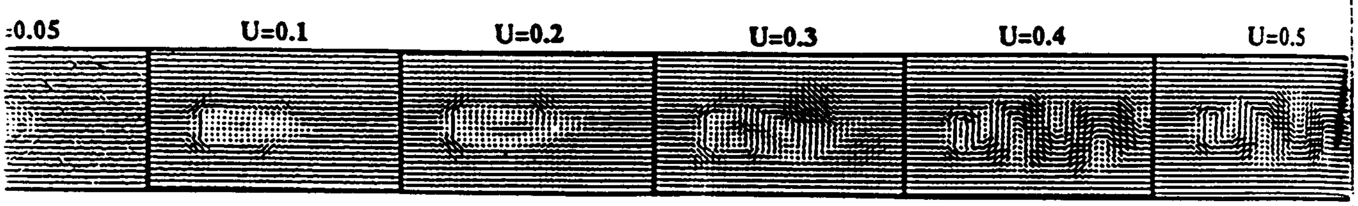


Figure 3. Hydrodynamic flows obtained after 10^5 time steps in the CA of figure 2, for various overall velocities U.

from 'thermodynamics & Hydrodynamics with CA,"
J. B. Salem, S. Wolfram, 1985.

CA15

2-d FHP Lattice gasses are computation universal

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COMPLEX SYSTEMS
7, pp. 297-307, 1993

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Abstract

We show that the FHP lattice gasses are computationally universal, implying that general questions about their behavior are undecidable. The proof embeds a universal 1-d cellular automaton in the 2-d FHP lattice gas. This provides evidence that general questions about fluid behavior are undecidable.

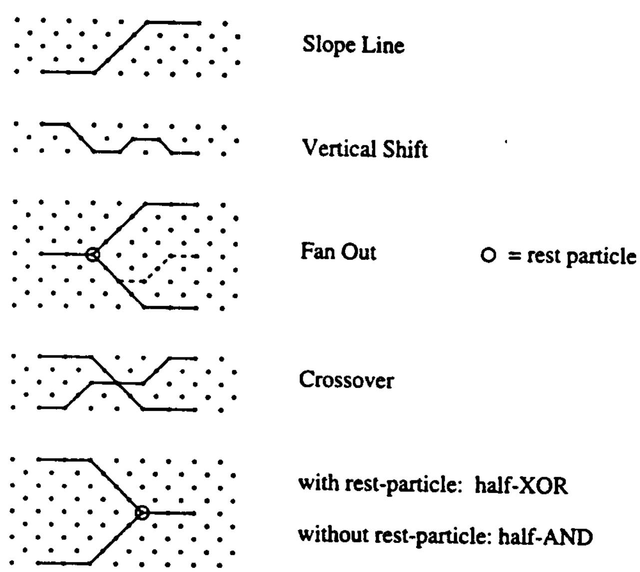
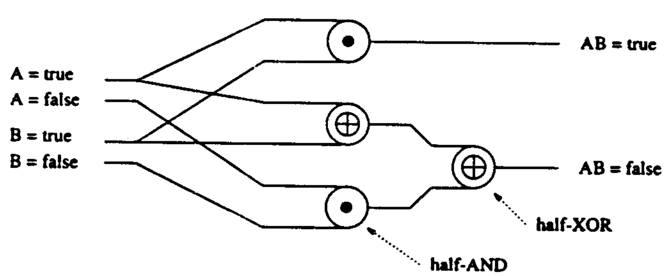


Figure 8.

The primitive devices. Most are self-explanatory. The Fan-Out device shows an optional dashed path for one of the exit paths. This may be used if the fan out is one-sided.



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