Multiple Model Estimation : The EM Algorithm & Applications

Princeton University
COS 429 Lecture

Dec. 4, 2008

Harpreet S. Sawhney
hsawhney@sarnoff.com

Plan

- IBR / Rendering applications of motion / pose estimation
- The problem of Multiple Motion Estimation
- EM Algorithm for Multiple Model Fitting
- EM algorithm for Multiple Motion Estimation

Real-World Apps of IBR

- The Matrix
- What Dreams May Come
- Titanic

Application : Dynamic New View Rendering

The Matrix

Flow-based New View Rendering

Original 8 frames

Tweened 71 frames

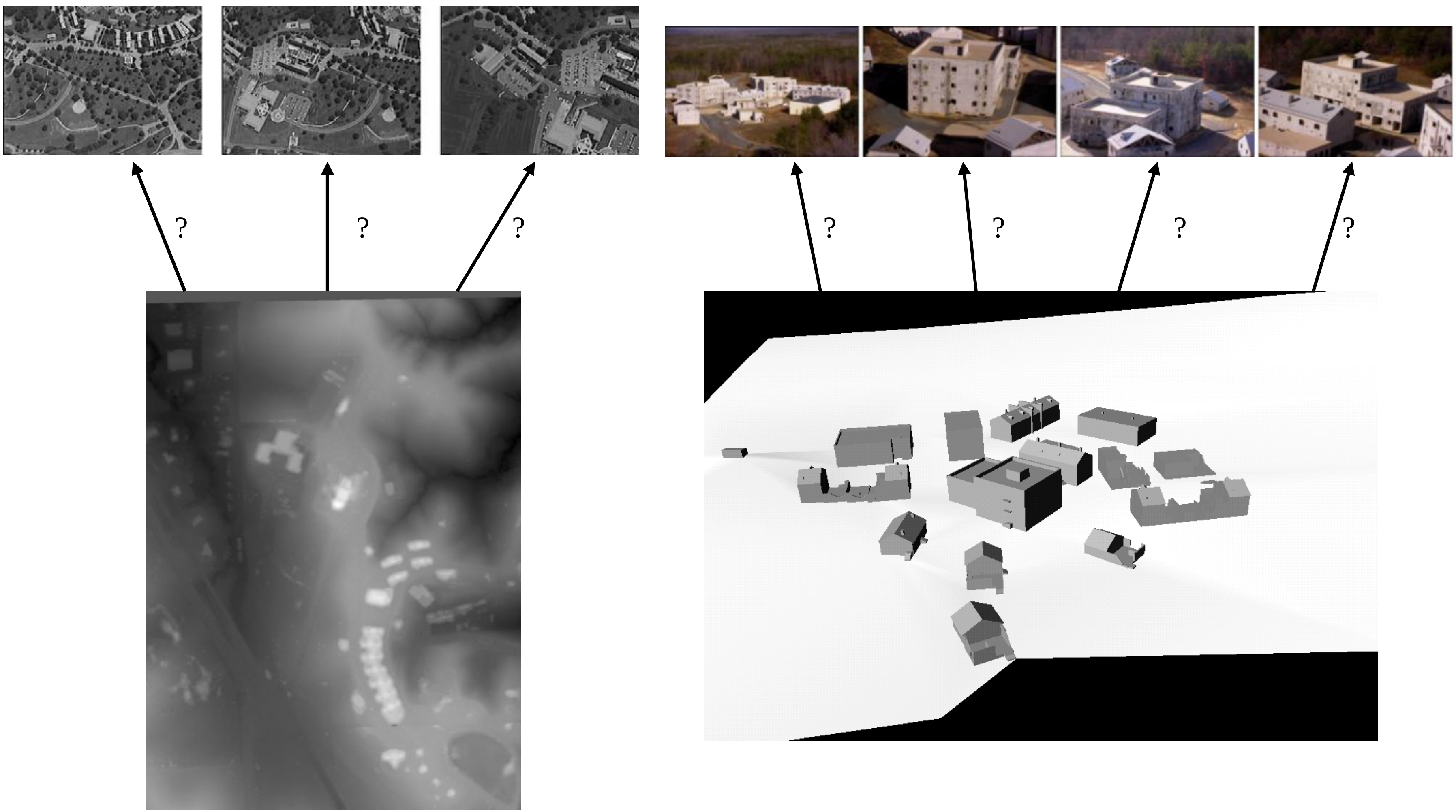
Enhanced Visualization

3D Model-based Direct Camera Pose Estimation and Video Visualization

[Hsu et al. ’00]

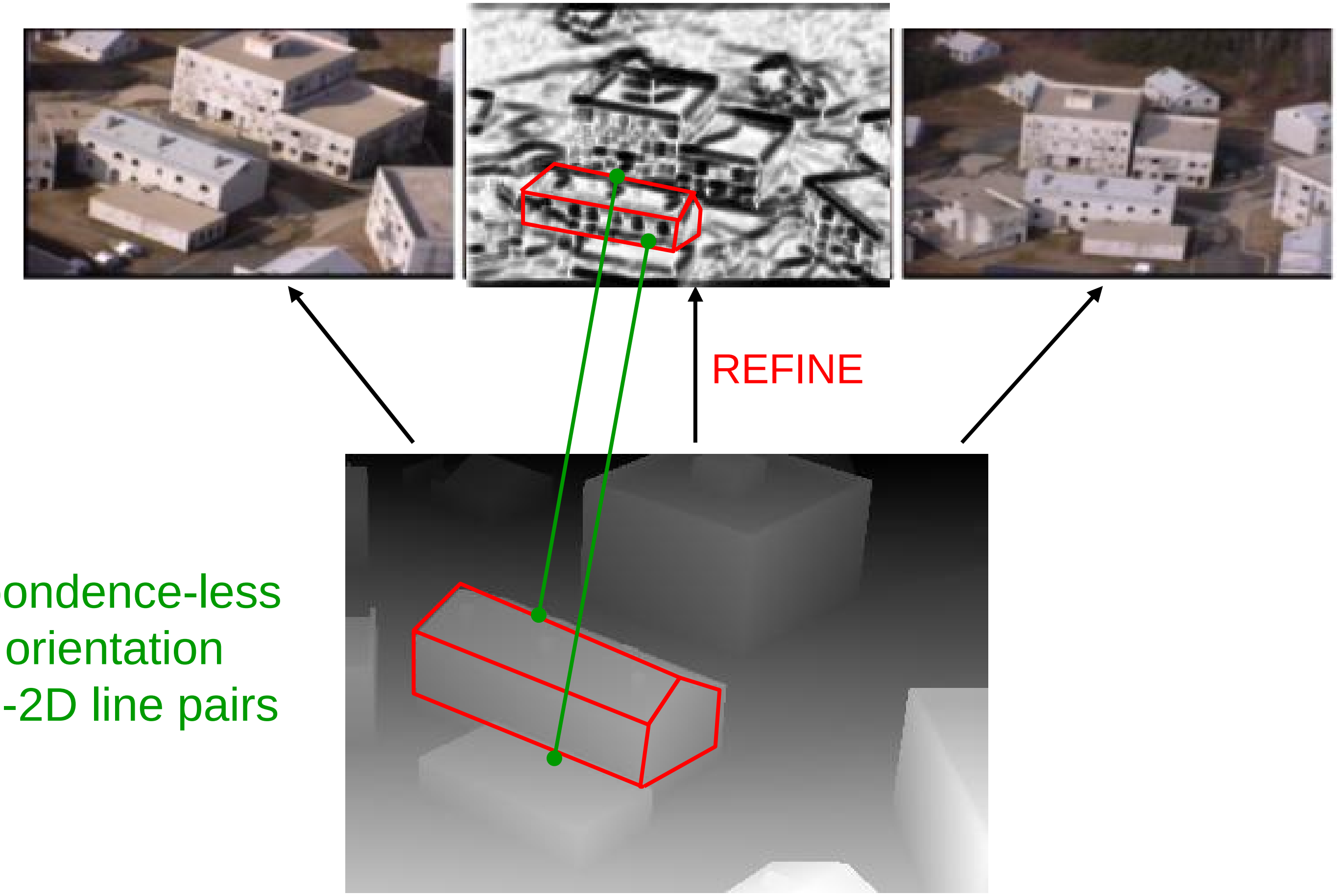
Pose Estimation

...when only shape of 3D scene is known ...



Video to Site Model Alignment

- **Model to frame alignment**



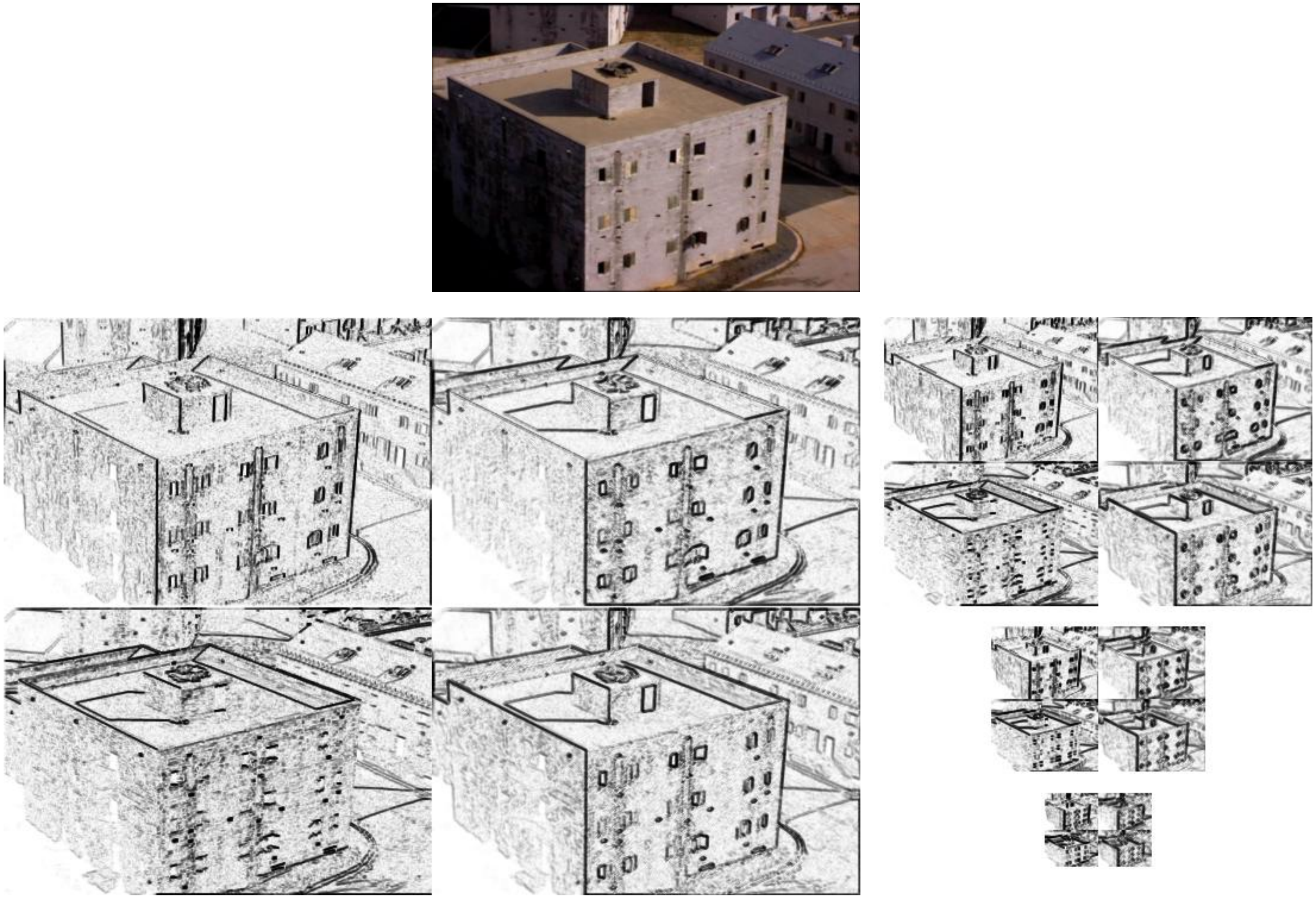
Correspondence-less exterior orientation from 3D-2D line pairs

The REGSITE Algorithm

… aligning site model edges to image edges…

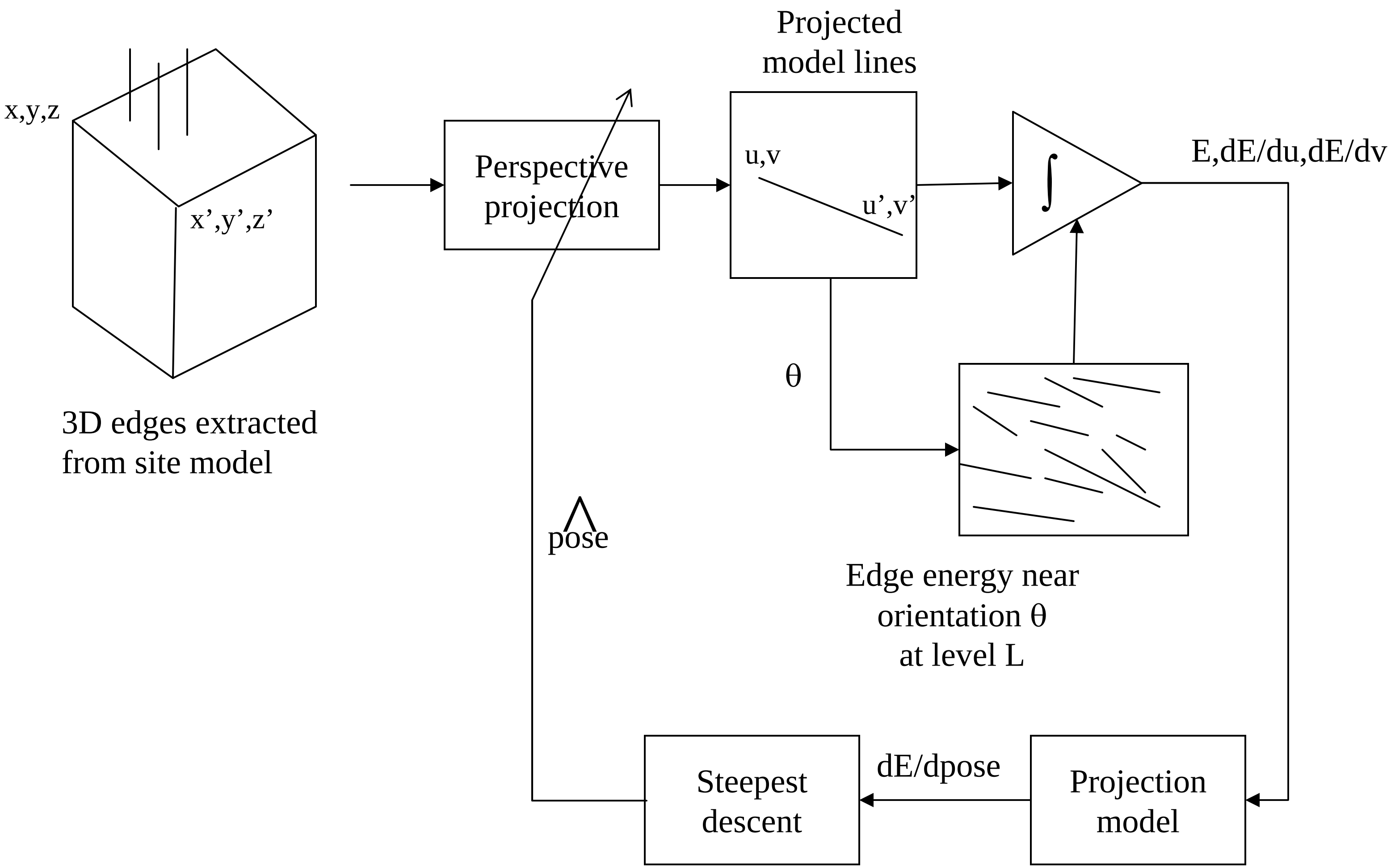
- Inputs:
  - Predicted pose of camera
  - Un-textured (Open Inventor) site model
  - Video frame
- Output:
  - Estimated pose of camera
- Premise
  - Discontinuities in 3D depth are correlated with brightness edges in the video frame (most of the time)
- Approach
  - Oriented energy image pyramids highlight image edges
  - Extract edges (depth discontinuities) from 3D site model
  - Adjust camera pose to maximize overlap of model and image edges
    - refinement is done using coarse to fine strategy over image pyramid

Oriented Energy Pyramid



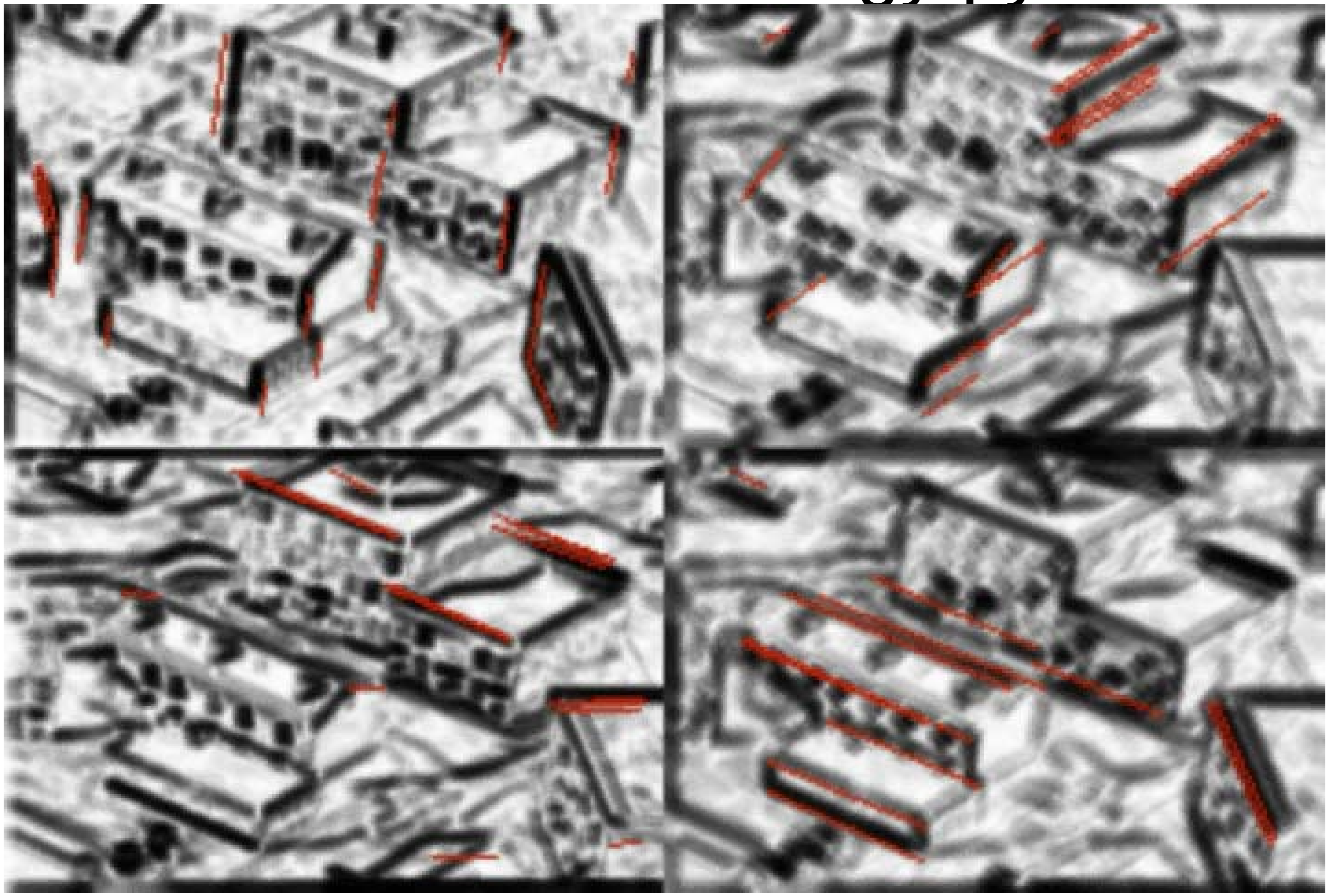
<u>Oriented Energy Pyramid: 4 Orientation Bands 0 deg., 45 deg., 90 deg., 135 deg.</u>

Pose Refinement Procedure



Pose Refinement Results

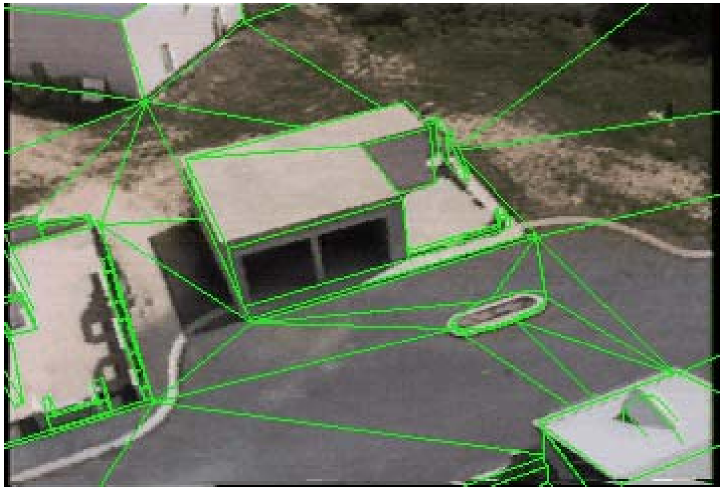
Iterative coarse-to-fine adjustment of pose over oriented energy pyramid



Geo-registration of Video Sequence from Draper Helicopter to 3D Site Models

Original Video

Model rendered from the pose of the helicopter sensor.
Pose recovered after geo-registration process



Overlay of site model on video

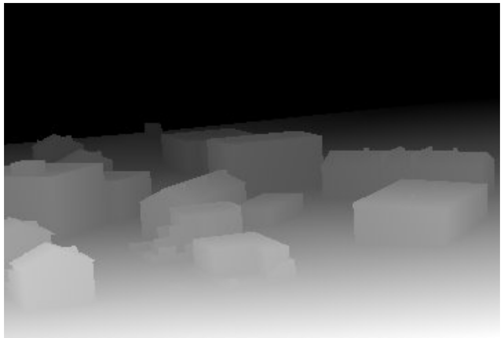
World as seen from the view-point of the runner

Re-projection & Enhanced Visualization of Video

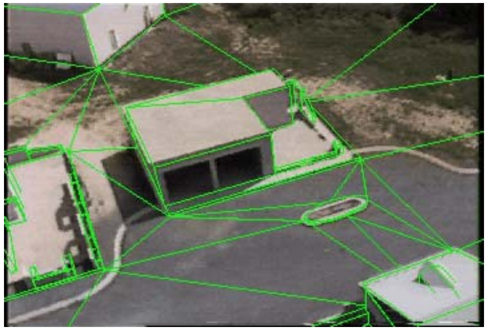
Geo-registration of video to site models

Original Video

Site model



Geo-registration of video to site model

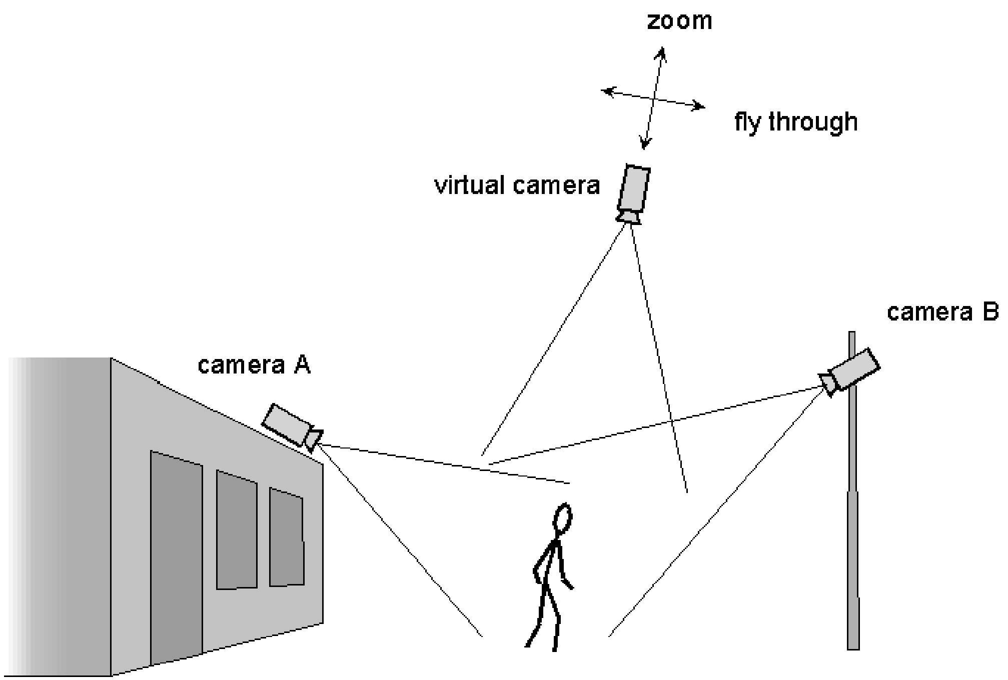


Re-projection of video after merging with model.

Application : Model-based Video Visualization

Immersive and Interactive Telepresence

Total Facilities Visualization



Multiple cameras are merged to form a unified 3D scene representation.

Each observer views the scene with his own “virtual” camera.

Video Flashlights Concept

[EGWR'02]

A tool for Global Visualization of Dynamic Environments

- **2D Video Flashlights:**
  - Project multiple 2D videos on a site model.
- **Moving Object Cued Video Flashlights:**
  - Project multiple 2D videos with automatically detected moving objects on a site model.
- **3D Video Flashlights:**
  - Project automatically extracted dynamic object models from multiple videos on a site model.

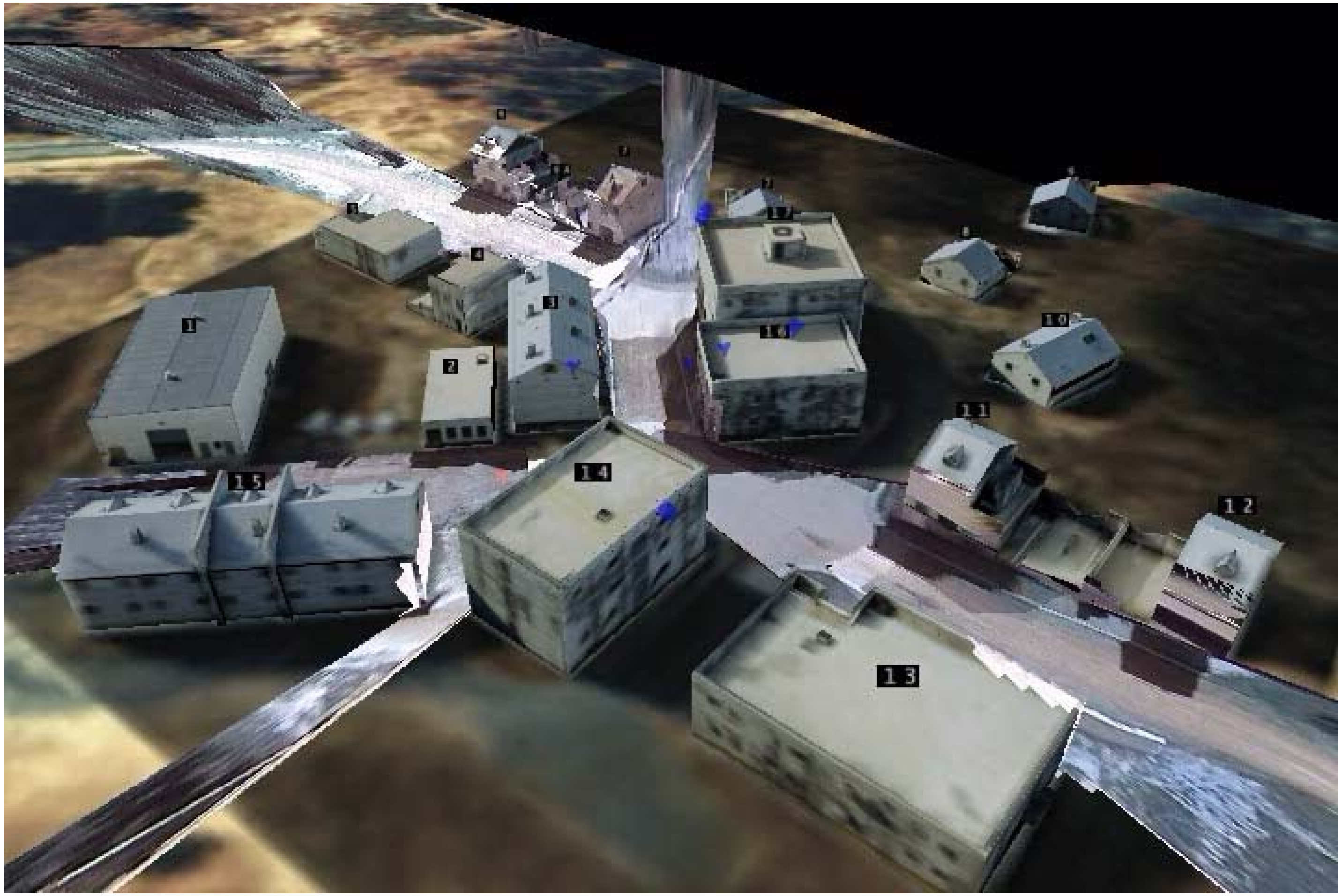
VideoFlashlights: Integration of video from many cameras

Live video streams are draped over a site model in real-time
Live videos are being viewed in the context of the model from a bird's eye view

Source videos





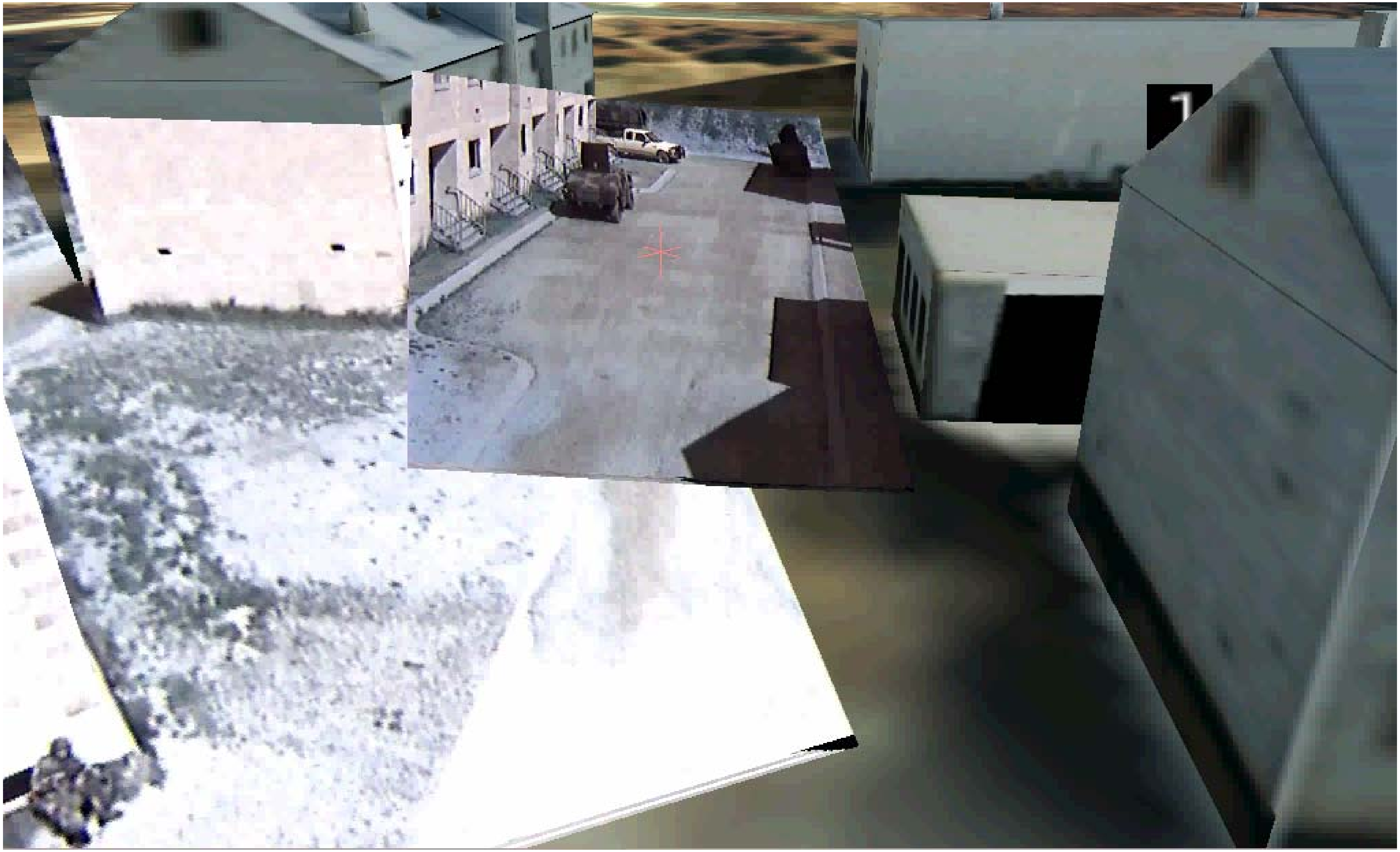



Accurate Projection of multiple video streams onto the site model

Enables interpretation of visual action in the global context of the model
Provides photo-realistic sky-to-street views at arbitrary scales and viewpoints

Video Flashlights

Close up view of multiple video streams draped over the site model
Close up view allows zooming onto action that is happening over multiple video cameras



Handling Moving Objects in 2D Parametric Alignment & Mosaicing

Multiple Motions : Robust Regression

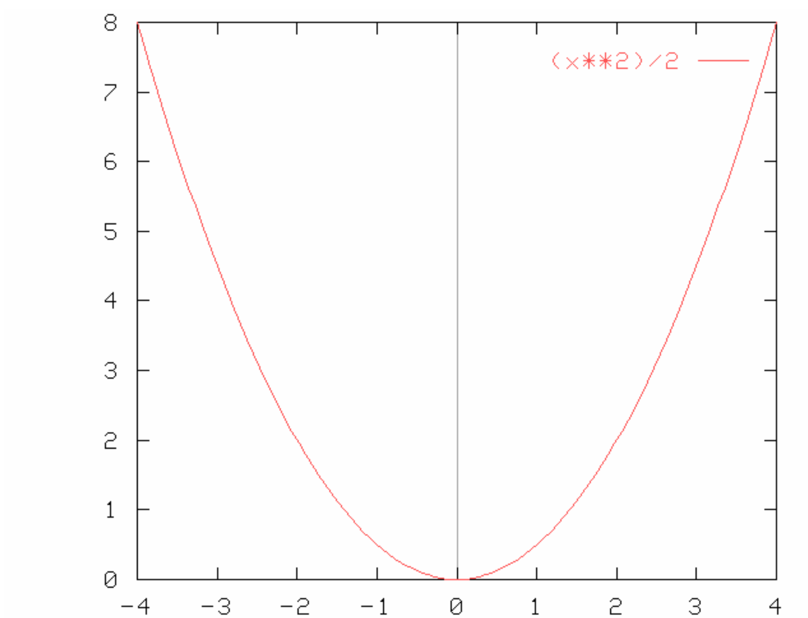
Find the dominant motion while rejecting outliers

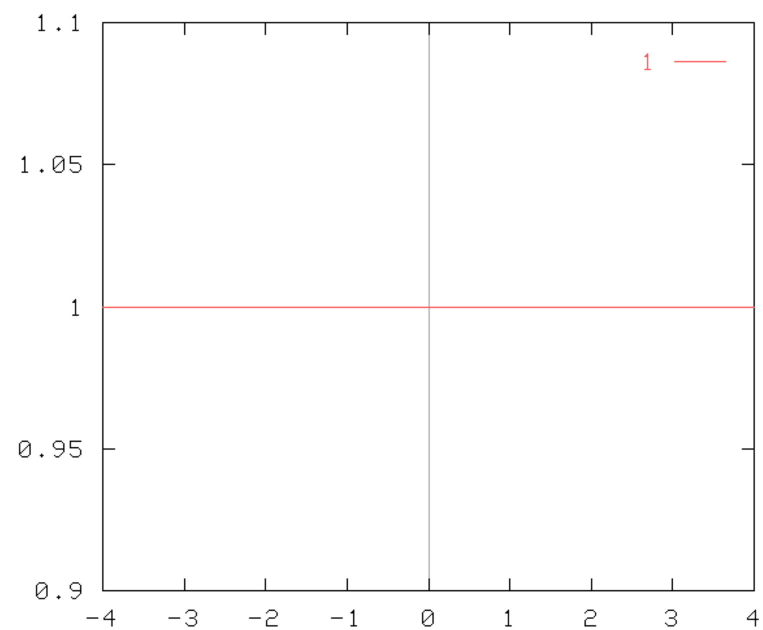
Estimating the Mean

$$r_i = d_i - \mu \qquad \min_{\mu} \sum_i \left(\rho(r_i) = r_i^2\right) \qquad \mu = \frac{1}{N}\sum_i d_i$$

- Mean is the "least squares" estimate
- But each measurement is given the same weight
- Influence of an outlier: $\frac{\delta}{N}$ → Can be arbitrarily large

Influence is given by : $\frac{\rho(r)}{r}$





Generalized M-Estimation

$$\min_{\Theta} \sum_{i} \rho(r_i;\sigma), \qquad r_i = I_2(p_i) - I_1(p_i - u(p_i;\Theta))$$

• Given a solution $\Theta^{(m)}$ at the *m*th iteration, find $\delta\Theta$ by solving :

$$\sum_{l}\sum_{i} \underbrace{\frac{\dot{\rho}(r_i)}{r_i}}_{w_i} \frac{\partial r_i}{\partial \theta_k} \frac{\partial r_i}{\partial \theta_l} \partial\theta_l = -\sum_{i} \underbrace{\frac{\dot{\rho}(r_i)}{r_i}}_{w_i} r_i \frac{\partial r_i}{\partial \theta_k} \quad \forall k$$

w_i

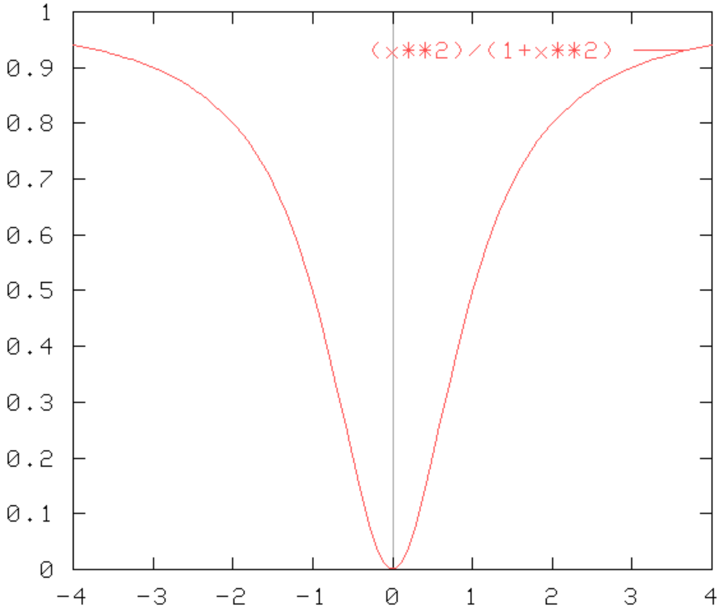
• w_i is a weight associated with each measurement. Can be varied to provide robustness to outliers.

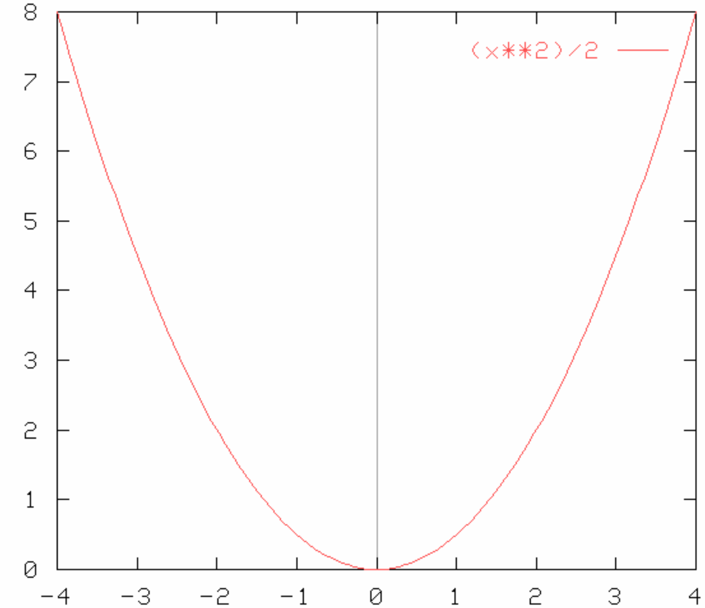
Choices of the $\rho(r_i;\sigma)$ function:

$$\rho_{SS} = \frac{r^2}{2\sigma^2} \qquad \rho_{GM} = \frac{r^2/\sigma^2}{1 + r^2/\sigma^2}$$

$$\frac{\dot{\rho}_{SS}(r)}{r} = \frac{1}{\sigma^2} \qquad \frac{\dot{\rho}_{GM}(r)}{r} = \frac{2\sigma^2}{(\sigma^2 + r^2)^2}$$

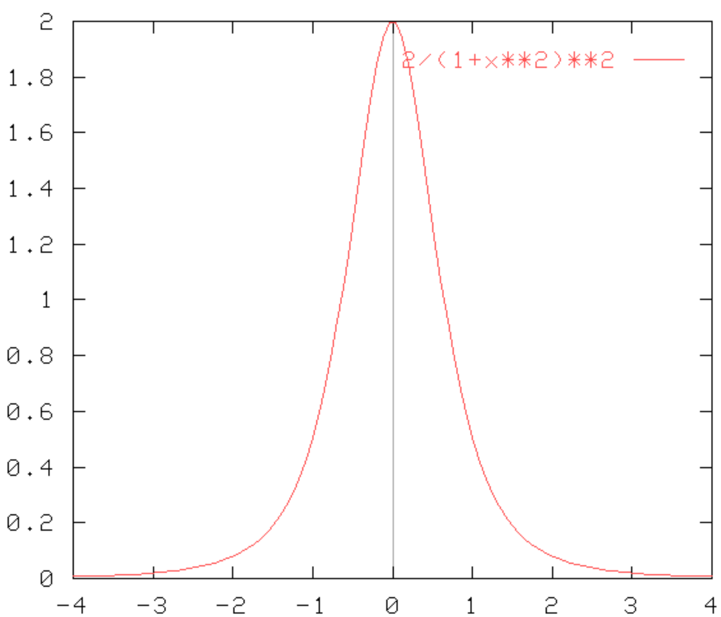
Optimization Functions & their Corresponding Weight Plots

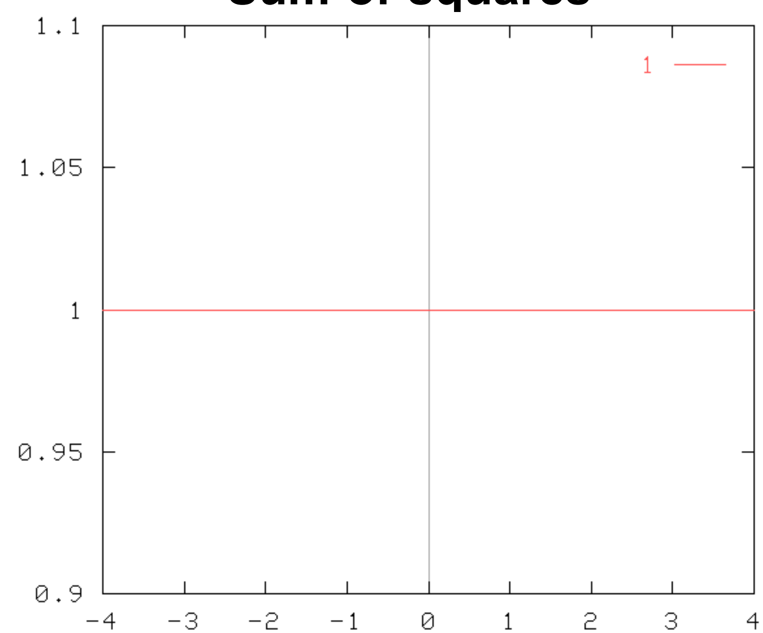




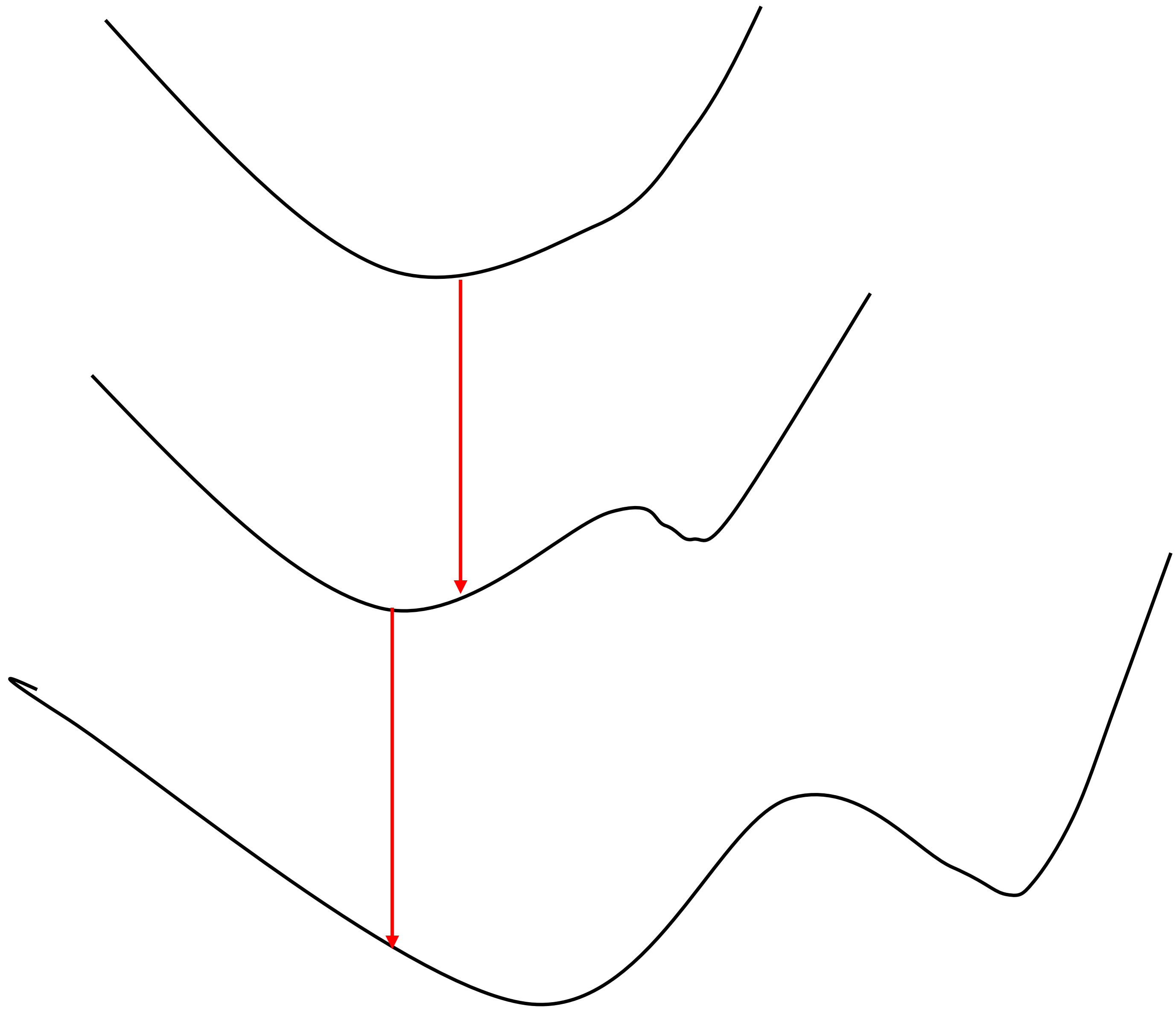
Geman-Mclure

Sum-of-squares





Continuation Method: Coarse-to-fine



With Robust Functions Direct Alignment Works for Non-dominant Moving Objects Too

Original two frames

Background Alignment

Object Deletion with Layers

Original Video

Video Stream with deleted moving object

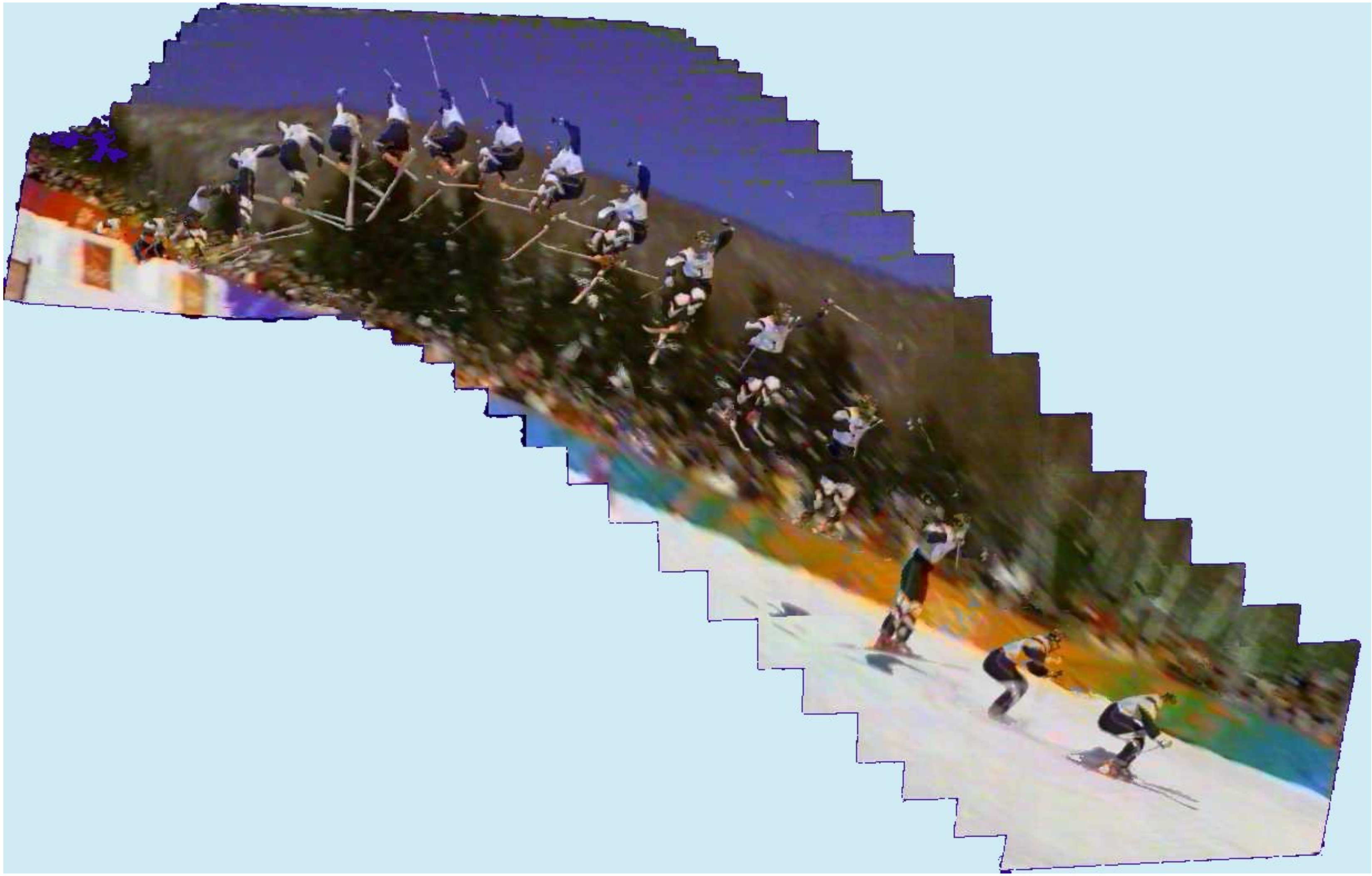
DYNAMIC MOSAICS

Original Video

Video Stream with deleted moving object

Dynamic Mosaic Video

SYNOPISIS MOSAICS



Problem

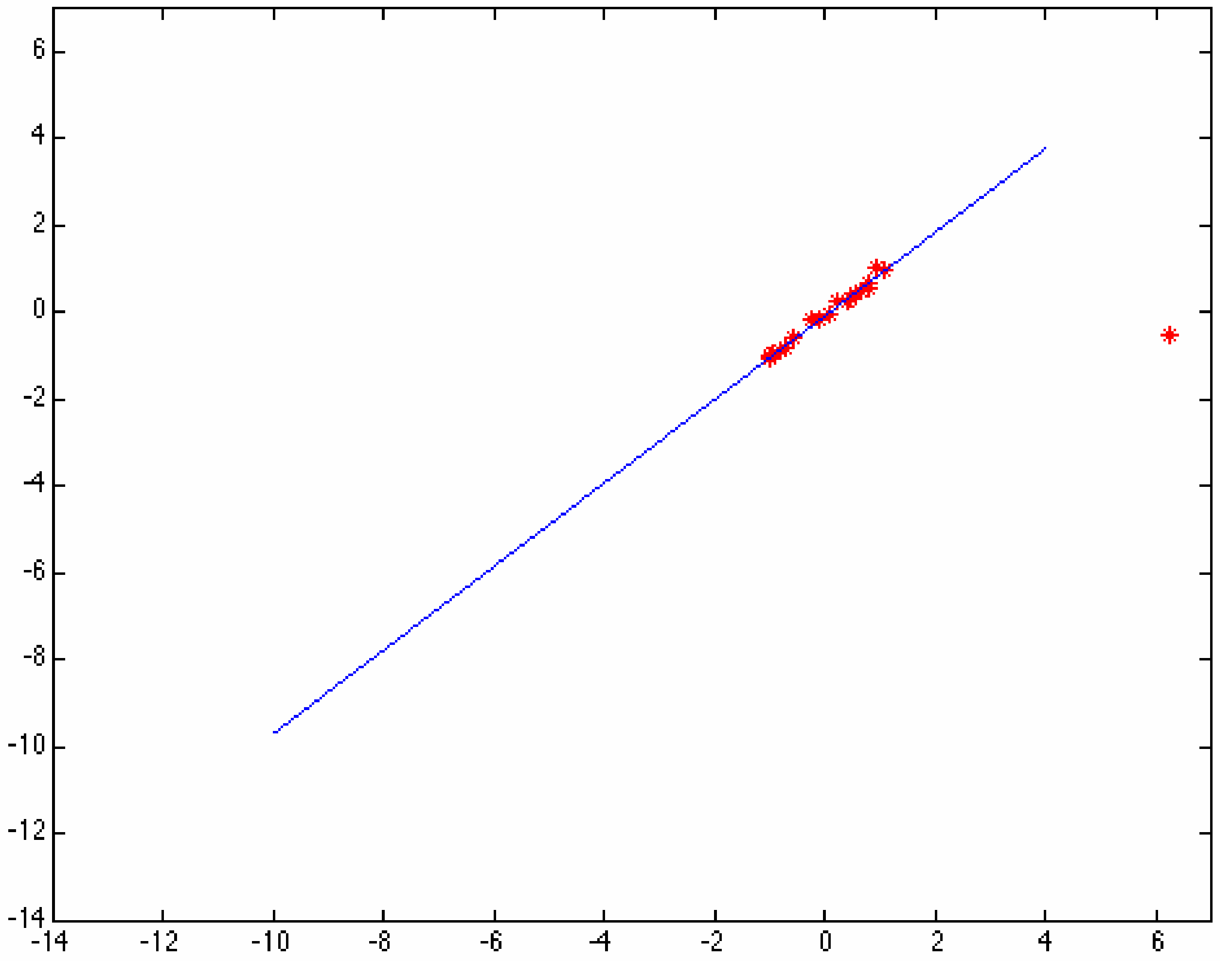
- Assumption:
  - Constraints that do not fit the dominant motion are treated as outliers : Extreme noise

- Problem:
  - But they are not noise
  - There indeed are multiple motions present in the scene

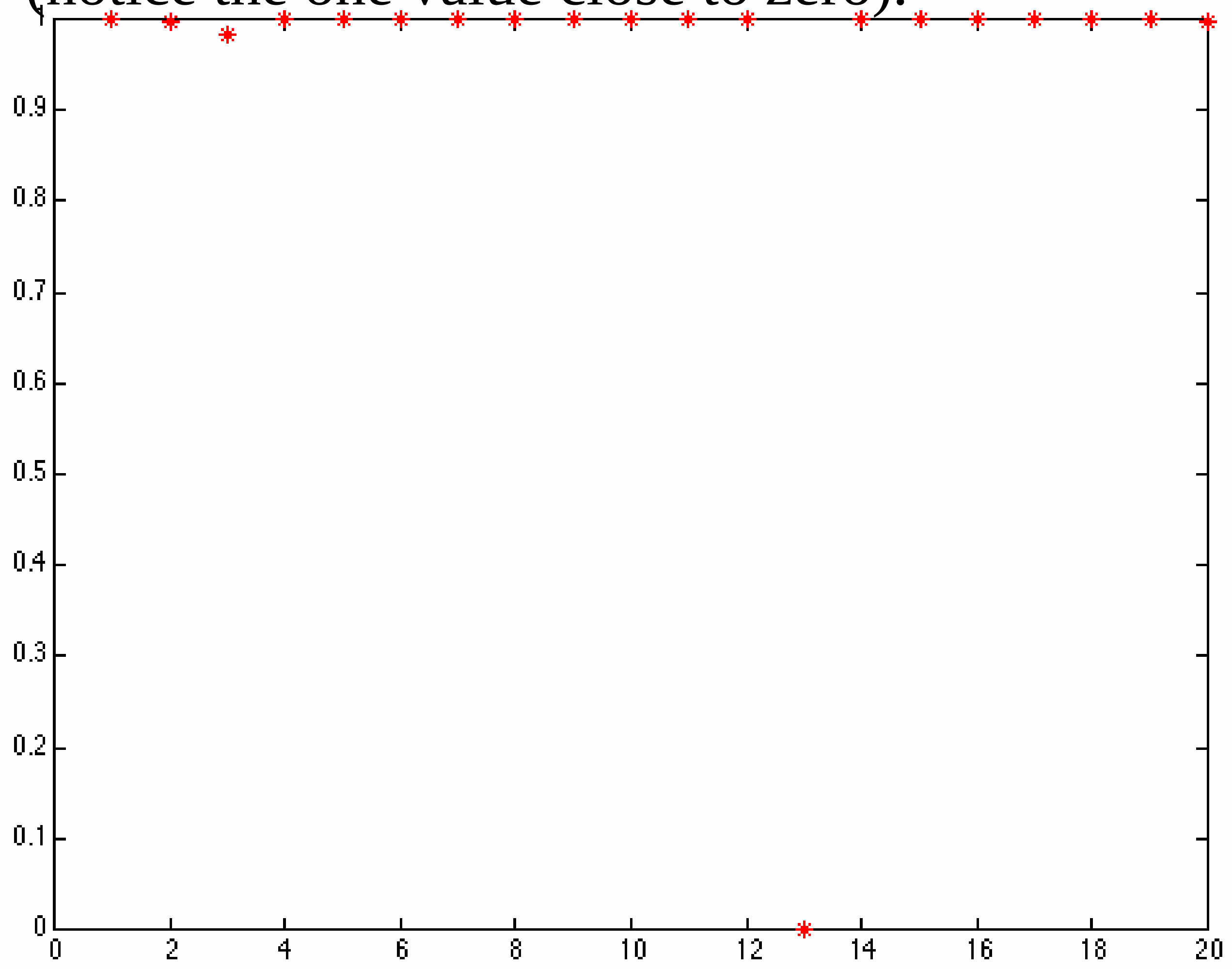
Motivate Simultaneous Multiple Model Estimation

Motivating Multiple Models

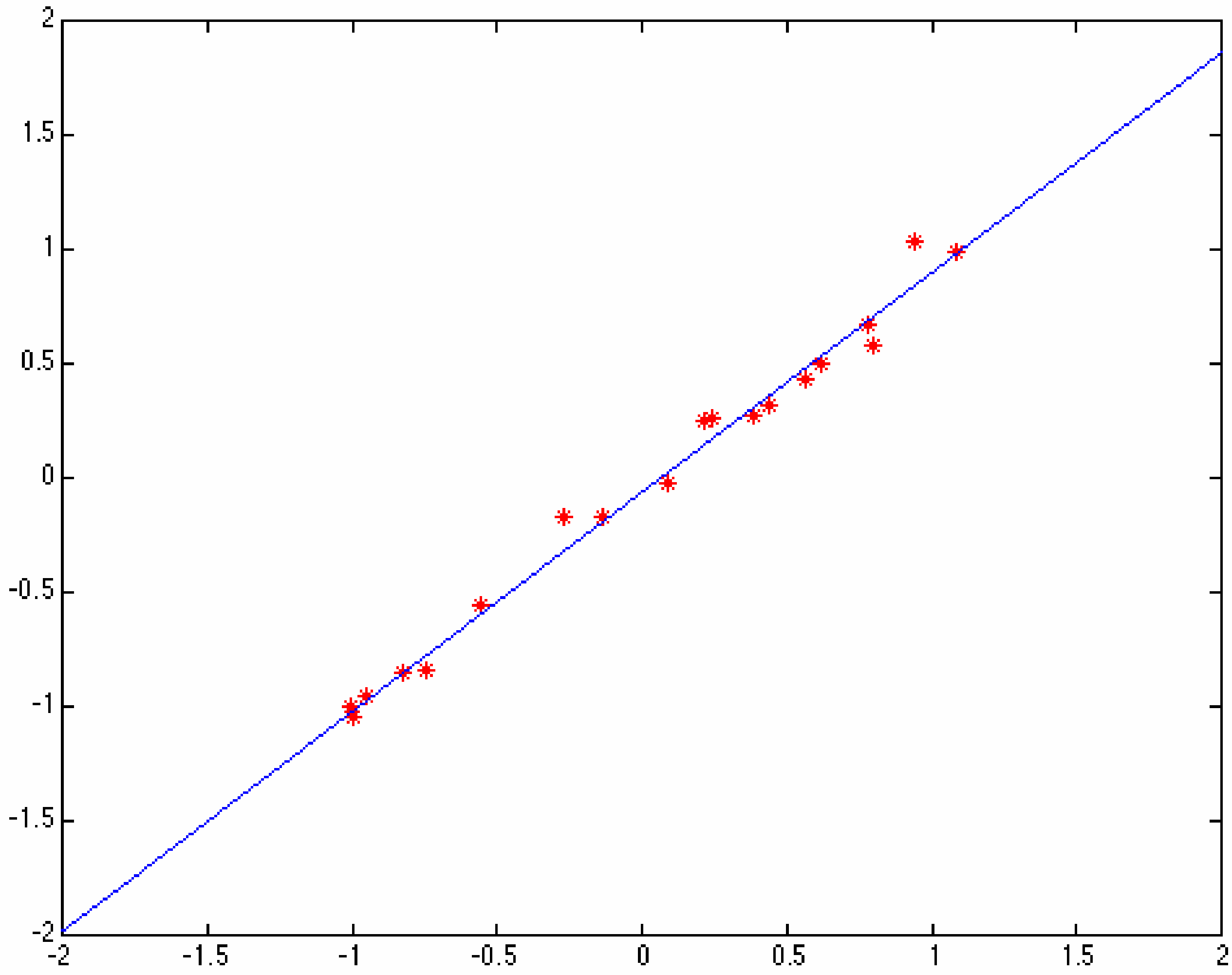
Line Fitting


6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-14
-12
-10
-8
-6
-4
-2
0
2
4
6

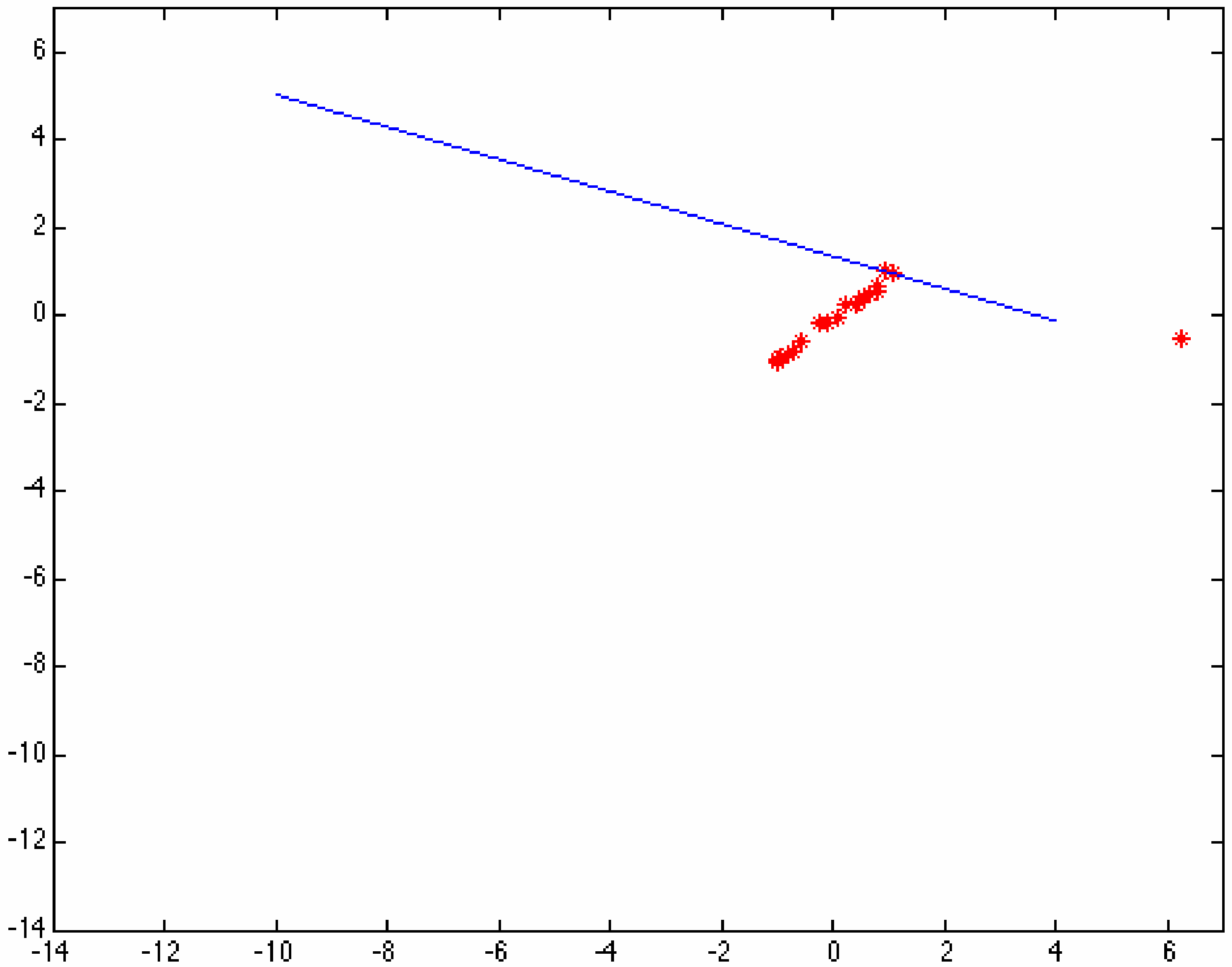
The expected values of the deltas at the maximum (notice the one value close to zero).



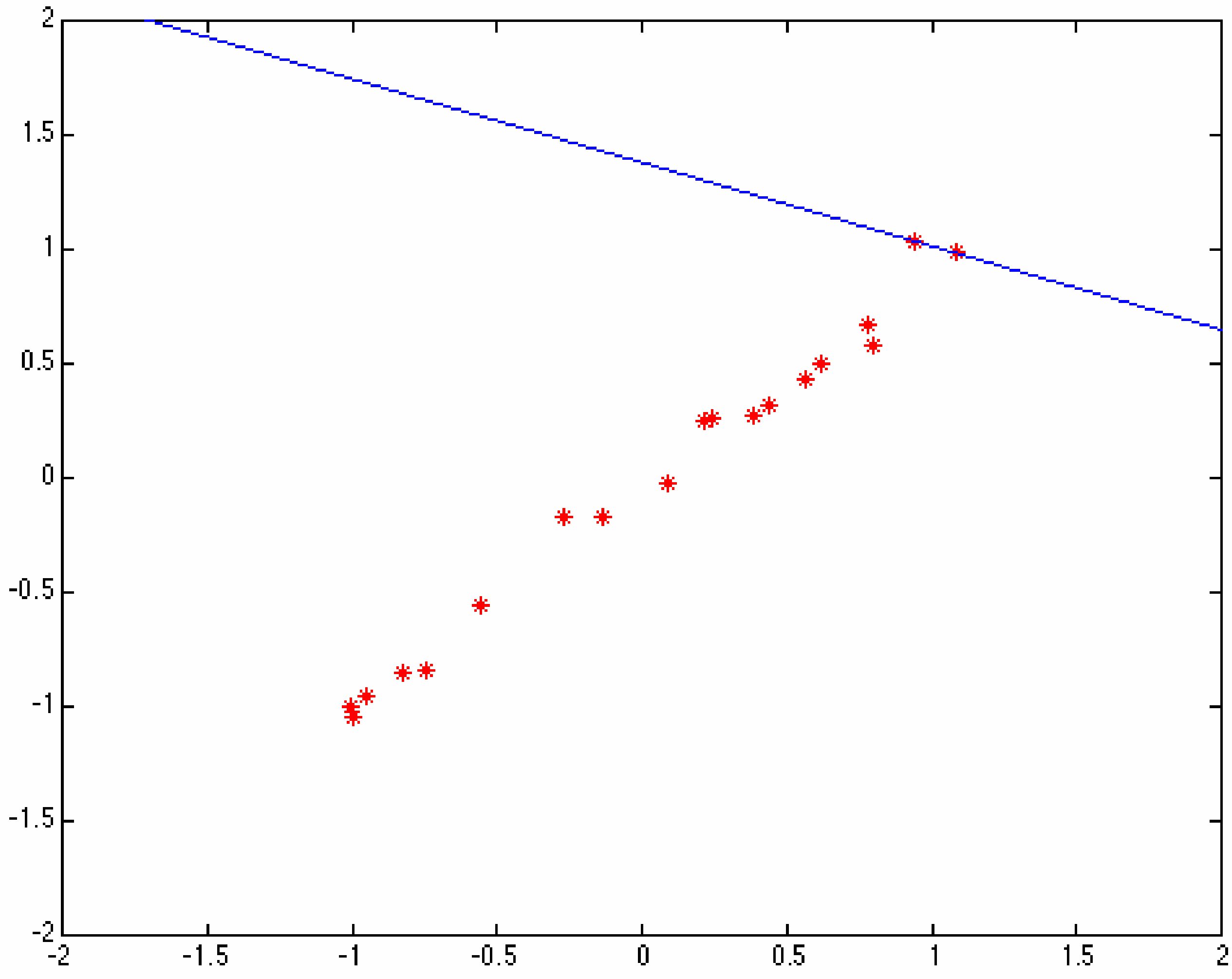
Closeup of the fit



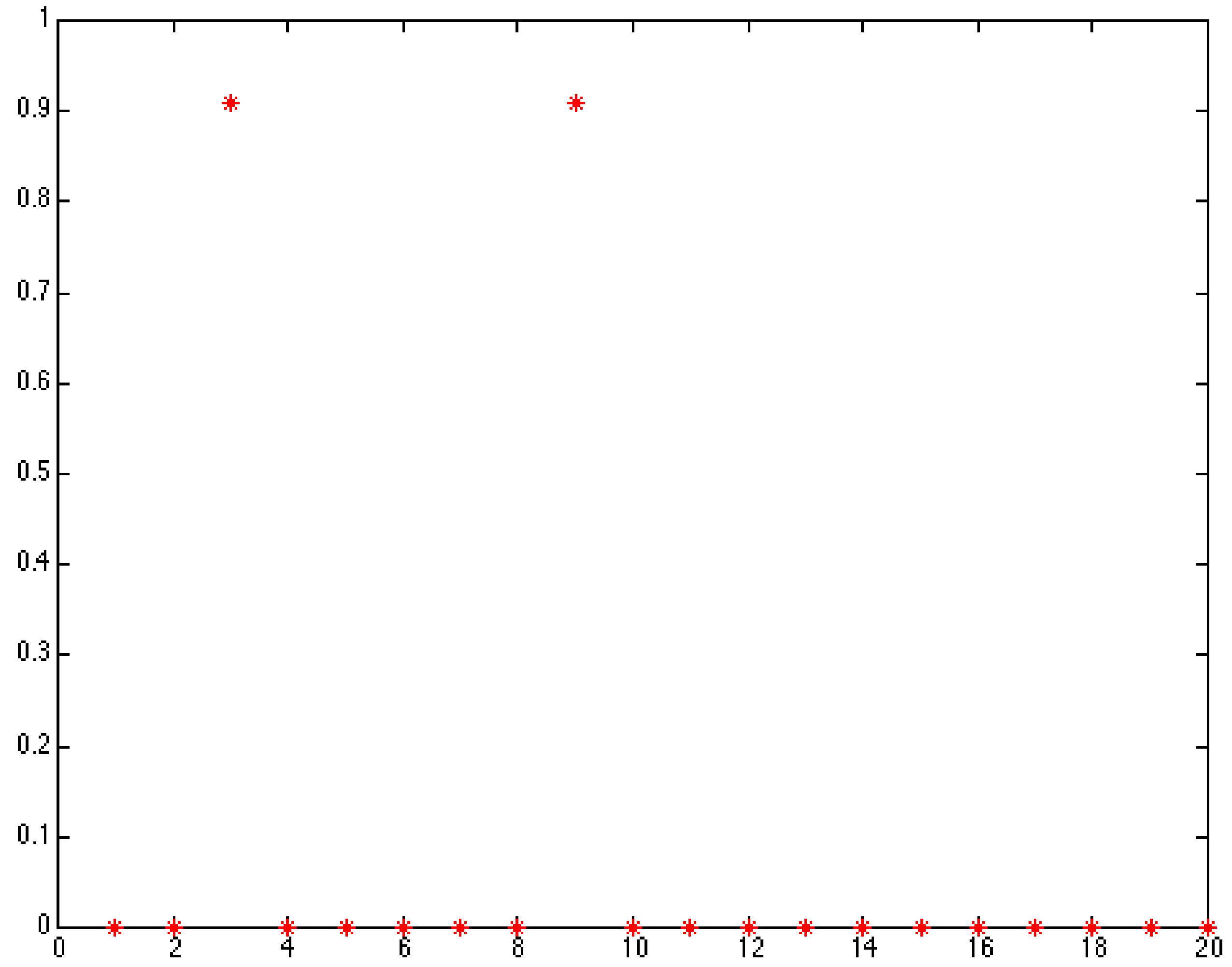
Local maximum



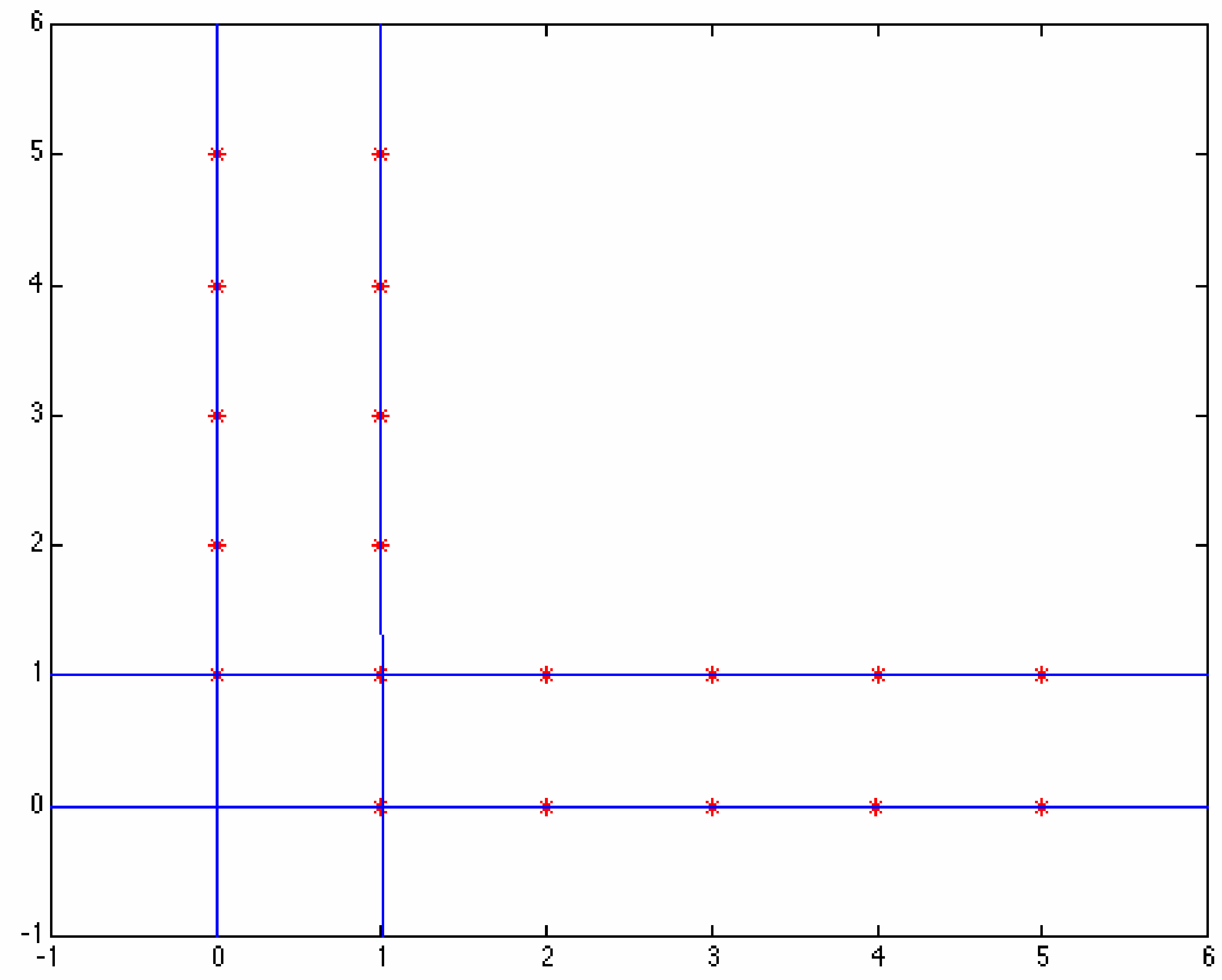
which is an excellent fit to some points



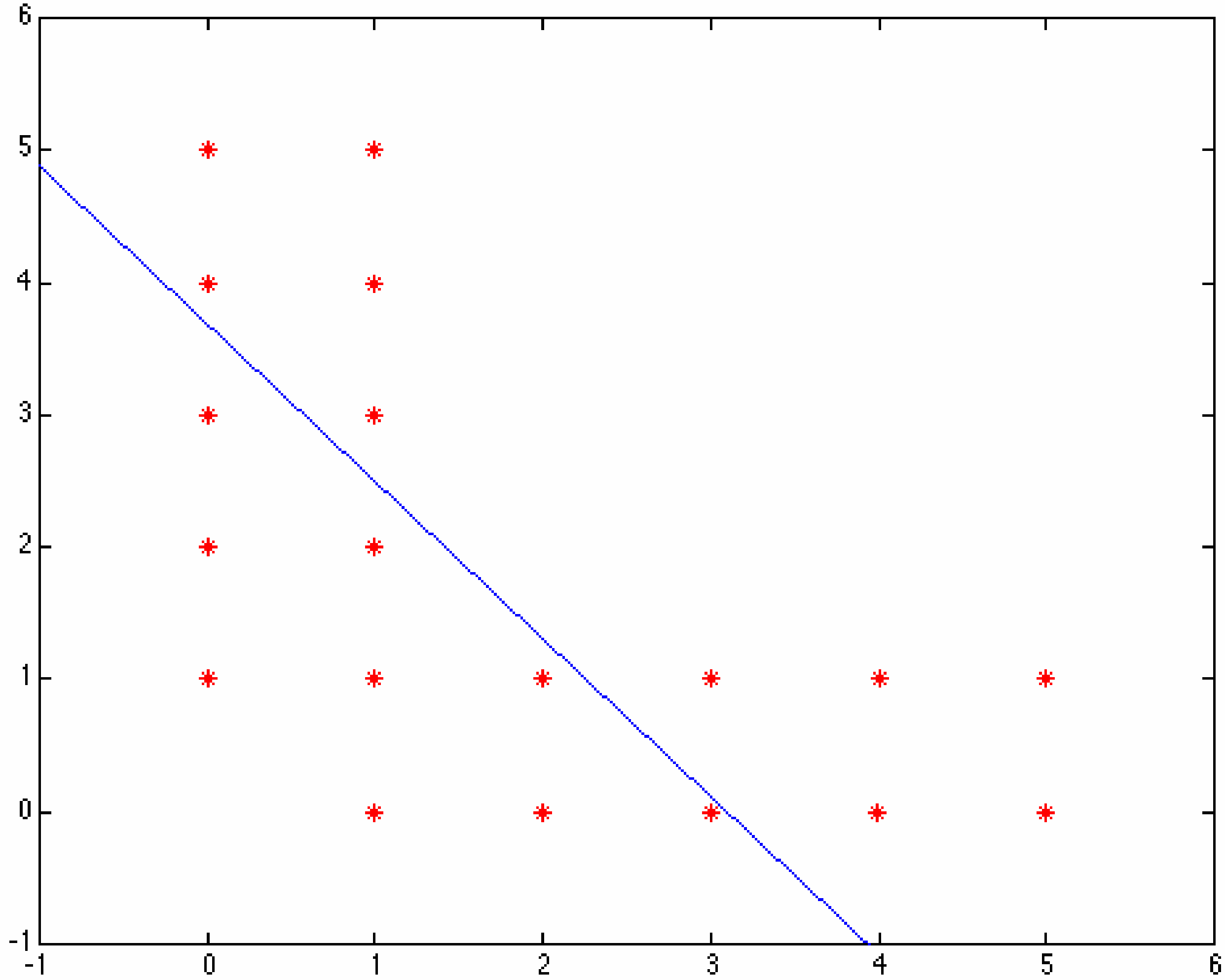
and the deltas for this maximum



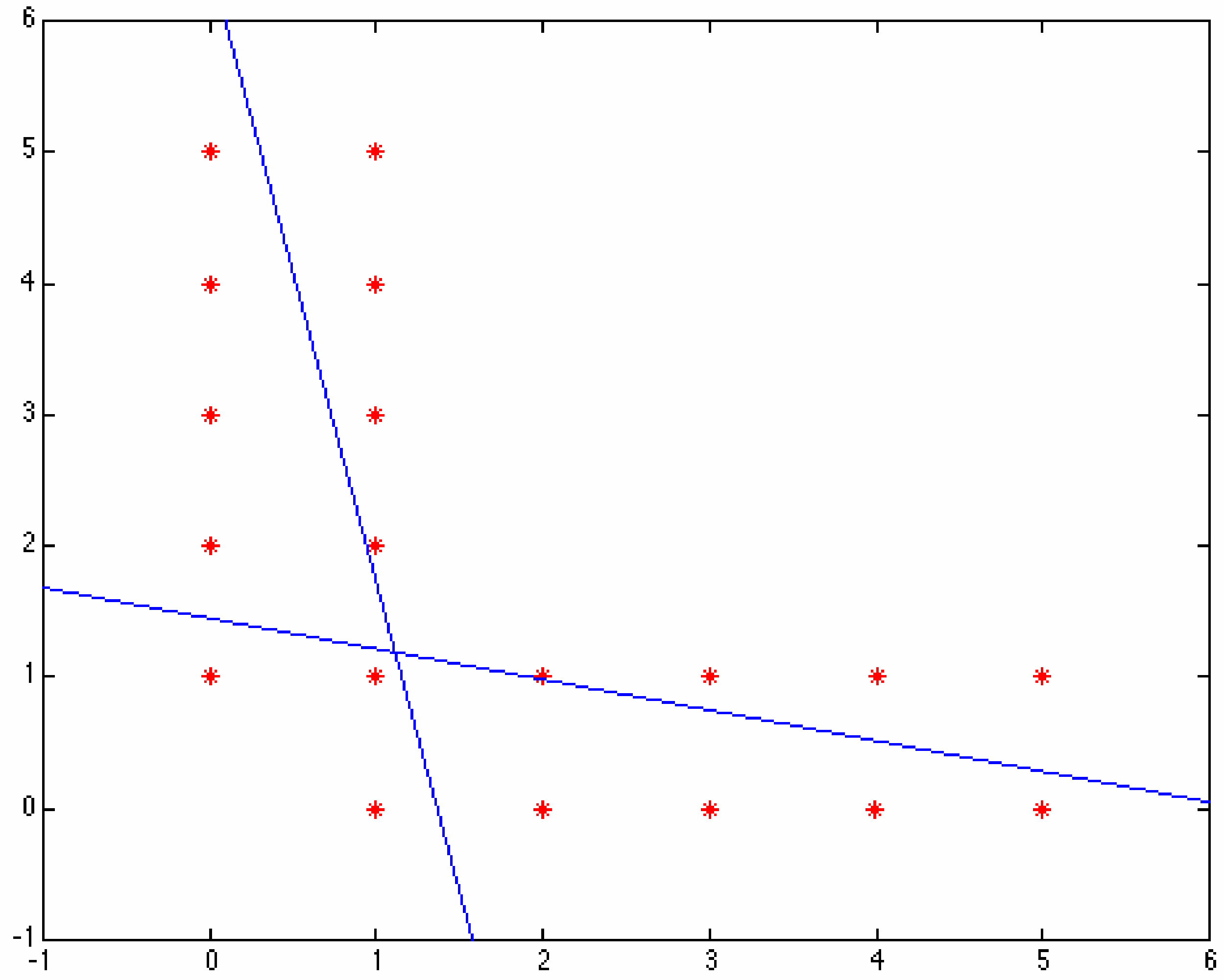
A dataset that is well fitted by four lines



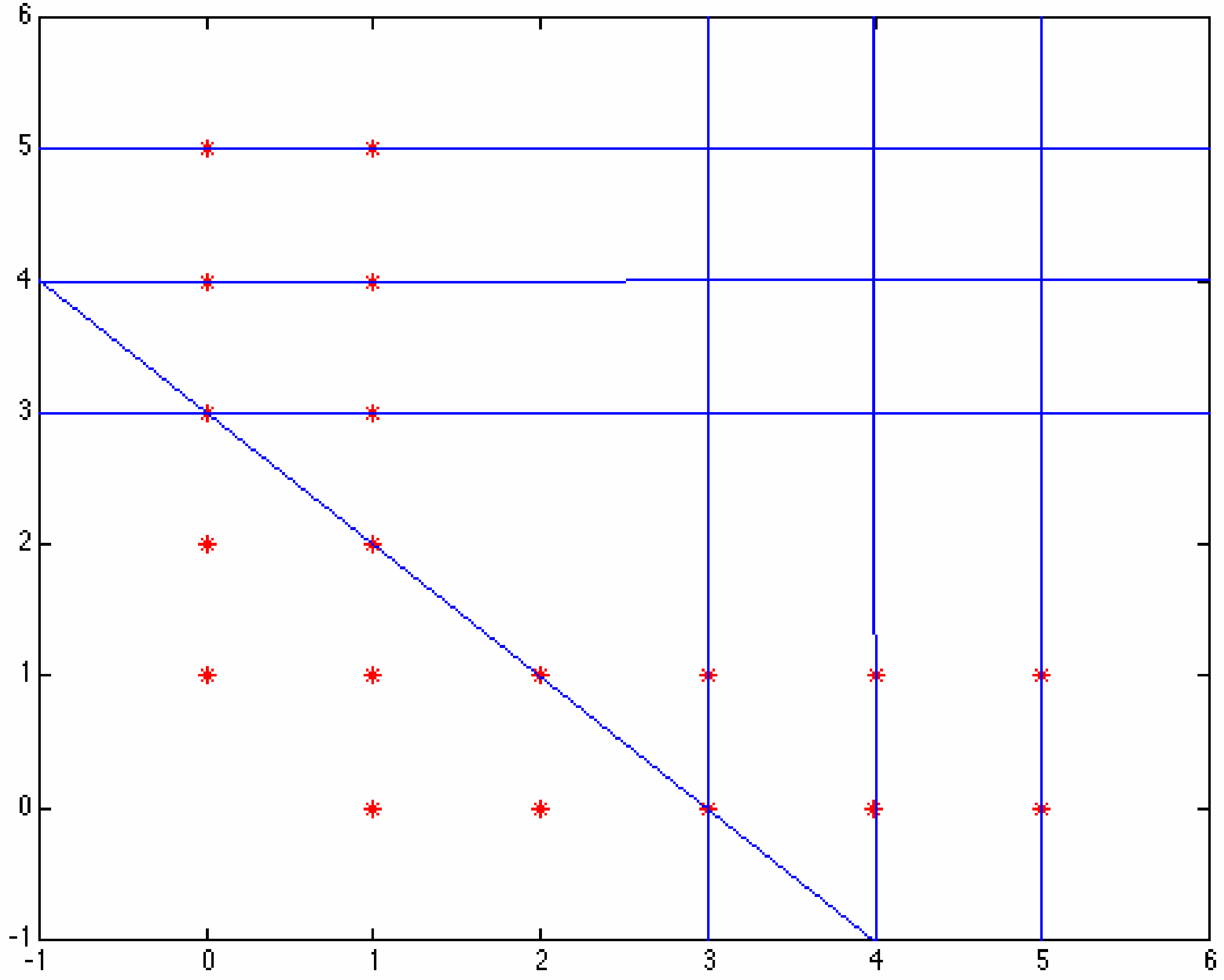
Result of EM fitting, with one line (or at least, one available local maximum).



Result of EM fitting, with two lines (or at least, one available local maximum).



Seven lines can produce a rather logical answer

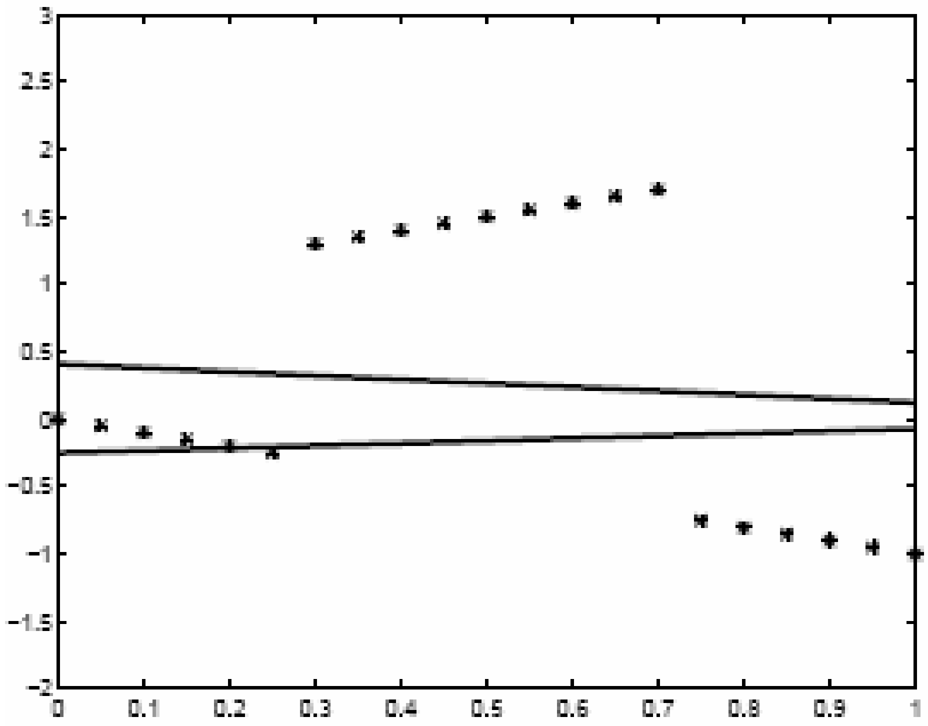


Example : Multiple Line Fitting with the Expectation-Maximization (EM) Algorithm

Fitting Two Lines

Suppose we have two line models:

(1) $y = x + 3$ and (2) $y = 2x - 1$



The Intuition

- We need to estimate two things:

  (1) the parameters (slope and intercept) of the two lines and

  (2) the assignment of each datapoint to the process that generated it.

- The intuition behind EM is that each of these steps is easy assuming the other one is solved.
- That is, assuming we know the assignment of each datapoint, then we can estimate the parameters of each line by taking into consideration only those points assigned to it.
- Likewise, if we know the parameters of the lines we can assign each point to the line that it fits the best.

Basic Structure

This gives the basic structure of an EM algorithm:

- Start with random parameter values for the two models.
- Iterate until parameter values converge:

  { E step: assign points to the model that fits it best.

  { M step: update the parameters of the models using only points assigned to it.

E-Step

- In the E step we compute for each data point two weights $w_1(i)$ and $w_2(i)$ (the soft assignment of the point to models 1 and 2 respectively)

$$r_1(i) = a_1 x_i + b_1 - y_i \qquad r_2(i) = a_2 x_i + b_2 - y_i$$

- Suppose the *i*th data point is x = 1; y = 1.1

$$r_1^{\,2}(i) = (2.9)^2 \qquad r_2^{\,2}(i) = (0.1)^2$$

- Then

$$w_1(i) = \frac{e^{-r_1^{\,2}(i)/\sigma^2}}{e^{-r_1^{\,2}(i)/\sigma^2} + e^{-r_2^{\,2}(i)/\sigma^2}} \qquad w_2(i) = \frac{e^{-r_2^{\,2}(i)/\sigma^2}}{e^{-r_1^{\,2}(i)/\sigma^2} + e^{-r_2^{\,2}(i)/\sigma^2}}$$

- So the E step calculates two weights for every datapoint

M-Step

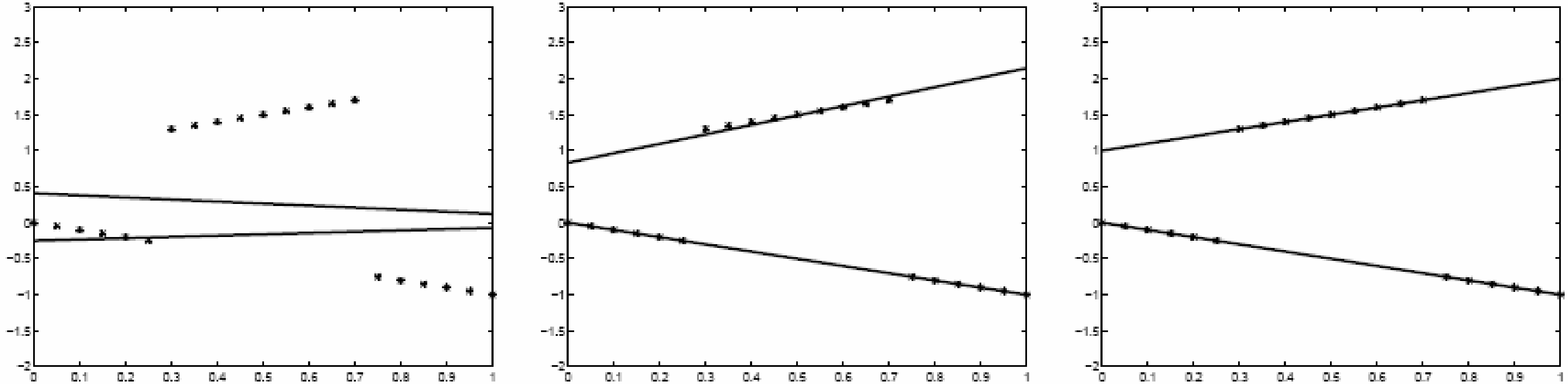
- In the M step we assume the weights are given, i.e. for each datapoint we know w1(i) and w2(i).
- To estimate the parameters of each process we just use weighted least squares.

$$\begin{pmatrix} \sum_i w_i x_i^2 & \sum_i w_i x_i \\ \sum_i w_i x_i & \sum_i w_i 1 \end{pmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_i w_i x_i y_i \\ \sum_i w_i y_i \end{bmatrix}$$

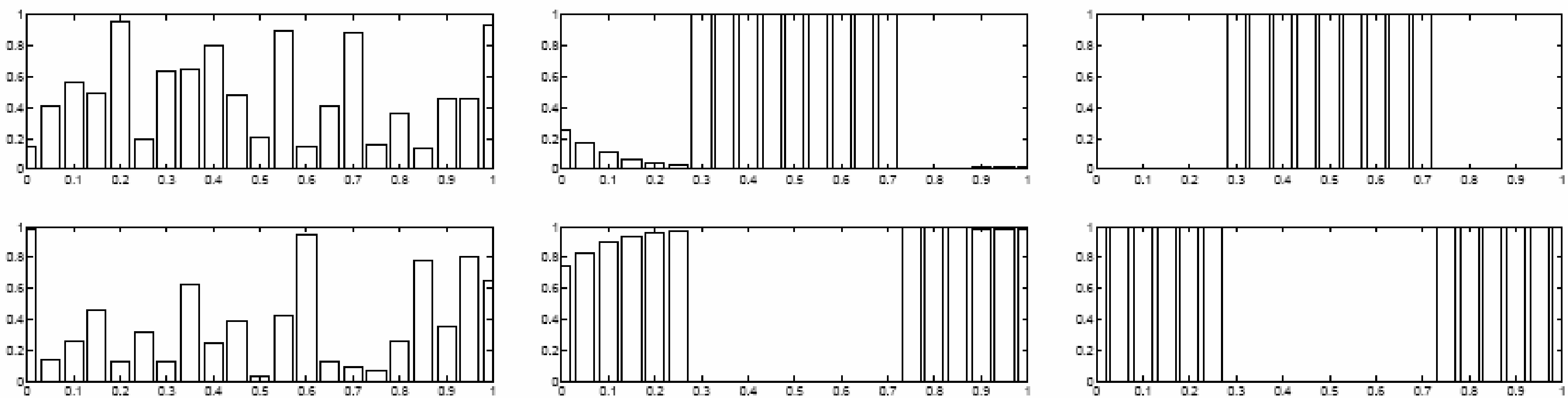
- So in the M step we solve the above equation twice.
  - First with $w_i = w_1(i)$ for the parameters of line 1 and
  - then with $w_i = w_2(i)$ for the parameters of line 2.
- In general, in the M step we solve two weighted least squares – one for each model, with the weights given by the results of the E step.

Example

Line fits



Weights



$t = 1$ $t = 2$ $t = 3$

Motivating Multiple Models

Multiple Motions

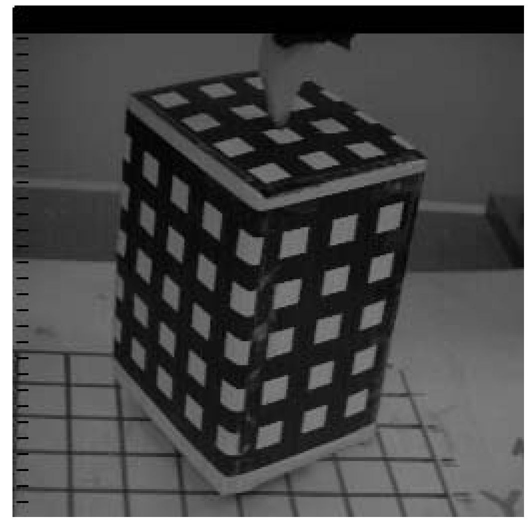
Independent Object Motion

Objects are the Focus
Camera is more or less steady

Independent Object Motion with Camera Pan

Most common scenario for capturing performances

Multiple Motions may not be due only to independent object motions but due to different surfaces Or " Motion Layers "



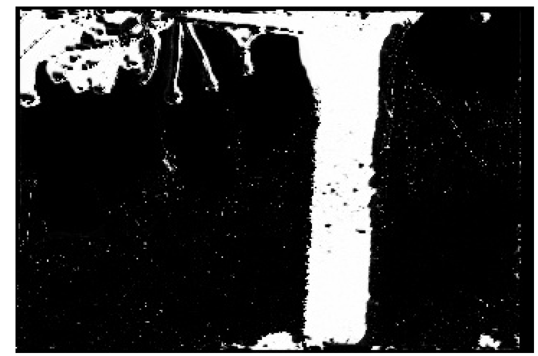
Multiple Motions as a Segmentation & Estimation Problem

- If we know which pixels go with what motion, can apply the now well-known methods of motion estimation to compute the motions
- Alternatively, given the motion parameters, potentially can label pixels corresponding to each of the motions.

Represent Multiple Motions as Layers

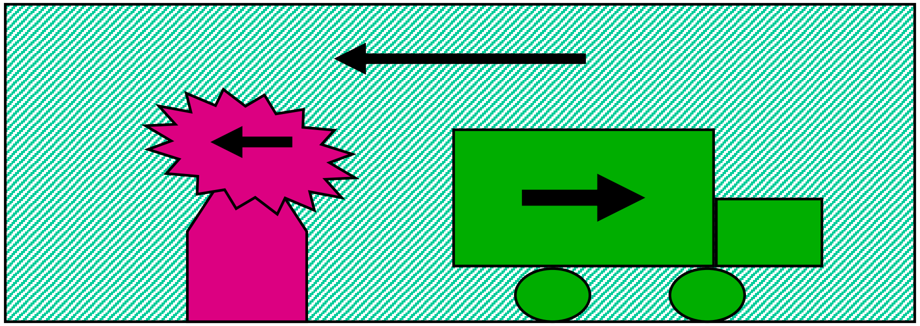
Input Sequence

Layers



Compact Video Representation

...motion and scene structure analysis...



Separate coherent & significant motion & structure components

- Coherence : Align images using 2D/3D models of motion and structure

  Separate backgrounds and moving objects with layers

- Significance : Regions of support for various motion & structure components

MULTIPLE 2D PARAMETRIC MODEL ESTIMATION

… layered scene representation …

THREE ISSUES

- How many models ?
- What are the model parameters ?
- What is the spatial support layer for each model ?

Competitive Multiple Model Estimation

[Ayer,Sawhney '95 '96]

- Model image motion in terms of a mixture of Gaussian models

- Layers of support represented as ownership probabilities

- Robust Maximum-Likelihood estimation of mixture and layer parameters using the Expectation-Maximization algorithm

- Minimum Description Length (MDL) encoding to select adequate number of models

Automatic Layer Extraction : Intuition

Input Sequence

Layer

Motion

Assume that the segmentation of pixels into layers is known,

then estimating the motion is easy.

$$E_{SSD}(u;A_i) = \sum_{p \in R} w_i(p)(\nabla I_1^T u(p;A_i) + \delta I(p))^2$$

Where do we get the weights from ?

Input Sequence **Layer** **Motion**

Model each pixel as potentially belonging to N layers each with its own motion model.

Assume that we know the motion model, but not the pixel ownership to the model

$$E_{SSD}(u; A_i) = \sum_{p \in R} w_i(p)(\nabla I_1^T u(p; A_i) + \delta I(p))^2$$

Where do we get the weights from ?

Input Sequence

Layer

Motion

Each pixel has a likelihood associated with a motion model and the two images

$$L(I_2(p) \mid I_1(p), A_i) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(\frac{-(\nabla I_1^T u(p; A_i) + \delta I(p))^2}{2\sigma^2}\right)$$

$$w_i(p) = \frac{L(I_2(p) \mid I_1(p), A_i)}{\sum_i L(I_2(p) \mid I_1(p), A_i)}$$

Multiple Models : Mixture Models

- Model an image as a density function created using a mixture of Gaussian models conditioned on the adjacent images :

No. of models

$$f(I(\mathbf{x},t)\,|\,I(\mathbf{x},t-1),\Phi)=\sum_{i=1}^{g}\pi_i\, p(I(\mathbf{x},t)\,|\,I(\mathbf{x}-\mathbf{u}(\mathbf{x},\Theta_i),t-1),\sigma_i)$$

The mixture model is parameterized by

$$\Phi=\left[\Pi,\Sigma,\Theta\right]$$

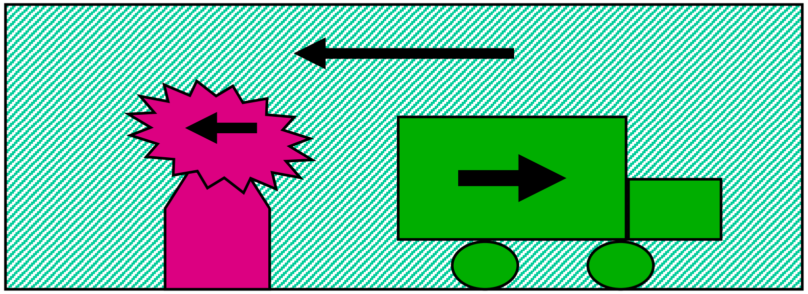
Θ : Ensemble of motion parameters for the g models

Σ : Ensemble of the Gaussian distribution parameters for the models

Π : Ensemble of the ownership layers for the models

Mixture Models

... represent layer ownerships as binary hidden variables ...



- $Z = \{z_i(p_j), i = 1..g, j = 1..N\}$

  is a set of binary indicator variables representing the model labels.

- The stochastic model for the complete data, measurements and hidden variables is:

$$f(I, Z \mid \Phi) = \underbrace{f(I \mid Z, \Phi)}\underbrace{p(Z \mid \Phi)}$$

Observation Likelihood

Prior on the labels

Mixture Model Estimation

The Expectation-Maximization (EM) Algorithm

- Maximize the negative log-likelihood of the parameters given the observations :

$$L(\Phi \mid I, Z) = -\log(f(I, Z \mid \Phi))$$

- Define the expectation of the likelihood :

$$Q(\Phi \mid \hat{\Phi}^{(k)}) = E\left[L(\Phi \mid I, Z) \mid I, \hat{\Phi}^{(k)}\right]$$

Mixing Proportions

Ownership

$$Q(\Phi \mid \hat{\Phi}^{(k)}) = \sum_{i=1}^{g} \sum_{x} \log(\pi_i) p\left(i \mid x, \hat{\Phi}^{(k)}\right) +$$

Data Likelihood

$$\sum_{i=1}^{g} \sum_{x} \log\left(p_l(I(x) \mid \hat{\Phi}^{(k)})\right) p\left(i \mid x, \hat{\Phi}^{(k)}\right)$$

The EM Algorithm

… iterate between layer and motion estimation …

Starting with an initial estimate $\hat{\Phi}^{(0)}$ repeat :

- **E-step :** Compute the function $Q(\Phi \mid \hat{\Phi}^{(k)})$

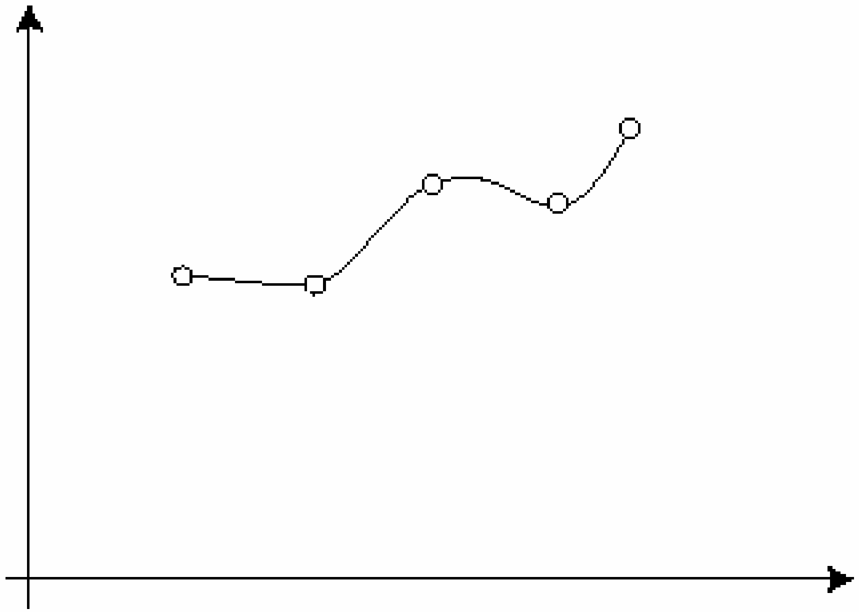
  Given the current estimate of the alignment parameters, compute the layer ownerships.

- **M-step :** Compute $\hat{\Phi}^{(k+1)} = \arg\max_{\Phi} Q(\Phi \mid \hat{\Phi}^{(k)})$

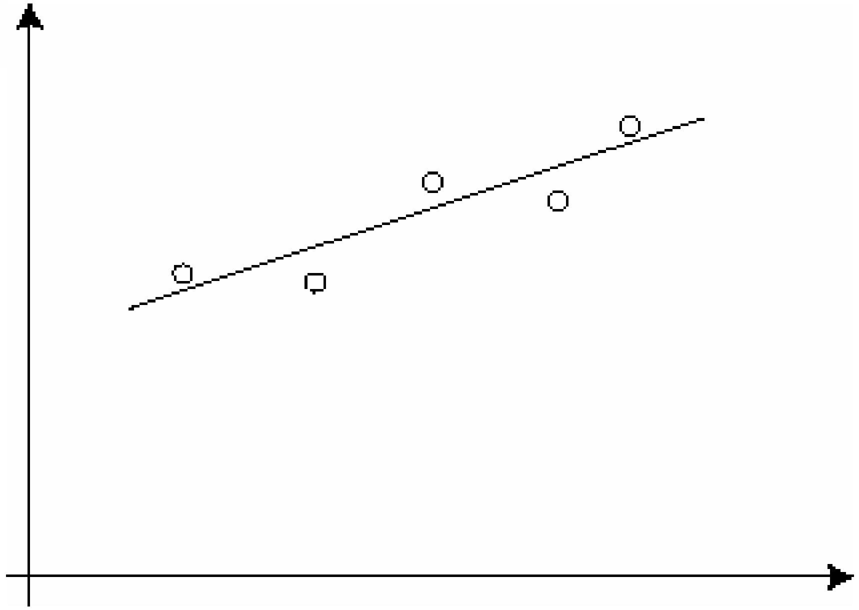
  Given the layer ownerships compute the alignment parameters.

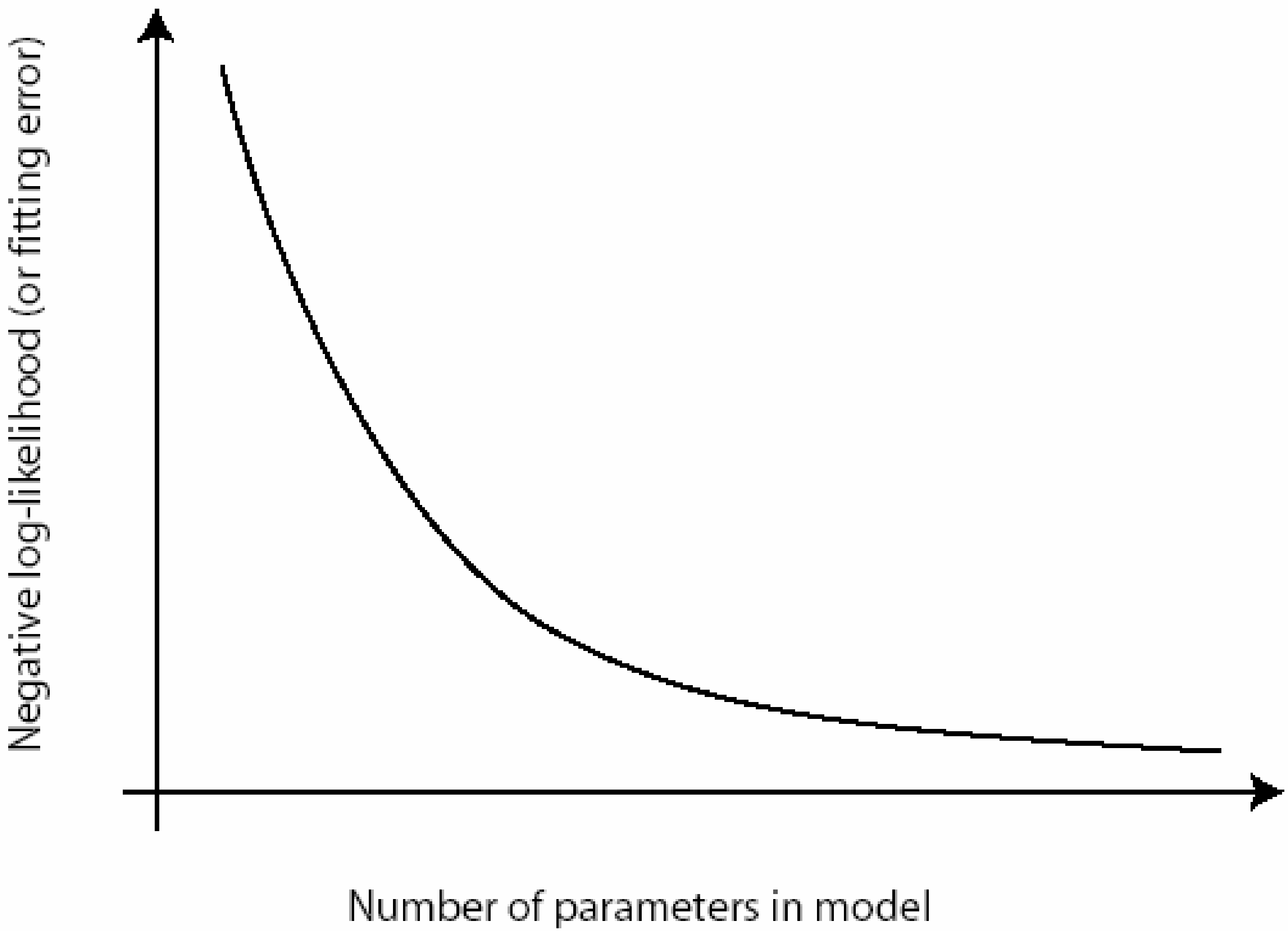
Model Selection

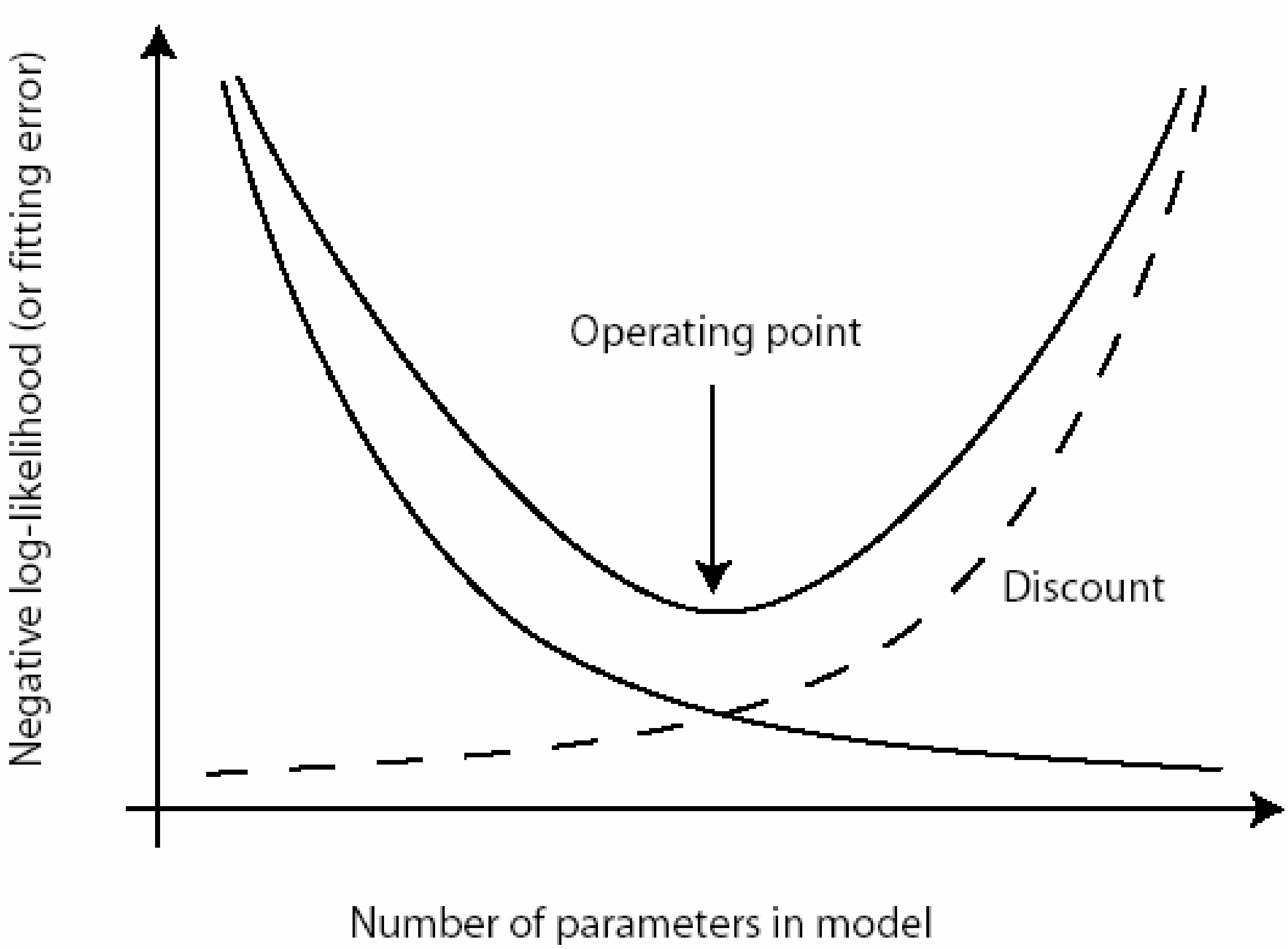
- We wish to choose a model to fit to data
  - e.g. is it a line or a circle?
  - e.g is this a perspective or orthographic camera?
  - e.g. is there an aeroplane there or is it noise?
- Issue
  - In general, models with more parameters will fit a dataset better, but are poorer at prediction
  - This means we can't simply look at the negative log-likelihood (or fitting error)



Top is not necessarily a better fit than bottom
(actually, almost always worse)




Negative log-likelihood (or fitting error)
Number of parameters in model



We can discount the fitting error with some term in the number of parameters in the model.

How Many Models Are Adequate ?

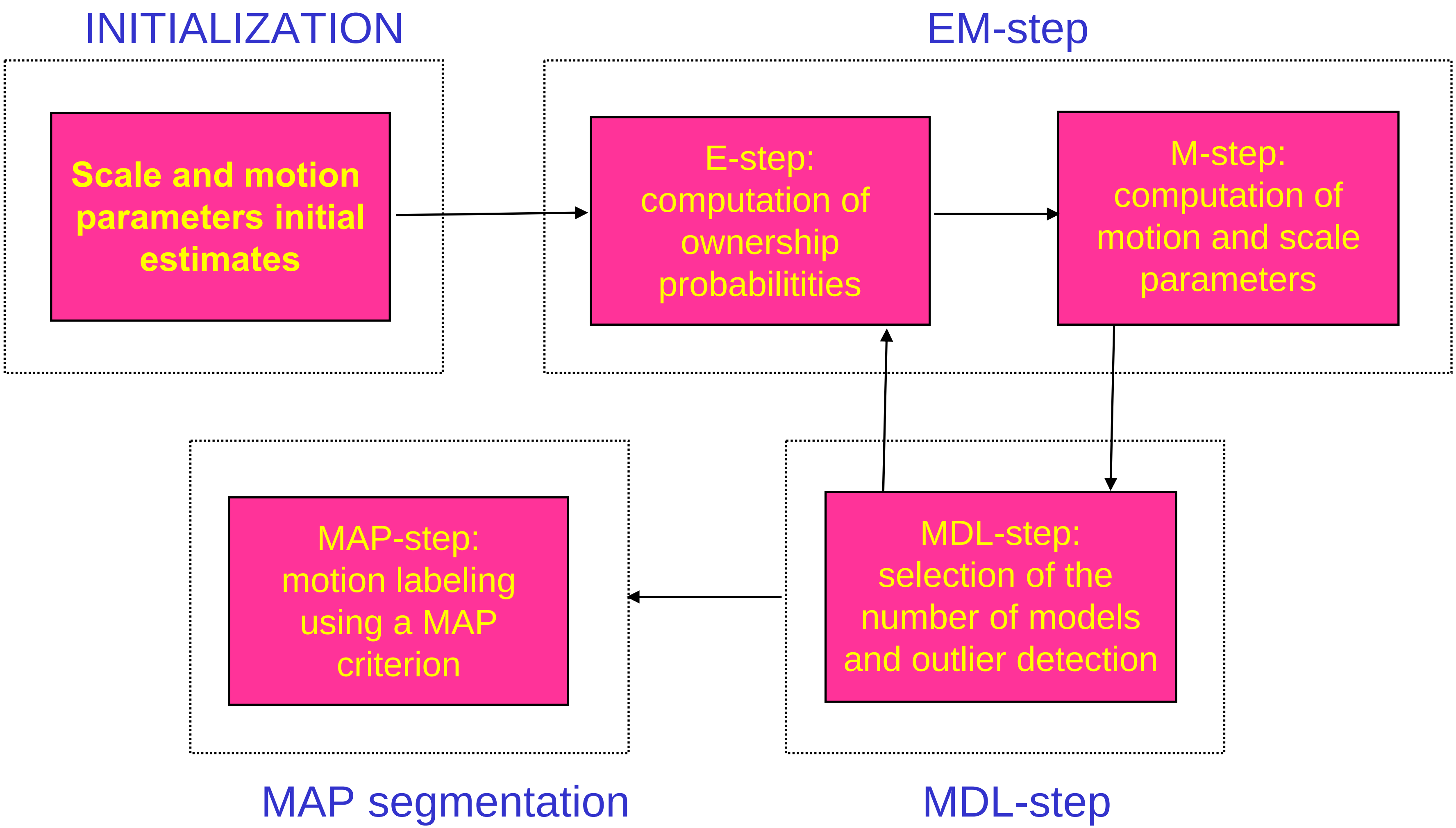
Minimum Description Length (MDL) encoding
for
Optimizing Modeling Complexity

- Define model complexity as the total number of bits needed to encode the data and the models:

$$L(\{I(p_j),\Phi\}) = \underbrace{L_M(\Phi)}_{\substack{\text{Model Encoding Length}\\ \text{(including pixel ownerships)}}} + \underbrace{L_D(\{I(p_j)\}|\Phi)}_{\substack{\text{Data Encoding Length}\\ \text{(residuals after alignment)}}}$$

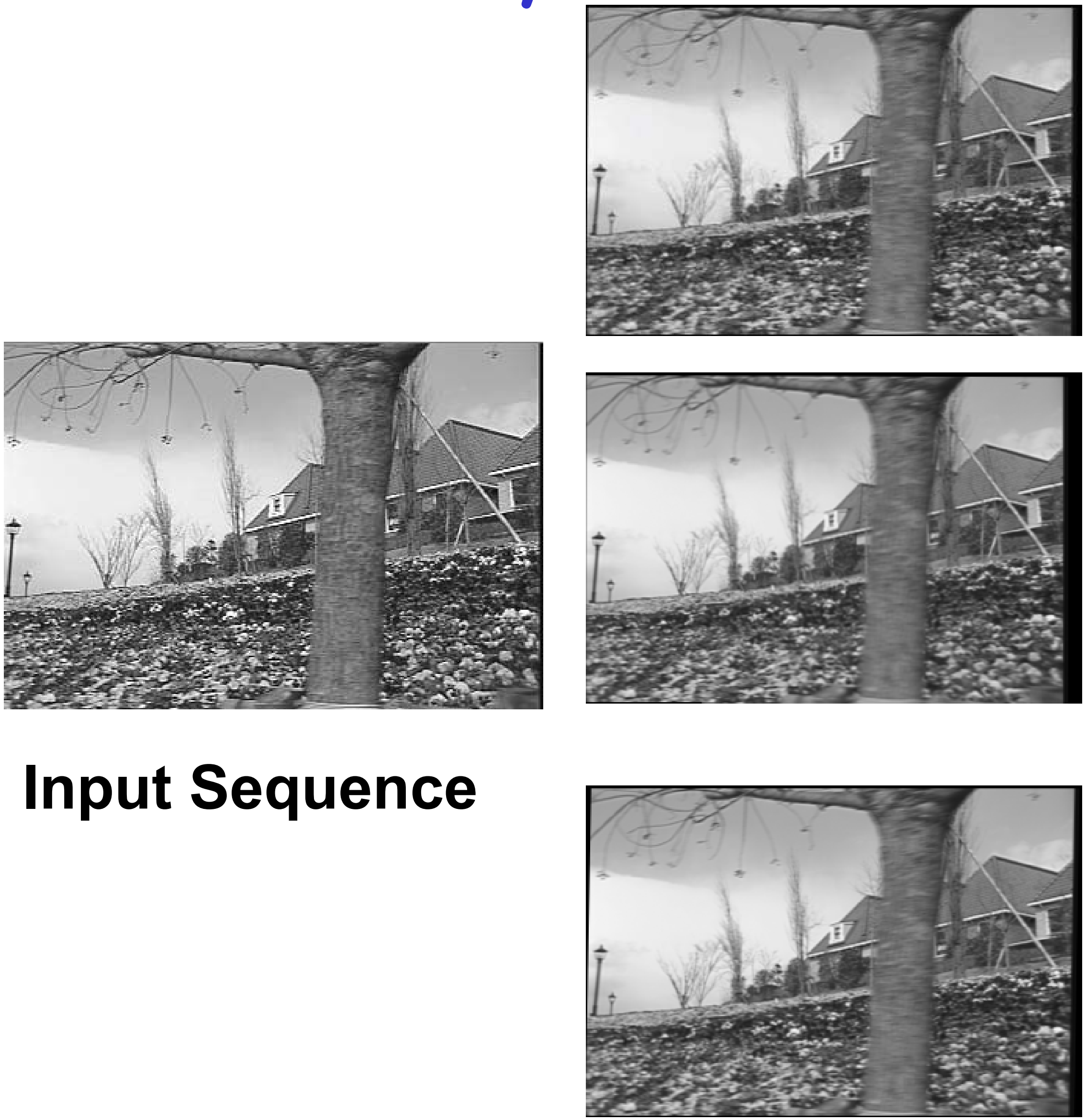
- Find the optimum number of models and the model parameters that minimize the total encoding complexity.

The Complete Algorithm



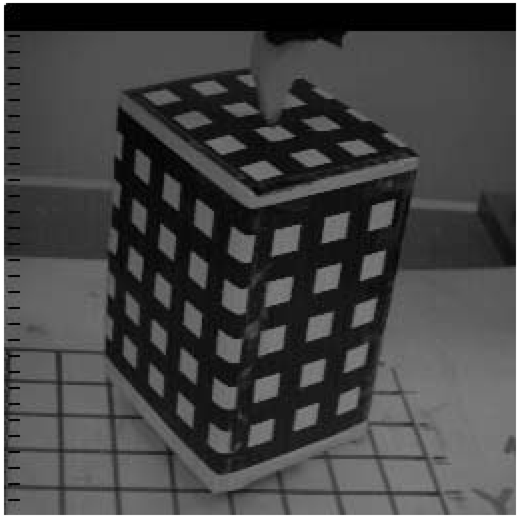
Automatic Extraction of 2D Layers

Layers

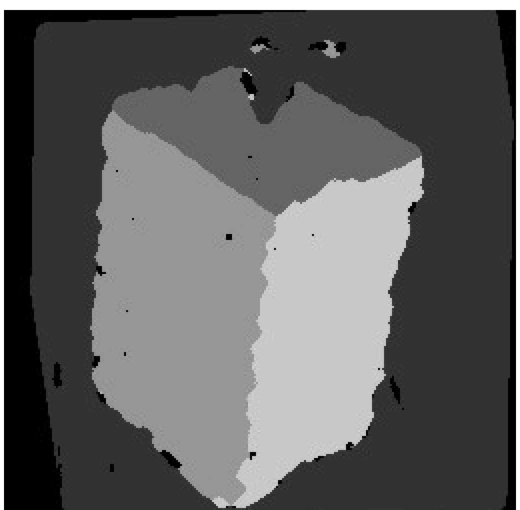


Input Sequence

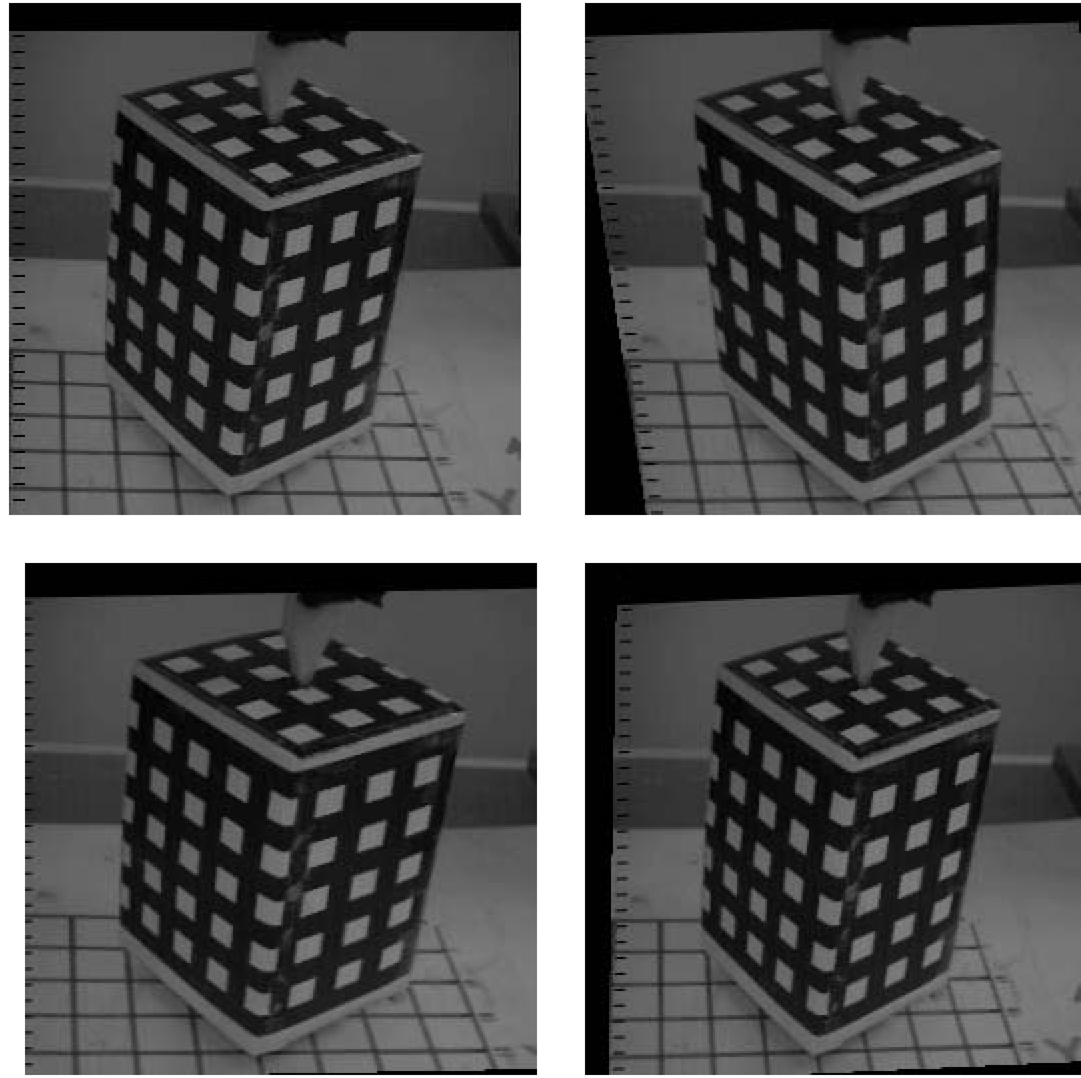
Automatic Extraction of 2D Layers



Input Sequence



Layers



The End