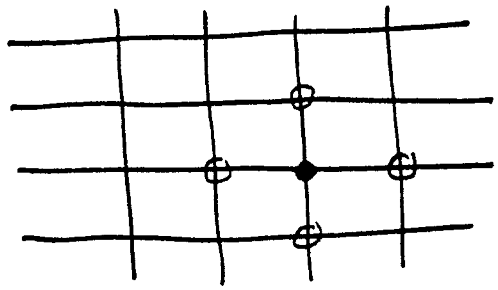
Example Spatial Epidemic Models

D [Dur 95] $\begin{cases} \text{discrete-time} \\ \text{discrete-space} \\ \text{discrete-state} \end{cases}$

3 States:
- susceptible
- infected
- removed



each site has 4 neighbors

1 individual at each site

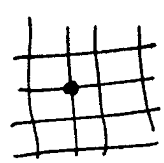
Dynamics: (a) susceptible individuals become infected at rate proportional to # of infected neighbors

(b) infected individuals become healthy at a fixed rate δ

(c) removed individuals become susceptible at a fixed rate α

Simulation, Case 1 $\alpha = 0$ (no return from removed)

Simple initial condition



single infected individual

theorem (!)

if δ is big, epidemic dies out

for $\delta < \delta_c$ = some critical δ, epidemic spreads linearly with time & approaches fixed shape

[Dur 95]

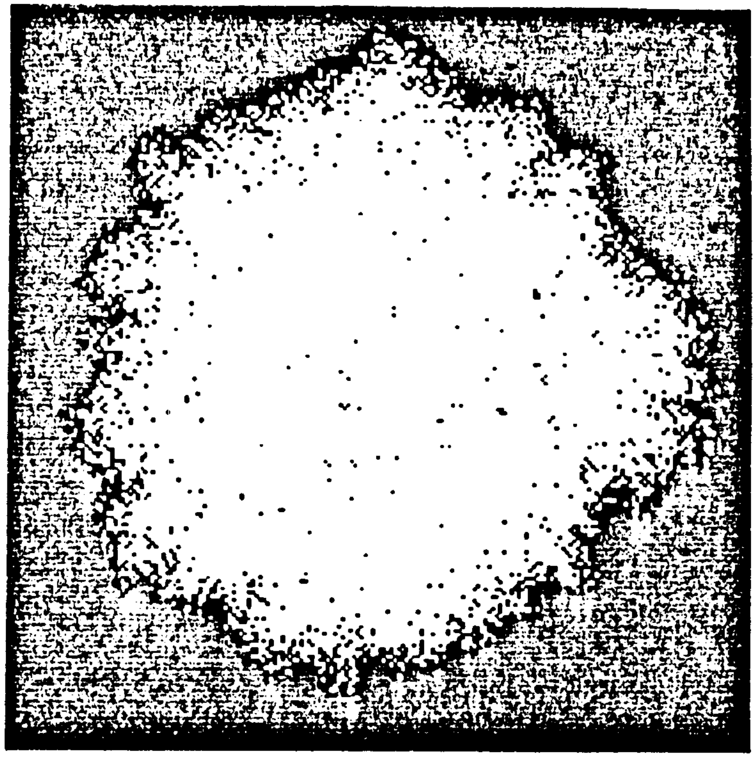


Figure 1:

SPREAD

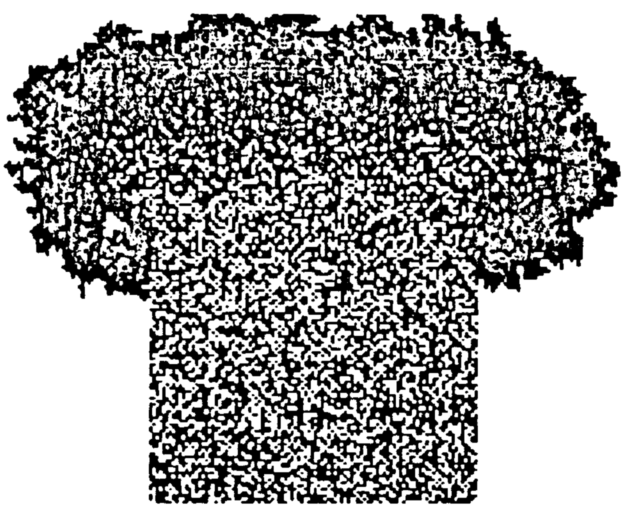


Figure 7:

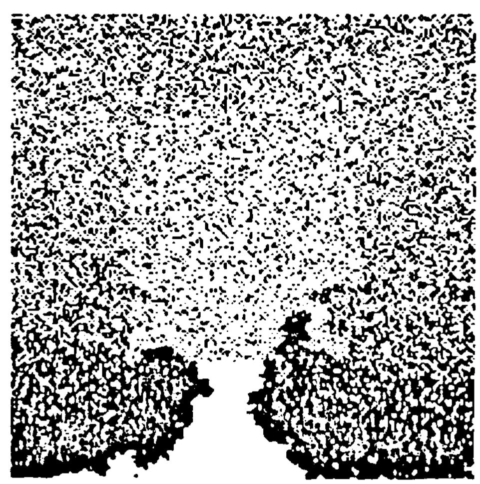


Figure 8:

FLEXIBLE EXPERIMENTS

[Dur95] gives data from 1929:

→ measles took 24 weeks to spread over Glasgow,

$\sim$ 1/12 mile/week = 440 ft./week.

also confirmation of linear spread rate with racoon rabies in NY

$\alpha > 0$ (return from removed) is a more complicated problem.

if $\delta < \delta_c$ (supercritical) $\Longrightarrow$ approaches steady-state distribution

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An Ordinary differential equation model

$$\begin{cases} u = \text{fraction of infected individuals} \\ v = \text{" " removed individuals} \\ 1-u-v = \text{" " susceptible individuals} \end{cases}$$

large, mixed, homogeneous population model

$$\boxed{\begin{aligned} \frac{du}{dt} &= u\cdot(1-u-v) - \delta\cdot u \\ \frac{dv}{dt} &= \delta u - \alpha v \end{aligned}}$$

~~removed~~ removed

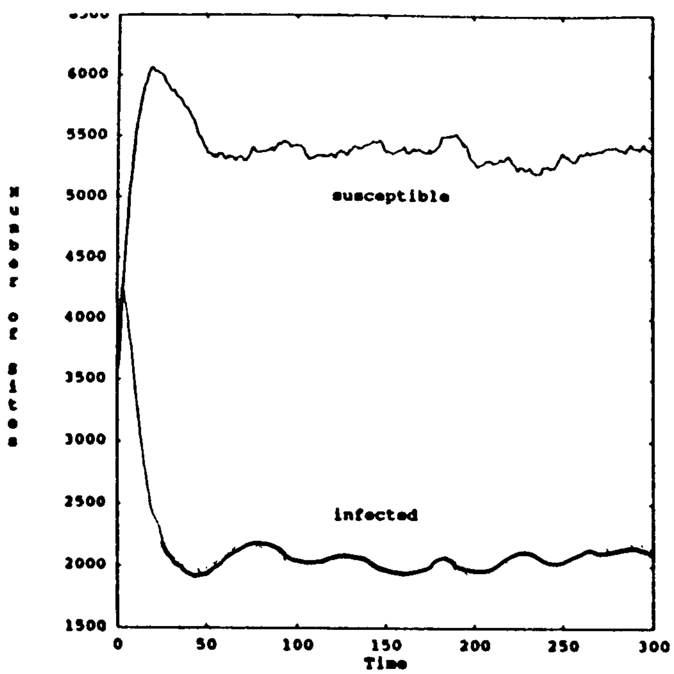
become susceptible

~~removed~~ removed

[Dur95]

0.1.10

$\alpha > 0$



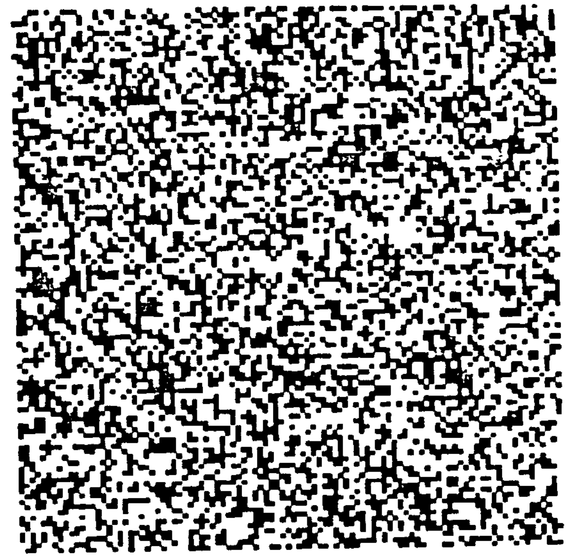
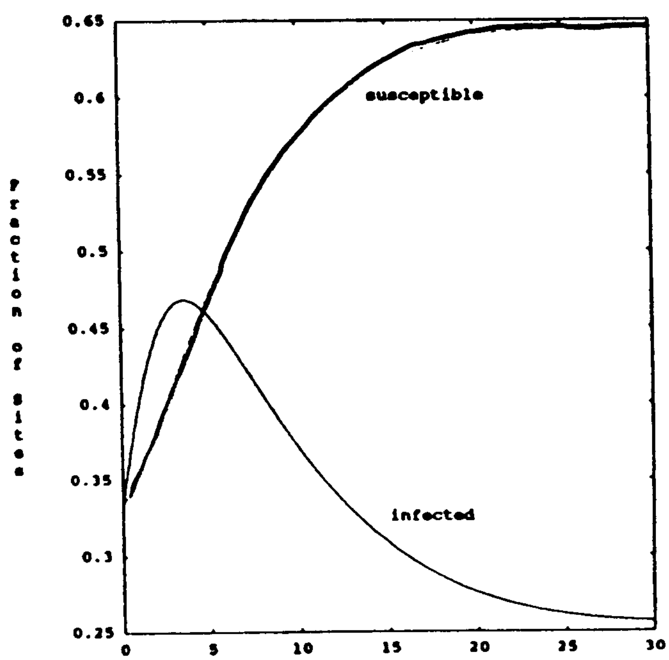


Figure 3:

ODE Model:

$$\frac{du}{dt} = u(1 - u - v) - \delta u$$
$$\frac{dv}{dt} = \delta u - \alpha v$$



not accurate.
why not?

Partial Differential Equation Models

models similar phenomena - but takes spatial distribution into account.

Example from [E-K88]:

$$P = P(x,t) = \text{population density}$$

$$\frac{\partial P}{\partial t} = D \cdot \nabla^2 P + \alpha P$$

also leads to linear spread rate.

approximates random walk.

Muskrats Escape in 1905...

K88]

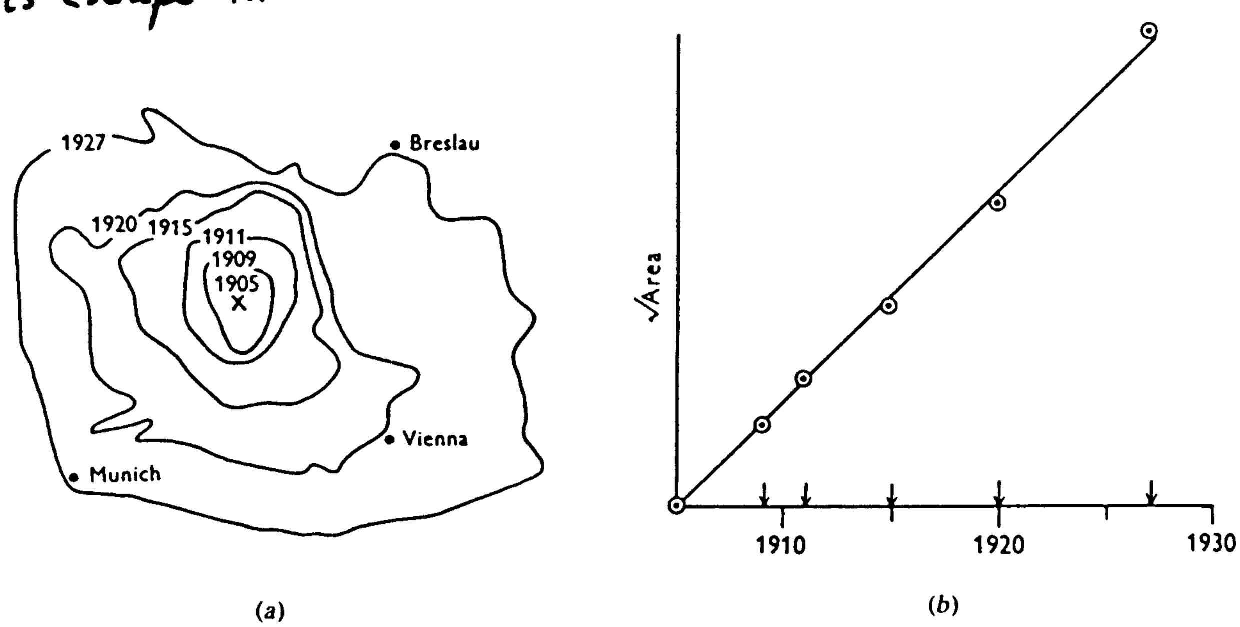
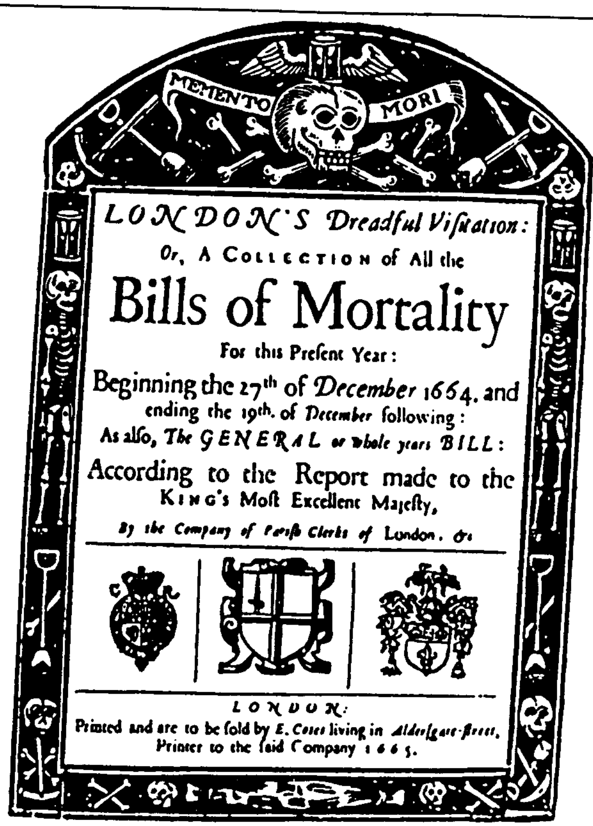


Figure 10.1 Spread of muskrats over central Europe during a period of 27 years described by Skellam (1951) as a random dispersal. (a) Equipopulation contours (level curves of p(x, t) for the lowest detectable muskrat population. A graph of (area)$^{1/2}$ of the regions enclosed by these curves reveals linear dependence on time t, as predicted by the growth-dispersal model of equation (1). [From Skellam J.G. (1951). Random dispersal in theoretical populations. Biometrika, 38, *figs. 1 and 2, p. 200. Reprinted with permission of the Biometrika Trustees.]*



Mortality from a variety of afflictions, only some of which were caused by disease, were systematically recorded as early as the 1600s in the Bills of Mortality published in London. Reproduced here is the title page of the London Bills of Mortality for 1665, the year of the great plague. The people of the city followed with anxiety the rise and fall in the number of deaths from the plague, hoping always to see the sharp decline which they knew from past experience indicated that the epidemic was nearing its end. When the decline came the refugees, mostly from the nobility and wealthy merchants, returned to the city, and then for a time the mortality rose again as the disease attacked these new arrivals. The plague of 1665 started in June; its peak came in September and its decline in October. The secondary rise occurred in November and cases of the disease were reported as late as March of the following year. *[From H. W. Haggard (1957),* Devils, Drugs, Doctors, *Harper & Row, New York.]*

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***The Diseases, and Casualties this year being* 1632.**

Abortive, and Stilborn	445	Grief	11
Affrighted	1	Jaundies	43
Aged	628	Jawfaln	8
Ague	43	Impostume	74
Apoplex, and Meagrom	17	Kil'd by several accidents	46
Bit with a mad dog	1	King's Evil	38
Bleeding	3	Lethargie	2
Bloody flux, scowring, and flux	348	Livergrown	87
Brused, Issues, sores, and ulcers,	28	Lunatique	5
Burnt, and Scalded	5	Made away themselves	15
Burst, and Rupture	9	Measles	80
Cancer, and Wolf	10	Murthered	7
Canker	1	Over-laid, and starved at nurse	7
Childbed	171	Palsie	25
Chrisomes, and Infants	2268	Piles	1
Cold, and Cough	55	Plague	8
Colick, Stone, and Strangury	56	Planet	13
Consumption	1797	Pleurisie, and Spleen	36
Convulsion	241	Purples, and spotted Feaver	38
Cut of the Stone	5	Quinsie	7
Dead in the street, and starved	6	Rising of the Lights	98
Dropsie, and Swelling	267	Sciatica	1
Drowned	34	Scurvey, and Itch	9
Executed, and prest to death	18	Suddenly	62
Falling Sickness	7	Surfet	86
Fever	1108	Swine Pox	6
Fistula	13	Teeth	470
Flocks, and small Pox	531	Thrush, and Sore mouth	40
French Pox	12	Tympany	13
Gangrene	5	Tissick	34
Gout	4	Vomiting	1
		Worms	27

Christened	Males....4994 Females..4590 In all....9584	Buried	Males....4932 Females..4603 In all....9535	Whereof, of the Plague.8

Increased in the Burials in the 122 Parishes, and at the Pest-house this year 993

Decreased of the Plague in the 122 Parishes, and at the Pest-house this year .... 266 [10]